

# HISTORICAL VIEW ON FLUTTER ANALYSIS AND RECENT RESEARCH IN AEROELASTICITY

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**Abstract.** Methodology to analytically investigate flutter phenomena for aircraft is reviewed. It spans from traditional typical section airfoil systems to modern three-dimensional aeroelastic analysis with the CFD code based on the Navier-Stokes equations. In the latter part, some current topics in the aeroelasticity field are referred to.

**Keywords.** Flutter Analysis, Unsteady Aerodynamics, Flutter Prediction, LCO

## 1 Introduction

Middle of 19's after fifty years of powered flight, wings of aircraft became slender as flight speed increased. The trend sometimes led to significant aeroelastic problems. The requirement for accurate analysis in the initial stage of design on aeroelastic phenomena prompted and developed the methodology to predict the flutter speed. Three excellent textbooks were written those days as listed in References [1]-[3]. Scanlan's book introduced practical methodology for calculations of flutter speeds by formulating the aeroelastic system with the generalized coordinates. Fung wrote the compact book of aeroelasticity including the subjects of buffet and stall. The third book by Bisplinghoff, Ashley, and Halfman was comprehensively written on the aeroelastic phenomena and is yet the bible in this field. Methodology has kept progressing. The finite element method has been adopted for the structural analysis and so called the panel method and CFD for the unsteady aerodynamic forces. Books [4] and [5] published later referred to these matters. Recent research in aeroelasticity is well summarized in the articles of references [6] and [7]. In the present paper, the history of methodology of flutter analysis is reviewed, especially stressed on unsteady aerodynamic calculations. Furthermore, some current studies are introduced.

## 2 Flutter Analysis of a Typical Section

A basic mathematical model is given by the simple two-degrees-of-freedom system as shown in

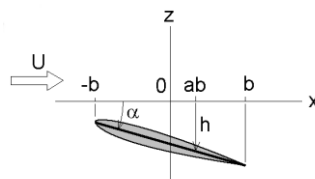


Figure 1: Typical section

Figure 1. This type of approximation for a real three-dimensional wing is known to be practical if we choose its properties to match those at a station of 70-75% span of a slender wing [8]. The linear potential flow theory gives its lift force and moment as [9]

$$L = L_{inst} + L_C \quad (1)$$

$$L_{inst} = \pi \rho b^2 [\ddot{h} + U\dot{\alpha} - ba\ddot{\alpha}]$$

$$L_C = 2\pi \rho U b C(k) [\dot{h} + U\alpha + b(\frac{1}{2} - a)\dot{\alpha}]$$

$$M = M_{inst} + M_C \quad (2)$$

$$M_{inst} = \pi \rho b^2 [ba\ddot{h} + Ub(\frac{1}{2} - a)\dot{\alpha} - b^2(\frac{1}{8} + a^2)\ddot{\alpha}]$$

$$M_C = 2\pi \rho U b^2 (a + \frac{1}{2}) C(k) [\dot{h} + U\alpha + b(\frac{1}{2} - a)\dot{\alpha}]$$

where the complex function  $C(k)$  is so called the Theodorsen function defined by

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)} = \frac{K_1(ik)}{K_0(ik) + K_1(ik)} \quad (3)$$

$H_n^{(2)}(k)$ : Hankel functions of the second kind,  $K_n(ik)$ : Bessel function of the second kind

The suffix *inst* in equations (1) and (2) indicates an instantaneous force due to the virtual mass  $\pi \rho b^2$  and/or the virtual mass couple  $\pi \rho (\frac{b}{2})^2 b$ . The force  $L_C$  denotes the running circulatory lift which acts at the quarter chord location. This force and its corresponding moment  $M_C$  depend solely on the upwash at the three-quarter chord point of the airfoil,

$$w_{\frac{3}{4}} = -[\dot{h} + U\alpha + b(\frac{1}{2} - a)\dot{\alpha}] \quad (4)$$

It is interesting to see that this part of the force can be expressed as the following equation when you think of arbitrary motion in the time domain.

$$L_C(s) = -2\pi \rho U b \left[ w_{\frac{3}{4}}(0)\phi(s) + \int_0^s \frac{dw_{\frac{3}{4}}(\sigma)}{d\sigma} \phi(s - \sigma) d\sigma \right], \quad (5)$$

