CHAPTER 1

Solar Radiation

The sun's structure and characteristics determine the nature of the energy it radiates into space. This chapter notes the characteristics of this energy outside of the earth's atmosphere and the effects of the atmosphere in attenuating the radiation. Then the characteristics of the resulting energy resource available at the earth's surface are outlined, that is, its intensity, spectral distribution, and its directional characteristics. We are concerned primarily with radiation in a wavelength range of 0.3 to 3.0 μm, the portion of the spectrum that includes most of the energy radiated by the sun.

In general, it is not practical to start from knowledge of extraterrestrial radiation and predict the intensity and spectral distribution to be expected on the ground. Adequate meteorological data for such calculations are seldom available, and recourse usually is made to measurements. However, an understanding of the nature of extraterrestrial radiation, atmospheric attenuation, and the effects of orienting a receiving surface is important in understanding and using solar radiation data.

1.1 THE SUN

The sun is a sphere of intensely hot gaseous matter with a diameter of 1.39×10^9 m and is, on the average, 1.5×10^{11} m from the earth. As seen from the earth, the sun rotates on its axis about once every four weeks. However, it does not rotate as a solid body; the equator takes about 27 days and the polar regions take about 30 days for each rotation.

The sun has an effective blackbody temperature of 5762 K.* The temperature in the central interior regions is variously estimated at 8×10^6 to 40×10^6 K and the density at about 100 times that of water. The sun is, in effect, a continuous fusion reactor with its constituent gases as the "containing vessel" retained by gravitational forces. Several fusion reactions have been suggested to supply the

* This effective blackbody temperature of 5762 K is the temperature of a blackbody radiating the same amount of energy as does the sun. Other effective temperatures can be defined, for example, that corresponding to the blackbody temperature giving the same wavelength of maximum radiation as solar radiation (about 6300 K).

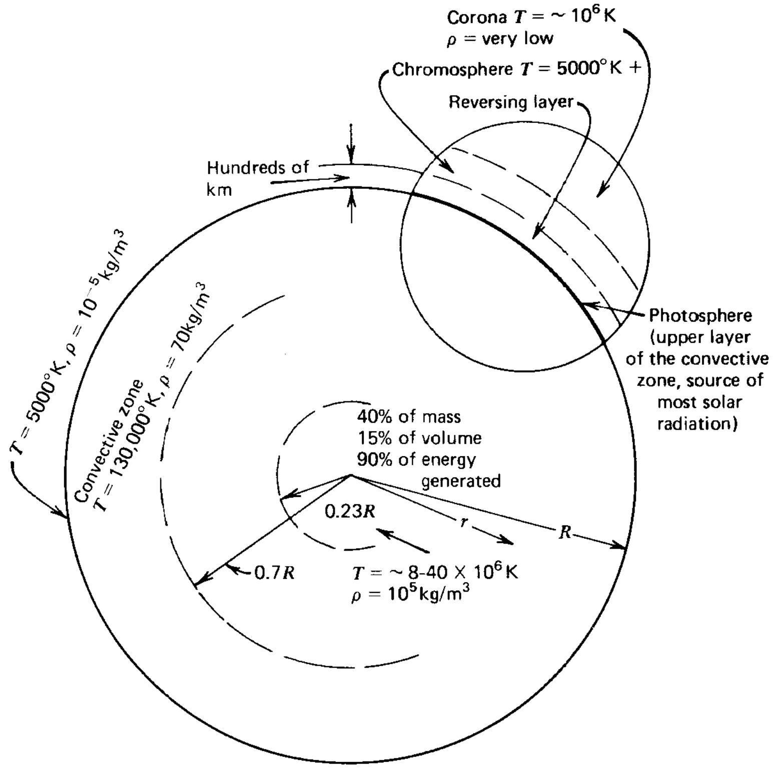

Figure 1.1.1 The structure of the sun.

energy radiated by the sun; the one considered the most important is a process in which hydrogen (i.e., four protons) combines to form helium (i.e., one helium nucleus); the mass of the helium nucleus is less than that of the four protons, mass having been lost in the reaction and converted to energy.

This energy is produced in the interior of the solar sphere, at temperatures of many millions of degrees. It must be transferred out to the surface and then be radiated into space. A succession of radiative and convective processes must occur, with successive emission, absorption, and reradiation; the radiation in the sun's core must be in the x-ray and gamma-ray parts of the spectrum with the wavelengths of the radiation increasing as the temperature drops at larger radial distances.

A schematic of the structure of the sun is shown in Figure 1.1.1. It is estimated that 90% of the energy is generated in the region of 0 to $0.23R$ (where R is the radius of the sun), which contains 40% of the mass of the sun. At a distance $0.7R$ from the center, the temperature has dropped to about 130,000 K and the density has dropped to 70 kg/m^3; here convection processes begin to become important and the zone from 0.7 to $1.0R$ is known as the *convective zone.* Within this zone, the temperature drops to about 5000 K and the density to about 10^{-5} kg/m^3.

The sun's surface appears to be composed of granules (irregular convection cells), with dimensions of cells from 1000 to 3000 km and with cell lifetime of a few minutes. Other features of the solar surface are small dark areas called pores, which are of the same order of magnitude as the convective cells, and larger dark areas called sunspots, which vary in size. The outer layer of the convective zone is called the *photosphere*. The edge of the photosphere is sharply defined, even though it is of low density (about 10^{-4} that of air at sea level). It is essentially opaque, as the gases of which it is composed are strongly ionized and able to absorb and emit a continuous spectrum of radiation. The photosphere is the source of most solar radiation.

Outside of the photosphere is a more or less transparent solar atmosphere, which is observable during total solar eclipse or by instruments that occult the solar disk. Above the photosphere is a layer of cooler gases several hundred kilometers deep called the *reversing layer*. Outside of that is a layer referred to as the *chromosphere*, with a depth of about 10,000 km. This is a gaseous layer with temperatures somewhat higher than that of the photosphere and with lower density. Still further out is the *corona*, of very low density and of very high (10^6 K) temperature. For further information on the sun's structure see Thomas (1958) or Robinson (1966).

This simplified picture of the sun, its physical structure, and its temperature and density gradients, will serve as a basis for appreciating that the sun does not, in fact, function as a blackbody radiator at a fixed temperature. Rather, the emitted solar radiation is the composite result of the several layers that emit and absorb radiation of various wavelengths. The resulting extraterrestrial solar radiation and its spectral distribution have now been measured by various methods in several experiments; the results are noted in the following two sections.

