Sensitivity Factors of Aircraft Mass for the Conceptual Design

Abstract

Purpose –The purpose of this work is to further develop the methodology for calculating the aircraft take-off-mass and its main functional components for the conceptual analysis and synthesis of new projects.

Design/methodology/approach –The method is based on the assessment of changes in the take-off gross mass (TOGM) of the already developed project or already existing a basic version of the aircraft when making local mass changes for its modification or for the numerical researches to create a more advanced project. The method is based on the "Sensitivity factors of mass" (SFM) of aircraft, which represents the ratio of TOGM to initial (local) mass changes of its main functional components. The method of analytical refined calculation of SFM for the initial mass change and the main aerodynamic characteristics is given.

Findings– In comparison with the long-known method based on weight (mass) growth factors, which were considered constant, this method takes into account the dependence from the value of the initial local mass change and its functional purpose.

Practical implications – This method allows the designer to calculate more strictly the final changes in the TOGM on the initial stages of conceptual design when finding new project solutions. Numerical calculations are given on the example of passenger aircraft. The dependence of SFM, and TOGM and its functional masses on the value of the initial change of the structure mass is shown.

Originality/value– The considered method based on SFM is simple and convenient, and more accurate for conducting project research on many project parameters when analyzing and synthesizing a new project.

KeywordsConceptual Design, Sensitivity Factors, Aircraft Take-off mass

Paper type research paper

Introduction

One of the most important parameters of any aircraft is its maximum take-off gross mass (TOGM). How important has been "weight (mass)" in aviation at all times speaks that in 1939, the Society of Aviation Weight Engineers was organized in Los Angeles, California, which continues to exist today (since 1973, the Society of Allied Weight Engineers, Inc.). In October 1954, at the meeting of this Society, a report “Clear design thinking using the aircraft grow factor” (Ballhays, 1954)was made which opened the bright direction in conceptual design that will be also considered in this work. The discussion of this topic continues to this day (Scholz, 2020).

A passenger plane contains millions of parts (for example, a Boeing 747 has about 6 million ones) ranging from light rivets to heavy load-bearing elements, and each contributes its part to TOGM. And any change in the mass of each of these parts will affect the weight of the aircraft and all its main components and this effect will be like a snowball if a designer wants to keep the flight characteristics unchanged, for example. For conceptual design on the initial design steps, semi-empirical weight formulas are usually used in the evaluation of the mass of the aircraft and its components: (Torenbeek, 2013), (Raymer, 2018),(Poghosyan,2018). However, these formulas work in fairly narrow weight corridors for a certain class of aircraft and can not always successfully assess possible changes.

As a rule, the main criterion for evaluating the aircraft project, if all the requirements are met, is first considered also its TOGM – m_{TO}. Mathematically TOGMis a continuous function of n variable parameters q_j that are of direct interest to the designer. The total mass differential for an infinitesimal change its parameters can be written in the following form

$$dm_{TO} = \sum_{j=1}^{n} \frac{\partial m_{TO}}{\partial q_j} dq_j \quad (1)$$

For finite but small values of the parameters Δq_j, the increment of the function can be approximately replaced by its main (linear) part, then

$$\Delta m_{TO} = \sum_{j=1}^{n} \frac{\partial m_{TO}}{\partial q_j} \Delta q_j \quad (2)$$

The transition to a new design solution will be reduced to the formation of initial (partial) changes in the mass of the functional parts of the aircraft of the basic project separately for each new technical solution. Any new technical solution for any part of the aircraft begins to be worked out with the "frozen" values of the remaining parts, including the entire mass of the aircraft. Changes in the properties of the aircraft and its functions at first with the initial mass of the aircraft will require partial changes in the masses of the functional elements of the system, which will then develop into a General (final) change in the mass of the aircraft, as a system formed of the interconnected elements.

