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Identifying Wave Drag for the Generic Drag Polar Equation – Unveiling Polars of 16 Passenger Aircraft

The ultimate drag equation for aircraft preliminary design, aircraft performance, and flight operations.

The drag polar of an aircraft shows its aerodynamic behavior. Drag consists of zero lift drag, wave drag (due to shock waves), and induced drag (due to lift). The drag coefficient is expressed primarily as a function of the lift coefficient. The result is plotted or expressed in the well-known equation, $C_D = C_{D,0} + C_{D,W} + C_L^2/(\pi A e)$. The next important parameter is the Mach number. All three drag components depend on Mach number. The drag coefficient is plotted versus lift coefficient with Mach number as a parameter. Alternatively, drag coefficient is plotted versus Mach number with lift coefficient as a parameter. For the drag polar equation, the graphical representation does not matter. Mach number influences induced drag when compressibility becomes noticeable (beyond compressibility Mach number, typically beyond 0.3). Wave drag starts beyond the critical Mach number, M_{crit} (where shock waves start to form). Per definition, the wave drag coefficient is 0,0020 (20 drag counts) at drag divergence Mach number, M_{DD}. Desired is a difference $M = M_{DD} - M_{crit}$ as large as possible. Passenger jet aircraft are usually designed such that they cruise at M_{DD} and hence with 20 drag counts of wave drag. This gives a good design compromise. The generic drag polar equation must have a convenient structure (made up of different terms, functions, and parameters) to describe all this and more.

PURPOSE

This work systematically derives the best form of a generic drag polar equation together with the optimum numerical values of its parameters to unveil the drag coefficient of 16 passenger aircraft as a function of lift coefficient and Mach number. The parameters are selected such that they can be estimated also for other aircraft mainly from their geometry.

METHODOLOGY

Drag polars in graphical form from Obert (2009 [1] are the starting point. Numerical values of the drag coefficient are obtained with the WebPlotDigitizer. In the generic equation, zero lift drag is assumed constant, the term representing induced drag is taken from Niţă (2012) [2]. For the wave drag term, seven functions of Mach number are investigated. The difference between Mach number and critical Mach number to the power of 4 is the classic approach based on Lock (1951) [3]. Two more general power functions, tan, tanh, sinh, and an exponential function are looked at. Parameters are optimized by minimizing the Root Mean Squared Percentage Error (RMSPE). Optimization is done with the Solver in Excel using the Generalized Reduced Gradient (GRG2) code supplied by Frontline Systems.

FINDINGS

Based on all 16 investigated aircraft, a generic drag polar using the hyperbolic tangent (tanh) to express wave drag is best with mean RMSPE of only 0.68% (Figure 1 and 2). The second best is the most general power function with mean RMSPE of 0.75%. Its special case, the often quoted but unflexible function from Lock comes out last here with a mean RMSPE of 0.95%. Nevertheless, all seven functions can be used to represent wave drag. The zero lift drag coefficient is identified between 0.013 (B777) and 0.020 (A320). The Mach dependence of the drag coefficient comes not only from wave drag, but also from induced drag and its Mach dependence beyond 0.3 Mach. Calculated parameters are plausible and come close to reference values from literature.

$$C_D = C_{D0} + \frac{C_L^2 \cdot d}{-e\cdot\left(\frac{M}{M_{comp}} - 1\right)^f + 1} + a \cdot \left(1 + \tanh\left(b \cdot \frac{M}{M_{crit}} - c\right)\right)$$

Figure 1: In the generic drag polar equation the first term is the identified zero-lift drag coefficient, the second term identifies the Mach-dependent induced drag coefficient, and the third term identifies the wave drag coefficient (here with the hyperbolic tangent).

RESEARCH LIMITATIONS

Aerodynamic data is generally confidential. Therefore, public drag data is limited. The extension of the method to other aircraft yields a drag estimate.

PRACTICAL IMPLICATIONS

The generic equation can be used in preliminary aircraft design as well as in calculations in aircraft performance and flight operations.

ORIGINALITY

This project formulates a generic drag polar equation with a choice of new wave drag terms some based on a historic precursor. The new approach with a hyperbolic tangent function is recommended.