1.2 THE SOLAR CONSTANT

Figure 1.2.1 shows schematically the geometry of the sun-earth relationships. The eccentricity of the earth's orbit is such that the distance between the sun and the earth varies by 1.7%. At a distance of one astronomical unit, 1.495×10^{11} m, the mean earth-sun distance, the sun subtends an angle of 32′. The radiation emitted by the sun and its spatial relationship to the earth result in a nearly fixed intensity of solar radiation outside of the earth's atmosphere. The *solar constant*, G_{sc}, is the energy from the sun, per unit time, received on a unit area of surface perpendicular to the direction of propagation of the radiation, at the earth's mean distance from the sun, outside of the atmosphere.

Until recently, estimates of the solar constant had to be made from ground-based measurements of solar radiation after it had been transmitted through the atmosphere, and thus in part absorbed and scattered by components of the atmosphere. Extrapolations from the terrestrial measurements, which were

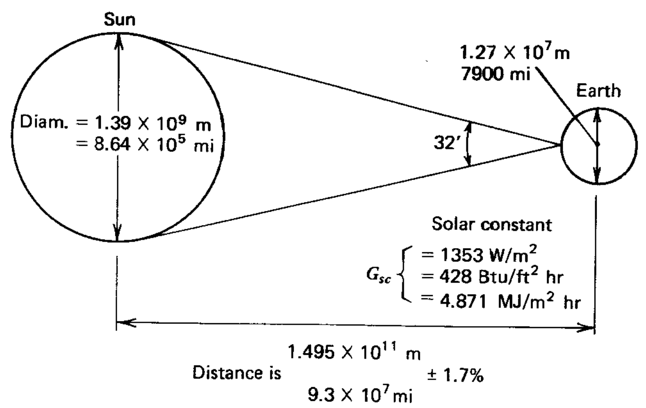

Figure 1.2.1 Sun-earth relationships.

made from high mountains, were based on estimates of atmospheric transmission in various portions of the solar spectrum. Pioneering studies were done by C. G. Abbot and his colleagues at the Smithsonian Institution. These studies and later measurements from rockets were summarized by Johnson (1954); Abbot's value of the solar constant of 1322 W/m^2 was revised upward by Johnson to 1395 W/m^2.

More recently, the availability of very-high-altitude aircraft, balloons, and spacecraft has permitted direct measurements of solar radiation outside most or all of the earth's atmosphere. These measurements were made with a variety of instruments in nine separate experimental programs. They resulted in a value of the solar constant, G_{sc}, of 1353 W/m^2 (1.940 cal/cm^2 min, 428 Btu/ft^2 hr, or 4.871 MJ/m^2 hr). The estimated error was ± 1.5 percent. For discussions of these experiments, see Thekaekara (1976) or Thekaekara and Drummond (1971). This standard value was accepted by NASA [see NASA (1971)] and by the American Society for Testing Materials.

The data on which the 1353 W/m^2 value was based, have been reexamined by Frohlich (1977), and reduced to a new pyrheliometric scale* based on comparisons of the instruments with absolute radiometers. Data from Nimbus and Mariner satellites have also been included in the analysis, and as of 1978, Frohlich recommends a new value of the solar constant of $G_{sc} = 1373$ W/m^2, with a probable error of 1 to 2 percent. This is 1.5 percent higher than the earlier value, and 1.2 percent higher than the best available determination of the solar constant by integration of spectral measurements. Thus there remains some uncertainty about the value of G_{sc}, but the uncertainty is of the order of 1 percent. (As will be seen in Chapter 2, uncertainties in most terrestrial solar radiation measurements are an order of magnitude larger than that.) In this book we use the value of 1353 W/m^2.

* Pyrheliometric scales are discussed in Section 2.2.

1.3 SPECTRAL DISTRIBUTION OF EXTRATERRESTRIAL RADIATION

In addition to the total energy in the solar spectrum (i.e., the solar constant) it is useful to know the spectral distribution of the extraterrestrial radiation, that is, the radiation that would be received in the absence of the atmosphere. A standard spectral irradiance curve has been compiled, based on high altitude and space measurements. This NASA/ASTM standard is shown in Figure 1.3.1. The averaged energy $G_{sc,\lambda}$ over small bandwidths centered at wavelength λ and the integrated fraction of the energy, $f_{0-\lambda}$, at wavelengths less than λ for the standard curve are indicated in Table 1.3.1. This is a condensed table; more detailed tables are available [e.g., see Thekaekara (1976)].

Example 1.3.1

Calculate the fraction of the extraterrestrial solar radiation and the amount of that radiation in the ultraviolet ($\lambda < 0.38$ μm), the visible (0.38 μm $< \lambda < 0.78$ μm), and the infrared ($\lambda > 0.78$ μm) portions of the spectrum.

Solution

From Table 1.3.1, the fractions $f_{0-\lambda}$ corresponding to wavelengths of 0.38 and 0.78 μm are 0.0700 and 0.5429 (interpolated). Thus, the fraction in the ultraviolet is 0.0700, the fraction in the visible range is (0.5429 − 0.0700) = 0.4729, and the fraction in the infrared is (1.0 − 0.5429) = 0.4571. Applying these fractions to a solar constant of 1353 W/m^2 and tabulating the results, we have:

Wavelength range (μm)	0–0.38	0.38–0.78	0.78–∞
Fraction in range	0.0700	0.4729	0.4571
Energy in range (W/m^2)	95	640	618

■

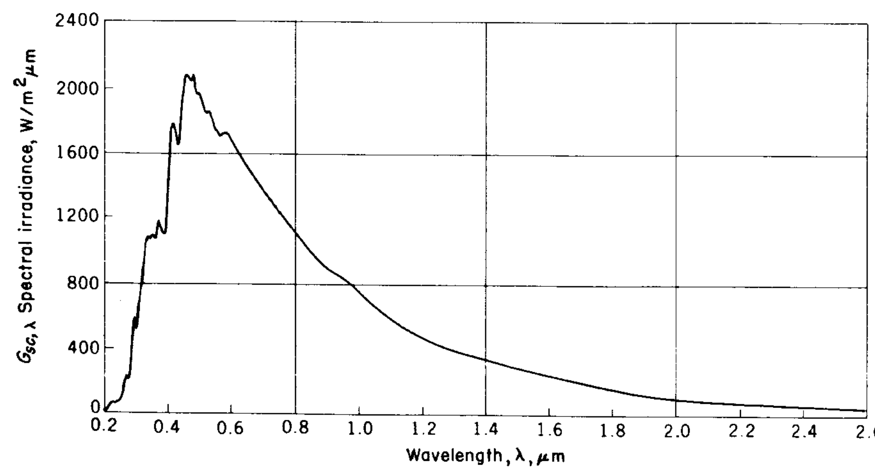

Figure 1.3.1 The NASA/ASTM standard spectral irradiance at the mean sun-earth distance and a solar constant of 1353 W/m^2.