$s = Ut / b$  : nondimensional time

This equation shows that the circulatory force can be calculated by a convolution integral of the indicial function  $\phi(\sigma)$  and the derivative of upwash at the three-quarter chord station. This indicial function is called as Wagner's function. The decaying part of the function changes almost in proportion to  $1/(t + c)$  as can be seen in Figure 2, where a half of the parameter  $s$  coincides with the non-dimensional time normalized by the full chord length [10]. This property is stemmed from the fact that the influence of wake behind wings is decaying as the fluid flows downstream. It is known that neglecting the initial term in equation (5) for the steady sinusoidal motion the frequency response function corresponds to the imaginary axis of the transfer function in the Laplace domain. If we compare equation (1) with equation (5), and if we think that the Laplace transform of the convolution integral becomes a product of the two functions, it can be seen that the Theodorsen function coincides with  $(ik)\Phi(ik)$  where  $\Phi(p)$  is the Laplace transform of  $\phi(s)$  [11]. This is why the Theodorsen function is not an algebraic but a transcendental function. However, it is often approximated by the polynomial functions in order to apply the integral and/or differential operators in the Laplace domain.

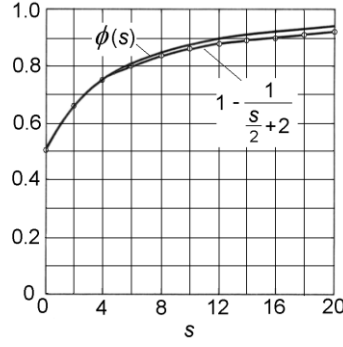


Figure 2 : Time varying functions

The classical flutter analysis can be made by applying these forces to the typical section airfoil, which is supported by the vertical and torsion springs. Since the unsteady forces are given by the linear combinations of variables corresponding to two degrees-of-freedom, the system equation becomes homogeneous. This leads us to a complex eigenvalue problem to be solved for roots depending upon flow speeds in the complex plane. For flutter analysis, a point of interest is the critical speed at which the root locus crosses the boundary from stable to unstable region, i.e. crossing point on an imaginary axis of the Laplace plane. Usually, the U-g method [1] is used to figure out this critical speed.

### 3 Unsteady Lifting Surface Theory

In 1940, Kussner derived an integral equation that relates the upwash and the distribution of pressure jump as follows [12].

$$w(x, y, z, t) = \frac{1}{4\pi} \iint_{R_a} \left[ \int_{-\infty}^{x_0} \left\{ -\sin \gamma(\eta) \frac{\partial}{\partial y} + \cos \gamma(\eta) \frac{\partial}{\partial z} \right\}^2 \right. \\ \left. \times \frac{\Delta p(\xi, \eta, t + \frac{x_0}{U} + \frac{v}{U\beta^2} - \frac{M\sqrt{v^2 + \beta^2(y_0^2 + z_0^2)}}{U\beta^2})}{\rho U \sqrt{v^2 + \beta^2(y_0^2 + z_0^2)}} dv \right] d\xi d\eta \quad (6)$$

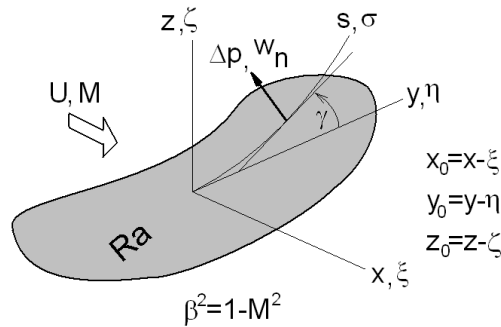


Figure 3: Lifting surface

The symbols used in this notation can be shown in Figure 3 where  $\Delta p$  is the pressure jump corresponding to lift distribution. If we put  $\gamma=0$ , then equation (6) becomes for a planar wing. Since the integral equation is based on the acceleration potential, the integration area  $R_a$  is limited within the wing, excluding the wake region. It should be noted that the integral in the brackets is not a physical integration but just an expression for the function of  $x_0$  in an integral form. Although the equation is valid for arbitrary motion in the time domain, it looks too complicated to be solved as it stands.

