Different Fidelity Computational Models in Aeroelastic Design of Aircraft and WT Models

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ABSTRACT

Using of different fidelity computational models and relations between them in the multidisciplinary design system is considered. A structural design procedure and optimization methods for aeroelastic design of aircraft and wind tunnel models are discussed. An application of the topology-based optimization together with the two-level structural sizing method is considered. Main stages of the approach to synthesis of structural layouts of aircraft components are described. Some numerical examples of analysis and aero-structural optimization of aircraft wings are considered to demonstrate the proposed methods and algorithms. The accuracy, reliability and efficiency of using of the considered structural models at design studies are discussed.

KEYWORDS: *aircraft structure, computational model, aeroelasticity, strength, optimization*

NOMENCLATURE

Latin
C – Vector of generalized coordinates
F – Objective function
G – Constraint function
GJ – Torsion stiffness
G^0 – Reduced stiffness matrix
K – Global stiffness matrix
M – Global mass matrix
M^r – Reduced mass matrix
Q^0 – Vector of generalized forces
R – Vector of applied forces
T – Torsion moment
U – Vector of nodal displacements
X – Vector of design variables
f – Polynomial term
p, q – Integer polynomial powers
r – Vector of nodal displacements
t - Time
x, y, z – Coordinates
u, v, w – Displacement components
Greek
Π – Transformation matrix
α – correction factor
λ – eigenvalue

1 INTRODUCTION

The design of an aircraft structure and its wind tunnel model is very complex problem. This is due to that many operating constraints arising from different technical disciplines, determining the performance of the aircraft, should be taken into account. An increase of requirements to the lift-to-drag ratio, aeroelasticity characteristics, the stability and the controllability of aircraft causes a necessity of the experimental validation of the static aeroelasticity characteristics on elastically-similar models in a high-speed wind tunnel (WT). When using modern advanced technologies for manufacturing of the models the accuracy of the modeling of the aeroelasticity characteristics depends mainly on the accuracy of a computational model of the aeroelastic wind tunnel model (AWTM) with chosen similarity scales. Therefore, nowadays the development and research of the AWTM computational models of different fidelity is an urgent and important problem. These models are intended for an aeroelasticity and strength analysis, for the analytical support of the lab and WT tests and for the purposes of design optimization.

In recent years, the multidisciplinary design optimization approach is often used in aircraft design to solve the problem involving a highly large number of design variables and constraints. This process is very time-consuming and, and often in practice, it is necessary to simplify the design problem by using mathematical structural models of different fidelity for different design problems and levels. Structural optimization can be performed by using the structural models both of the global and the local levels. Many technical papers are devoted to the development of the multilevel methods for analysis and optimization [1-6]. Two-level approach to structural optimization with stress and panel buckling constraints has been developed in the paper [5]. There the optimization problem is solved by using different level models in the multidisciplinary design optimization environment. The multilevel approach for structural analysis and optimization with taking into account strength, buckling and aeroelasticity requirements is presented in [6]. In this case, the considered models are a global finite element model for stress/aeroelasticity analysis and a local panel model for buckling analysis and optimization.
The purpose of this paper is to present an approach for aeroelastic analysis and optimization based on two-level modeling. Development of agreed mathematical models of different levels is discussed. In this method the sizing problem is to determine the structural sizes that will ensure a minimum weight while satisfying the numerous constraints which are of different types for many load conditions in disciplines such as linear static analysis, normal modes, and static and dynamic aeroelasticity. The responses in the disciplines can be analyzed by programs which use the structural models of different fidelity. In the developed multidisciplinary design system the problems of aeroelasticity and loads are solved by using the discrete-continual model of prescribed forms. The finite element model is used for detailed evaluation of stresses and displacements of a structure. Also, the application of topology optimization for the determination of a reasonable structural layout together with optimization by using the structural models of different level is discussed. Three numerical examples on using of the different fidelity computational models for structural analysis and optimization are presented.

2 MULTIDISCIPLINARY SYSTEM WITH DIFFERENT FIDELITY COMPUTATIONAL MODELS

2.1 Flowchart of multidisciplinary design system

Integrated structural design systems play the important role in the design of aircraft structures [7-10]. They give an opportunity to increase a number of numerical investigations and improve the weight, aerodynamic, strength and aeroelastic performances of aircraft. The automated multidisciplinary system ARGON [10] was developed in Central Aero-Hydrodynamic Institute (TsAGI) for efficient solution of the problems related with the design of aircraft structures made of metallic and composite materials. Mostly, this software package is intended for airframe structural optimization, prediction of stiffness/stress/mass distributions, aerodynamic characteristics, loads and aeroelastic characteristics of aircraft at the preliminary design stages. It is based on the common initial data for the structural models of different fidelity and it integrates the following disciplines: 1) linear aerodynamics; 2) flight loads with account of structural elasticity; 3) structural analysis by using the models of prescribed form method (PFM) and finite element method (FEM); 4) structural optimization; 5) modal analysis; 6) aircraft static aeroelasticity; 7) flutter; 8) aeroservoelasticity.

