Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

AIRCRAFT DESIGN AND SYSTEMS GROUP (AERO)

Understanding the
Aircraft Mass Growth and Reduction Factor

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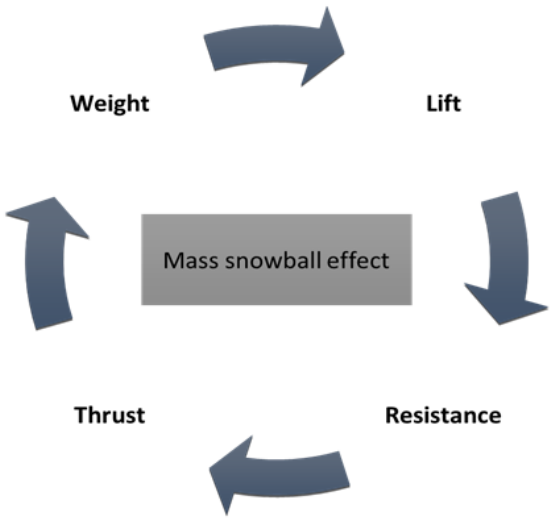

Abstract

Purpose – This project work shows a literature survey, clearly defines the mass growth factor, shows a mass growth iteration, and derives an equation for a direct calculation of the factor (without iteration). Definite values of the factor seem to be missing in literature. To change this, mass growth factors are being calculated for as many of the prominent passenger aircraft as to cover 90% of the passenger aircraft flying today. The dependence of the mass growth factor on requirements and technology is examined and the relation to Direct Operating Costs (DOC) is pointed out.

Methodology – Calculations start from first principles. Publically available data is used to calculate a list of mass growth factors for many passenger aircraft. Using equations and the resulting relationships, new knowledge and dependencies are gained.

Findings – The mass growth factor is larger for aircraft with larger operating empty mass ratio, smaller payload ratio, larger specific fuel consumption (SFC), and smaller glide ratio. The mass growth factor increases much with increasing range. The factor depends on an increase in the fixed mass, so this is the same for the payload and empty mass. The mass growth factor for subsonic passenger aircraft is on average 4.2, for narrow body aircraft 3.9 and for wide body aircraft (that tend to fly longer distance) 4.9. In contrast supersonic passenger aircraft show a factor of about 14.

Practical implications – The mass growth factor has been revisited in order to fully embrace the concept of mass growth and may lead to a better general understanding of aircraft design.

Social implications – A detailed discussion of aircraft costs as well as aircraft development requires detailed knowledge of the aircraft. By understanding the mass growth factor, consumers can have this discussion with industry at eye level.

Originality/value – The derivation of the equation for the direct calculation of the mass growth factor and the determination of the factor using the method for 90% of current passenger aircraft was not shown.

Acknowledgment

This presentation is based on the project of

John Singh Cheema

prepared at

Hamburg University of Applied Sciences

Aircraft Design and Systems Group (AERO)

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Referenced here as:

Cheema 2020

HAW HAMBURG

Project

The Mass Growth Factor –
Snowball Effects in Aircraft Design

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Submitted: 31.03.2020

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Background

- The aircraft mass growth factor is fundamental to aircraft preliminary design. Due to the fact that mass during some aircraft design phases seems rather to increase than to decrease compared to initial estimates, the factor is called mass growth factor.
- However, a mass reduction factor is mathematically the same. The mass reduction factor can lead to even substantial mass reduction and is as such the secret to efficient aircraft design. For simplicity and tradition we may just talk about mass growth.
- It is usually defined as the ratio of an increase in the total mass (take-off mass) due to an arbitrary increase in local mass (empty mass) determined after a full iteration in aircraft design to achieve the original performance requirements (payload and range).
- The aircraft design iteration sees after each loop another increment in the take-off mass, so that an initial (local) mass increase aggravates the situation like a snow ball transforming into an **avalanche**. Hence the pseudonym **snowball factor**.
- The concept of the mass growth factor is probably as old as aviation. It has been discussed heavily from the 1950th to the 1970th and has continued to be mentioned until today. Nevertheless, it seems not to be well enough understood today. Maybe its importance has declined due to modern computing power providing quite accurate mass estimates in each design phase, but detaching the engineer from the feel for the numbers.