## 4 Kernel Function Method

In 1955, a mile stone paper was published as a NACA Report [13] on the kernel function mentioned above.

$$w(x, y) = \frac{1}{4\pi} \int_{R_a} \Delta p(\xi, \eta) K(x_0, y_0) d\xi d\eta \quad (7)$$

$$\begin{aligned} K(x_0, y_0) = & \frac{k^2 e^{-ikx_0}}{l^2} \left\{ -\frac{1}{k|y_0|} K_1(k|y_0|) - \frac{i\pi}{2k|y_0|} [I_1(k|y_0|) - L_1(k|y_0|)] \right. \\ & + \frac{i}{k|y_0|} - \frac{kx_0}{(ky_0)^2 \sqrt{(kx_0)^2 + \beta^2 (ky_0)^2}} e^{\frac{i}{\beta^2} [kx_0 - M \sqrt{(kx_0)^2 + \beta^2 (ky_0)^2}]} \\ & \left. + \frac{i}{k|y_0|} \int_0^{\frac{1}{\beta^2 |y_0|} (x_0 - M \sqrt{(kx_0)^2 + \beta^2 (ky_0)^2})} \frac{\tau}{\sqrt{1 + \tau^2}} e^{-ik|y_0|\tau} d\tau \right\} \end{aligned} \quad (8)$$

In the equation above, the symbol  $l$  denotes a reference length, and the  $I_1$  and  $L_1$  are the modified Bessel function of the first kind and the modified Struve function of the first order, respectively. Although equation (8) looks still complicated, it enables us to calculate numerical values of the kernel function. This kernel was applied to the calculation of load distribution on a finite wing for the first time by Reference [14]. The success depended much on the proper treatment [15] on a strong singularity  $(-1/y_0^2)$  in the kernel function, i.e. the integration being treated as a finite part of the infinite integral. Later, this method of calculation for the unsteady aerodynamic forces has become called as the kernel function method among the aeroelasticians.

## 5 Panel Method

### 5.1 DLM

Alberno and Roddon[15,16] have developed a practical tool called the double lattice method to calculate unsteady forces of the finite wing by using the extended integral equation for non-planar wings apart from the planar kernel of Watkins et al.. Since their method descritizes the surface into trapezoidal elements as can be seen in Figure 4, it can be applied to an arbitrary planform of a thin wing. In the calculations, however, they failed to handle the singularity of the kernel function numerically, they had to resort to the vortex lattice method [17] for the steady state portion, which corresponds to zero reduced frequency.

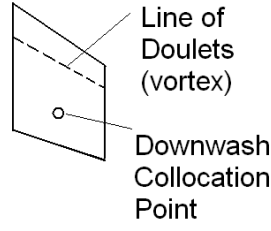


Figure 4: DLM element

## 5.2 Green Function Method

In 1977, Morino et al.[18] proposed a general formula to calculate unsteady flow around body surfaces in 3-dimensional space where the potential flow theory holds. Applying the Green function of the potential theory, they formulated the governing equation as a Fredholm integral equation of the second kind,

$$2\pi\hat{\phi} = -\oint_S \frac{\partial\hat{\phi}}{\partial N} G dS + \oint_S \hat{\phi} \frac{\partial G}{\partial N} dS - \iint_W \Delta\hat{\phi} \frac{\partial\hat{\phi}}{\partial N} dW \quad (9)$$

where the Green function can be given by  $G = e^{-i\Omega R} / R$  and  $\Omega = \omega / (a_\infty \beta)$ .  $W$  denotes the wake area of the flow. The equation is formulated in the distorted space transformed by the Prandtl-Glauert similarity rule to include the compressibility effects and can be applied to the three dimensional bodies including wings with finite thickness. However, since the variable governed by the equation is the velocity potential, it requires numerical differentiation, after solved, to obtain the pressure distribution, i.e. unsteady forces. This method is firstly called a panel method but later the Green function method because the former term has become including the acceleration potential method when the finite difference method appeared. The method is not so widely used for unsteady aerodynamic calculations because neither the effect of thickness nor the body is essential for flutter.