Table 1.3.1 Extraterrestrial Solar Irradiance (Solar Constant = 1353 W/m²)

λ	$G_{sc,\lambda}$	$f_{0-\lambda}$[b]	λ	$G_{sc,\lambda}$[a]	$f_{0-\lambda}$[b]	λ	$G_{sc,\lambda}$[a]	$f_{0-\lambda}$[b]
0.24	63.0	0.0014	0.47	2033	0.1817	1.0	748	0.6949
0.25	70.9	0.0019	0.48	2074	0.1968	1.2	485	0.7840
0.26	130	0.0027	0.49	1950	0.2115	1.4	337	0.8433
0.27	232	0.0041	0.50	1942	0.2260	1.6	245	0.8861
0.28	222	0.0056	0.51	1882	0.2401	1.8	159	0.9159
0.29	482	0.0081	0.52	1833	0.2538	2.0	103	0.9349
0.30	514	0.0121	0.53	1842	0.2674	2.2	79	0.9483
0.31	689	0.0166	0.54	1783	0.2808	2.4	62	0.9586
0.32	830	0.0222	0.55	1725	0.2938	2.6	48	0.9667
0.33	1059	0.0293	0.56	1695	0.3065	2.8	39	0.9731
0.34	1074	0.0372	0.57	1712	0.3191	3.0	31	0.9783
0.35	1093	0.0452	0.58	1715	0.3318	3.2	22.6	0.9822
0.36	1068	0.0532	0.59	1700	0.3444	3.4	16.6	0.9850
0.37	1181	0.0615	0.60	1666	0.3568	3.6	13.5	0.9872
0.38	1120	0.0700	0.62	1602	0.3810	3.8	11.1	0.9891
0.39	1098	0.0782	0.64	1544	0.4042	4.0	9.5	0.9906
0.40	1429	0.0873	0.66	1486	0.4266	4.5	5.9	0.9934
0.41	1751	0.0992	0.68	1427	0.4481	5.0	3.8	0.9951
0.42	1747	0.1122	0.70	1369	0.4688	6.0	1.8	0.9972
0.43	1639	0.1247	0.72	1314	0.4886	7.0	1.0	0.9982
0.44	1810	0.1373	0.75	1235	0.5169	8.0	0.59	0.9988
0.45	2006	0.1514	0.80	1109	0.5602	10.0	0.24	0.9994
0.46	2066	0.1665	0.90	891	0.6337	50.0	3.9×10^{-4}	1.0000

[a] $G_{sc,\lambda}$ is the solar spectral irradiance in W/m² μm averaged over a small bandwidth centered at λ.
[b] $f_{0-\lambda}$ is the fraction of the solar constant associated with wavelengths shorter than λ.
From Thekaekara (1974).

1.4 VARIATION OF EXTRATERRESTRIAL RADIATION

Two sources of variation in extraterrestrial radiation must be considered. The first is the variation in the radiation emitted by the sun. There are conflicting reports in the literature on periodic variations of intrinsic solar radiation. It has been suggested that there are small variations (less than ± 1.5 percent) with different periodicities and variation related to sunspot activities. Others consider the measurements to be inconclusive or not indicative of regular variability. Measurements from Nimbus and Mariner satellites over periods of several months showed variations within limits of ± 0.2 percent over a time when sunspot activity was very low [Frohlich (1977)]. See Coulson (1975) or Thekaekara (1976) for further discussion of this topic. For engineering purposes, in view of the uncertainties and variability of atmospheric transmission, and until reliable

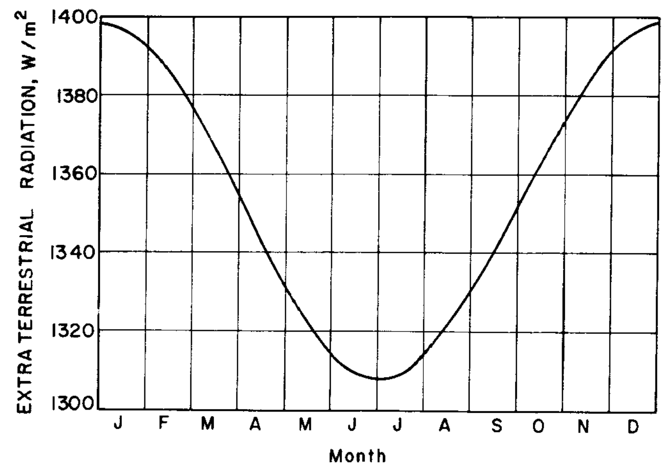

Figure 1.4.1 Variation of extraterrestrial solar radiation with time of year.

measurements indicate otherwise, the energy emitted by the sun can be considered to be fixed.

Variation of the earth-sun distance, however, does lead to variation of extraterrestrial radiation flux in the range of $\pm 3\%$. The dependence of extraterrestrial radiation on time of year is indicated by Equation 1.4.1 and Figure 1.4.1.

$$\boxed{G_{on} = G_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right)} \tag{1.4.1}$$

where G_{on} is the extraterrestrial radiation, measured on the plane normal to the radiation on the nth day of the year.

1.5 DEFINITIONS; SOLAR TIME

Several definitions will be useful in understanding the balance of this chapter.

Zenith Angle, θ_z The angle subtended by a vertical line to the zenith (i.e., the point directly overhead) and the line of sight to the sun.