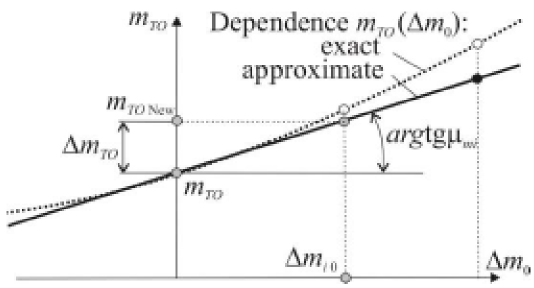
So, any changes to the project will be implemented through initial changes to the corresponding functional mass Δm_{i0}(the indexes "i" is the i^{th} mass component; "0" indicates the initial mass changes). Just this problem, i.e. $\Delta q_j=\Delta m_{i0}$, will be discussed in more detail in this paper, i.e.

$$\Delta m_{TO} = \frac{\partial m_{TO}}{\partial m_i} \Delta m_i = \mu_{mi} \Delta m_i \quad (3)$$

where $\mu_{m\,i}$ is the sensitivity factors of mass (SFM) of aircraft, which represents the ratio of TOGM to an initial (local) mass change of the i^{th} element. The term SFM more accurately reflects the physical meaning of this value than the "Weight (mass) grow factor" which was usually used in the literate (Roskam, 1985), Hays A. P. (1993),(Grebenkov, 1999), etc.

By this, we can only consider minor initial changes in mass (no more than 10% -15%). Otherwise, a significant error may be made when calculating the final mass – Figure 1.

Figure 1 Dependence of the take-off mass on the initial changes Δm_{i0}.



Following this idea, it is better to present the take-off mass as follows

$$m_{TO} = m_{Dep\,TO} + m_{Indep\,TO} \tag{4}$$

where m_{DepTO} and $m_{IndepTO}$ are mass components accordingly dependent and independent from the take-off mass.

To calculate $\mu_{m\,i}$ we add Δm_{i0} to Eqn(4) and differentiate it concerning the m_{TO}

$$1 = dm_{Dep\,TO} / dm_{TO} + d\Delta m_{i0} / dm_{TO} \tag{5}$$

And if we take the linear dependence of m_{Dep} on m_{TO}, we'll get

$$\mu_{mi} = 1/(1 - dm_{Dep\,TO} / dm_{TO}) \approx 1/(1 - \overline{m}_{Dep\,TO}) \tag{6}$$

Here and further, the relative masses $\overline{m}_i$ of the mass components are used, calculated relative to the take-off mass $\overline{m}_i = m_i / m_{TO}$.

There are two approaches to using SFM: 1 – despite changes in the initial mass of the base project, the payload and flight characteristics must remain unchanged; 2 – the task is to change the consumer properties of the base project aircraft (payload, flight-take-off, and landing characteristics, indicators of technical excellence, component solutions), which will require initial changes in functional parts. SFM can be the main tool to solve both of these problems. In the synthesis of a new project, any changes will be considered as equivalent to a change in mass. But in this work, the main focus is on the consideration of the first direction of application – that is, any changes in the basic project are performed within the framework of the constant payload mass and the preservation of the aircraft's flight characteristics.

Calculation of the Sensitivity Factors

Initial mass change near the aircraft mass center

Initially, an airplane is considered as a material point that coincides with its center of gravity (CG). For the mass analysis, we will take the following representation of the dependent and independent parts of the take-off mass by the functional components

$$m_{Dep\,TO} = m_{str} + m_{eng.s} + m_{fuel.s}, \quad m_{Indep\,TO} = m_{target} \tag{7}$$

where m_{str} is the mass of structures (mass subsystem implements the aerodynamic principle of flight: wing, fuselage, tail, landing gear, control system); $m_{eng.s}$ – the mass of the engine subsystem, which provides the creation of thrust (engines, pylons, nacelles); $m_{fuel.s}$ is the mass of the fuel subsystem which provides fuel for the engines (fuel and fuel storage and submit system); m_{target} – the mass of the target load that is associated with the appointment of aircraft: commercial load (payload) and service load, included the equipment payload, the equipment providing reliable operation of the aircraft, crew, thus $m_{target} = m_{payload} + m_{servic}$. The coefficients $\bar{m}_{str}$, $\bar{m}_{eng,s}$, $\bar{m}_{fuel.s}$ are determined by the requirements of the technical specification for flight properties, as well as the level of technology development. Thus, changes in the technical perfection indicators of the aircraft of the basic design or the selected prototype through changes in $\bar{m}_{str}$, $\bar{m}_{eng.s}$, $\bar{m}_{fuel.s}$ will lead to a change in the mass of the aircraft.

