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Generic Fuel Consumption Equations
Based on Breguet Range Equation

Fuel Consumption from Breguet Range Equation – Getting Started

Basic derivation from the lecture.

$$R = \underbrace{\frac{E \cdot V}{c \cdot g}}_{B} \ln \frac{m_1}{m_2} \qquad m_F = m_1 - m_2$$

$$e^{\frac{R}{B}} = \frac{m_1}{m_2}$$

$$m_1 = m_2 \, e^{R/B}$$

$$m_2 = m_1 \, e^{-R/B}$$

$$m_F = m_1 - m_1 e^{-R/B}$$

$$\boxed{m_F = m_1 \left(1 - e^{-R/B}\right)}$$

$$m_F = m_2 e^{R/B} - m_2$$

$$\boxed{m_F = m_2 \left(e^{R/B} - 1\right)}$$

An aircraft with range, *R* can fly a stage length, s with $0 \le s \le R$.

Stage length is the term used to describe **the length of** the flight from take-off to landing in **a single leg**. If an aircraft has multiple stops along its route, then it would have multiple **s**tage lengths.

https://www.stratosjets.com/glossary/charter-flight-stage-length

An aircraft with range, *R* can fly a relative stage length, $x = s/R$ with $0 \le x \le 1$.

Unity Equation / First Law of Aircraft Design

$$m_{MTO} = m_{PL} + m_F + m_{OE} \quad .$$

$$m_{MTO} - m_F - m_{OE} = m_{PL}$$

$$m_{MTO} \cdot \left(1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}} \right) = m_{PL}$$

$$m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \quad .$$

$$1 = \frac{m_{OE}}{m_{MTO}} + \frac{m_{PL}}{m_{MTO}} + \frac{m_F}{m_{MTO}}$$

$$\frac{m_F}{m_{MTO}} = 1 - \frac{m_{OE}}{m_{MTO}} - \frac{m_{PL}}{m_{MTO}}$$

$$\frac{m_{OE}}{m_{MTO}} \approx 0.4 \ \dots \ 0.5$$

(long-range to short-range)

$$0 \ \leq \frac{m_{PL}}{m_{MTO}} \leq 0.6 \, , \ 0 \ \leq \frac{m_F}{m_{MTO}} \leq 0.6$$

Fuel Consumption from Breguet Range Equation – Carrying as Little Fuel as Possible

$$\boxed{m_F = m_2 \left(e^{R/B} - 1 \right)}$$

This equation is used when only that much fuel is put into the tank as required for the flight. This is how it is done in the real world. The aim is to fly as light as possible and to carry only the fuel (weight) which is really needed.

$\frac{m_F}{m_L} = e^{R/B}$ - 1 for (maximum) range, *R*

using the index "L" (landing) instead of the more general index "2"

$\frac{m_{F,s}}{m_L} = e^{s/B}$ - 1for any stage length, *s*

$\frac{m_{F,s}}{m_L} = e^{\frac{s R}{R B}}$ - 1 written more complicated

Relative fuel consumption with respect to m_L, y_L

$y_L = \frac{m_{F,s}}{m_L} = e^{x \frac{R}{B}}$ - 1

as function of relative stage length, *x* for carrying as little fuel as possible.

With respect to m_{MTO}:

$$\frac{m_L}{m_{MTO}} = \frac{m_{OE}}{m_{MTO}} + \frac{m_{PL}}{m_{MTO}} = 1 - \frac{m_F}{m_{MTO}}$$

$$y = \frac{m_{F,s}}{m_{MTO}} = \frac{m_F}{m_L} \cdot \frac{m_L}{m_{MTO}} = \frac{m_L}{m_{MTO}} \cdot \frac{m_F}{m_L} = \left(1 - \frac{m_F}{m_{MTO}}\right)\left(e^{x\frac{R}{B}} - 1\right)$$

Relative fuel consumption with respect to m_{MTO}, y

$$y = \frac{m_{F,s}}{m_{MTO}} = \left(1 - \frac{m_F}{m_{MTO}}\right)\left(e^{x\frac{R}{B}} - 1\right)$$

as function of relative stage length, x for **carrying as little fuel as possible**.