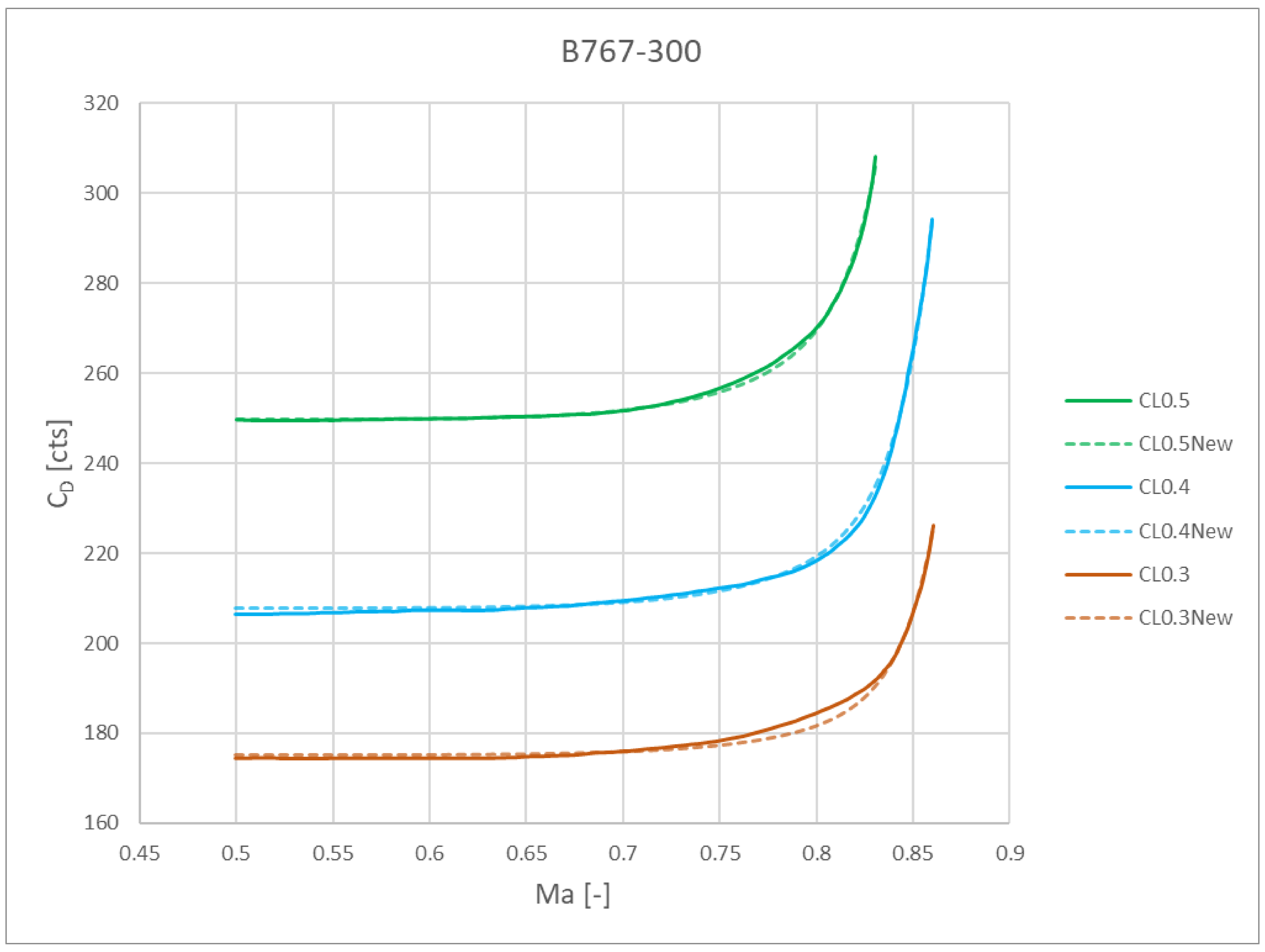

Figure 2: The drag polar for the Boeing 767. The drag coefficient is plotted versus Mach number with lift coefficient as parameter. Obert (2009) gives the drag polar for three lift coefficients (0.3, 0.4, and 0.5), solid line. The polar is identified with the equation from Figure 1, dashed line.

References

[1] https://ebooks.iospress.nl/book/aerodynamic-design-of-transport-aircraft
[2] https://nbn-resolving.org/urn:nbn:de:101:1-201212176728
[3] https://reports.aerade.cranfield.ac.uk/handle/1826.2/3677

All details in the Master Project of Krull (2025):

https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2025-03-07.011

Software and Data in Harvard Dataverse:

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