Thus SFM can be expressed in terms of functional mass components in the following representation

$$\mu_{mi} = 1/(1 - \bar{m}_{str} - \bar{m}_{eng.s} - \bar{m}_{fuel.s}) = 1/\bar{m}_{target} \tag{8}$$

However, when calculating the relative mass of a component in which the mass was changed $m_i^* = m_i + \Delta m_{i0}$, it must also be taken into account

$$\bar{m}_i^* = \bar{m}_i + \Delta \bar{m}_{i0} \tag{9}$$

Then, $1 = \bar{m}_{str} + \bar{m}_{eng.s} + \bar{m}_{fuel.s} + \bar{m}_{target} - \Delta \bar{m}_{i0}$ and

$$\mu_{mi} = 1/(1 - \bar{m}_{str} - \bar{m}_{eng.s} - \bar{m}_{fuel.s}) = 1/(\bar{m}_{target} - \Delta \bar{m}_{i0}) \tag{10}$$

As we can see, in contrast to the traditional form of representation μ_{mi}, when it was constant, in this case, there is another term that depends on the value of the initial mass change Δm_{i0}. For the first time, this peculiarity was discovered by (Gogolin, 1973). Another feature is related to the fact that, according to Eqn (10), we must exclude from the denominator all relative masses that do not depend on the take-off mass change. If the project is carried out according to the first scenario (the payload that is in the fuselage does not change), then the cost of fuel and thrust to overcome the aerodynamic drag of the fuselage will also not change, since its dimensions will remain the same. To do this, the relative masses can be written as follows

$$\overline{m}_{eng.s} + \overline{m}_{fuel.s} = (\overline{m}_{eng.s} + \overline{m}_{fuel.s})C_{D\,fus} / C_D + (\overline{m}_{eng.s} + \overline{m}_{fuel.s})(1 - C_{D\,fus} / C_D) \quad (11)$$

where $C_{D\,fus}$ and C_D are drag coefficients of the fuselage and the entire aircraft. Let the thrust of the power plant is determined by the cruising mode. Then the first component of the right part Eqn(11) determines the mass cost for the thrust and fuel during transporting of the fuselage of the original (unchanged) dimensions in cruising flight, so it does not change when implementing Δm_{i0} and, accordingly, $d((m_{eng}+m_{fuel})C_{D\,fus}/C_D)/d(m_{TO})=0$, in contrast to the second member. And then Eqn (10) will take the form

$$\mu_{mi} = 1/[\overline{m}_{\text{target}} - \Delta\overline{m}_{i0} + (\overline{m}_{eng.s} + \overline{m}_{fuel.s})C_{D\,fus} / C_D] \quad (12)$$

The General formula for determining the SFM of an aircraft as a result of initial changing the masses of all four possible functional components is as follows

$$\mu_{m\Sigma} = \Delta m_{TO} / \sum_{i-1}^{4} \Delta m_{i0} = 1/[\overline{m}_{\text{target}} - \Delta\overline{m}_{str0} - (\Delta\overline{m}_{eng.s0} + \Delta\overline{m}_{fuel.s0})(1 - C_{D\,fus} / C_D) + (\overline{m}_{eng.s} + \overline{m}_{fuel.s})C_{D\,fus} / C_D] \quad (13)$$

The final changes of the aircraft mass and its functional components will be

$$\Delta m_{TO} = \mu_{m\Sigma} \sum_{i-1}^{4} \Delta m_{i0} \quad (14)$$

$$\Delta m_{\text{target}} = \Delta m_{\text{target}\,0} \quad (15)$$

$$\Delta m_{str} = \Delta m_{str0} + (\overline{m}_{str} + \Delta\overline{m}_{str0})\Delta m_{TO} \quad (16)$$

$$\Delta m_{eng.s} = \Delta m_{eng.s0} + (\overline{m}_{ens.s} + \Delta\overline{m}_{ens.s0})(1 - C_{D\,fus} / C_D)\Delta m_{TO} \quad (17)$$

$$\Delta m_{fuel.s} = \Delta m_{fuel.s0} + (\overline{m}_{fuel.s} + \Delta\overline{m}_{fuel.s0})(1 - C_{D\,fus} / C_D)\Delta m_{TO} \quad (18)$$