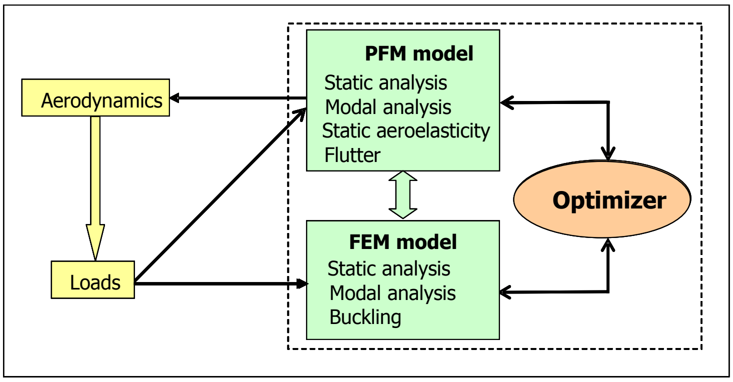


Figure 1: Interaction between the system modules

The main modules of the system and interaction between them are presented in Fig. 1. The aeroelasticity/strength design cycle starts with calculation of the aerodynamic and inertial loads for the various maneuver parameters of aircraft. Structural and aeroelastic analyses are performed with using of the model of PFM [10]. The optimization under both strength constraints for obtained loads and aeroelasticity constraints is performed. The loads for the optimized elastic structure are calculated again, and new optimization is carried out for the loads on elastic aircraft. Optimization results for the first level model give: the extreme load cases for structural parts with their corresponding load distribution, the preliminary structural sizes, the stiffness requirements as the constraints on the generalized displacements, etc.

The results from optimized PFM model are used to form initial data for the detailed structural design by the finite element (FE) model. Based on the FEM the stresses and strains can be evaluated more accurately. Optimization of the design parameters under strength, stiffness, frequency and buckling constraints can also be performed using this model. Finally, the finite element stiffness/mass matrices for the optimal design variables can be transformed into corresponding matrices of the PFM model. This makes it possible to verify the aeroelastic characteristics of aircraft. The design cycle is completed if the strength, buckling and aeroelasticity constraints are satisfied.

2.2 Different fidelity computational models

The PFM structural model of aircraft consists of flat elastic surfaces that model flexibility of aircraft components such as wing, tails, fuselage, etc. The structural displacements of the aircraft components are represented as polynomial functions of spatial coordinates. The stiffness and mass parameters are set by the user for each elastic surface by using elements of different types. These elements are concentrated mass, isotropic, orthotropic and laminated panel, plate, beam etc. The local coordinate system for an elastic surface is such that its plane $x0y$ coincides with the surface plane. Normal displacements $w(x,y,t)$ of the elastic surface can be expressed by follows:

$$w(x,y,t)=\sum_{k=1}^{N} f_k(x,y)C_k(t) \quad \text{, where} \quad f_k(x,y)=x^{p_k}y^{q_k}, \quad p_k,q_k=0,1,... \tag{1}$$

The coefficients $C_k(t)$ are the generalized coordinates of the PFM. The polynomial function f_k can be chosen for an elastic surface by different ways. Also the generalized coordinates can be added to describe the in-plane displacements. The elastic surfaces are combined into one analytical model by using springs and damping elements. They give possibility to model different attachment conditions between elastic surfaces. The Ritz method for minimization of total energy is employed to form the linear system of equations for static and modal analysis. Stiffness matrix and structural damping matrix of the PFM model in common case are fully filled ones while mass matrix consists of the diagonal matrix blocks. The paper [10] describes using of stiffness, structural damping and mass matrices for aeroelasticity analysis in details. Also, knowing the structural displacements the stresses and strains can be simply computed from equations for beams and plates. The particular cases of the PFM model are equivalent plate model and equivalent beam model.

