Parameter Estimation of a Mini Aerial Vehicle using Multiple Trim Flight Data

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ABSTRACT

In this work, a novel flight test approach for accurate aerodynamic parameter estimation is designed. As low aspect ratio mini aerial vehicles have highly nonlinear aerodynamics, 3-2-1-1 doublets are used for elevator deflection to perturb aircraft at two different trim conditions, at low angle-of-attack (high velocity) and high angle-of-attack (low velocity). A combination of these two flight data is used to estimate the parameters. Realistic flight data is generated using complete nonlinear aircraft model. Equation-error estimation technique is used for parameter estimation. Parameter estimates using multiple trim flight data are compared with parameter estimates using different trim flight data. This comparison shows that multiple trim flight data is effective to get accurate estimates for a nonlinear aircraft model, even by using simple least square estimation technique.

KEYWORDS: *Parameter Estimation, Mini Aerial Vehicle, Maneuvers*

NOMENCLATURE

MAC – Mean Aerodynamic Chord
MAV – Mini Aerial Vehicle
NAL – National Aerospace Laboratory
OBES – On Board Estimation System
J – Cost function
N – No. of samples
S – Wing area
T – Thrust
V– Velocity
Z – Measurements
b – Wingspan
$\bar{c}$ – Mean Aerodynamic Chord
g – Gravitational acceleration
m – Mass
n – No. of parameters
q – Pitch rate
u – Velocity component in body x-axis
w – Velocity component in body z-axis
α – Angle-of-attack
ρ – Air density
δ_e – Elevator deflection angle
ϵ - Error
Θ – Parameters
θ – Pitch angle
C_D – Drag coefficient
C_L – Lift coefficient
C_M – Pitching moment coefficient
F_X – Force component in body x-axis
F_Z – Force component in body z-axis
I_{yy} – Moment of inertia about y-axis
M_Y – Pitching moment
a_x – Acceleration component in body x-axis
a_z – Acceleration component in body z-axis
x_E – x-coordinate of aircraft's location in Earth-fixed frame
z_E – x-coordinate of aircraft's location in Earth-fixed frame

1. INTRODUCTION

Applications of mini aerial vehicles (MAVs) are increasing day by day and they are widely used for surveillance. For completing these tasks, the MAVs must be flying in a controlled manner. Controlling any system requires accurate model of the system. Parameter estimation techniques are essential for getting an accurate mathematical model used for simulation and control law development. MAVs are incapable of carrying accurate sensors due to sensors' weights; it brings in additional challenge in parameter estimation. In addition to that, MAV aerodynamics may be different at low airspeed (high angle-of-attack) and high airspeed (low angle-of-attack).
Research area of exciting aircrafts with various perturbations for quality flight data is considerably explored. Doublets, 3-2-1-1 doublets, improvised 3-2-1-1 doublets are used widely for time domain open loop parameter estimation [1] [2]. Sine waves of varied amplitude and frequency or frequency sweeps are also explicitly used for frequency domain parameter estimation. [3] Dryden space research center has developed single-surface inputs which are sequential inputs to different control devices, each control device at a time. They also developed a system to merge these sequential inputs with pilot inputs or the controller inputs which was named on board estimation system (OBES) for closed loop parameter estimation [4]. Eugene Morelli developed optimized orthogonal multi-step inputs which are also merged with the pilot or controller inputs [5]. But perturbations like frequency sweep, sine waves or optimized inputs are either extremely hard to perform or they require additional hardware system. 3-2-1-1 doublets are relatively easy inputs to apply and excites wide range of frequency [6] [7] but may not provide good estimates for a highly nonlinear aircraft. Instead of designing maneuvers for quality flight data or using any nonlinear estimation technique, a novel multi-trim flight test approach is proposed.
It is observed that accurate parameter estimation can be done even with simple 3-2-1-1 doublets if multiple trim flight data is used. Black-kite MAV is taken as test vehicle which has highly nonlinear aerodynamic model. Least square estimation technique is used for parameter estimation of Black-kite MAV. The results show that ordinary least square can provide accurate parameter estimates if multiple trim flight data is used.

2. BLACK-KITE MAV

In the present work, Black-kite MAV data is used which is designed, fabricated and tested in a wind tunnel by National Aerospace Laboratory (NAL), Bangalore. Its dimensional characteristics are mentioned in Table 1.

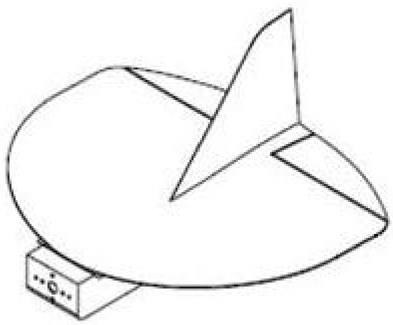

Figure 1: Black-kite MAV [8]

Table 1: Dimensional Characteristics of Black-kite MAV

MAV Characteristics	Values
Mass (kg)	0.3
Mean Aerodynamic Chord (MAC) (m)	0.083
Wingspan (m)	0.3
Wing area (m^2)	0.042
Moment of Inertia about y-axis (kgm^2)	5.6345×10^{-4}

From the wind tunnel test data provided by NAL, lift coefficient C_L drag coefficient C_D and pitching moment coefficient C_M are expressed as functions of angle-of-attack α and elevator deflection δ_e. These aerodynamic coefficients are formulated as Eq. 1, Eq. 2 and Eq. 3 respectively.