Fuel Consumption from Breguet Range Equation – Always Going with a Full Tank

$$\boxed{m_F = m_1\left(1 - e^{-R/B}\right)}$$

This equation is used when the tank is filled to maximum level for every flight (no matter what the stage length is). This is not as it is done in the real world, because it means more weight than necessary and higher fuel consumption. Reserves are necessary, but only calculated based on regulations plus in some cases additions based on the decision of the pilot ("just in case"). To go always with a full tank can be done for a trip with a car (where fuel burn for carrying fuel is neglectable) but not with an aircraft.

Relative fuel consumption, y

$$y = \frac{m_{F,s}}{m_{MTO}} = 1 - e^{-x\frac{R}{B}}$$

as function of relative stage length, x for **always going with a full tank**.

Calculating R/B from Typical Parameters

Long-range:

$$\frac{m_{OE}}{m_{MTO}} = 0.4\,, \quad \frac{m_{PL}}{m_{MTO}} = 0.1, \quad \frac{m_F}{m_{MTO}} = 0.5$$

Short-range:

$$\frac{m_{OE}}{m_{MTO}} = 0.5\,, \quad \frac{m_{PL}}{m_{MTO}} = 0.25, \quad \frac{m_F}{m_{MTO}} = 0.25$$

$$\boxed{m_F = m_1\left(1 - e^{-R/B}\right)}$$

$$\frac{m_F}{m_1} = 1 - e^{-R/B}$$

$$\frac{m_F}{m_1} - 1 = -e^{-R/B}$$

$$1 - \frac{m_F}{m_1} = e^{-R/B}$$

$$\frac{1}{1 - \frac{m_F}{m_1}} = e^{R/B}$$

$$R/B = Ln\left(\frac{1}{1 - \frac{m_F}{m_1}}\right)$$

Long range

$$\frac{m_F}{m_{MTO}} \approx 0{,}5$$

$$R/B = ln\ 2$$

$$R/B = 0{,}69$$

Check with the other equation should give the same values for *R/B*

$$\boxed{m_F = m_2\left(e^{R/B} - 1\right)}$$

$$\frac{R}{B} = ln\left(\frac{m_F}{m_L} + 1\right)$$

Long range

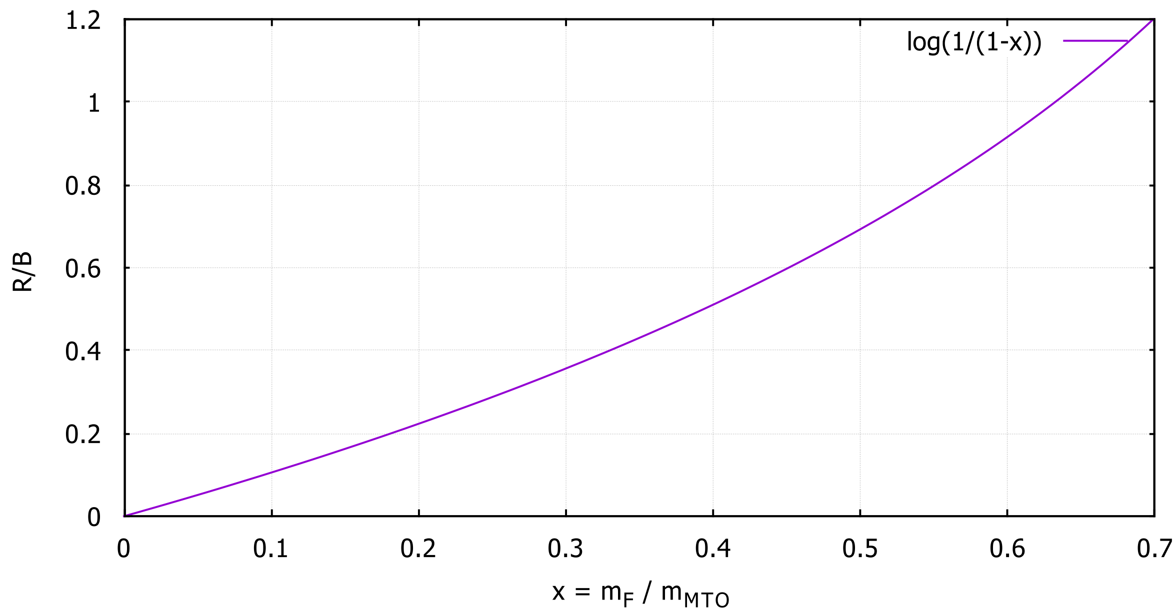
$$\frac{m_F}{m_L} = \frac{m_F/m_{MTO}}{m_L/m_{MTO}} = \frac{0.5}{0.5} = 1$$