The FEM structural model is most commonly employed for the detailed analysis of structural displacements and stresses. Structural analysis modules with using FEM contain a wide variety of isoparametric one- and two-dimensional finite elements: membranes, shells, beams and rods. Additional non-structural masses can be included in structural model. Isotropic, orthotropic and composite materials can be treated in the programs. Nodal displacements in static analysis are determined from solution of set of linear algebraic equations:

$$\mathbf{KU}=\mathbf{R} \tag{2}$$

where **K** is stiffness matrix, **U** are vectors of unknown nodal displacements and **R** are vectors of applied forces for several considered load cases. This sparse set of linear equations is solved by using the advanced direct sparse solver. Obtained displacements are used for determination of strains and stresses in finite elements. Computation of eigenvalues and vectors is performed for solution problems on free structural vibration and general buckling analysis. The program allows determining several eigenvectors that correspond to the first lowest eigenvalues by solving the problem: $\mathbf{KU}=\lambda\mathbf{MU}$, where **M** is the structural mass matrix for the vibration problem or the geometric stiffness matrix for the buckling problem.

The aerodynamic and inertial loads for flexible aircraft that are obtained by using the simpler PFM model can be automatically transferred to nodes of the finite element mesh.

2.3 Structural optimization methods

The structural design problem is formulated as a conventional problem of mathematical programming:

minimize $F(\mathbf{X})$ (3)

subjected to constraints $G_j(\mathbf{X}) \leq 0,\ j=1,\ldots,\ M$ (4)

$x_i^L \leq x_i \leq x_i^U,\ \ i=1,\ldots,\ N.$ (5)

The vector of the design variables **X** in structural design problem includes the transverse sizes of elements and the variables which define the shape of structure. The objective function $F(\mathbf{X})$ is the weight of structure. It is a linear function of the design variables when they are the transverse sizes of elements and a nonlinear function in the case of the shape design variables. The constraints Eq. 4 can include constraints on the stresses, displacements, natural frequencies, buckling of panels, aeroelastic lift effectiveness, aileron effectiveness, and flutter velocity. They are highly nonlinear and implicit functions with respect to the design variables. The values x_i^L and x_i^U in the constraints Eq. 5 are correspondingly the lower and upper bounds of the design variables. Two major numerical approaches for solving structural optimization problem Eq. 3 - Eq. 5 are used in the multidisciplinary system. The first approach is based on the optimality criteria and the second one is based on the mathematical programming methods.
The main advantage of the optimality criteria methods is their efficiency in obtaining the near-optimal solutions independently of the number of design variable. Often they are closely related with a physical nature of the structural response. For example, the stresses in an optimal homogeneous structure under only one load case have a maximum allowable value. The obtaining algorithms in the optimality criteria approaches are based on simple recurrence relationships for the different types of the constraints. The following optimality criteria algorithms have been implemented in the multidisciplinary system:

- Fully-stressed design (FSD) algorithm;
- Algorithm with compensation of violated stress constraints;
- Algorithms with stress and displacement constraints;
- Algorithm of equal-stability structural panel;
- Topology optimization based on the FSD algorithm;
- Topology optimization based on the criterion of uniform specific strain energy;

Three mathematical programming methods have been implemented in the multidisciplinary system:

- Gradient projection method;
- Sequential quadratic programming method;
- Modified Pshenichny method.

The brief description of these methods is given in the paper [11] where the particular attention is paid to the modification of the Pshenichny method.

2.4 Relations between matrices of FEM and PFM models

Here we represent how the stiffness and mass matrices for the equivalent plate model as a particular case of PFM model can be calculated from the corresponding matrices of FE models. It gives possibility to form the data for the aircraft lifting surfaces to perform aeroelasticity analysis by using first level model. For more common cases the solution method of this problem is described in the paper [12]. In static analysis the displacements $u(x, y, z)$ and $v(x, y, z)$ of the equivalent plate in accordance with the Kirchhoff hypothesis are determined through the transverse displacements $w(x, y)$:

$$u(x,y,z) = -z\frac{\partial w}{\partial x},\quad v(x,y,z) = -z\frac{\partial w}{\partial y}. \qquad (6)$$

Taking into account Eq. 1 the transverse displacements $w(x, y)$ in static analysis can be represented as polynomial function of the non-dimensional coordinates $\bar{x}$ and $\bar{y}$:

$$w(x,y)=\sum_{k=1}^{N}C_k\overline{x}^{p_k}\overline{y}^{q_k}, \qquad (7)$$

where p_k and q_k are integer polynomial powers, N is the number of the polynomial terms, C_k are unknown coefficients.