Avalanche – Snow Ball Effect

Avalanche (Dahu1, wikimedia.org, CC BY-SA)

Literature Review (Overview)

Ballhaus 1954	(SAWE Paper)	elaborate the principle
Saelman 1973	(SAWE Paper)	
Fürst 1999	(LTH, Germany)	
Sinke 2019	(Lecture Notes, TU Delft)	
SAWE 2019	(SAWE Book)	

Aircraft Design Books

Torenbeek 1976	talk about design aspects
Roskam 1989	
Jenkinson 1999	
Howe 2000	
Müller 2003	

See **Cheema 2020** for details.

SAWE: Society of Allied Weight Engineers
LTH: Luftfahrttechnisches Handbuch

Literature Review (Overview)

CLEAR DESIGN THINKING USING THE AIRCRAFT GROWTH FACTOR

By

Wm. F. Ballhaus

TO BE PRESENTED AT THE SAE NATIONAL AERONAUTICAL MEETING

Los Angeles, California

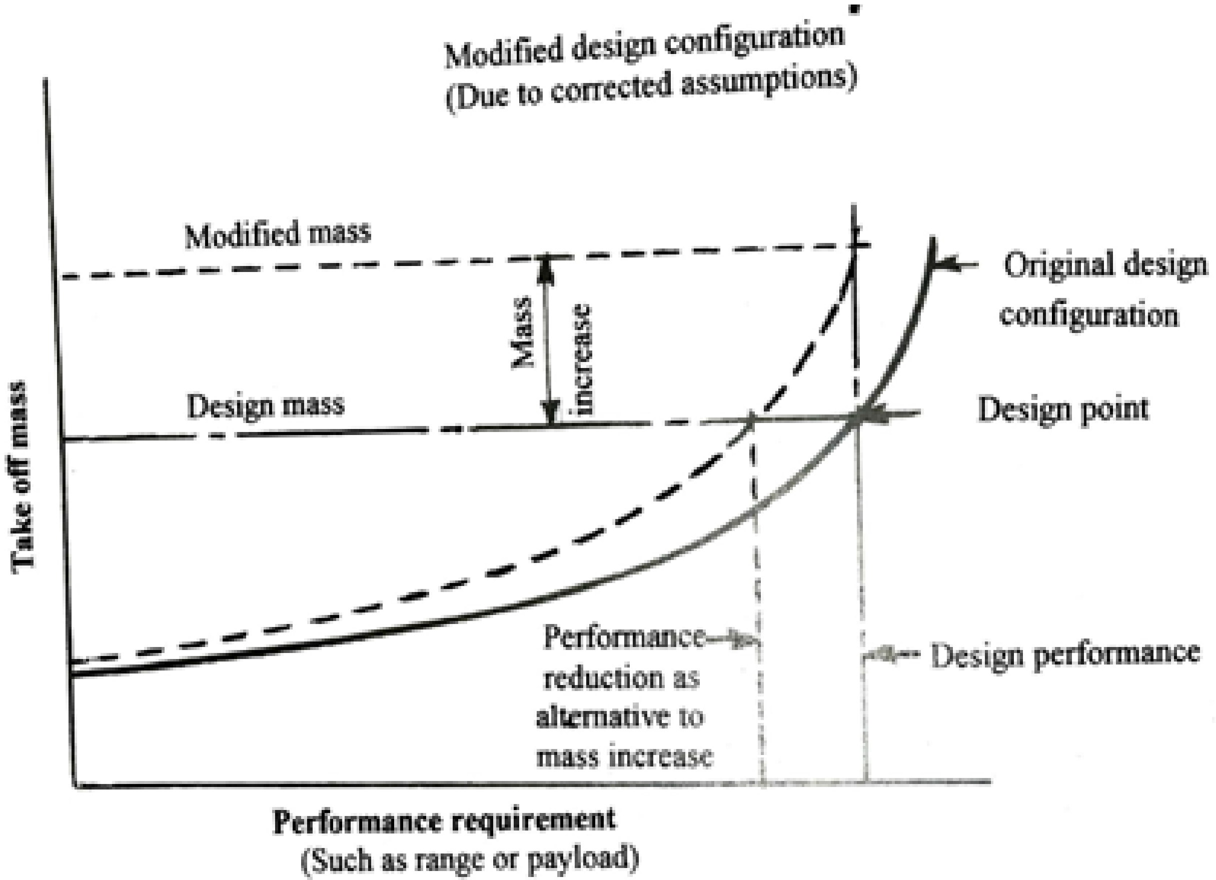
October 8, 1954

Definition

$$G = \frac{\text{Change in Gross Weight}}{\text{Fixed Weight Added or Change in Fixed Weight}}$$

