Deutscher Luft- und
Raumfahrtkongress 2005

Hochschule für Angewandte Wissenschaften Hamburg
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Mach Number, Relative Thickness, Sweep and Lift Coefficient of the Wing –

An Empirical Investigation of Parameters and Equations

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Background:
Preliminary Sizing in Aircraft Design

Preliminary sizing requires quick estimates of e.g.:

- **maximum lift coefficient**
- **zero lift drag, induced drag, wave drag**
- **buffet onset boundary**
- **aircraft mass, CG position**
- **floation (ACN)**
- **...**
- **relative thickness of the wing**

Introduction (Motivation)

Wing design requirements:

- High lift requirements (takeoff and landing)
- Cruise Mach number
- Buffet-free high altitude flight
- Low wing weight
- High wing stiffness
- Sufficient fuel volume in the wing
- ...

Wing parameters:

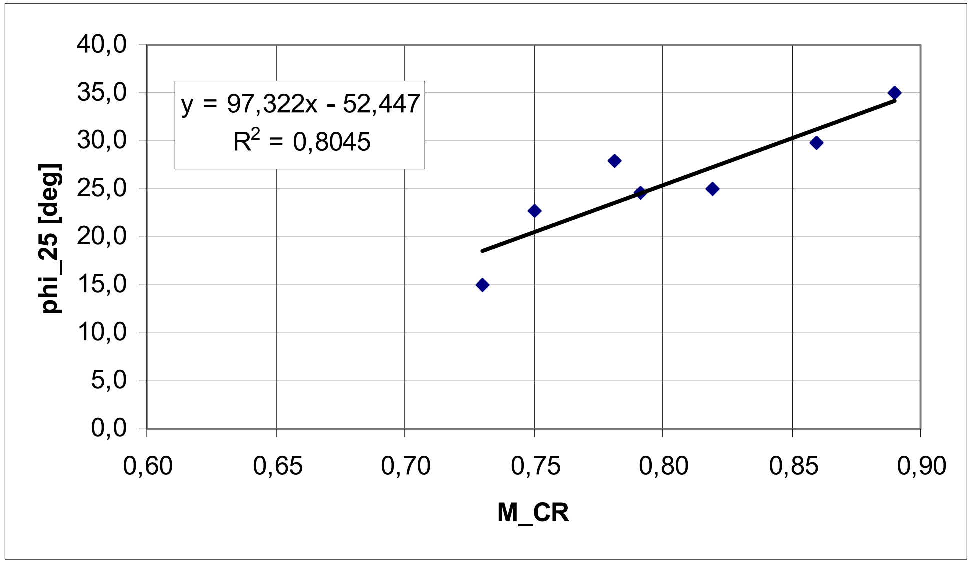
- relative thickness *t/c*, sweep, cruise lift coefficient
- taper ratio, dihedral angle, incidence angle, ...

Introduction (Motivation)

- Suitable sequence to obtain parameters
  1. Lift coefficient

$$C_L = \frac{2\, m_{MTO}\, g}{\rho\, V_{CR}{}^2\, S_W}$$

  2. Sweep



  3. Relative thickness *t/c*

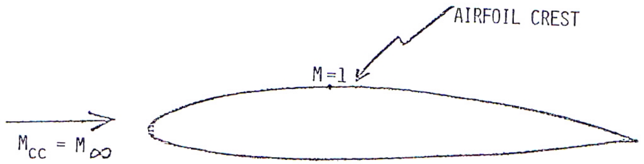
Introduction (Literature)

- There are a number of equations presented in the literature trying to establish a relationship among the parameters that are of interest in this paper. 12 equations have been investigated.

- No reference has been found in the literature that
  - a) extensively compares these equations with one another or
  - b) tries to check the equations against a large set of statistical data.

Fundamentals (1)

- Mach number, M
  - *"The ratio of the true airspeed to the speed of sound under prevailing atmospheric conditions."*
- Free stream Mach number, M
  - The Mach number of the moving body. $M = v/a$ with v being the true airspeed and a the speed of sound.
- Critical Mach number, M_{cr}
  - That freestream Mach number at which sonic flow is first obtained *somewhere on the airfoil*.
- Crest critical Mach number, M_{CC}
  - That freestream Mach number at which sonic flow is first obtained *at the airfoil crest.*