To build a correspondence between the models, we write the displacements in the nodes of a finite element mesh (vector **U**) in the form of polynomial functions as follows:

$$\mathbf{U}=\mathbf{\Pi C}, \qquad (8)$$

where the components of matrix $\mathbf{\Pi}$ in accordance with Eqs. 6 and 7 are calculated by the following formulas:

$$\Pi_{3i-2,j}=-z_i\frac{\partial\overline{x}}{\partial x}p_j\overline{x}_i^{(p_j-1)}\overline{y}_i^{q_j},\ \ \Pi_{3i-1,j}=-z_i\frac{\partial\overline{y}}{\partial y}q_j\overline{x}_i^{p_j}\overline{y}_i^{(q_j-1)},\ \ \Pi_{3i,j}=\overline{x}_i^{p_j}\overline{y}_i^{q_j}. \qquad (9)$$

Now, at the established relationship between the nodal displacements **U** and the generalized coordinates **C** (Eqs. 8 and 9) the stiffness matrix of FE model can be replaced by the equivalent PFM stiffness matrix from the condition of equality of the strain energies for the considering models. We have the following equation:

$$\mathbf{C}^{\mathbf{T}}\mathbf{G}^{\mathbf{0}}\mathbf{C}=\mathbf{r}^{\mathbf{T}}\mathbf{K}\mathbf{r}, \qquad (10)$$

where $\mathbf{G^0}$ is the reduced stiffness matrix for the equivalent plate model.
Substituting Eq. 7 into Eq. 10 we obtain $\mathbf{C^TG^0C}=\mathbf{C^T\Pi^TK\Pi C}$. From here we find the expression for the reduced stiffness matrix

$$\mathbf{G^0}=\mathbf{\Pi^TK\Pi}. \qquad (11)$$

The vector of generalized forces $\mathbf{Q^0}$ in the prescribed form method can be found from the condition of equality of the work done by external forces: $\mathbf{R^Tr}=\mathbf{Q^0}^{\mathbf{T}}\mathbf{C}=\mathbf{R^T\Pi C}$, or

$$\mathbf{Q^0}=\mathbf{\Pi^TR}. \qquad (12)$$

The above relations (Eq. 11 and Eq. 12) are derived under the assumption of modelling the lifting surface by a thin symmetrical plate. Therefore, when implementing the presented matrix operations the structural layout of the lifting surface should be symmetrised. Note that in the structural dynamics problem it is also necessary to have the mass matrix. If we have the finite element mass matrix **M** for a lifting surface then it is easy to derive the reduced mass matrix $\mathbf{M^r}$ for PFM (equivalent plate model) from the equality of kinetic energies. This matrix can be calculated analogously to Eq. 11:

$$\mathbf{M^r}=\mathbf{\Pi^TM\Pi}.$$

It should be noticed that the assembled stiffness matrix of FE model for a lifting surface may be singular without excluding the degrees of freedom providing absence of displacement of the structure as a rigid body. That is because the prescribed forms of displacements in the PFM are chosen to satisfy to some specified boundary conditions. Unknown generalized coordinates are determined from the solution of the equation of equilibrium for generalized forces: $\mathbf{G^0C}=\mathbf{Q^0}$. The reduced stiffness $\mathbf{G^0}$ and mass $\mathbf{M^r}$ matrices can be used in the multidisciplinary system for efficient solution of the static and dynamic aeroelasticity problems.
The comparative analysis of numerical results for the above procedure for reducing the stiffness/mass matrices both for test examples and aircraft lifting surfaces shows that the developed method provides reliable results on the modeling of elastic and mass properties. Minor differences in displacements of lifting surfaces by using the FEM and the proposed method based on the PFM are due to that the relatively small number of polynomial terms is usually used in the prescribed form method and also the equivalent plate model, unlike the FEM, assumes symmetry in thickness. The displacements at the tip part of realistic lifting surfaces for the reduced models are usually 5-7% less than for the FE model. Nevertheless, such a difference can be reduced by correcting the reduced stiffness matrix based on the results of finite element calculations.