Air Mass, m The ratio of the optical thickness of the atmosphere through which beam radiation passes to the optical thickness if the sun were at the zenith. Thus at sea level, $m = 1$ when the sun is at the zenith, and $m = 2$ for a zenith angle θ_z, of 60°. For zenith angles from 0° to 70° at sea level,

$$\boxed{m = (\cos \theta_z)^{-1}}$$

For higher zenith angles, the effect of the earth's curvature becomes significant and must be taken into account. For a more complete discussion of air mass, see Robinson (1966) or Kondratyev (1969).

Beam Radiation The solar radiation received from the sun without having been scattered by the atmosphere. (Beam radiation is often referred to as direct solar radiation; to avoid confusion between subscripts for direct and diffuse, we use the term beam radiation.)

Diffuse Radiation The solar radiation received from the sun after its direction has been changed by scattering by the atmosphere. (Diffuse radiation is referred to in some meteorological literature as sky radiation or solar sky radiation; the definition used here will distinguish the diffuse solar radiation from radiation emitted by the atmosphere.)

Total Solar Radiation The sum of the beam and the diffuse radiation on a surface.* (The most common measurements of solar radiation are total radiation on a horizontal surface, often referred to as *global radiation*).

Additional radiation terminology used in this book includes the following terms:

Irradiance, W/m^2 The rate at which radiant energy is incident on a surface, per unit area of surface. The symbol G is used, with appropriate subscripts, for beam or diffuse radiation.

Irradiation or Radiant Exposure, J/m^2 The incident energy per unit area on a surface, found by integration of irradiance over a specified time, usually an hour or a day. (*Insolation* is a term applying specifically to solar energy irradiation.) The symbol H is used for insolation for a day (or other period if specified). The symbol I is used for insolation for an hour. H and I can be beam, diffuse, or total and can be on surfaces of any orientation.†

Radiosity or Radiant Exitance, W/m^2 The rate at which radiant energy leaves a surface, per unit area, by combined emission, reflection, and transmission.

Emissive Power or Radiant Self Exitance, W/m^2 The rate at which radiant energy leaves a surface per unit area, by emission only. The symbol E is used, with appropriate subscripts.

Any of these terms, except insolation, can apply to any specified wavelength range (such as the solar energy spectrum) or to monochromatic radiation. Insolation refers only to irradiation in the solar energy spectrum.

* Total solar radiation is sometimes used to indicate quantities integrated over all wavelengths of the solar spectrum.

† Subscripts on G, H, and I are as follows: o refers to radiation above the earth's atmosphere, referred to as extraterrestrial radiation; b and d refer to beam and diffuse radiation; T and n refer to radiation on tilted or normal planes. If neither T nor n appear, the radiation is on a horizontal plane.

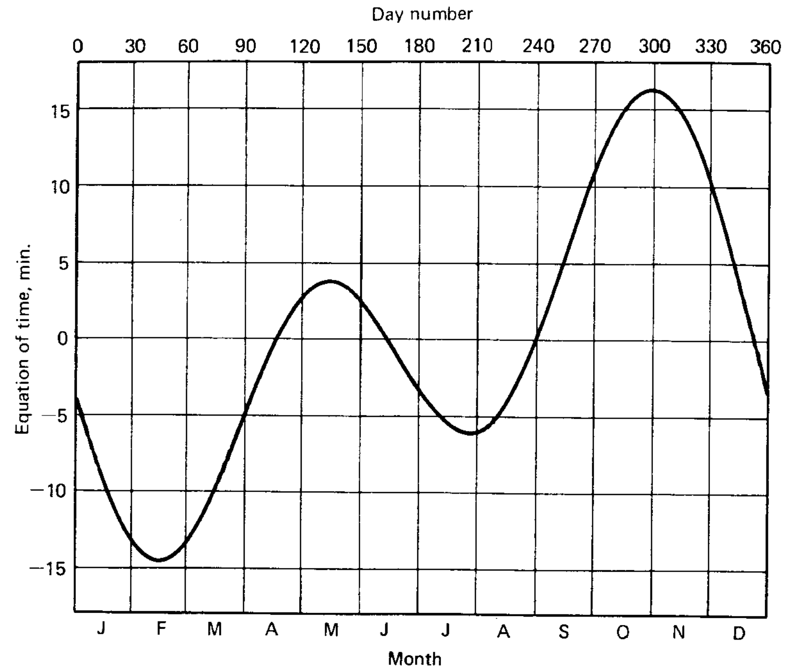

Figure 1.5.1 The equation of time, E, in minutes, as a function of time of year.

Solar Time Time based on the apparent angular motion of the sun across the sky, with *solar noon* the time the sun crosses the meridian of the observer.

Solar time is the time specified in all of the sun angle relationships; it does not coincide with local clock time. It is necessary to convert standard time to solar time by applying two corrections. First, there is a constant correction for the difference in longitude between the observer's meridian location and the meridian on which the local standard time is based*; the sun takes four minutes to transverse 1° of longitude. The second correction is from the *equation of time*, which takes into account the perturbations in the earth's rate of rotation, which affect the time the sun crosses the observer's meridian.† Solar time is related to standard time by

$$\boxed{\text{solar time} = \text{standard time} + 4\,(L_{st} - L_{loc}) + E} \qquad (1.5.1)$$ 3

where E is the equation of time from Figure 1.5.1 or Equation 1.5.2 [from Whillier (1979)] in minutes, L_{st} is the standard meridian for the local time zone, and L_{loc} is the longitude of the location in question in degrees west.

$$\boxed{E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B} \qquad (1.5.2)$$ 2

* Standard meridians for continental U. S. time zones are Eastern, 75°W; Central, 90°W; Mountain, 105°W; and Pacific, 120°W.

† There may also be an additional 1 hr correction for daylight saving time.

where

1

$$\boxed{B = \frac{360(n-81)}{364}}$$ 1.5.3

$$n = \text{day of the year, } 1 \le n \le 365$$

Example 1.5.1

At Madison, WI, what is the solar time corresponding to 10:30 A.M. central standard time on February 2?

Solution

In Madison, where the longitude is 89.4°, Equation 1.5.1 gives

$$\begin{aligned} \text{solar time} &= \text{standard time} + 4(90 - 89.4) + E \\ &= \text{standard time} + 2.48 + E \end{aligned}$$

On February 2, E is -13.5 min, so the correction to standard time is -11 min. Thus 10:30 A.M. central standard time is 10:19 A.M. solar time. ■

In this book, all times are assumed to be solar times unless indication is given otherwise.