Using Eqns (15-18) we can write the SFM for each functional mass

$$\varphi_{\text{target}} = \Delta m_{\text{target}} / \sum_{i-1}^{4} \Delta m_{i0} = \Delta m_{\text{target}\,0} / \sum_{i-1}^{4} \Delta m_{i0} \quad (19)$$

$$\varphi_{str} = \Delta m_{str} / \sum_{i-1}^{4} \Delta m_{i0} = \Delta m_{str0} / \sum_{i-1}^{4} \Delta m_{i0} + \overline{m}_{str}\mu_m \quad (20)$$

$$\varphi_{eng.s} = \Delta m_{eng.s} / \sum_{i=1}^{4} \Delta m_{i\,0} = \Delta m_{eng.s\,0} / \sum_{i=1}^{4} \Delta m_{i\,0} + \overline{m}_{eng.s} \mu_m (1 - C_{D\,fus} / C_D) \quad (21)$$

$$\varphi_{fuel.s} = \Delta m_{fuel.s} / \sum_{i=1}^{4} \Delta m_{i\,0} = \Delta m_{fuel.s\,0} / \sum_{i=1}^{4} \Delta m_{i\,0} + \overline{m}_{fuel.s} \mu_m (1 - C_{D\,fus} / C_D) \quad (22)$$

where φ_i are the SFM of the aircraft functional masses.

Three special cases simplify this expression

1. If the sum of partial changes is relatively small

$$\Delta \overline{m}_{str\,0} + (\Delta \overline{m}_{eng.s\,0} + \Delta \overline{m}_{fuel.s\,0})(1 - C_{D\,fus} / C_D) << \overline{m}_{\text{target}} + (\overline{m}_{eng.s} + \overline{m}_{fuel.s}) C_{D\,fus} / C_D] \quad (23)$$

Then

$$\mu_{m\Sigma} = \mu_m = 1 / [\overline{m}_{\text{target}} + (\overline{m}_{eng.s} + \overline{m}_{fuel.s}) C_{D\,fus} / C_D] \quad (24)$$

and

$$\mu_m = \mu_{m_{\text{target}}} = \mu_{m_{eng.s}} = \mu_{m_{fuel.s}} = \mu_{m_{str}} \quad (25)$$

2. If the thrust of the power plant is determined by take-off mode, and not by cruising mode

$$\mu_{m\Sigma} = \Delta m_{TO} / \sum_{i=1}^{4} \Delta m_{i\,0} = 1 / [\overline{m}_{\text{target}} + \overline{m}_{eng.s} + \overline{m}_{fuel.s} C_{D\,fus} / C_D - \Delta \overline{m}_{str\,0} - \Delta \overline{m}_{fuel.s\,0} (1 - C_{D\,fus} / C_D)] \quad (26)$$

3. If the aircraft is being designed for a given engine

$$\mu_{m\Sigma} = \Delta m_{TO} / \sum_{i=1}^{4} \Delta m_{i\,0} = 1 / [\overline{m}_{\text{target}} + \overline{m}_{eng.s} + \overline{m}_{fuel.s} C_{D\,fus} / C_D - \Delta \overline{m}_{str\,0} - \Delta \overline{m}_{fuel.s\,0} (1 - C_{D\,fus} / C_D)] \quad (27)$$

Accounting for distance from the center of gravity

SFM was obtained for the aircraft, which was considered as a material point. But the initial change in mass at a significant distance from the center of mass of the aircraft will cause the appearance of concomitant changes in the mass of parts of the aircraft, which will also have an initial private character. These changes can be caused by changes in the load, aerodynamics, displacement of the CG, and as a result, changes in the parameters of the tail. This task will be relevant, for example, when assessing the impact of cargo rearrangement on the weight of the aircraft. In this case, it is necessary to go to the dimensional model of the aircraft, in particular, the simplest aircraft to consider as a system of intersecting beams in the center of mass of the aircraft: the fuselage and two wing beams.