Consider the calculating transverse displacements at n selected nodal points for N_{lc} load cases: v_F^{il} are for the FEM model and v_R^{il} are for the reduced PFM model. We introduce the correction coefficient α for the reduced stiffness matrix, such that the corrected reduced stiffness matrix is calculated by the formula $\mathbf{G}^c = \alpha \mathbf{\Pi}^T \mathbf{K} \mathbf{\Pi}$. Due to the linearity of the static problem, the displacements on the reduced model will change by a factor of $1/\alpha$. We apply the least squares method to find the minimum of the sum of the squares of the displacements $S(\alpha)$:

$$S(\alpha) = \sum_l^{N_{lc}} \sum_i^n (w_F^{il} - \frac{1}{\alpha} w_R^{il})^2 \ .$$

Solving this problem we obtain the following expression for the correction factor:

$$\alpha = \sum_l^{N_{lc}} \sum_i^n (w_R^{il})^2 / \sum_l^{N_{lc}} \sum_i^n w_F^{il} w_R^{il} \ . \tag{13}$$

Application of the described method for individual aircraft components makes it possible to generate PFM analysis models of significantly smaller dimensions for the solution of aeroelasticity problems and to obtain reliable results when performing multidisciplinary design studies on models of different levels.

2.5 Analysis models in topology-based structural design procedure

It is well-known that the application of topology optimization for determination of reasonable structural layout usually results in advanced designs. In this case computational model is built on the basis of 3D finite elements. But weight reducing for the realistic structure is achieved after the sizing optimization of some interpreted structural layout for which shell/beam model is used. Below we discuss a novel approach which combines modern topology optimization methods with two-level sizing optimization technique.

The general process of the structural design procedure is presented in Fig. 2 and it includes the topology optimization module that allows to search of reasonable structural layouts subjected to several load cases. The first stage is topology optimization. The procedure begins from specifying of geometric outlines that serve as initial data for aerodynamic analysis and topology optimization. For topology optimization they define design domain, some part of which is supposed to be fixed and another part is subjected by external loads. The first step is preparing of the solid FEM model for topology optimization and the aerodynamic model for calculation of loads of some extreme load cases. Then, a set of topology optimizations with different control parameters to reveal where load-bearing material should be located in global sense is performed. The optimization results are interpreted to find out the location of the primary structural elements. The second stage of the procedure is design of the structural elements in the interpreted thin-walled structural layouts. It includes shape and sizing optimization with the aim to minimize structural weight under stress/buckling/flutter constraints. The auxiliary optimization algorithms in this approach are based on optimality criteria and mathematical programming methods. The final stage is to identify the best structural layout by comparison of the obtained structural weights.

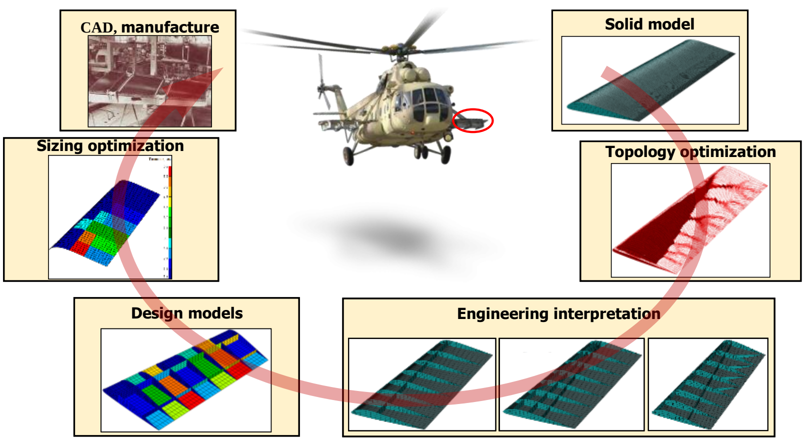


Figure 2: Design procedure

The first stage is characterized by the following aspects. It is important to correctly transfer pressure loads from aerodynamic model to the FEM model. It can be performed by interpolation of the obtained pressures with using polynomial function of nodal coordinates on outer surfaces of the model. A set of topology optimizations with different control parameters and different load cases should be performed to reveal adequately in global sense where load-bearing material is reasonably to locate. A design engineer should use some conventional technical solutions for missing information at interpretation of topology optimization results.
On the second stage the structural optimization problem can be solved by different methods and approaches. It can be solved sequentially by introduction of additional constraints during optimization process or by application of the developed global-local methods with using structural models of different fidelity. Another important aspect is account of aeroelasticity requirements. Note that some extra design cycles can be needed to take into consideration effect of structural elasticity.