$$\frac{R}{B} = \ln(2) = 0.69314718056 \approx 0.69$$

Here again the equation necessary to get the non-dimensional fuel economy *R*/*B* of an aircraft at a given range (not stage length) at a particular payload and operating empty mass, without going into detail with $E = L/D$ or SFC. It follows from a single parameter, the fuel fraction m_F / m_{MTO} :

Calculating the fuel economy *R*/*B* for given range, *R*

$$\frac{R}{B} = ln\left(\frac{1}{1-\frac{m_F}{m_{MTO}}}\right)$$



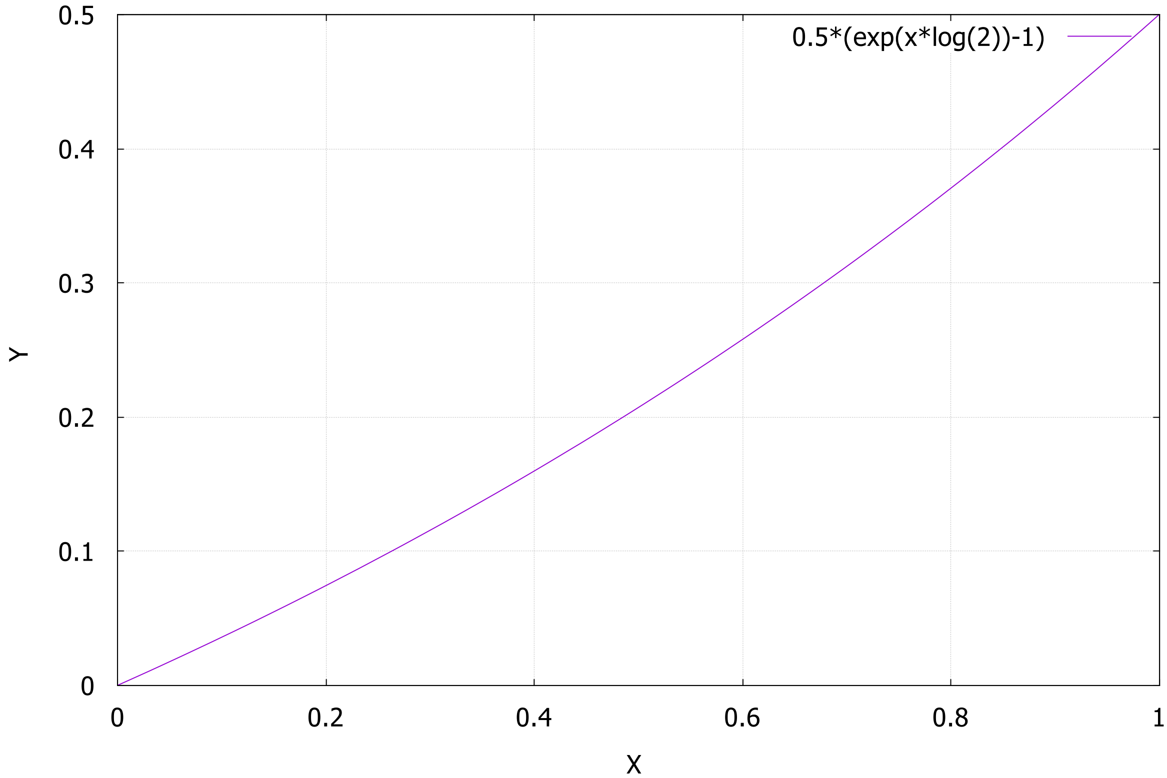
Plotting Fuel Consumption – Carrying as Little Fuel as Possible