## 5.3 DPM

What do you think is the meaning of the kernel function in the singular integral equation (7)? We can find the answer if the pressure distribution is assumed as Dirac's delta function representing a 'point' lift. Yes, it is corresponding to the upwash distribution due to a point-lift. This clear concept has led to a convenient method called Doublet-Point Method[20]. The unsteady lift, which mathematically corresponds to the doublet of acceleration potential, on a similar trapezoidal element to DLM, is represented with one point-lift force at a quarter chord location of each element on the center of width. The method is simple and sound so that it has been developed also for supersonic flow[21]. Furthermore, the calculation in the frequency domain has extended to the entire complex plane of the Laplace domain[22], which can provide the exact linear force for the analysis of stability margin of the system. These results are summarized in Reference [23]. In these investigations, it is pointed out that the following function belongs a kind of extended Bessel function and can be given by the exact expansion series. It plays an essential role in the unsteady lift calculation.

$$B_\nu(p, r, X) \equiv \int_{-\infty}^X e^{pu} (u^2 + r^2)^{-\nu-\frac{1}{2}} du, \quad p: \text{complex}, \quad \nu: \text{integer} \quad (10)$$

## 6 Linear Subsonic and Supersonic Aerodynamics

Almost all the aerodynamic engineers know the thin airfoil theory that in incompressible flow the aerodynamic center is located at a quarter chord point and in supersonic flow at the mid-chord point. And it is also a common knowledge that the lift coefficients of 2-D airfoils become infinite at sonic flow. Since the actual airfoil has thickness effects yielding local shock wave and make the governing equation essentially nonlinear, less attention has been paid how it varies from subsonic to super sonic flow in the linear theory. For finite span wings, it should be continuous mathematically. The answer is revealed in Reference [24] by the calculation of DPM. Figure 5 quoted from the reference shows the

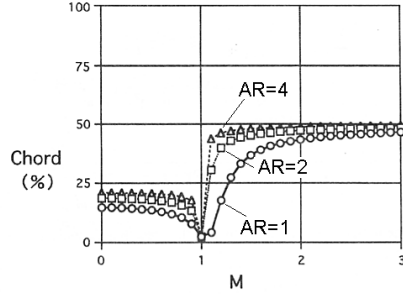


Figure 5: Location of aerodynamic center

variation of the aerodynamic center depending upon the Mach number for rectangular wings with various aspect ratios. As can be seen in the figure, every wing at  $M = 1$  has the aerodynamic center at the leading edge, irrespective of aspect ratios, where the flow becomes two dimensional in the perpendicular plane to the uniform flow direction. The infinite aspect ratio correspond to the 2-D airfoil. Three different kernels for subsonic, sonic, and supersonic flows are listed in Table 1. Considering a limit of  $\lim_{M \rightarrow 1} R = |x_0|$ , we can easily find that they are continuous.

Table 1 : Kernel functions depending on the Mach number

| $M < 1$                                       | $M = 1$                                   | $M > 1$                                |
|-----------------------------------------------|-------------------------------------------|----------------------------------------|
| $\frac{1-M^2}{R(R-x_0)} = \frac{R+x_0}{Rr^2}$ | $0 (x_0 < 0), \frac{2x_0}{r^2} (x_0 > 0)$ | $\frac{2x_0}{Rr^2}$ (inside Mach cone) |

where  $r = |y_0|$  and  $R = \sqrt{x_0^2 + (1-M^2)r^2}$

## 7 Nonlinear Transonic Analysis

Isogai[25] analysed so-call the transonic dip with the aid of his CFD code based on the small perturbation potential, which could capture the motion of shock waves on the airfoil surface. This pioneering work has led to the full simulation using the Navier-Stokes equation as briefly referred to in the following. After this work, flutter analysis combining CFD and FEM become common. Essential works were reviewed in the arecticles of Journal of Aircafet [26, 27].

Nakamichi et al. have succeeded to simulate the aeroelastic response of a wing including flow separation during the oscillation[28]. They combined the Navier-Stokes CFD code using the moving grid with the structural dynamic analysis using the Raileigh-Ritz method. On the wing surface, the displacement and the velocity of the grid related to the flow momentum were connected to the

structural parameters of displacements and elastic forces which were computed by superposing the vibration modes of linear analysis. Typical example of the simulation at  $M=0.75$  is shown in Figure 6.