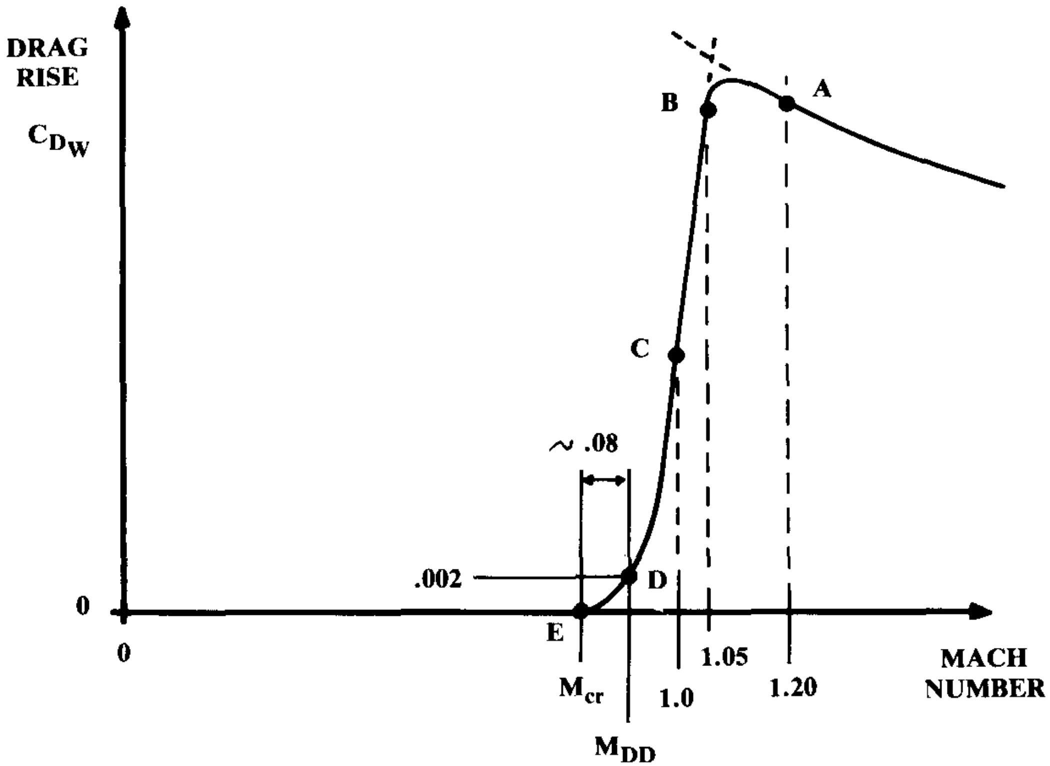


Fundamentals (2)

- **Drag rise Mach number**
  - The Mach number beyond which a rapid increase in compressibility drag occurs.
- **Drag divergence Mach number, M_{DD}**
  - At Airbus and Boeing M_{DD} is that Mach number where the wave drag coefficient reaches 20 drag counts ($\Delta C_{D,wave}$ = 0.002).
- **Drag divergence Mach number, M_{DIV}**
  - At Douglas M_{DIV} was defined as that Mach number at which the rate of change in compressibility drag with Mach number is $dC_D / dM = 0.1$

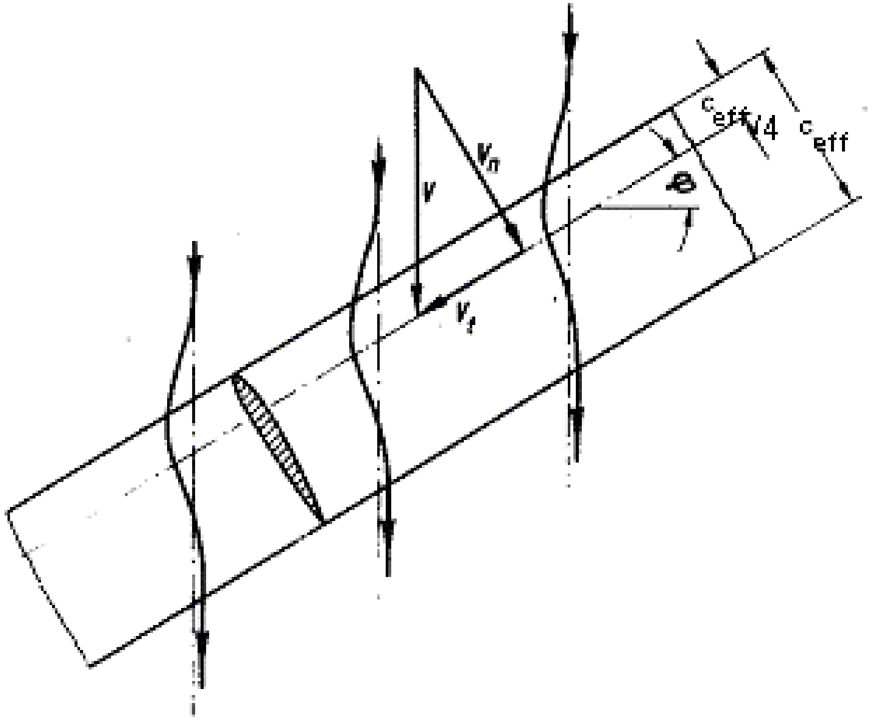
Fundamentals (3)