In this case, the initial change in mass Δm_{i0} at a distance from the CG will cause the appearance of concomitant initial changes in the mass of parts of the aircraft $\Sigma\Delta m_{i0\,com}$ from the condition of maintaining the flight characteristics of the aircraft and payload at the same level (the 1st scenario). Because of this, the total initial mass change will already be

$$\Delta m_{00} = \Delta m_0 + \Sigma\Delta m_{i0\,com} \tag{28}$$

If we again use the previously obtained parameter SFM then the final change in the aircraft mass can be written as follows

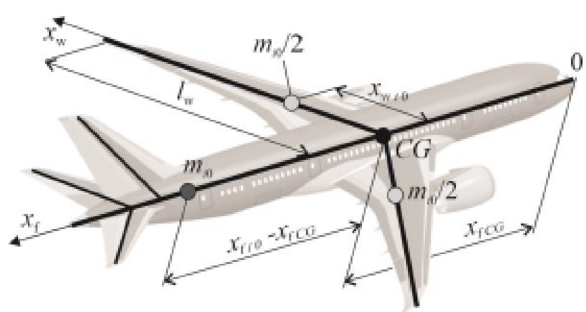
$$\Delta m_{TO} = \mu_m \Delta m_{i00} = \mu_m (\Delta m_{i0} + \Sigma\Delta m_{i0\,com}) = \mu_m^* \,\Delta m_{i0} \tag{29}$$

where $\mu^*_{m\,i}$ is corrected SFM taking into account the location of the place where the initial mass change occurred

$$\mu_m^* = \Delta m_{TO} / \Delta m_{i0} = \mu_m (1 + \Sigma\Delta m_{i0\,com} / \Delta m_{i0}) = \mu_m k_{\mu\,mi} \tag{30}$$

Of course, we can only estimate very roughly the value of the correction coefficient $k_{\mu\,mi}$ depending on the coordinates of the location of the initial mass change Δm_{i0}. But this assessment will generally answer a fundamental question about such an impact. We will consider the fuselage as a beam directed along the x_f axis, while the beginning of this axis is more convenient to place on the nose of the fuselage. And each wing cantilever is considered as a beam directed along the x_w axis – Figure 2. At the same time, we assume that these beams intersect at a point close to CG.

Figure 2 Aircraft – as a system of intersecting beams in the center gravity.



When Δm_{i0} is on or in the wing due to the symmetry of the aircraft, we consider the mass location on each console $\Delta m_{i0}/2$ at a distance of $x_{w\,i\,0}$ from the plane of symmetry (this may be an additional fuel tank or simply pumping fuel to unload the wing, a new engine with a different mass, the application or Vice versa removal of the cross joint in the wing, etc.). First of all, this will lead to a change in the bending moment from the force $\Delta m_{i0}gn/2$, where n is the load factor. For a more rigorous estimation of the correction coefficient, the third z coordinate perpendicular to the $x_f x_w$ plane must also be taken into account. In particular, if the mass addition is on the wing, it can make a difference as to whether the mass addition is inboard or outboard.

The correction coefficient is $k_{\mu mi}$<1 for a positive additional mass, so the calculation case with a positive overload will be decisive. An approximate estimate, taking into account the change in the bending moment and the corresponding concomitant change in the mass of the force elements that perceive this bend (Grebenkov, 1999).

When considering the coefficient k_{μ} in the case of the location of the initial mass Δm_{i0} on the fuselage with the $x_{f\,i\,0}$ coordinate (for example, the installation of a fuel tank in the keel, additional equipment, etc.), in addition to changing the bending moment, the position of the CG and the tail arm of the $x_{f\,CG\,Old}$ will change. At $x_{f\,CG\,Old} < x_0$ (in the rear fuselage), and if Δm_{i0}>0 then CG will move backward and there will be associated costs mass as a result of increasing the area of the vertical tail, and this, in turn, will lead to increased aerodynamic drag, and therefore need to increase thrust and fuel costs. It is obvious that in this case $k_{\mu\,mi}$>1.