1.6 DIRECTION OF BEAM RADIATION

The geometric relationships between a plane of any particular orientation relative to the earth at any time (whether that plane is fixed or moving relative to the earth) and the incoming beam solar radiation, that is, the position of the sun relative to that plane, can be described in terms of several angles (Benford and Bock (1939)). These angles and the relationships between them are as follows:

- ϕ Latitude, that is, the angular location north or south of the equator, north positive. $-90° \le \phi \le 90°$.
- δ Declination, that is, the angular position of the sun at solar noon with respect to the plane of the equator, north positive. $-23.45° \le \delta \le 23.45°$.
- β Slope, that is, the angle between the plane surface in question and the horizontal. $0 \le \beta \le 180°$ ($\beta > 90°$ means that the surface has a downward facing component).
- γ Surface azimuth angle, that is, the deviation of the projection on a horizontal plane of the normal to the surface from the local meridian, with zero due south, east negative, west positive.* $-180° \le \gamma \le 180°$.

* The sign convention used here for γ and ω is the reverse of that in *Solar Energy Thermal Processes*. This convention is consistent with Hottel and Woertz (1942) and other authors in the solar energy literature. Either is correct; it is necessary to be consistent.

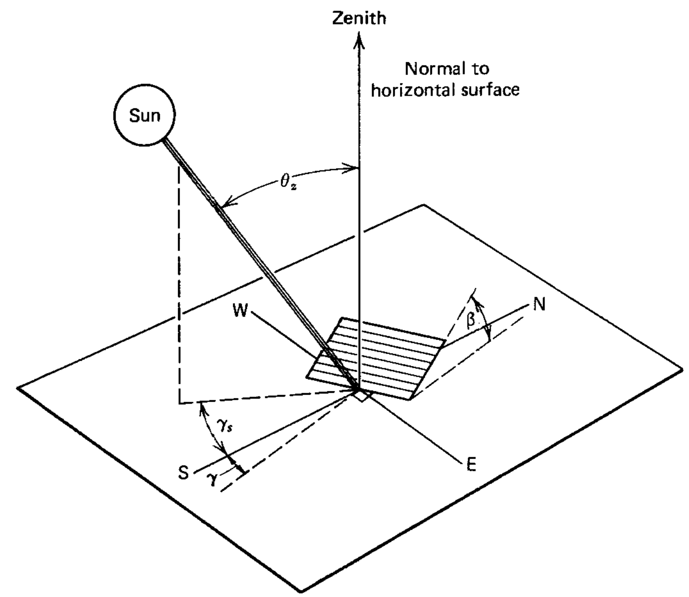

Figure 1.6.1 Zenith angle, slope, surface azimuth angle and solar azimuth angle for a tilted surface.

$\omega = (t_s - 12\,\text{hr}) \cdot 15°/\text{hr}$ t_s: solar time (1.6.?)

- ω Hour angle, that is, the angular displacement of the sun east or west of the local meridian due to rotation of the earth on its axis at 15° per hour, morning negative, afternoon positive.
- θ Angle of incidence, that is, the angle between the beam radiation on a surface and the normal to that surface.

Zenith angle, slope, and surface azimuth angle are shown in Figure 1.6.1. The declination, δ, can be found from the equation of Cooper (1969):

$$\boxed{\delta = 23.45 \sin\left(360\,\frac{284+n}{365}\right)} \qquad (1.6.1)$$

where n is the day of the year; n can be conveniently obtained with the help of Table 1.6.1 (or from Figure 1.6.3).

Thus at 2:30 P.M. solar time on October 15 at Madison (latitude 43°N) for a surface tilted 45° from the horizontal and facing 20° west of south, $\phi = 43°$, $n = 288$, $\delta = -9.60$ (from Equation 1.6.1), $\beta = 45°$, $\gamma = 20°$, and $\omega = 37.5°$.

The equation relating the angle of incidence of beam radiation, θ, and the other angles is:

$$\boxed{\begin{aligned} \cos\theta = {} & \sin\delta \sin\phi \cos\beta - \sin\delta \cos\phi \sin\beta \cos\gamma \\ & + \cos\delta \cos\phi \cos\beta \cos\omega \\ & + \cos\delta \sin\phi \sin\beta \cos\gamma \cos\omega \\ & + \cos\delta \sin\beta \sin\gamma \sin\omega \end{aligned}} \qquad (1.6.2)$$

Table 1.6.1 Recommended Average Day[a] for Each Month [from Klein (1976)] and Values of n by Months

Month	n for ith Day of Month[b]	For the Average Day of the Month: Date	n, Day of Year[b]	δ, Declination
January	i	17	17	−20.9
February	$31 + i$	16	47	−13.0
March	$59 + i$	16	75	−2.4
April	$90 + i$	15	105	9.4
May	$120 + i$	15	135	18.8
June	$151 + i$	11	162	23.1
July	$181 + i$	17	198	21.2
August	$212 + i$	16	228	13.5
September	$243 + i$	15	258	2.2
October	$273 + i$	15	288	−9.6
November	$304 + i$	14	318	−18.9
December	$334 + i$	10	344	−23.0

[a] The average day is that day which has the extraterrestrial radiation closest to the average for the month. See Section 1.8.
[b] These do not account for leap year; values of n from March onward for leap years can be corrected by adding 1. Declination values will also shift slightly.

Example 1.6.1

Calculate the angle of incidence of beam radiation on a surface located at Madison, WI at 10:30 (solar time) on February 13, if the surface is tilted 45° from the horizontal and is pointed 15° west of south.

Solution

Under these conditions, the declination is −14°, the hour angle is −22.5°, and the surface azimuth angle is 15°. Using the slope of 45° and Madison's latitude of 43°N, Equation 1.6.2 is

$$\begin{aligned}\cos\theta = {} & \sin(-14)\sin 43\cos 45\\ & -\sin(-14)\cos 43\sin 45\cos 15\\ & +\cos(-14)\cos 43\cos 45\cos(-22.5)\\ & +\cos(-14)\sin 43\sin 45\cos 15\cos(-22.5)\\ & +\cos(-14)\sin 45\sin 15\sin(-22.5)\\ \cos\theta = {} & -0.117+0.121+0.464+0.418-0.068\\ \cos\theta = {} & 0.817\\ \theta = {} & 35^\circ\end{aligned}$$

■

Additional angles are also defined. The *solar azimuth angle* γ_s, is the angular displacement from south of the projection of the beam radiation on the horizontal plane, as shown in Figure 1.6.1. In architectural and illumination practice, other angles are defined, such as the profile angle and the solar altitude angle $(90 - \theta_z)$. Care must be exercised in the use of any source of information on these angles so that definitions and sign conventions are understood and followed.