- **Drag divergence Mach number, M_{DD}**



Fundamentals (4)

- **Effective parameters of swept wings (cosine-rule)**



$$v_{eff} = v \cos \varphi_{25} \qquad M_{eff} = M \cos \varphi_{25}$$

$$c_{eff} = c \cos \varphi_{25}$$

$$t_{eff} = t$$

$$(t/c)_{eff} = (t/c) / \cos \varphi_{25}$$

Fundamentals (5)

- **Effective Mach number (real flows)**
  - The real flow does not necessarily follow the cosine-rule. More generally it can be said that

$$M_{eff} = M \left(\cos\varphi_{25}\right)^{x}$$

  - $0 < x < 1$.
  - Standard: $x = 0.5$,
  - STAUFENBIEL: $x = 0.75$,
  - cosine-rule: $x = 1.0$.

$$\boxed{M_{DD,eff} = M_{DD}\sqrt{\cos\varphi_{25}}}$$

Fundamentals (6)

- **Airfoils for transonic flow**
  - **Conventional airfoils**
    - NACA 64-series airfoils. Originally designed to encourage laminar flow, turned out to have relative high values of M_{cr} in comparison with other NACA shapes.
  - **Peaky airfoils**
    - A peaky pressure distribution intentionally creates supersonic velocities and suction forces close to the leading edge. Drag rise is postponed to high speeds.
  - **Supercritical airfoils**
    - The supercritical airfoil has a relatively flat top in turn, the terminating shock is weaker, thus creating less drag.
    - This paper distinguishes arbitrarily between older supercritical airfoils (1965-1987) and modern supercritical airfoils (1988-today).

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Equations for the calculation of the relative thickness

- **Equation based on TORENBEEK**

$$\frac{t}{c} = 0.30 \left\{ \left[1 - \left\{ \frac{5 + M^2}{5 + (M^*)^2} \right\}^{3,5} \right] \frac{\sqrt{1 - M^2}}{M^2} \right\}^{2/3}$$

M^* depending on airfoil

$$\boxed{\frac{t}{c} = 0.3 \cos \varphi_{25} \left\{ \left[1 - \left\{ \frac{5 + M_{DD,eff}^{\;2}}{5 + (M^* - 0.25\, C_L)^2} \right\}^{3,5} \right] \frac{\sqrt{1 - M_{DD.eff}^{\;2}}}{M_{DD,eff}^{\;2}} \right\}^{2/3}}$$

Equations for the calculation of the relative thickness

- **Equations from Aerodynamic Similarity based on ANDERSON**
  - Similarity Parameter *K*

$$K = \frac{1 - M_\infty}{\tau^{2/3}}$$

  - Solved for relative thickness

$$\boxed{t/c = \left(\frac{1 - M_{DD}}{K}\right)^{3/2}}$$

$$\boxed{t/c = \left(\frac{1 - M_{DD,eff}}{K_{eff}}\right)^{3/2}}$$

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Equations for the calculation of the relative thickness

- **Equation from SHEVELL**

$$\frac{M_\infty^2 \cos^2\Lambda}{\sqrt{1-M_\infty^2\cos^2\Lambda}}\cdot\left[\left(\frac{\gamma+1}{2}\right)\frac{2.64\,(t/c)_\infty}{\cos\Lambda}+\left(\frac{\gamma+1}{2}\right)\frac{2.64\,(t/c)_\infty(0.34C_L)}{\cos^3\Lambda}\right]$$
$$+\frac{M_\infty^2\cos^2\Lambda}{1-M_\infty^2\cos^2\Lambda}\cdot\left[\left(\frac{\gamma+1}{2}\right)\left[\frac{1.32\,(t/c)_\infty}{\cos\Lambda}\right]^2\right]$$
$$+M_\infty^2\cos^2\Lambda\cdot\left[1+\left(\frac{\gamma+1}{2}\right)\frac{(0.68C_L)}{\cos^2\Lambda}+\frac{\gamma+1}{2}\left(\frac{0.34C_L}{\cos^2\Lambda}\right)^2\right]-1=0$$