Ballhaus 1954

Aircraft Design Mass Growth (Overall Picture)



New curve may become asymptotic at a mass less than the design value

Howe 2000

Iteration of the Mass Growth Factor (Equations)

$$\Delta m_L = 1 \text{ kg}$$

$$m_{MTO,0} = m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F$$

$$m_{MTO,1} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,0} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,0}$$

...

$$\Delta m_G = m_{MTO,X} - m_{MTO}$$

$$k_{MG} = \frac{\Delta m_G}{\Delta m_L}$$

m_F	Fuel Mass
m_{MPL}	Maximum Payload
m_{PL}	Payload
m_{MTO}	Maximum take-off mass
m_{OE}	Operating empty mass
Δm_L	Local mass growth
Δm_G	Global mass growth
k_{MG}	Mass growth factor

Iteration of the Mass Growth Factor (Excel Table)

Mass Growth Factor by Iteration

Local Mass Growth	DmL	1	kg
Maximum take-off mass	mMTO	73000	kg
Operating empty mass fraction	mOE/mMTO	0.5	
Operating empty mass	mOE	36500	kg
Fuel mass fraction	mF/mMTO	0.25	
Fuel mass fraction	mF	18250	kg
Maximum Payload	mMPL	18250	kg

1. For the iteration method, the masses must be researched and thus the mass fractions determined

2. Enter the values in the blue fields

3. The iteration starts automatically and the mass growth factor is calculated

Iteration

i	mMTO,i		Global mass growth: DmG	Change	DmG/DmL	kMGW
0	73001.000	kg	1.00000			4.00
1	73001.750	kg	1.75000	75.0000%	1.75000	
2	73002.313	kg	2.31250	32.1429%	2.31250	
3	73002.734	kg	2.73438	18.2432%	2.73438	
4	73003.051	kg	3.05078	11.5714%	3.05078	
5	73003.288	kg	3.28809	7.7785%	3.28809	
6	73003.466	kg	3.46606	5.4128%	3.46606	
7	73003.600	kg	3.59955	3.8512%	3.59955	
8	73003.700	kg	3.69966	2.7813%	3.69966	
9	73003.775	kg	3.77475	2.0295%	3.77475	
10	73003.831	kg	3.83106	1.4918%	3.83106	
11	73003.873	kg	3.87329	1.1024%	3.87329	
12	73003.905	kg	3.90497	0.8178%	3.90497	
13	73003.929	kg	3.92873	0.6084%	3.92873	
14	73003.947	kg	3.94655	0.4535%	3.94655	
15	73003.960	kg	3.95991	0.3386%	3.95991	
16	73003.970	kg	3.96993	0.2531%	3.96993	
17	73003.977	kg	3.97745	0.1893%	3.97745	
18	73003.983	kg	3.98309	0.1417%	3.98309	
19	73003.987	kg	3.98732	0.1062%	3.98732	
20	73003.990	kg	3.99049	0.0795%	3.99049	

Determination of mass fraction

m_{MTO}	73000	kg
m_{PL}	18250	kg
m_{OE}	36500	kg
m_F	18250	kg
m_{PL}/m_{MTO}	0.250	
m_{OE}/m_{MTO}	0.500	
m_F/m_{MTO}	0.250	