$$\boxed{m_F = m_2\left(e^{R/B} - 1\right)}$$

Typical long-range:

$\frac{m_{OE}}{m_{MTO}} = 0.4\,,\ \ \frac{m_{PL}}{m_{MTO}} = 0.1,\ \ \frac{m_F}{m_{MTO}} = 0.5,\ \ \frac{R}{B} = \ln{(2)}$

$$\boxed{y = \frac{m_{F,S}}{m_{MTO}} = (0.5)\left(e^{x\cdot\ln(2)} - 1\right)}$$



Plotting Fuel Consumption per Distance – Carrying as Little Fuel as Possible

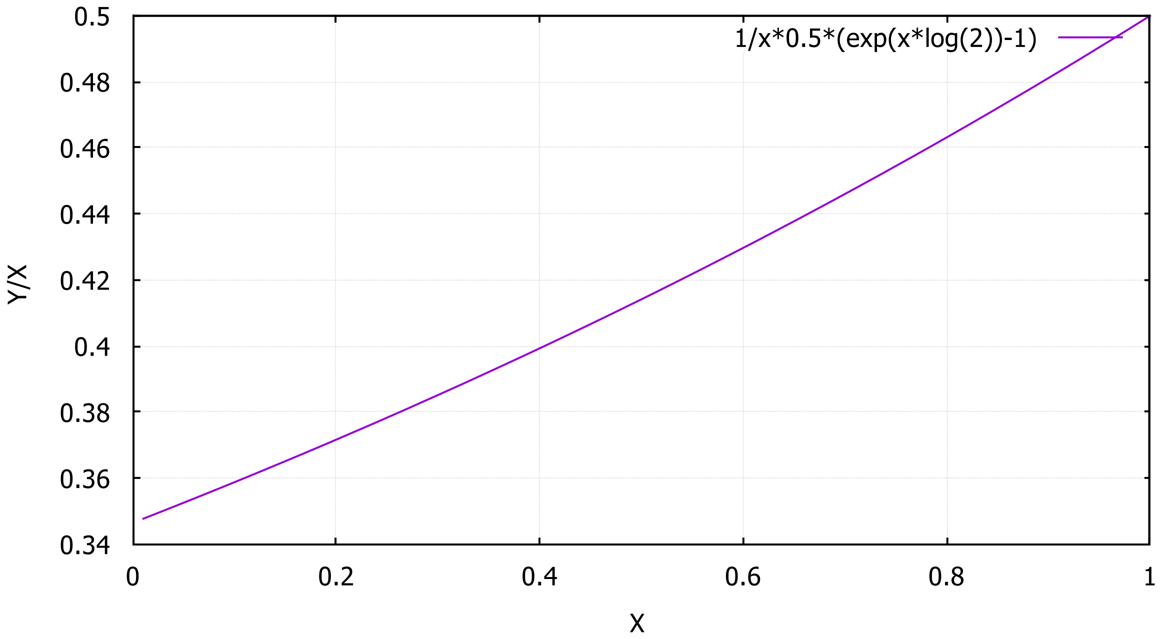
Relative fuel consumption, *y* per distance, *x* :

$$y/x = \frac{m_{F,s}}{m_{MTO}}/x = \frac{1}{x} \cdot \left(1 - \frac{m_F}{m_{MTO}}\right)\left(e^{x\frac{R}{B}} - 1\right)$$

as function of relative stage length, x for carrying **as little fuel as possible**.

