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Identifying Wave Drag for the Generic Drag Polar Equation – Unveiling Polars of 16 Passenger Aircraft

Purpose – This work systematically derives the best form of a generic drag polar equation together with the optimum numerical values of its parameters to unveil the drag coefficient of 16 passenger aircraft as a function of lift coefficient and Mach number. The parameters are selected such that they can be estimated also for other aircraft mainly from their geometry.

Methodology – Drag polars in graphical form from Obert (2009) are the starting point. Numerical values of the drag coefficient are obtained with the WebPlotDigitizer. In the generic equation, zero lift drag is assumed constant, the term representing induced drag is taken from Niţă (2012). For the wave drag term, seven functions of Mach number are investigated. The difference between Mach number and critical Mach number to the power of 4 is the classic approach based on Lock (1951). Two more general power functions, tan, tanh, sinh, and an exponential function are looked at. Parameters are optimized by minimizing the Root Mean Squared Percentage Error (RMSPE). Optimization is done with the Solver in Excel using the Generalized Reduced Gradient (GRG2) code supplied by Frontline Systems.

Findings – Based on all 16 investigated aircraft, a generic drag polar using the hyperbolic tangent (tanh) to express wave drag is best with mean RMSPE of only 0.68% (Figure 1 and 2). The second best is the most general power function with mean RMSPE of 0.75%. Its special case, the often quoted but unflexible function from Lock comes out last here with a mean RMSPE of 0.95%. Nevertheless, all seven functions can be used to represent wave drag. The zero lift drag coefficient is identified between 0.013 (B777) and 0.020 (A320). The Mach dependence of the drag coefficient comes not only from wave drag, but also from induced drag and its Mach dependence beyond 0.3 Mach. Calculated parameters are plausible and come close to reference values from literature.

Research Limitations – Aerodynamic data is generally confidential. Therefore, public drag data is limited. The extension of the method to other aircraft yields a drag estimate.

Practical Implications – The generic equation can be used in preliminary aircraft design as well as in calculations in aircraft performance and flight operations.

Originality – This project formulates a generic drag polar equation with a choice of new wave drag terms some based on a historic precursor. The new approach with a hyperbolic tangent function is recommended.

$$C_D = C_{D0} + \frac{C_L^2 \cdot d}{-e \cdot \left(\frac{M}{M_{comp}} - 1\right)^f + 1} + a \cdot \left(1 + \tanh\left(b \cdot \frac{M}{M_{crit}} - c\right)\right)$$

Figure 1: In the generic drag polar equation the first term is the identified zero-lift drag coefficient, the second term identifies the Mach-dependent induced drag coefficient, and the third term identifies the wave drag coefficient (here with the hyperbolic tangent).

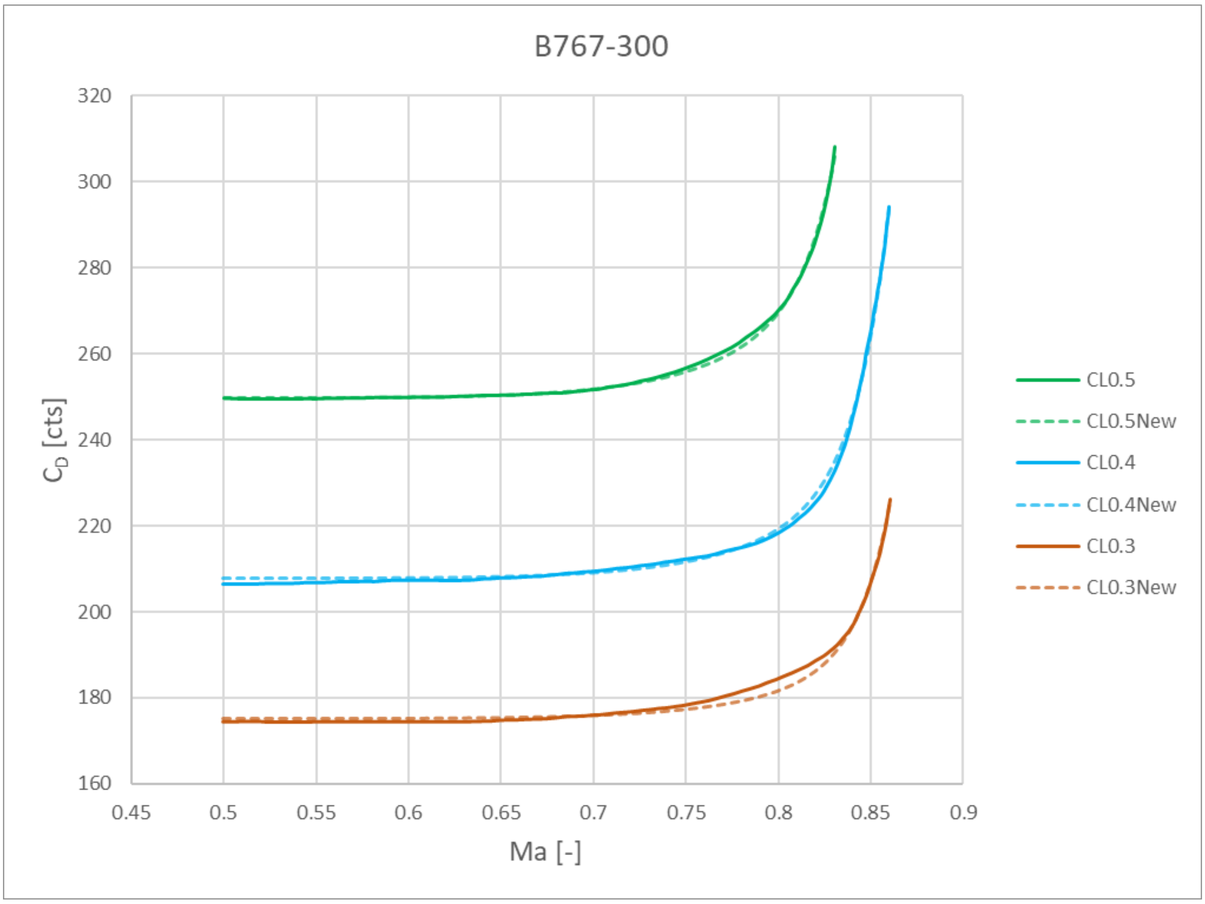

Figure 2: The drag polar for the Boeing 767. The drag coefficient is plotted versus Mach number with lift coefficient as parameter. Obert (2009) gives the drag polar for three lift coefficients (0.3, 0.4, and 0.5), solid line. The polar is identified with the equation from Figure 1, dashed line.

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