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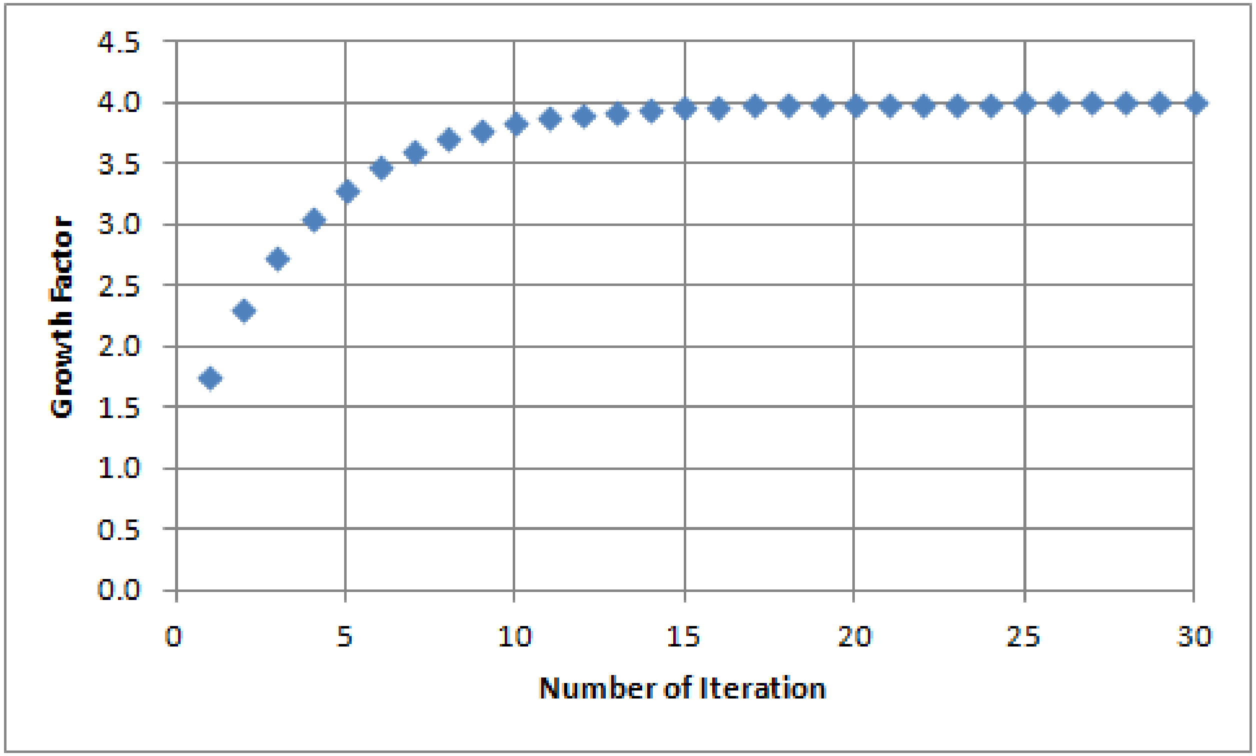
HARVARD Dataverse

Data and tools uploaded to Harvard Dataverse: https://doi.org/10.7910/DVN/6NHDDP

Iteration of the Mass Growth Factor (Convergence)



Convergence of the mass growth factor using the example of the Boeing 767-300

Considering 90% of Passenger Aircraft

World Fleet

Table A.1 90% of current aircraft (according to Robson 2019)

Aircraft Type	Number in Operation	Percent of total	Sum of most aircraft types in percent
Boeing 737-800	4804	16.83%	16.83%
A320	4135	14.48%	31.31%
A320neo	658	2.30%	33.61%
A321neo	160	0.56%	34.18%
A321	1650	5.78%	39.95%
Boeing 737 Max 8		0.00%	39.95%
A319	1249	4.37%	44.33%
Boeing 737-700	1005	3.52%	47.85%
ATR72	775	2.71%	50.56%
Boeing 777-300(ER)	829	2.90%	53.47%
Embraer 175	595	2.08%	55.55%
Boeing 787-9	451	1.58%	57.13%
A330-300	707	2.48%	59.61%
Boeing 767-300	622	2.18%	61.79%
A350-900	261	0.91%	62.70%
Boeing 757-200	600	2.10%	64.80%
A330-200	547	1.92%	66.72%
Boeing 737-900	550	1.93%	68.64%
De Havilland Canada Dash 8-400	502	1.76%	70.40%
Embraer 190	495	1.73%	72.14%
Boeing 737 Max TBD		0.00%	72.14%
Bombardier CRJ900	444	1.56%	73.69%
A220	77	0.27%	73.96%
Boeing 777-200	431	1.51%	75.47%
Embraer ERJ-145	422	1.48%	76.95%
Boeing 737 Max 10		0.00%	76.95%
Boeing 787-8	328	1.15%	78.10%

