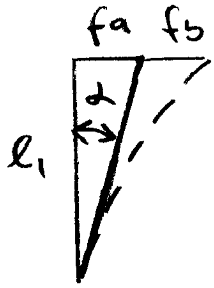
Klausur WS 02/03 TM 2

Aufgabe 1



$s = f_a + f_b \qquad c = \frac{F}{s}$

$f_a = \tan\alpha \cdot \ell_1$

$f_b = \frac{F \cdot \ell_1^3}{3EI}$

$\tan\alpha$ aus Biegefall 6 :

$M_2 = 0 \qquad M_1 = F \cdot \ell_1 \qquad x = \ell_2/2$

$$\tan\alpha = \tan\alpha_1 = \frac{\ell_2}{6EI} \cdot 2M_1 = \frac{\ell_2 F \ell_1}{3EI}$$

$$s = \frac{F\ell_1^2 \ell_2}{3EI} + \frac{F\ell_1^3}{3EI} = \frac{F\ell_1^3 \, \ell_2/\ell_1}{3EI} + \frac{F\ell_1^3}{3EI}$$

$$= \frac{F\ell_1^3}{3EI}\left[\frac{\ell_2}{\ell_1} + 1\right] = \frac{F\ell_1^3}{3EI} \, \frac{\ell_2 + \ell_1}{\ell_1} = \frac{F\ell_1^2}{3EI}(\ell_2 + \ell_1)$$

$$c = \frac{F}{s} = \frac{3EI}{\ell_1^2(\ell_2 + \ell_1)} \quad //$$

Aufgabe 2

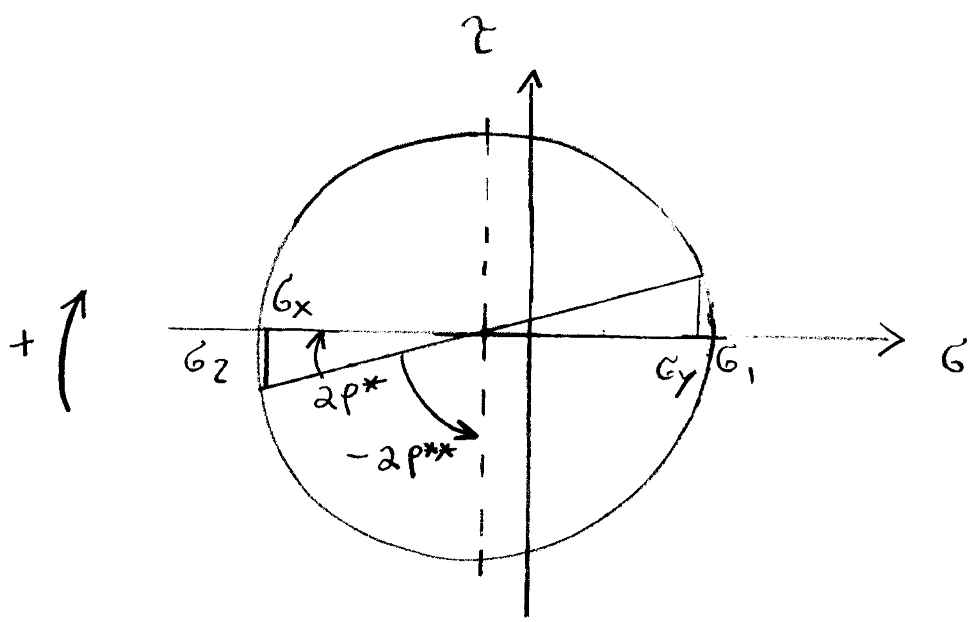
a) $$\sigma_{1,2} = \underbrace{\frac{1}{2}(\sigma_x + \sigma_y)}_{-20\ N/mm^2} \pm \underbrace{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{98.489\ N/mm^2}$$

$$\sigma_1 = (-20 + 98.489)\ N/mm^2 = 78.5\ N/mm^2 \ //$$

$$\sigma_2 = (-20 - 98.489)\ N/mm^2 = -118.5\ N/mm^2 \ //$$

$$\tan 2\varphi^* = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-80}{-180} \Rightarrow \varphi^* = 12^\circ \ //$$

b)



c) $$\tau_{max} = \pm \frac{1}{2}(\sigma_1 - \sigma_2) = \pm 98.25\ N/mm^2$$

$$\varphi^{**} = \varphi^* \pm 45^\circ = +12^\circ - 45^\circ = 33^\circ \ //$$

d) $$\sigma_v \overset{!}{=} \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}$$

$$= \sqrt{110^2 + 70^2 + 70 \cdot 110 + 3 \cdot 40^2}\ N/mm^2$$

$$\overset{!}{=} 171.8\ N/mm^2 \ //$$

e) $$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \cdot \sigma_y) = -6.24 \cdot 10^{-4}$$