$$(t/c)_\infty = t/c$$

$$M_\infty = M_{CC} \qquad \Lambda = \varphi_{25}$$

$$\boxed{t/c = f(M_{CC}, \varphi_{25}, C_l)}$$

Equations for the calculation of the relative thickness

- **Equation from SHEVELL (continued)**

$$t/c = f(M_{CC}, \varphi_{25}, C_l)$$

$$M_{CC} = \frac{M_{DIV,conventional}}{1.025 + 0.08(1-\cos\varphi_{25})}$$

$$M_{DIV,conventional} = M_{DIV,supercritical} - 0.06$$

$$M_{DIV} = M_{DD} - 0.02$$

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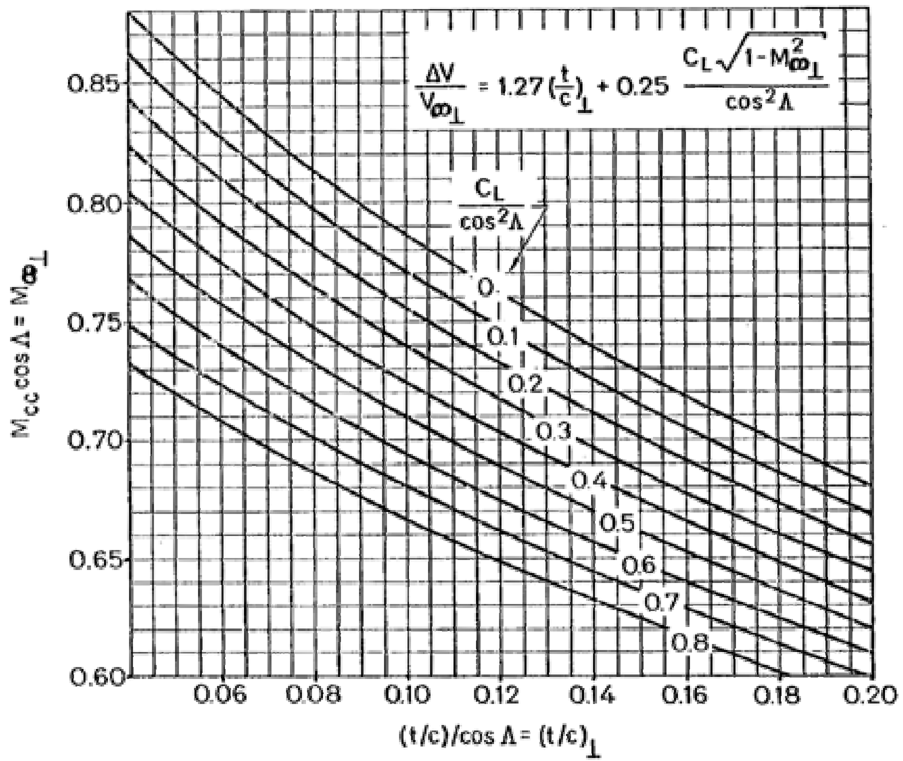
Equations for the calculation of the relative thickness

- **Equation based on KROO**



$$\boxed{t/c = x\cos\varphi_{25}}$$

$$x = \frac{-v-\sqrt{v^2-4uw}}{2u}$$

$$u = 2.8355$$

$$v = -1.9072 + 0.2\cdot 2.131y$$

$$w = 0.9499 - 0.2y - M_{CC}\cdot\cos\varphi_{25}$$

$$y = C_L/(\cos\varphi_{25})^2$$

Equations for the calculation of the relative thickness

- **Equation from HOWE**

$$M_{DD,eff} = A_F - 0.1\, C_L - t/c$$

  - **"A_F is a number, which depends upon the design standard of the aerofoil section. For older aerofoil A_F was around 0.8 but a value of 0.95 should be possible with an optimized advanced aerofoil."**
  - **We can think of as A_F being the effective drag divergence Mach number of an airfoil of zero thickness at zero lift coefficient.**

$$\boxed{t/c = A_F - 0.1\, C_L - M_{DD,eff}}$$

Equations for the calculation of the relative thickness

- **Equation from JENKINSON**

$$M_{DD} = 0.9965 - 1.387 \cdot t/c + 4.31 \cdot 10^{-5} \varphi_{25} - 0.18 \cdot C_L$$