There are several commonly occurring cases for which Equation 1.6.2 is simplified: For fixed surfaces sloped toward the south or north, that is, with a surface azimuth angle, γ, of 0° or 180° (a very common situation for fixed flat-plate collectors), the last term drops out. 1) For vertical surfaces, $\beta = 90°$ and the equation becomes

$$\boxed{\begin{aligned}\cos\theta = {} & -\sin\delta\cos\phi\cos\gamma+\cos\delta\sin\phi\cos\gamma\cos\omega\\ & +\cos\delta\sin\gamma\sin\omega\end{aligned}} \tag{1.6.3}$$

2.) For horizontal surfaces, $\beta = 0°$, and the angle of incidence is the zenith angle of the sun, θ_z. Equation 1.6.2 becomes

$$\boxed{\cos\theta_z = \cos\delta\cos\phi\cos\omega+\sin\delta\sin\phi} \tag{1.6.4}$$

Example 1.6.2

Calculate the zenith angle of the sun at Madison at 9:30 on February 13.

Solution

For this date, declination is −14°. From Equation 1.6.4

$$\begin{aligned}\cos\theta_z &= \cos(-14)\cos 43\cos(-37.5)+\sin(-14)\sin 43\\ \cos\theta_z &= 0.398\\ \theta_z &= 66^\circ\end{aligned}$$

■

$\gamma = 0°$

Useful relationships for the angle of incidence on surfaces sloped to the north or south can be derived from the fact that surfaces with slope β to the north or south have the same angular relationship to beam radiation as a horizontal surface at an artificial latitude of $(\phi - \beta)$. The relationship is shown in Figure 1.6.2, for the northern hemisphere. Modifying Equation 1.6.4,

$$\boxed{\cos\theta = \cos(\phi-\beta)\cos\delta\cos\omega+\sin(\phi-\beta)\sin\delta} \tag{1.6.5}$$

For the southern hemisphere the equation is modified by replacing $(\phi - \beta)$ by $(\phi + \beta)$, consistent with the sign conventions on ϕ and δ

$$\cos\theta = \cos(\phi+\beta)\cos\delta\cos\omega+\sin(\phi+\beta)\sin\delta \tag{1.6.6}$$

Equation 1.6.4 can be solved for the *sunset hour angle*, ω_s, when $\theta_z = 90°$

$$\cos\omega_s = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta}$$

$$\boxed{\cos\omega_s = -\tan\phi\tan\delta} \tag{1.6.7}$$

$\omega_s = -\omega$

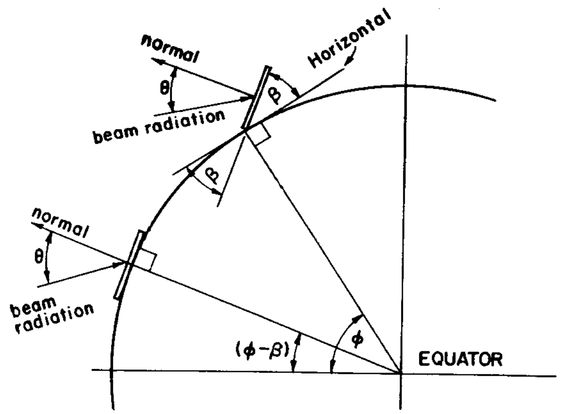

Figure 1.6.2 Section of Earth showing β, θ, ϕ and $(\phi - \beta)$ for a south-facing surface.

It also follows that the number of daylight hours is given by

$$\boxed{N = \tfrac{2}{15}\cos^{-1}(-\tan\phi\tan\delta)} \tag{1.6.8}$$

A convenient nomogram for determining day length has been devised by Whillier (1965), and is shown in Figure 1.6.3. Information on latitude and declination leads directly to times of sunrise and sunset and day length, for either hemisphere.

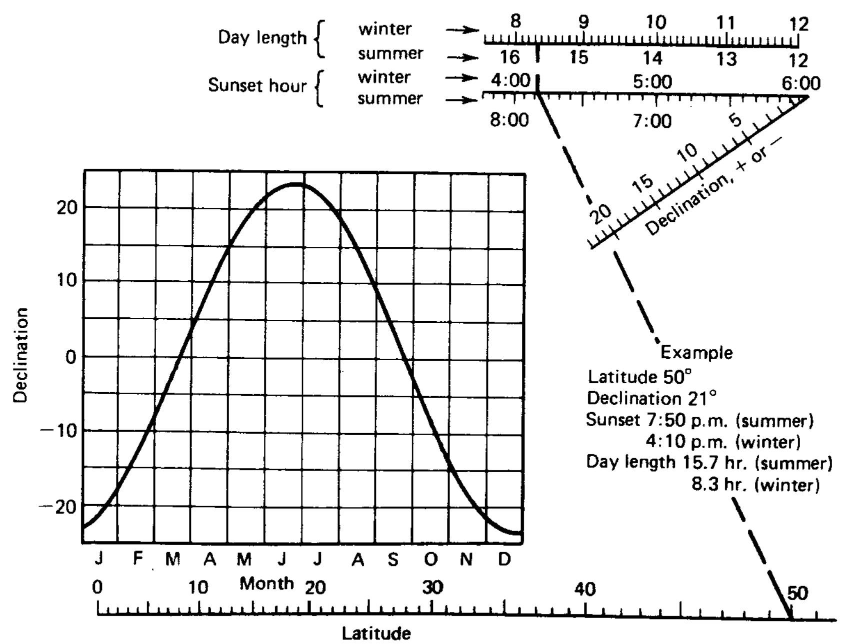

Figure 1.6.3 Nomogram to determine time of sunset and day length. Adapted from Whillier, Solar Energy 9, 164 (1965).

Solar azimuth and altitude angles are tabulated as functions of latitude, declination, and hour angle by the U. S. Hydrographic Office (1940). Information on the position of the sun in the sky is also available with less precision but easier access in various types of charts. Examples of these are the Sun Angle Calculator (1951) and diagrams in a paper by Hand (1948). (Care is necessary in interpreting information from sources such as these, since definitions of angles and sign conventions may vary from those used here.) Brooks and Miller (1963) also present a discussion of these geometrical relationships.