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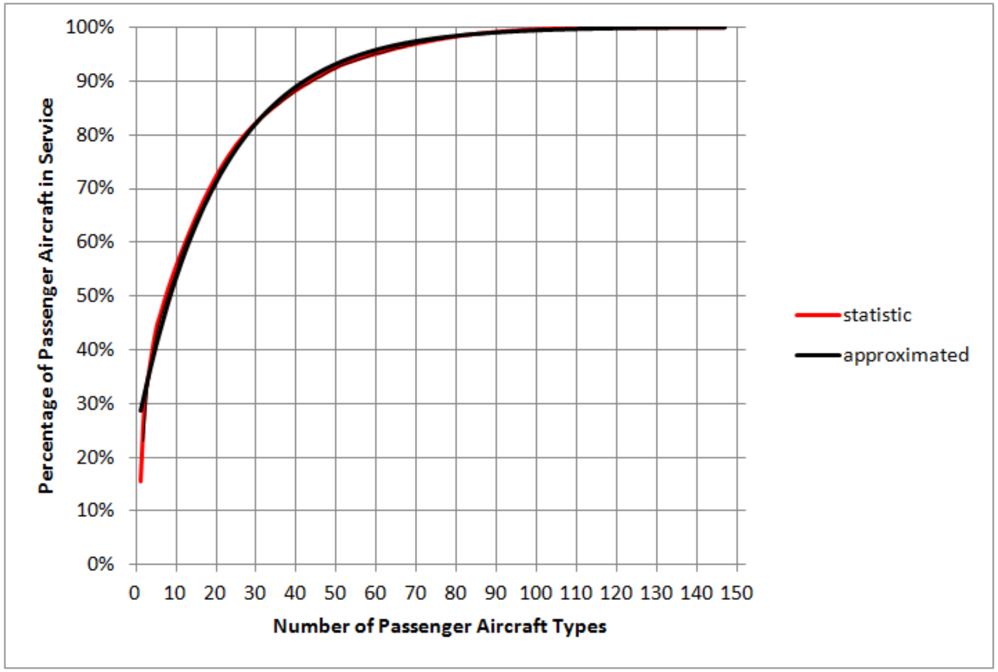
e.g. 27 aircraft, 78.1% of fleet

Considering 90% of Passenger Aircraft

$$\left(\frac{n}{n_{max}}\right)_{in\_service} = 1 - a \cdot e^{b\left(\frac{n}{n_{max}}\right)_{type}}$$

a	0.7480879
b	-0.047978



with data from Flight 2016

Mass Growth Factor – Considering 90% of Passenger Aircraft

Table 3.3 Evaluation of 90% of all current flying commercial aircraft including two supersonic aircraft

Aircraft	m_{MTO} [kg]	m_{OE} [kg]	m_{MPL} [kg]	k_{MGW}	Sources
Boeing 737-800	78220	41480	14690	**5.32**	Jenkinson 2019
A320-200	73500	42100	18633	**3.94**	Jackson 2011
A320neo	79000	44300	20000	**3.95**	Airbus 2005, Wiki 2020
A321neo	97000	50100	25500	**3.80**	Airbus 2005a, Wiki 2020
A321-200	89000	48000	22780	**3.90**	Jenkinson 2019
A319-100	64000	39200	17390	**3.68**	Jenkinson 2019
Boeing 737-700	69400	37585	11610	**5.97**	Jenkinson 2019
ATR 72-500	22500	12950	7350	**3.06**	Jackson 2011
Boeing 777-300 ER	299370	155960	68570	**4.36**	Jackson 2011
Embraer 175	37500	21810	9890	**3.79**	Jackson 2011
Boeing 787-9	244940	128850	52587	**4.65**	Boeing 2018, Wiki 2020c
A330-300	217000	118189	48400	**4.48**	Jenkinson 2019
Boeing 767-300	156489	87135	39140	**3.99**	Jenkinson 2019
A350-900	280000	142400	53300	**5.25**	Airbus 2005b, Wiki 2020a
Boeing 757-200	115900	58040	25690	**4.51**	Jenkinson 2019
A330-200	230000	120200	36400	**6.31**	Jenkinson 2019
Boeing 737-900	74389	42901	19831	**3.75**	Boeing 2013, Wiki 2020e
DHC Dash 8-400	24993	14968	7257	**3.44**	Lambert 1991
Embraer 190	50300	28080	13530	**3.71**	Jackson 2011
Bombardier CRJ900	36500	21430	10320	**3.53**	AirlinesInform 2020
A220-100	63049	35221	15127	**4.16**	Airbus 2019, Wiki 2020d
Boeing 777-200	242670	135875	54635	**4.44**	Jenkinson 2019

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90% of aircraft considered globally with more than 19 seats by looking at 44 aircraft types.