In the case of an initial mass change in the nose of the fuselage ($x_{f\,CG\,Old} > x_{f\,i\,0}$), the concomitant change in the mass of the fuselage coincides, but the change in the concomitant mass of the tail, powerplant, and fuel system will be the opposite sign.

The solution of the problem related to the study of the influence of the location of the additional mass $\pm\Delta m_{i0}$ on the final mass of the aircraft, allows us to consider another relevant problem of estimating the mass of the aircraft when the position of the cargo changes from one point to another along the x_w and x_f axes. This is the case when choosing a layout solution. A classic example of this problem is a comparative assessment of the position of engines on the tail of the fuselage with engines on the wing.

Sensitivity Factors of Mass by Aerodynamic Parameters

The introduced design solutions, along with changes in mass, can be associated with changes in aerodynamics and have a significant impact on the aerodynamic characteristics, in particular on drag. And, as a rule, these effects on mass and aerodynamics are opposite in their effect (for example, the descending nose of a supersonic aircraft). The concept based on SFM is convenient in such a problem of resolving contradictions between the mass and aerodynamic drag of aircraft parts.

SFM by the aerodynamic drag

Assume that the thrust of the power plant is determined by the cruising mode of flight – altitude, and speed of flight

$$T_{H,V} = D = C_D q S \tag{31}$$

where $T_{H,V}$ and D are the thrust and drag, q is dynamic pressure, S is the area of the wing planform. If we proceed from the constancy of the main flight properties of the aircraft and the specific characteristics of the power plant and consider the dependencies proportional to $(m_{eng.s}+m_{fuel.s})g$ from $T_{H,V}$, then we can accept that the initial change in aerodynamically drag ΔD_0will result in a change in thrust

$$\Delta D_0 = \Delta T_{H,V} = T_{H,V} / (m_{eng.s} + m_{fuel.s})(\Delta m_{eng.s\,0} + \Delta m_{fuel.s\,0}) \tag{32}$$

Given that $D= T_{H,V} =mg/E$, where *E is L/D* ratio, we get

$$\Delta D_0 = g / E(\overline{m}_{eng.s} + \overline{m}_{fuel.s})(\Delta m_{eng.s\,0} + \Delta m_{fuel.s\,0}) \tag{33}$$

According to definition SFM by drag is $\mu_D = \partial m_{TO} / \partial D_0$ and taking into account $\Delta m_{TO} = \mu_m (m_{eng.s} + m_{fuel.s})$ we receive

$$\mu_D = \Delta m_{TO} / \Delta D_0 = \mu_m E(\overline{m}_{eng.s} + \overline{m}_{fuel.s}) / g \tag{34}$$

SFM by the aerodynamic drag coefficient

As in the previous case, we assume a proportional relationship between the change in the drag coefficient and the change in the mass of the engine and fuel

$$C_D / (m_{eng.s} + m_{fuel.s}) = \Delta C_{D_0} / (\Delta m_{eng.s\,0} + \Delta m_{fuel.s\,0}) \tag{35}$$

and then we will obtain

$$\mu_{C_D} = \Delta m_{TO} / \Delta C_{D_0} = \mu_m (\overline{m}_{eng.s} + \overline{m}_{fuel.s}) m_{TO} / C_D \tag{36}$$

SFM by the Lift/Drag ratio

Similarly, given a proportional relationship $(m_{eng.s}+m_{fuel.s})\sim T_{H,V}\sim 1/E$. For fairly small changes, we can take

$$(\Delta m_{eng.s} + \Delta m_{fuel.s}) / \Delta E_0 = [(m_{eng.s} + m_{fuel.s})E]_{Old} / E^2_{Old} E \tag{37}$$

According to definition SFM by lift to drag ratio $\mu_K=dm/dE$

$$\mu_{L/D} = \Delta m_{TO} / \Delta E_0 = -\mu_m (\overline{m}_{eng.s} + \overline{m}_{fuel.s}) m_{TO} / E \tag{38}$$