2.6 Beam structural model for design studies of aeroelastic models

Modern commercial FE-based software is widely applied in design of elastic-similar and dynamically-similar models of aircraft which are used for testing in WTs. Often, in the development of such models, the beam structural model can be used where the stiffness properties of the structure are modeled using the given distributions of bending and torsion stiffness characteristics [13]. For example, such approach is relevant for design of AWTM fuselage, high-aspect ratio wing, helicopter tail boom, etc.
Calculation of the bending stiffness characteristics for beam sections is not difficult problem, but the determination of torsion stiffness is a rather complex problem consisting in solving partial differential equations. It is related with the classical Saint-Venant problem for torsion of prismatic rods with an arbitrary cross section and it is reduced to solution of two-dimensional Laplace equation for warping function or Poisson equation for the stress function. In the framework of the multidisciplinary system several methods are developed for calculation of the torsion stiffness of cross sections of arbitrary shape, including the presence of cavities in them. The methods and algorithms are presented in details in papers [14]. Below we consider only an example of application of them to form beam models and discuss accuracy, reliability and efficiency of them at design studies.

3 ILLUSTRATIVE EXAMPLES

3.1 Wing example

Let us demonstrate the above method for reducing the FEM model to the PFM model by the example of a wing of a supersonic passenger aircraft. The deformation of the wing structure is studied for two load cases at cruise flight with M=1.4 and M=2.05. The aerodynamic and mass models of the wing have been prepared in the framework of the multidisciplinary system. The FE model of the wing (Fig. 3) was previously created and it used for computing structural elasticity by application the reduction procedure. The FE model has 1556 elements and 456 nodes.

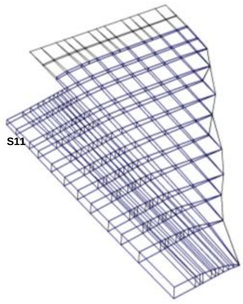


Figure 3: Finite element model of wing

The calculated aerodynamic and inertial loads on the first-level model of the ARGON system were transferred to the nodes of the upper wing panel of the FE model by a statically equivalent way, and static analysis was performed to determine the wing displacements. Then, the FE stiffness matrix of the wing structure was reduced to the stiffness matrix of the equivalent plate model, the deflections of which are determined by a polynomial of the 6th order along the wing span and 4th order in the stream-wise direction.

The reduced PFM stiffness matrix was used to calculate the loads on the wing taking into account its elasticity. These obtained loads iteratively used for the calculation of displacements for the elastic wing. It was required to carry out two iterations to correctly take into account of the effect of elasticity on the aerodynamic loads. The wing displacements for the load case at M=2.05 after the finite element analysis are shown in Fig. 4. The curves of deflections of the wing along the spar #11 (marked as S11 in Fig. 3) are given in Fig. 5 for the FEM structural models and the PFM models with the reduced stiffness matrices. As can be seen, the difference in the displacements at the wing tip of two models is less than 5% for both load cases. Moreover, the application of the correction factor of the stiffness matrix (Eq. 13) makes it possible to reduce this difference to less than 2%.

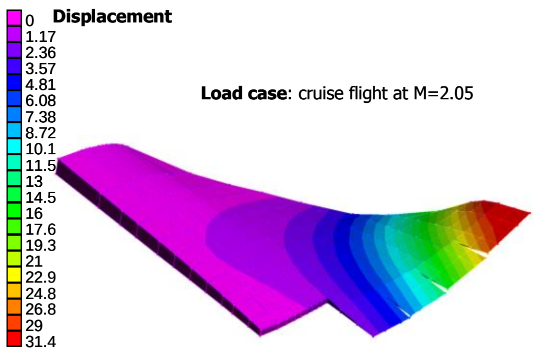


Figure 4: Wing displacements, cm

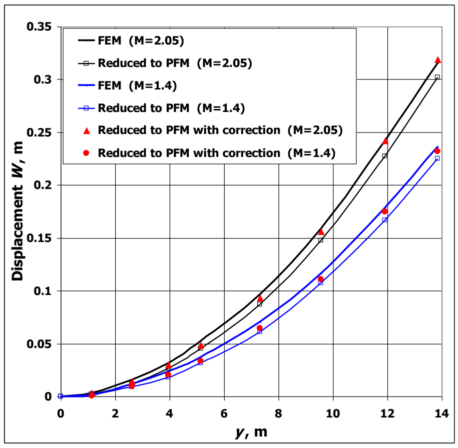


Figure 5: Displacements for different structural models

The resulting stiffness matrix for the PFM model was used to determine the aeroelastic characteristics of the supersonic passenger aircraft.