For typical long-range:

$$y/x = \frac{1}{x} \cdot (0.5)\left(e^{x \cdot \ln(2)} - 1\right)$$



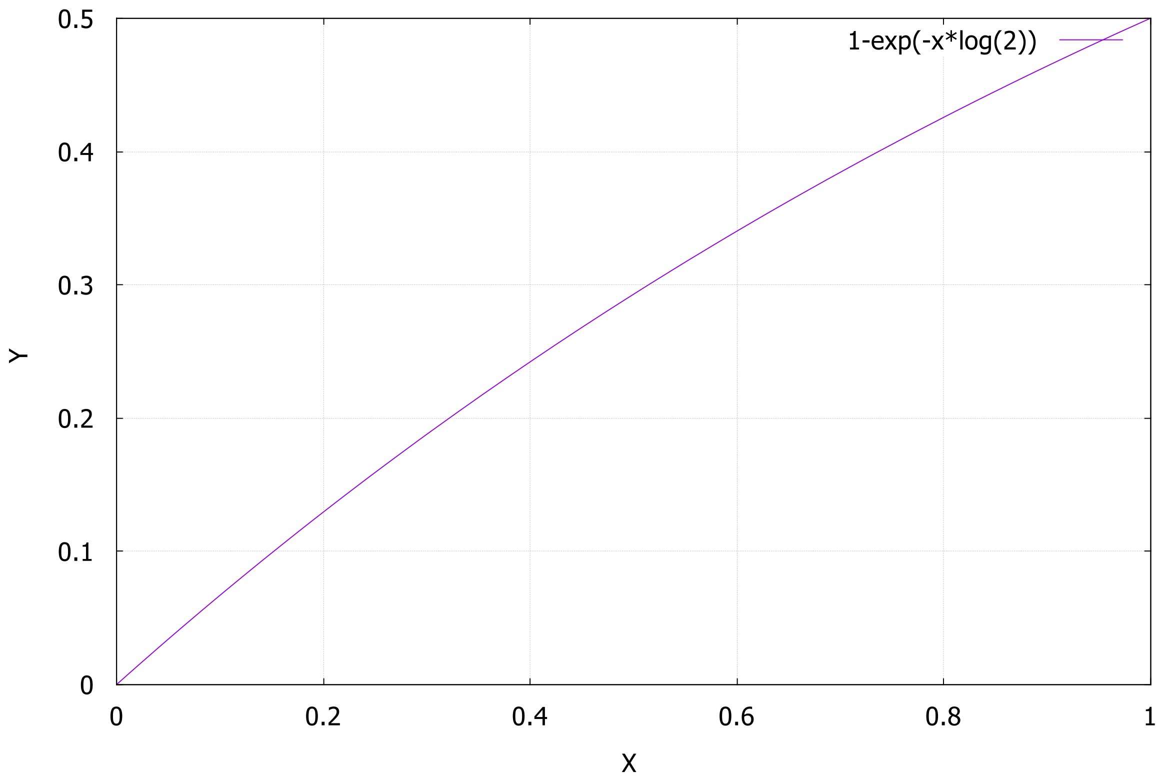
Plotting Fuel Consumption – Starting Always with a Full Tank

$$\boxed{m_F = m_1 \left(1 - e^{-R/B}\right)}$$

$$y = \frac{m_{F,S}}{m_{MTO}} = 1 - e^{-x\frac{R}{B}}$$

For typical long-range:

$$y = 1 - e^{-x \cdot \ln(2)}$$



Plotting Fuel Consumption per Distance – Starting Always with a Full Tank

Relative fuel consumption, *y* per distance, *x* :

$$y/x = \frac{m_{F,S}}{m_{MTO}}/x = \frac{1}{x}\cdot\left(1 - e^{-x\frac{R}{B}}\right)$$

as function of relative stage length, x for **always going with a full tank**.

For typical long-range:

$$y/x = \frac{1}{x}\cdot\left(1 - e^{-x\cdot\ln(2)}\right)$$