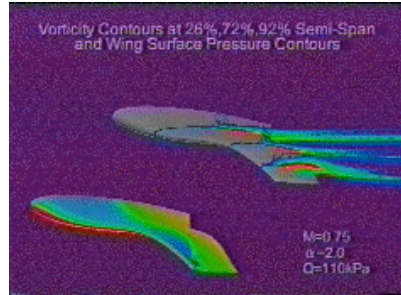


Figure 6: Flutter simulation with CFD

## 8 Some Recent Topics of Aeroelastic Research Around

### 8.1 T-tail Flutter Analysis

It is not a new problem but the T-tail flutter is analyzed by using the non-coplanar DPM to examine its assurance[29]. The numerical example illustrated in Figure 7 shows the effects of dihedral angles of the horizontal tail including those of incidence. It can be seen that the positive dihedral angles and positive incident angles decrease the flutter speed, as was observed in the wind tunnel tests[30].

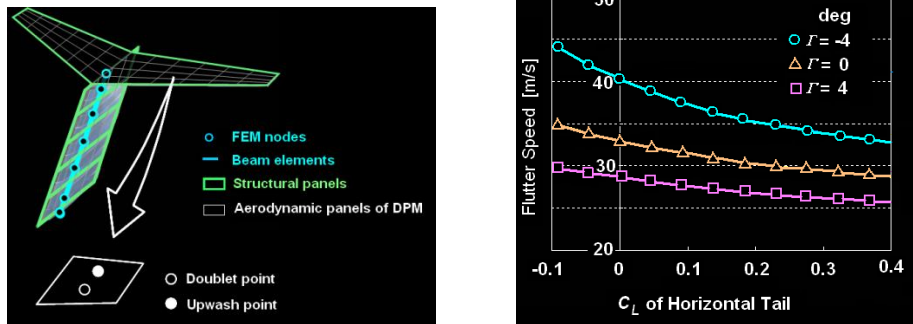


Figure 7: T-tail analytical mode and the effects of dihedral angels on flutter speeds

### 8.2 Analysis of Limit Cycle Oscillation

One of the recent research topics on Aeroelasticity in JAXA is the limit cycle oscillation (LCO) in transonic region[31]. LCO is caused by the non-linear aerodynamic forces depending the amplitudes due to the local shock wave on the airfoil. The research is being carried out in collaboration with DLR in Germany where they observed small amplitude pitching oscillation for a 2D airfoil in transonic wind tunnel tests. JAXA is trying to analyse the LCO with the thin layer NS equations and Baldwin-Lomax turbulence model. The grid for this analysis and the results are shown in Figure 8. As illustrated in the figures, the results are explaining the small amplitude LOCs.