Evaluation of aerodynamic changes in a project

Consider the scheme for solving the problem of whether it is appropriate to switch to a new aerodynamic version using the "mass" criterion. Suppose we consider the implementation of a new value of the drag $D+\Delta D_0$. For example, using Eqn (34), we can calculate the equivalent mass of this model from the point of view of aerodynamics

$$\Delta m_1 = \mu_D \Delta D_0 = \mu_m E(\overline{m}_{eng.s} + \overline{m}_{fuel.s}) \Delta D_0 / g \tag{39}$$

The physical implementation of this solution in the construction is expressed in terms of the initial massΔm_{str0}, which is eventually transformed into the mass $\Delta m_2 = \mu_{m\,str}\Delta m_{str0}$.

The condition for whether this option is appropriate is

$$\Delta m = \Delta m_1 + \Delta m_2 < 0 \quad (40)$$

Issues of the aerodynamic nature of the project should also be evaluated and by the fuel efficiency criterion. As it is easy to see, this scheme differs from the previous one only by the transition from Δm to Δm_{fuel}, so using the results of the previous problem and the mass structure after the implementation, we have

$$\Delta m_{fuel\,0} = m_{fuel}\,\Delta D_0 / D = m_{fuel}\,\Delta D_0 E/(m_{TO} g) = \overline{m}_{fuel} E\Delta D_0 / g \quad (41)$$

$$\Delta m_{fuel\,1} = \Delta m_{fuel\,0} + (m_{fuel} + \Delta\overline{m}_{fuel\,0})(1 - C_{D_{fus}} / C_D\,)\Delta m_1 \quad (42)$$

$$\Delta m_2 = \mu_{m_{str}} \Delta m_{str0}, \quad \Delta m_{fuel\,2} = \overline{m}_{fuel}(1 - C_{D_{fus}} / C_D\,)\Delta m_2 \quad (43)$$

Ignoring the second-order quantities of smallness ($\Delta\Delta$), we can find the algebraic sum $\Delta m_{fuel} = \Delta m_{fuel\,1} + \Delta m_{fuel\,2}$, substituting instead of $\Delta m_{fuel\,0}$, Δm_1 and Δm_2 their expanded expressions

$$\Delta m_{fuel} = \overline{m}_{fuel}\{[E/g + (1 - C_{D_{fus}} / C_D\,)\mu_D]\Delta D_0 + (1 - C_{D_{fus}} / C_D\,)\mu_{m_{str}}\Delta m_{str0}\,\} \quad (44)$$

According to the Δm_{fuel} sign, we conclude according to the "fuel mass" criterion that it is advisable to switch to a new version of the constructive solution.

From the condition Δm_{fuel}=0, we can determine the limit value of the change Δm^*_{str0} compensating for the change

$$\Delta m^*_{str\,0} = -\{1/[(1 - C_{D_{fus}} / C_D\,)\mu_{m_{str}}] + (\overline{m}_{eng.s} + \overline{m}_{fuel.s})\} E\Delta D_0 \quad (45)$$

Numerical Research

The important question is the analysis of the effect of the correction associated with taking into account the dependence of the SFM on the value of the initial mass change. As a basic project, the aircraft is considered with characteristics close to the Boeing 747. The initial data for the Boeing 747-200B is accepted as follows from table 1 (http://en.wikipedia.org/wiki/Boeing_747), and also, we'll assume that C_{Dfus}/C_D=0.3.

Table 1 Components of the mass of Boeing 747-200B.

Masses	m_{TO}	Main functional parts				Elements of structure		
		$m_{payload}+m_{servic}=m_{\text{target}}$	m_{str}	$m_{eng\cdot s}$	$m_{fuel.s}$	m_{fus}	m_{wing}	m_{tail}

Absolute masses m_i, t	377.8	68.18+44.9=113.08	97.3	28.4	139	29. 3	43.5	8.5
Relative masses $\bar{m}_i$	1	0.18+0.12=0.3	0.26	0.07	0.37	0.08	0.11	0.02

We can analyze the transition from the metal structure of the wing, fuselage, and tail, for which mass in the basic project was m_{fus}+ m_{wing} + m_{tail}=81.3 t, to the composite one. According to design experience, such a transition can reduce the initial mass of the construction by 20-30%, i.e.Δm_{str0}= –81.3×0.3= –24.4 t.