Annahme: $\nu = 0.3$

Aufgabe 3

a) Fall III : $F_K = \frac{EI\pi^2}{l_1^2} \cdot 2.04$

$$I = \left[\frac{60^4}{12} - \frac{52^4}{12}\right] mm^4 = 470.7 \cdot 10^3 \; mm^4$$

$$F_K = -S_1 \overset{!}{=} \frac{2.1 \cdot 10^5 \cdot 470.7 \cdot 10^3 \cdot \pi^2 \cdot 2.04}{2000^2} \, N$$

$$= 497.5 \; KN$$

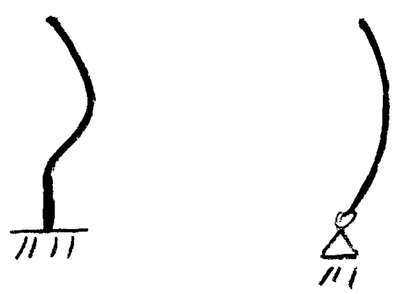
b) Fall II : $F_K = \frac{EI\pi^2}{l_2^2}$

$$l_2 = \sqrt{2} \cdot l_1$$

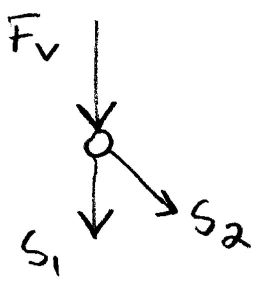
$$F_K = -S_2 \overset{!}{=} \frac{2.1 \cdot 10^5 \cdot 470.7 \cdot 10^3 \cdot \pi^2}{2 \cdot 2000^2} \, N$$

$$= 121.95 \; KN$$

c) a) b)



d)



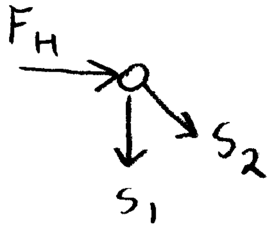
$(\rightarrow) \Rightarrow S_2 = 0$

$(\uparrow) \Rightarrow S_1 = -F_V$ (Druck)

Knicken bei $F_V = F_K$

$\overset{!}{=} 497.5 \, KN$

e)



$$(\rightarrow)\quad \frac{S_2}{\sqrt{2}} = -F_H$$

$$F_{HK} = -\frac{S_2}{\sqrt{2}}$$

$$(\downarrow)\quad S_1 + \frac{S_2}{\sqrt{2}} = 0$$

$$S_1 = -\frac{S_2}{\sqrt{2}}$$

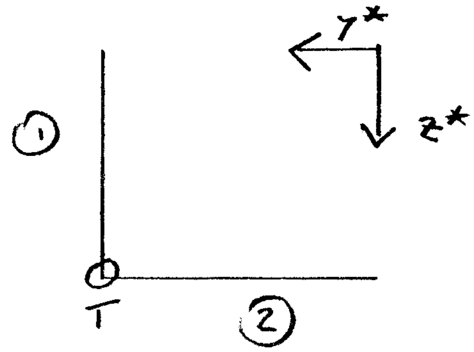
Also 2 wird auf Druck beansprucht 1 dagegen auf Zug.

Struktur knickt wenn $-S_2 = F_K = 121{,}95\,kN$

$$F_{HK} = -\frac{S_2}{\sqrt{2}} = \frac{121{,}95\,kN}{\sqrt{2}} = 86.23\ kN$$ //

Aufgabe 4

a) • Es handelt sich um ein dünnes Profil ⇒ Linienschwerpunkt

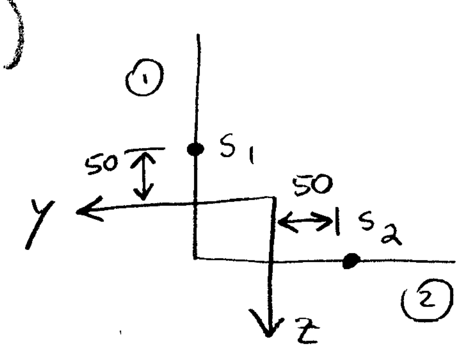


n	ℓ	y_{si}^*	z_{si}^*	$\ell \cdot y_{si}^*$	$\ell \cdot z_{si}^*$
1	200	200	100	40000	20000
2	200	100	200	20000	40000
Σ	400	×	×	60000	60000

$$\left.\begin{aligned} y_s^* &= \frac{60000}{400}\,\text{mm} = 150\,\text{mm} \\ z_s^* &= \quad -\text{"}- \quad = 150\,\text{mm} \end{aligned}\right\}\ \text{Schwerpunkt}$$

• Der Schubmittelpunkt liegt im Schnittpunkt der Schubkräfte, d.h. im
" der beiden Schenkel;
gekennzeichnet durch

b)



$$I_y = \frac{b_1 h_1^3}{12} + A_1 \cdot z_{s_1}^2 + \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot z_{s_2}^2$$

$$I_y = \left[\frac{8 \cdot 200^3}{12} + 1600 \cdot 50^2 + \frac{200 \cdot 8^3}{12} + 1600 \cdot 50^2\right] mm^4$$