ATR 72:
propeller aircraft,
short range

Cheema 2020

Wisdom Gained

1.) Larger aircraft do not necessarily have a higher mass growth factor.

2.) It does not matter how large the local mass growth is;
the mass growth factor remains unaffected.*

$$m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F$$

3.) It does not matter whether there is one kg more operating empty mass or one kg more payload on board. The mass growth factor for a growth in the operating empty mass is therefore the same factor as for a growth in the payload.

4.) Old long-range aircraft are more sensitive to local mass growth than new short-range aircraft.

* This as long as the local mass growth is much smaller than the fixed mass.
The position of the local mass growth does not matter, if the wing is not yet fixed and is positioned according to the new weight and balance situation.

Mass Growth Factor – Aircraft Categories

Table 4.1 Mass growth factor for different aircraft categories

	Aircraft categories			
	Wide-Body	Narrow-Body	} Subsonic	Supersonic
k_{MG}	4.91	3.85	4.23	13.82

Mass Growth Factor – Obtained from Payload Fraction

After a longer derivation (Cheema 2020), we find a simple equation:

$$\boxed{k_{MG} = \frac{m_{MTO}}{m_{MPL}}} = \frac{1}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

Table 4.2 Mass growth factor for typical mass fractions

	Range type		
	Short-range	Medium-range	Long-range
m_{OE}/m_{MTO}	0.60	0.525	0.45
m_F/m_{MTO}	0.15	0.3	0.45
$\boldsymbol{k_{MG}}$	**4**	**5.7**	**10**

Mass Growth Factor – Can Go to Infinity

$$k_{MG} = \frac{m_{MTO}}{m_{MPL}} = \frac{1}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

As soon as

$$\left(\frac{m_F}{m_{MTO}} + \frac{m_{OE}}{m_{MTO}}\right)$$

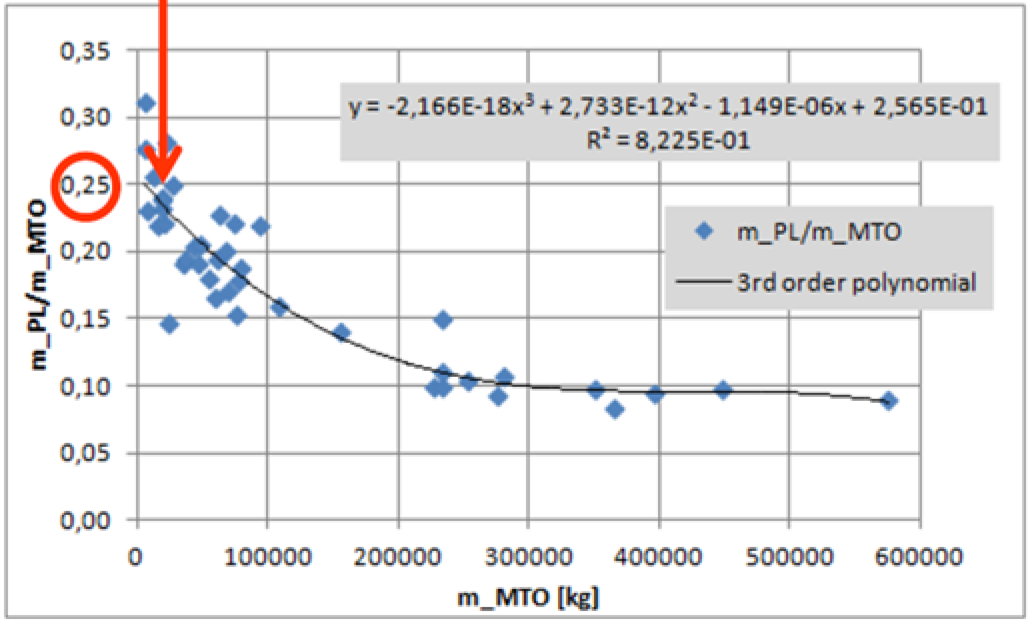
approaches 1.0 the mass growth factor, k_{MG} goes towards infinity!

This means the design task has no solution!