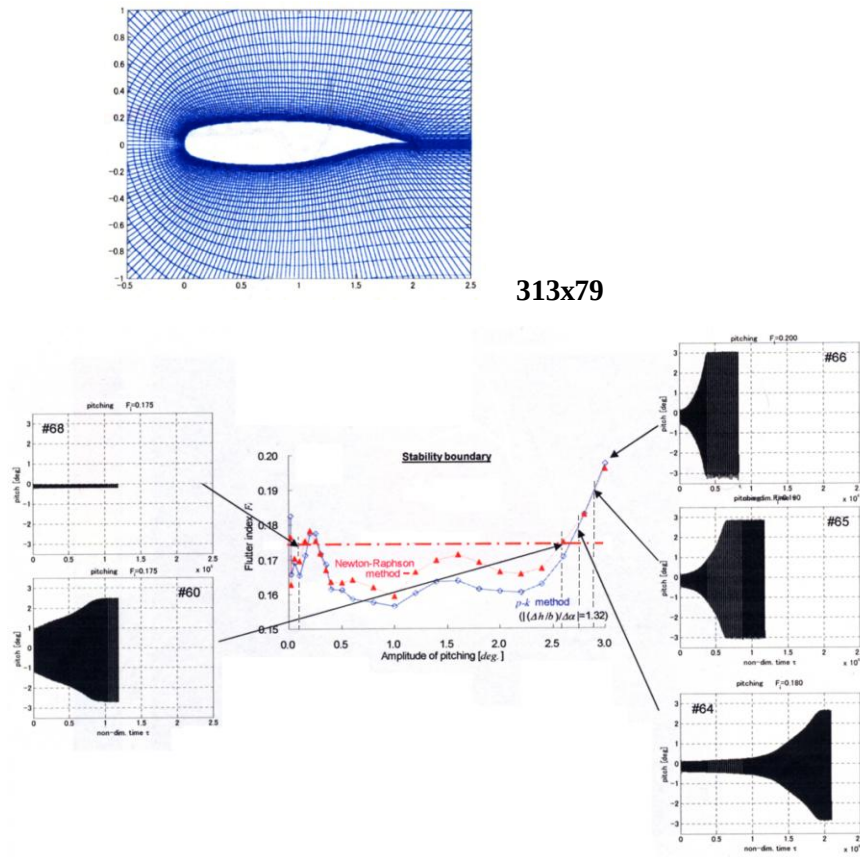


Figure 8: Grid for LCO analysis and its results

### 8.3 Flutter Prediction

In the actual flight tests as well as the wind tunnel tests, flutter prediction meets difficult but important technique. It is not an easy problem to accurately estimate the flutter boundary from signals below critical speeds. Many attempts have been made hitherto, there is not a reliable method in real circumstances. They are theoretically valid, but practically dependent. They include methods using the damping parameters, envelope functions, margin parameters, or certain coefficients in the autoregressive moving average model. The most useful clues for the onset of flutter may be lying in the frequency trends that can be visualized by applying Fourier transform. Flutter is generally caused by the coalescence of two different natural-frequencies as the flow speed increases. Theoretically, it must be detected in the frequency domain. It is, however, difficult to find the coalescence in the sub-critical response of wings or wind-tunnel models in actual cases. A novel technique has been proposed to predict flutter by using the wavelet transform[32]. The wavelet was originally introduced by J. Morlet, a French geophysicist, as a tool for signal analysis in the applications for the seismic phenomena. It has been now developed in various fields of sciences and applied in practical engineering. The transform has two parameters, one of which is the scale that corresponds to frequencies and the other is the position that corresponds to time. If we use the Fourier transform, the signal is completely transformed into the frequency domain. The representation gives only the frequency content, but information concerning time-localization cannot be read off from the transformed data whereas the wavelet provides a tool for time-frequency localization. For flutter, we may need its frequency content locally in time. Figure 9 shows an example application to wind tunnel

experiments. The left figure is the time-frequency plot of the signal and the right varying index values calculated from the signal versus flow speeds for flutter prediction.

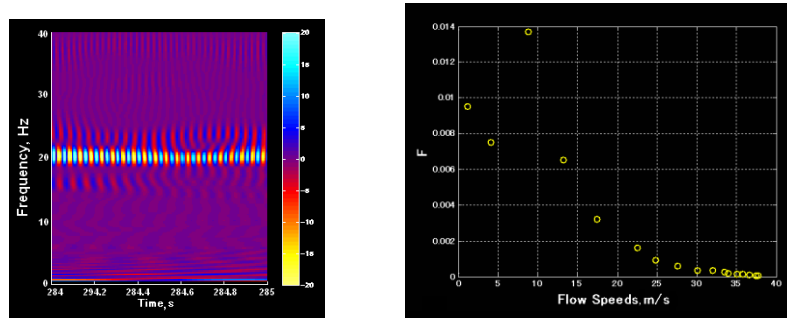


Figure 9: Output of the Wavelet transform and F-value based on it.

## 9 Conclusion

The history of methodology for analyzing wing flutter was briefly reviewed and some recent activities in the aeroelasticity field were introduced. It can be said that the configuration of modern transport aircraft has matured fifty years after the first manned flight in 1903. The design technology is, however, still progressing. Due to uncertainty of the discrepancy between full scale aircraft and the analytical model, flutter margin has been required 20% above the flight envelop for the certification in previous decades. Nowadays, improvement in the accuracy of the analysis and the confidence in reliability have reduced the flutter margin down to 15%. Furthermore, the accurate analysis can lead us to more efficient structural design before it comes to the final configuration. Technology is progressing. As can be seen in the recent innovative composite structures, new challenging issues may also be expected in the aeroelasticity field.

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