Equation 1.6.2 is a generally applicable relationship for the angle of incidence of beam radiation on a surface of any orientation, and reduces to simpler forms for special cases such as horizontal surfaces (Equation 1.6.4) and vertical surfaces (Equation 1.6.3). By far the most common case is a surface tilted toward the equator (Equations 1.6.5 and 1.6.6). There are other special cases of interest, for example, the angle of incidence on planes which are moved in prescribed ways. Some collectors are moved to track the beam radiation to varying degrees.* In this section we show the forms of Equation 1.6.2 applicable to some of the more common modes of tracking.

Tracking systems can be classified by the mode of their motion. Motion can be about a single axis (which can be oriented east-west, north-south, or parallel to the earth's axis), or it can be about two axes. The following equations are derivable from Equation 1.6.2, and apply to planes moved as indicated [see Eibling et al. (1953)].

For a plane rotated about a horizontal east-west axis with a single daily adjustment so that its surface-normal coincides with the solar beam at noon each day,

$$\cos\theta = \sin^2\delta + \cos^2\delta\cos\omega \tag{1.6.9}$$

For a plane rotated about a horizontal east-west axis with continuous adjustment to minimize the angle of incidence,

$$\cos\theta = (1 - \cos^2\delta\sin^2\omega)^{1/2} \tag{1.6.10}$$

For a plane rotated about a horizontal north-south axis with continuous adjustment to minimize the angle of incidence

$$\cos\theta = [(\sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega)^2 + \cos^2\delta\sin^2\omega]^{1/2} \tag{1.6.11}$$

For a plane rotated about a north-south axis parallel to the earth's axis, with continuous adjustment

$$\cos\theta = \cos\delta \tag{1.6.12}$$

A two-axis tracking surface continuously oriented to face the sun will at all times have

$$\cos\theta = 1 \tag{1.6.13}$$

* Tracking solar collectors are almost always of the concentrating type, which are discussed in Chapter 8.

of the two variables and found the best correlation to be with $\bar{n}/\bar{N}$, that is, Equation 2.7.2. Cloud cover data are estimated visually, and there is not necessarily a direct relationship between the presence of partial cloud cover and solar radiation at any particular time. Thus there may not be as good a statistical relationship between $\bar{H}/\bar{H}_0$ and $\bar{C}$ as there is between $\bar{H}/\bar{H}_0$ and $\bar{n}/\bar{N}$. Many surveys of solar radiation data [e.g., Bennett (1965) and Löf et al. (1966)] have been based on correlations of radiation with sunshine hour data. However, Paltridge and Proctor (1976) have used cloud cover data to modify clear sky data for Australia and derived therefrom monthly averages of $\bar{H}_0$ which are in good agreement with measured average data.

2.8 ESTIMATION OF CLEAR SKY RADIATION

The effects of the atmosphere in scattering and absorbing radiation are variable with time, as atmospheric conditions and air mass change. It is useful to define a standard "clear" sky, and calculate the hourly and daily radiation which would be received on a horizontal surface under these standard conditions.

Hottel (1976) has presented a convenient method for estimating the beam radiation transmitted through clear atmospheres which takes into account zenith angle and altitude for a standard atmosphere and for four climate types. The atmospheric transmittance for beam radiation, τ_b, is G_{bn}/G_o and is given in the form

4

$$\boxed{\tau_b = a_o + a_1 e^{-k/\cos\theta_z}} \tag{2.8.1}$$

The constants a_o, a_1, and k for the standard atmosphere with 23 km visibility are found from a_o^*, a_1^*, and k^*, which are given for altitudes less than 2.5 km by

1

$$a_o^* = 0.4237 - 0.00821\,(6 - A)^2 \tag{2.8.2}$$

$$a_1^* = 0.5055 + 0.00595\,(6.5 - A)^2 \tag{2.8.3}$$

$$k^* = 0.2711 + 0.01858\,(2.5 - A)^2 \tag{2.8.4}$$

where A is the altitude of the observer in kilometers. Plots of these coefficients are shown in Figure 2.8.1. (Hottel also gives equations for a_o^*, a_1^*, and k^* for a standard atmosphere with 5 km visibility.)

2a

Table 2.8.1 Correction Factors for Climate Types

Climate Type	r_o	r_1	r_k
Tropical	0.95	0.98	1.02
Mid-Latitude Summer	0.97	0.99	1.02
Subarctic Summer	0.99	0.99	1.01
Mid-Latitude Winter	1.03	1.01	1.00

[a] From Hottel (1976).

or

2b

$r_o = 1.0 - 0.04 \cdot \delta/23.45°$

$r_1 = 1.0 - 0.015 \cdot \delta/23.45°$

$r_k = 1.01 + 0.015 \cdot \delta/23.45°$

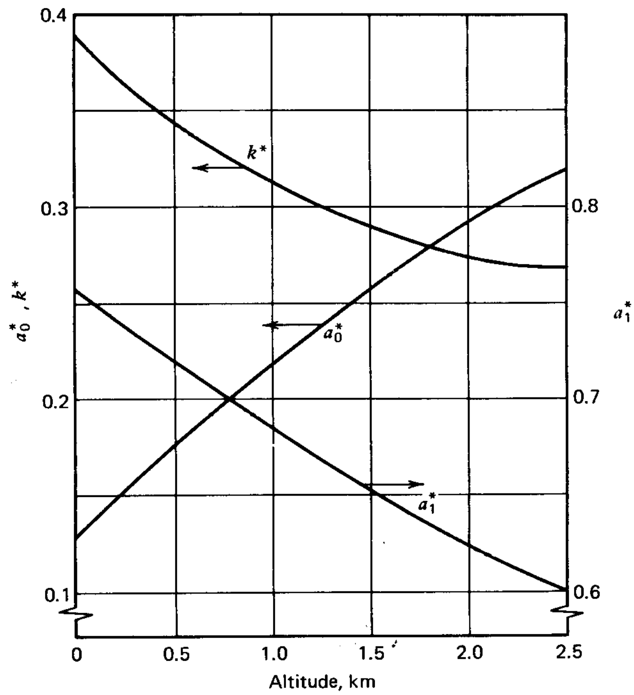

Figure 2.8.1 Constants a_0^*, a_1^*, and k^* for the 23 km visibility standard atmosphere. Adapted from Hottel (1976).