Mass Growth Factor – Payload Fraction from Statistics

$$k_{MG} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}}$$

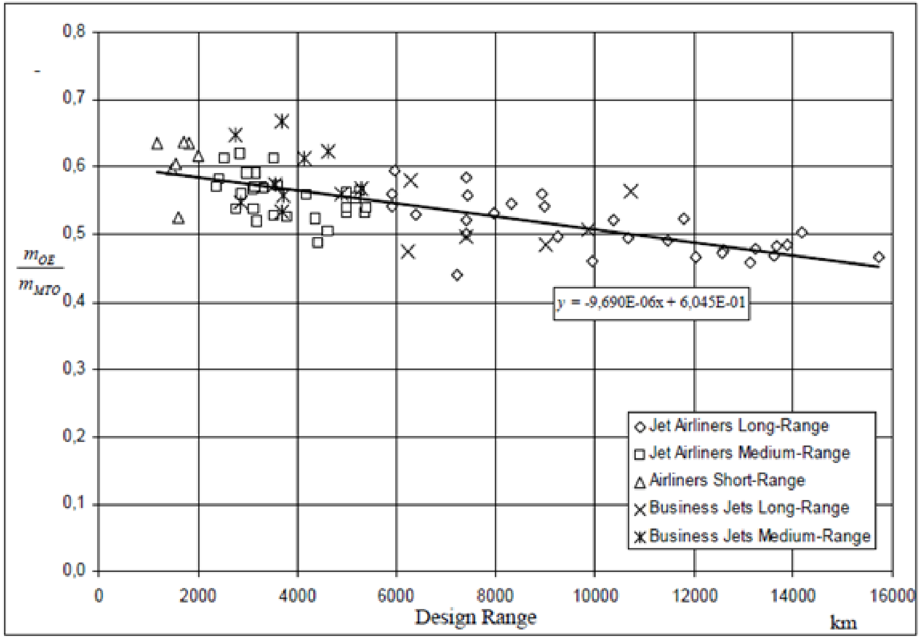
small A/C; short range



$$k_{MG} = 4 \ldots 10$$

Scholz 2018

Mass Growth Factor – Obtained from Technology and Range



Scholz 2015

c	Specific fuel consumption
R	Range
B	Breguet factor
E	Glide ratio in cruise flight
V	Cruising speed

$$B = \frac{E \cdot V}{c \cdot g}$$

$$\frac{m_F}{m_{MTO}} = 1 - e^{-\frac{R}{B}}$$

$$\frac{m_{OE}}{m_{MTO}} = 0.5967 - 0.0000166(1/\mathrm{NM}) \cdot R$$

Lehnert 2018

$$k_{MG} = \frac{1}{1 - (0.5967 - 0.0000166(1/\mathrm{NM}) \cdot R) - \left(1 - e^{-\frac{R}{B}}\right)}$$

Mass Growth Factor – Linked to Direct Operating Costs (DOC)

Seat-Mile-Costs:

$$C_{s,m} = \frac{C_{a/c,t}}{R \cdot n_s} = \frac{C_{a/c,a}}{R \cdot n_s \cdot n_{t,a}}$$

$$DOC = C_{DOC} = C_{a/c,a}$$

$n_{t,a}$ ← depends on cruise speed (given)

R ← range (given)

$$DOC \approx m_{MTO} = m_{OE} + m_F + m_{PL}$$

m_{PL} ← payload (given, constant)

m_F → fuel costs

m_{OE} → depreciation, maintenance costs

$$\frac{DOC}{n_s} \approx \frac{m_{MTO}}{m_{PL}} = \frac{1}{\frac{m_{PL}}{m_{MTO}}} = k_{MG}$$

$C_{a/c,a}$	Aircraft annual costs
$n_{t,a}$	Number of Flights per year
$C_{s,m}$	Seat-mile costs
$C_{a/c,t}$	Aircraft trip cost

Summary

- The mass growth (and reduction) factor is well known, but not well understood:
  - Derivation missing => added
  - Numerical values missing => added

$$k_{MG} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}}$$

- Average value for subsonic passenger aircraft: 4.2
  range from 3.1 (ATR-72) via 6.2 (A380) and 6.5 (B747-400) to 15.6 (Concorde)
- The mass growth factor important for the design phase (snow ball effect) and also a good indicator to quickly understand the aircraft's economy.
- The mass growth factor can be estimated from basic parameters:
  range, *R; E = L/D; c* = SFC; cruise speed, *V* :

$$k_{MG} = \frac{1}{1 - (0.5967 - 0.0000166(1/\mathrm{NM}) \cdot R) - \left(1 - e^{-\frac{R}{B}}\right)}$$

$$\uparrow$$

$$B = \frac{E \cdot V}{c \cdot g}$$

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