$A_1 = A_2 = 1600\ mm^2$

$$= 13{,}34 \cdot 10^6\ mm^4$$

$$I_z = I_y = 13.34 \cdot 10^6\ mm^4 \ //$$

$$I_{yz} = \underbrace{I_{yz_1}}_{0} - y_{s_1} \cdot z_{s_1} \cdot A_1 + \underbrace{I_{yz_2}}_{0} - y_{s_2} \cdot z_{s_2} \cdot A_2$$

$$= -(+50)(-50) \cdot 1600\ mm^4 - (-50)(+50) \cdot 1600\ mm^4$$

$$= 8 \cdot 10^6\ mm^4 \ //$$

c) I: $\sigma_x = \frac{N}{A} = \frac{-6000\ N}{3200\ mm^2} = -1{,}875\ N/mm^2 \ //$

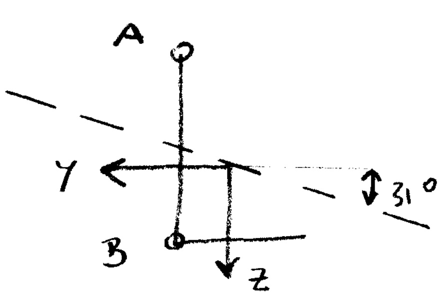
$A = 200 mm \cdot 8 mm \cdot 2 = 3200 mm^2$

II: $M_y = 3\ KNm \qquad M_z = 0$

$$\tan \alpha_0 = \frac{\underbrace{M_z}_{0} I_y - M_y I_{yz}}{M_y I_z - \underbrace{M_z}_{0} I_{yz}}$$

$$= -\frac{3000 \cdot 8 \cdot 10^6}{3000 \cdot 13.34 \cdot 10^6} = -0.60$$

$$\Rightarrow \alpha_0 = -31^\circ$$



$P(y, z)$: $A(50, -100)$

$B(50, 50)$

mit $M_z = 0$

$$\sigma_x(y;z) = \frac{1}{\Delta}\left[M_y \cdot I_z \cdot z + M_y I_{yz} \cdot y\right]$$

$$\Delta = I_y \cdot I_z - I_{yz}^2 = \left[\left(13.34 \cdot 10^6\right)^2 - \left(8 \cdot 10^6\right)^2\right] mm^8$$

$$= 1.139 \cdot 10^{14} \; mm^8$$

<u>A:</u> $$\sigma_x = \frac{3000 \cdot 1000 \; Nmm}{1.139 \cdot 10^{14} \; mm^8}\left[13.34 \cdot 10^6 \cdot 50 + 8 \cdot 10^6 (-100)\right] mm^5$$

$$= -3{,}50 \; N/mm^2 \; //$$

<u>B:</u> $$\sigma_x = \frac{3 \cdot 10^6 \; Nmm}{1.139 \cdot 10^{14} \; mm^8}\left[13.34 \cdot 10^6 \cdot 50 + 8 \cdot 10^6 \cdot 50\right] mm^5$$

$$= 28.10 \; N/mm^2 \; //$$

Aufgabe 5

a) Schubfluß : $\tau \cdot t$

1. BREDTsche Formel :

$$\tau \cdot t = \frac{M_t}{2 \cdot A_m}$$

$D = 150\,mm$ $\qquad A_m = \frac{A}{2} = \frac{D^2 \pi}{4 \cdot 2} = 8836\,mm^2$

$$\tau \cdot t = \frac{4 \cdot 10^6\,N\,mm}{2 \cdot 8836\,mm^2} = 226\,N/mm \;//$$

b) $\tau_{zul} = 80\,N/mm^2$

$$\tau_{zul} \cdot t = \tau t$$

$$t = \frac{\tau t}{\tau_{zul}} = \frac{226\,N \cdot mm^2}{mm \cdot 80\,N}$$

$$= 2{,}83\,mm \;//$$

c) $$G = \frac{E}{2(1+\nu)} = \frac{0.7 \cdot 10^5}{2(1+0.34)}\,N/mm^2 = 2.612 \cdot 10^4\,N/mm^2$$

$$\vartheta = \frac{M_t}{G \cdot I_t} = \frac{4 \cdot 10^6\,N\,mm^3}{2.612 \cdot 10^4\,N \cdot 24.3 \cdot 10^5\,mm^4} = \text{s.u.}$$

$$I_t = \frac{4 A_m^2}{\oint \frac{ds}{t(s)}} = \frac{4 A_m^2}{\frac{D + \frac{\pi D}{2}}{t}} = \frac{4 \cdot 8836^2}{\frac{150}{3}\left(1 + \frac{\pi}{2}\right)}\,mm^4$$

$$= 24.30 \cdot 10^5\,mm^4$$

$\vartheta = 3.61\,°/m \;//$