3.2 Optimization of wing structural layout

In this numerical example structural optimization of helicopter wing is considered. The baseline wing with traditional wing-box structure has mean aerodynamic chord of 1.3 m and the wingspan of 5.7 m. The extreme load case corresponds to a flight of helicopter at maximum angle of attack and flight with M of 0.3. The weight of the baseline structure, satisfying to strength, buckling and aeroelasticity requirements, is 69.5 kg. The purpose is to reduce the structural weight by using the topology-based optimization approach. To determine aerodynamic forces in extreme load cases and to perform aeroelasticity analyses an aerodynamic model of the wing is created by using the wing outer geometry. This geometry also serves for generation of a solid finite element model that is used in topology optimization. Topology optimization is carried out to minimize compliance at saving 30 percent of initial solid model weight in the final design. The obtained pattern where the load-bearing material should be located is presented in Fig. 6. This optimization result is interpreted as the traditional thin-walled structural layout with spars, ribs, skins. Seven interpretations with different number of spars, with/without additional sloped ribs were considered at design studies. One of such interpretations is shown in Fig. 6 by the bold lines; the shell/beam FEM model is shown in Fig. 7. For all the interpreted layouts the global shell/beam FEM models were generated. The optimization results on the global FEM models with taking into consideration of strength/buckling/aeroelasticity constraints were obtained. The best structural layout is the three-spar layout with weight of 42.4 kg. This weight is less by 37% than the weight of the baseline wing structure.
For some wing upper panels the buckling constraints become active. This is due to the fact that upper panels have smooth skin (no stringers). It is obvious that the stiffened panel is more effective to resist buckling. So addition of stringer elements is necessary to include. Addition of such elements in global finite element significantly increases both the number of degrees of freedom and the number of design variables (DV). Some of new design variables related with stringer elements are geometric and they define stringer shape. Therefore, it is necessary to consider the design problem with using local models of panels.

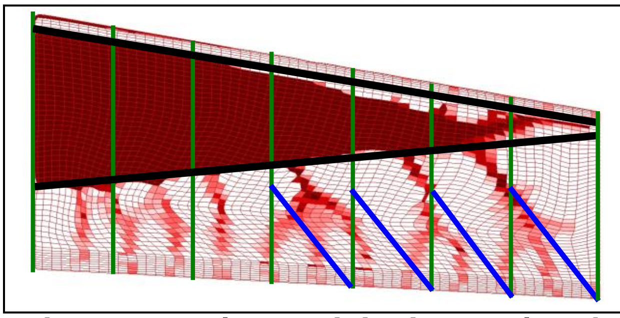


Figure 6: Topology optimization result and its interpretation

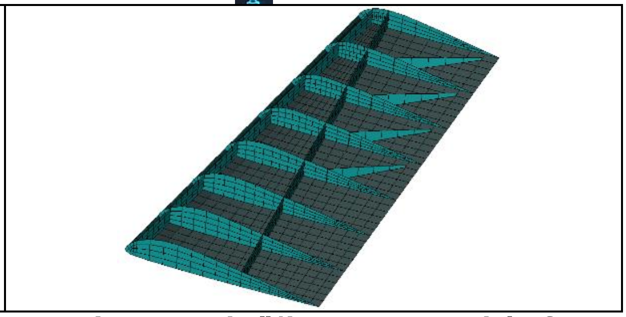


Figure 7: Shell/beam FEM model of interpreted structural layout

In the local optimization problem the design variables are the number of stringers, thicknesses of stringer elements, the stringer depth and skin thickness. In this research the shape of stringer section is rectangular, and we have four DV for each panel. Only upper wing panels are considered because they are under the action of compression loads. Figure 8 shows relation between global FE model and local one for a separate panel.

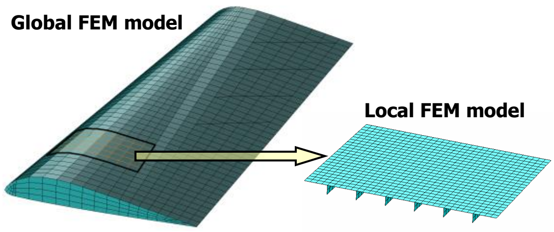


Figure 8: Global and local FEM models

Iterative global-local optimization was performed when stringer sizes are treated as design variables in local panel model. Some details of this procedure are following. The stress resultants in the elements of the local model are taken from the global model. The stringer and skin sizes are defined by parametric change of the number of stringers and the stringer depth in the local optimization. So it is needed re-meshing of the local model of panel for different parameters. It can be noted that the optimal number of stringer and stringer depth can be very different for adjacent panels. It is difficult to include such manufacturing constraint into design procedure. Finally the best layout after global-local optimization is the two-spar wing with weight of 36.3 kg. The weight was decreased by 16.8% in comparison with weight after global optimization. In summary, the weight benefit is 47.8 percent if the obtained two-spar wing with additional ribs at the end part compare with the conventional wing structural design.