  - We can think of M_{DD} = 0.9965 for a wing with zero relative thickness at zero lift coefficient and with zero sweep

$$\boxed{t/c = 0.7185 + 3.107 \cdot 10^{-5} \varphi_{25} - 0.1298 \cdot C_L - 0.7210 \cdot M_{DD}}$$

Equations for the calculation of the relative thickness

- **Equation from WEISSHAAR**

$$M_{DD} = \frac{K_A}{\cos\varphi_{25}} - \frac{t/c}{\cos^2\varphi_{25}} - \frac{C_L}{10\cos^3\varphi_{25}}$$

  - K_A is approximately 0.80 ... 0.90.
  - We can think of K_A as being the drag divergence Mach number of an unswept wing of zero thickness at zero lift coefficient

$$\boxed{t/c = K_A \cos\varphi_{25} - M_{DD}\cos^2\varphi_{25} - \frac{C_L}{10\cos\varphi_{25}}}$$

Equations for the calculation of the relative thickness

- **Equation based on BÖTTGER**

$$t / c = \frac{27}{30}\left[a(C_L - b)^d + c + 0.00288(\varphi_{25} - 29.8°) - M_{DD}\right] + 0.113$$

with

a = -1.147
b = 0.200
c = 0.838
d = 4.057

Equations for the calculation of the relative thickness

- **Equation based on RAYMER**

$$\boxed{M_{DD} = M_{DD}(C_L = 0)\ LF_{DD} - 0.05 \cdot C_L}$$

$$M_{DD}(C_L = 0) =$$

$$1 + k_{M,DD}\left(u\left(90° - \varphi_{25}\right)^3 + v\left(90° - \varphi_{25}\right)^2 + w\left(90° - \varphi_{25}\right)\right)$$

with

u = 8.029·10^-7 1/deg^3
v = -1.126·10^-4 1/deg^2
w = 8.437·10^-4 1/deg

Equations for the calculation of the relative thickness

- **Equation based on RAYMER (continued)**

$$k_{M,DD} = 1317 \cdot (t/c)^3 - 324.3 \cdot (t/c)^2$$
$$+ 28.948 \cdot (t/c) - 0.0782$$

$$LF_{DD} = k_{LF,DD} \left(a\, C_L{}^2 + b\, C_L \right) + 1$$

with

a = -0.1953

b = -0.1494

$$k_{LF,DD} = 23.056 \cdot (t/c)^2 + 3.889 \cdot (t/c)$$

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Equations for the calculation of the relative thickness

- **Equation based on Linear Regression**

$$t/c = a\,M_{DD} + b\,\varphi_{25} + c\,C_L + d\,k_m$$

or knowing that

$$M_{DD,eff} = M_{DD}\,\sqrt{\cos\varphi_{25}}$$

better

$$\boxed{t/c = a\,M_{DD,eff} + b\,C_L + c\,k_m}$$

Equations for the calculation of the relative thickness

- **Equation based on Nonlinear Regression**

$$\boxed{t / c = k_t \cdot M_{DD}^{\,t} \cdot \cos \varphi_{25}^{u} \cdot c_L^{v} \cdot k_M^{w}}$$

The parameters k_t, t, u, v, w are fit to given aircraft data

Investigation, comparison and adaptation of equations

- Input from aircraft data covers a range of different values
  - sweep: from 0° to 35°
  - drag divergence Mach numbers M_{DD}: from 0.65 to 0.88
  - average relative wing thickness t/c: from 9% to 13.4%
  - cruise lift coefficient C_L: from 0.22 to 0.73
  - type of airfoil:
    - conventional (NACA)
    - peaky
    - older supercritical airfoils (1965-1987)
    - modern supercritical airfoils (1988-today)

Investigation, comparison and adaptation of equations

- Aircraft considered with conventional (NACA) airfoils
  - IAI 1124A Westwind 2
  - Sud Aviation Caravelle
  - VFW 614
  - HFB 320
  - Gates Lear Jet Model 23
  - Lockheed C-141 Starlifter
  - Lockheed Jetstar II
  - Dassault Falcon 20

Investigation, comparison and adaptation of equations

- Aircraft considered with peaky airfoils
  - BAC One-Eleven Series 500
  - McDonnell Douglas DC-9 Series 30
  - Vickers VC-10 Super VC-10
  - McDonnell Douglas DC-8 Series 63
  - McDonnell Douglas DC-10 Series 10
  - Lockheed C-5A