Correction factors are applied to a_0^*, a_1^*, and k^* to allow for changes in climate types. The correction factors $\boxed{r_o \equiv a_o/a_o^*,\ r_1 \equiv a_1/a_1^* \text{ and } r_k \equiv k/k^*}$ are given in Table 2.8.1. Thus, the transmittance of this standard atmosphere for beam radiation can be determined for any zenith angle and any altitude up to 2.5 km. The clear sky beam normal radiation is then 3 ⇒ a_o a_1 k

$$G_{cnb} = G_{on}\tau_b \tag{2.8.5}$$

where G_{on} is obtained from Equation 1.4.1. The clear sky horizontal beam radiation is

$$\boxed{G_{cb} = G_{on}\tau_b \cos\theta_z} \tag{2.8.6}$$

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For periods of an hour, the clear sky horizontal beam radiation is

$$I_{cb} = I_{on}\tau_b \cos\theta_z \tag{2.8.7}$$

Example 2.8.1

Calculate the transmittance for beam radiation of the standard clear atmosphere at Madison (altitude 270 m) on August 22 at 11:30 a.m. solar time. Estimate the intensity of beam radiation at that time and its component on a horizontal surface.

Solution

On August 22, n is 234, the declination is 11.4°, and from Equation 1.6.4 the cosine of the zenith angle is 0.846.

The next step is to find the coefficients for Equation 2.8.1. First, the values for the standard atmosphere are obtained from Equations 2.8.2–2.8.4 for an altitude of 0.27 km:

$$a_o^* = 0.4237 - 0.00821\,(6 - 0.27)^2 = 0.154$$
$$a_1^* = 0.5055 + 0.00595\,(6.5 - 0.27)^2 = 0.736$$
$$k^* = 0.2711 + 0.01858\,(2.5 - 0.27)^2 = 0.363$$

The climate-type correction factors are obtained from Table 2.8.1 for mid-latitude summer. Equation 2.8.1 becomes

$$\tau_b = 0.154(0.97) + 0.736(0.99)e^{-0.363(1.02)/0.846}$$
$$= 0.62$$

The extraterrestrial radiation is given by Equation 1.4.1. For the solar constant of 1353 W/m² G_o is 1325 W/m². The beam radiation is then

$$G_{cbn} = 1325 \times 0.62 = 822 \text{ W/m}^2$$

The component on a horizontal plane is

$$822 \times 0.846 = 695 \text{ W/m}^2$$ ■

It is also necessary to estimate the clear sky diffuse radiation on a horizontal surface to get the total radiation. Liu and Jordan (1960) developed an empirical relationship between the transmission coefficient for beam and diffuse radiation for clear days:

$$\boxed{\tau_d = 0.2710 - 0.2939\tau_b} \qquad (2.8.7)$$

where τ_d is G_d/G_o (or I_d/I_o) the ratio of diffuse radiation to the extraterrestrial radiation on a horizontal plane. The equation is based on data for three stations. The data used by Liu and Jordan predated that used by Hottel and may not be entirely consistent with it; until better information becomes available, it is suggested that Equation 2.8.7 be used to estimate diffuse clear sky radiation, which can then be added to the beam radiation predicted by Hottel's method to obtain a clear day total. (For purposes of correlating radiation data, it is necessary to have a well-defined standard (clear) day. This definition of a standard clear sky radiation is used in later sections.)

These calculations can be repeated for each hour of the day, based on the midpoints of the hours, to obtain the standard clear day's radiation, H_c.

Example 2.8.2

Estimate the standard clear day radiation on a horizontal surface, for Madison, on August 22.

Solution

For each hour, based on the midpoints of the hour, the transmittances of the atmosphere for beam and diffuse radiation are estimated. The calculation of τ_b is illustrated for the hour 11–12 (i.e., at 11:30) in Example 2.8.1, and the beam radiation for a horizontal surface for the hour is 2.50 MJ/m^2.

The calculation of τ_d is based on Equation 2.8.7

$$\tau_d = 0.2710 - 0.2939(0.62) = 0.089$$

The diffuse irradiance on the horizontal plane is obtained from G_{on} which on August 22 is 1325 W/m^2, and $\cos\theta_z$ for 11:30 is 0.846.

$$G_{cd} = 1325 \times 0.089 \times 0.846 = 100 \text{ W/m}^2$$

$$\boxed{G_{cd} = G_{on} \cdot \tau_d \cdot \cos\theta_z}$$

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Then the diffuse radiation for the hour is 0.36 MJ/m^2. The total radiation on a horizontal plane for the hour is $2.50 + 0.36 = 2.86$ MJ/m^2. These calculations are repeated for each hour of the day. The result is shown in the tabulation.

Hours	τ_b	I_{cb} MJ/m^2 normal	I_{cb} MJ/m^2 horizontal	τ_d	I_{cd} MJ/m^2	I_c MJ/m^2
11–12, 12–1	0.620	2.96	2.50	0.089	0.36	2.86
10–11, 1–2	0.608	2.90	2.31	0.092	0.35	2.66
9–10, 2–3	0.580	2.77	1.95	0.101	0.34	2.29
8–9, 3–4	0.531	2.53	1.44	0.115	0.31	1.75
7–8, 4–5	0.445	2.12	0.87	0.140	0.27	1.14
6–7, 5–6	0.290	1.38	0.31	0.186	0.20	0.51
5–6, 6–7	0.150	0.72	0.03	0.227	0.04	0.07

The beam for the day, H_{cb}, is twice the sum of column 4, giving 18.8 MJ/m^2. The day's total radiation, H_c, is twice the sum of column 7, or 22.6 MJ/m^2. ■

A simpler method for estimating clear sky radiation by hours is to use data for the ASHRAE standard atmosphere. Farber and Morrison (1977) provide tables of beam normal radiation and total radiation on a horizontal surface as a function of zenith angle. These are plotted on Figure 2.8.2. For a given day, hour-by-hour estimates of I can be made, based on midpoints of the hours.

Example 2.8.3

For August 22 for Madison, estimate the hour by hour insolation on a horizontal surface for the ASHRAE clear sky.