3.3 Torsion stiffness of AWTM wing cross section

In this example a geometrically complex cross-section of the high-aspect ratio wing of aeroelastic model is considered. To determine its torsion stiffness two different analysis approaches are applied. In the first approach the solid FE model used for the beam-like structure shown in Fig. 9.

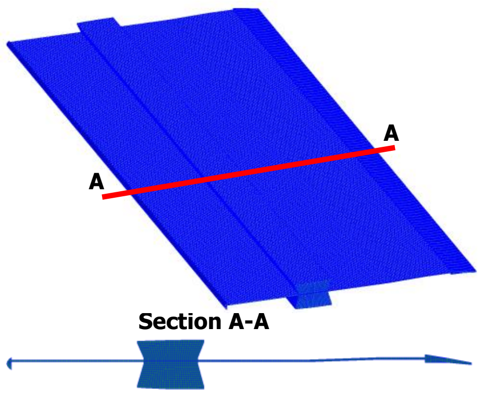


Figure 9: Solid FEM model

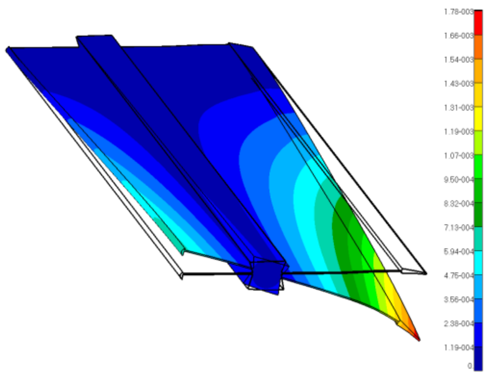
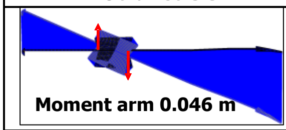
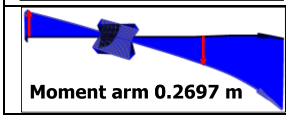


Figure 10: Deformed structure (2nd load case)

The span of the structure is 2 m, its width is 0.394 m and minimum thickness of the structural section is 2 mm. Finite element model has more than 1 million tetrahedral elements with quadratic approximation and more than 3.5 millions degrees of freedom. Structural material is steel. All displacements of the root part is fixed and at the tip part the torsion moment is applied as pair of forces of 1000 N. Two load cases are considered. In the first one the moment arm is 0.046 m, and in the second one is 0.2697 m. The location of the applied forces is shown in Table 1. The deformed structure at acting forces of the second load case is shown in Fig. 10. It can be seen that the tip section is slightly curved under the action of such a load. So in this case it is difficult to uniquely define what is considered a twist angle. For the FEM model we define twist angle as arctangent of the ratio of the difference in vertical displacements for the nodes, in which the forces are applied, and distance between these nodes.
In the second approach the torsion stiffness for the considered section was calculated by using the finite element method with triangles of quadratic approximation and the method of the fundamental solutions for the two-dimensional problem. The twist angle per length of the beam can be calculated by formula: $\theta' = T/GJ$, where T is the torsion moment and GJ is torsion stiffness. The obtained values of the torsion stiffness by two methods are very close, 56459 N·m^2 and 56500 N·m^2, correspondingly.

Table 1: Comparison of twist angles, radian

Load case	Solid FEM model	Beam model	Difference, %
	$8.7111 \cdot 10^{-4}$	$8.8255 \cdot 10^{-4}$	1.3
	$4.9440 \cdot 10^{-3}$	$5.1744 \cdot 10^{-3}$	4.5

Comparison of the twist angles at the tip section obtained by the FEM and the beam models is given in Table 1. The results for a fairly complex cross section computed by both models are in good agreement. Note that the computation time for the FEM model takes 2609 seconds, while for the beam model it is a few seconds.

4 CONCLUSION

The paper proposes the aeroelastic design approaches with using different fidelity structural models. Using of the reduced stiffness matrix for the equivalent plate model allows computing the elastic displacements of wing in the multidisciplinary design system with good accuracy. Application of the models of different level in the topology-based structural optimization gives possibility to obtain optimal structural layout of wing with taking into consideration of strength, buckling and aeroelasticity constraints. The developed multidisciplinary approaches used different fidelity models shows its efficiency and usefulness in the design process of complex aerospace structures and further they can be used in modern design practice.

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