Investigation, comparison and adaptation of equations

- Aircraft considered with older supercritical airfoils (1965-1987)
  - Mitsubishi Diamond I
  - Airbus A300-600
  - Boeing 767-200
  - Cessna 650 Citation VI
  - Airbus A310-300
  - Raytheon Hawker 800XP
  - Raytheon Beechjet 400A
  - Beriev Be-40

Investigation, comparison and adaptation of equations

- Aircraft considered with modern supercritical airfoils (1988-today)
  - Bombardier Global Express
  - Bombardier Challenger CRJ 200 LR
  - Tupolev Tu-204-300
  - BAe RJ85
  - Embraer EMB-145
  - Airbus A321-200
  - Airbus A340-300

Investigation, comparison and adaptation of equations

- M_{DD} was taken as M_{MO} (following Boeing and Airbus design principles) if M_{MO} was known.
- M_{DD} was taken as a Mach number (calculated from V_{MO} and a known or assumed altitude *h* up to which V_{MO} is flown) if M_{MO} was unknown.
- Average relative thickness of the wing *t/c* from JENKINSON:

$$t/c = \frac{3\,(t/c)_{tip} + (t/c)_{root}}{4}$$

Investigation, comparison and adaptation of equations

- **Standard Error of Estimate** *SEE*

$$SEE = \sqrt{\frac{\sum \left(y_{estimate} - y\right)^2}{n}}$$

- **Optimization**
  - Optimized values of the free parameters determined
  - Leads to a minimum Standard Error of Estimate *SEE*
  - Calculated with EXCEL and the modified Newton method of the "Solver"

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Investigation, comparison and adaptation of equations

- **Comparison of the SEE of the equations**

ranking	Method	SEE	optimized
1	nonlinear regression	0.75 %	yes
2	TORENBEEK (with term C_L)	0.80 %	yes
3	linear regression	1.18 %	yes
4	similarity with sweep	2.43 %	yes
5	HOWE	3.67 %	yes
6	similarity without sweep	3.71 %	yes
7	WEISSHAAR	3.95 %	yes
8	JENKINSON	4.23 %	no
9	BÖTTGER	4.32 %	no
10	RAYMER	4.54 %	no
11	KROO	4.59 %	no
12	SHEVELL	8.06 %	no
	average SEE	3.25 %	

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Investigation, comparison and adaptation of equations

- **TORENBEEK's equation optimized**

$$\frac{t}{c} = k_T \cos\varphi_{25} \left\{ \left[1 - \left\{ \frac{5 + M_{DD,eff}^{\;2}}{5 + (M^* - 0.25\, C_L)^2} \right\}^{3,5} \right] \frac{\sqrt{1 - M_{DD.eff}^{\;2}}}{M_{DD,eff}^{\;2}} \right\}^{E}$$

parameter	standard	optimized
M* for conventional	1.000	0.907
M* for peaky	1.050	1.209
M* for older supercritical	1.135	4.703
M* for modern supercritical	1.135	1.735
k_T	0.300	0.130
E	0.667	0.038

Investigation, comparison and adaptation of equations

- Equation from **nonlinear regression** optimized

$$\boxed{t/c = k_t \cdot M_{DD}^{\,t} \cdot \cos\varphi_{25}^{\,u} \cdot c_L^{\,v} \cdot k_M^{\,w}}$$

k_T = 0.127	k_M for conventional ...	0.921
t = -0.204	k_M for peaky ...	0.928
u = 0.573	k_M for older supercritical ...	1.017
v = 0.065	k_M for modern supercritical ...	0.932
w = 0.556	... airfoils	

Investigation, comparison and adaptation of equations

- **HOWE's equation optimized**

$$\boxed{t/c = A_F - 0.1\,C_L - M_{DD,eff}}$$

A_F	standard	optimized
A_F for conventional	0.80	0.861
A_F for peaky	0.85	0.935
A_F for older supercritical	0.90	0.907
A_F for modern supercritical	0.95	0.926

Summary and conclusions

- Goal: Relate the parameters Mach number, relative thickness, sweep, and lift coefficient to one another
- 12 equation were found in the literature
- Some equations draw strongly from *aerodynamic theory* but other equations are purely based on *statistical considerations*
- Data from 29 transport aircraft was used
- The equation based on nonlinear regression and TORENBEEK's equation can be recommended
- Many equation in the literature lead to unacceptable results!