Figure3 shows the values of the coefficient μ_m in the range of the initial change in the mass of the structure $\Delta m_{str0}$from –25 t to +25 t (solid line), which was 2.0<μ_m< 2.7. The dotted line corresponds to the constant value μ_m = 2.31, which is obtained by Eqn (24) accordingly. The dashed line corresponds to the value μ_m = 3.33, which is obtained by Eqn (8).

Figure 3 Values of SFM for the Boeing 747.

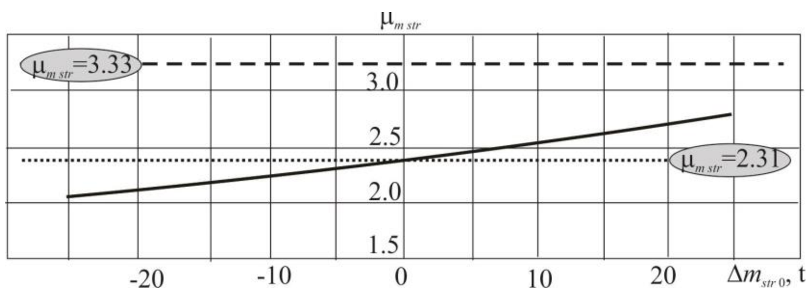
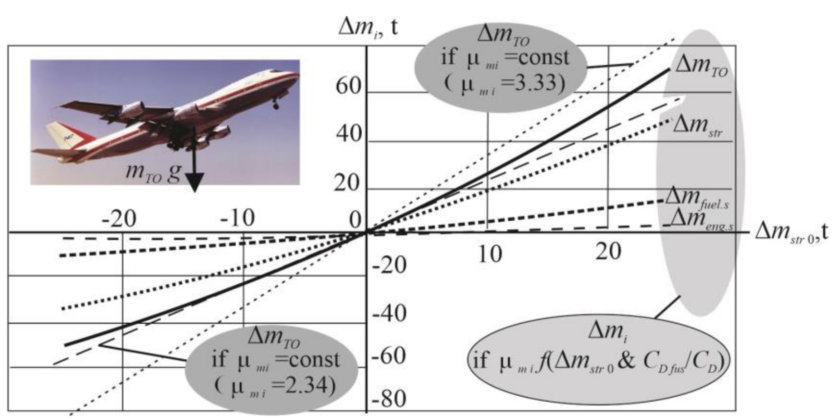


Figure 4 shows the final changes in take-off and functional masses with the initial change in the mass of the structure –25 t <Δm_{str0}<+25 t. The thin lines correspond to the take-off mass change calculated by the Eqn (8) and Eqn(24), which are usually used in the designing.

Figure 4 Change in take-off mass of the aircraft Boeing 747 and its components.



And two numerical examples related to the estimation of SFM to the initial change in aerodynamic characteristics: 1 – to the aerodynamic drag; 2 – to *L/D* ratio. If we assume that for cruising mode *L/D*=18, then by Eqn (34) μ_D =1.76 (kg/N), and by Eqn (38) $\mu_{L/D}$ =–18000 (kg/per unit *L/D*). These numbers mean that the change of the1 (N)aerodynamic drag will cause a change in the total mass of 1.76 (kg), and the change of L/D factor by one unit, will lead to a change of the mass of the aircraft by 18 (tons)with a different sign.

Conclusion

The methodology presented in this paper for refined evaluating of the aircraft take-off mass and its functional parts and checking it on the example of calculating changes masses of passenger aircraft convincingly shows the great convenience of using this approach for rapid evaluation of new design solutions. Numerical studies have shown that when using the presented algebraic refined sensitivity analysis method with a change in the initial total mass by 6%, the refinement of the change in the final mass can be 20-40% compared to the previously using weight(mass) growth factors.

Further Work

D.Scholz and A.Kretov "Application of sensitivity factors of mass for the synthesis of a passenger aircraft project". It is preparing the publishing in "Aircraft Design" with a Continuous Open Access Special Issue (2020)

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