$$C_L = 0.1784 + 2.453\alpha - 1.691\alpha^2 + 29.986\alpha^3 - 49.245\alpha^4 + 0.7405\delta_e - 0.3638\delta_e^2 \tag{1}$$

$$C_D = 0.08712 - 0.05593\alpha + 3.4825\alpha^2 + 0.1471\delta_e + 0.2258\delta_e^2 \tag{2}$$

$$C_M = 0.0385 - 0.59977\alpha - 1.27402\alpha^2 - 0.4106\delta_e + 0.1587\delta_e^2 \tag{3}$$

3. FLIGHT DATA GENERATION

Only longitudinal flight has been considered in this paper. Longitudinal flight data is generated using 3-degree-of-freedom simulation using nonlinear aerodynamic model in MATLAB. Equations of motion used in simulation are given in subsequent sections.

3.1. Equations of motion

Aircraft velocity components u and w in body-fixed x-axis and z-axis are generated by using differential Eq. 4 and Eq. 5 respectively.

$$\dot{u} = \frac{F_X + T}{m} - g\sin\theta - qw \tag{4}$$

$$\dot{w} = \frac{F_Z}{m} + g\cos\theta + qu \tag{5}$$

where T is thrust, g is gravitational acceleration, θ is pitch angle and q is pitch rate. Pitch rate q and pitch angle θ are calculated for simulations using Eq. 6 and Eq. 7 respectively.

$$\dot{q} = \frac{M_Y}{I_{yy}} \tag{6}$$

$$\dot{\theta} = q \tag{7}$$

where I_{yy} is aircraft's moment of inertia about y-axis. Aircraft's location with respect to Earth-fixed frame is calculated by using differential Eq. 8 and Eq. 9 respectively.

$$\dot{x_E} = u\cos\theta + w\sin\theta \tag{8}$$

$$\dot{z_E} = -u\sin\theta + w\cos\theta \tag{9}$$

Airspeed V and angle-of-attack α are calculated using Eq. 10 and Eq. 11 respectively.

$$V = \sqrt{u^2 + w^2} \tag{10}$$

$$\alpha = \tan^{-1}\frac{w}{u} \tag{11}$$

3.2. Forces and pitching moment calculation

Aerodynamic force components in x-axis, z-axis and pitching moment are calculated using Eq. 12, Eq. 13 and Eq. 14 respectively.

$$F_X = \frac{1}{2}\rho V^2 S C_X \tag{12}$$

$$F_Z = \frac{1}{2}\rho V^2 S C_Z \tag{13}$$

$$M_Y = \frac{1}{2}\rho V^2 S\bar{c} C_M \tag{14}$$

where F_X and F_Z are forces in direction of x-axis and z-axis of body-fixed frame respectively. M_Y is pitching moment about y-axis of body-fixed frame. ρ is sea-level air density, V is airspeed, S is wing area and $\bar{c}$ is mean aerodynamic chord (MAC) of wing. Body-fixed frame is fixed at center of gravity of aircraft with x-axis pointing towards nose of aircraft and z-axis is pointed towards center of Earth. y-axis can be found using right hand rule. C_X and C_Z are calculated using Eq. 15 and Eq. 16 respectively.

$$C_X = C_L \sin\alpha - C_D \cos\alpha \tag{15}$$

$$C_Z = -C_L \cos\alpha - C_D \sin\alpha \tag{16}$$

3.3. Measurement noise

Flight data is generated through simulations. It is made similar to real measured flight data by adding artificial noise. Noise in each sensor has been assumed to be Gaussian white noise. Airspeed$V$, pitch rate$q$, acceleration components a_x and a_z and angle-of-attack α are measured quantities. Measurement ranges and standard deviations of Gaussian white noises of sensors are mentioned in Table 2.

Table 2: Sensors noise data

Measured quantity	Sensor type	Range	Standard deviation
V	Pressure sensor	40 Pa	0.8081 m s^{-1}
q	Rate gyro	600 deg s^{-1}	6 deg s^{-1}
a_x	Accelerometer	4 g	0.04 g
a_z	Accelerometer	4 g	0.04 g
α	α-sensor	50 deg	5 deg

3.4. Flight data sets

Simulations are done for 100 s durations with step size 0.001s. Two flight data sets are generated such that one contains low angle-of-attack and high velocity and other one contains high angle-of-attack and low velocity. Trim values of velocity and angle-of-attack are mentioned below for flight data sets I and II.

Table 3: Trim states for different flight data

Flight data	Trim velocity (ms^{-1})	Trim angle-of-attack (deg)
I	9	20.67
II	20	-1.80

Time history of angle-of-attack and velocity for three different flight data sets are shown in Fig. 2 and Fig. 3 respectively. It can be observed that outputs are very noisy. In flight data set I and II, 3-2-1-1 doublets with magnitude of$2°$ and total time duration of 7 seconds are used after initial 5 seconds. In an attempt to excite more frequencies of MAV so that quality flight data is acquired, a step with magnitude of $2°$ and time duration of 15 seconds is used as elevator deflection after 50 seconds. The third flight data is combination of first two flight data sets. In this third flight data, MAV is flown at two different trim conditions for 50 seconds each. Previously mentioned 3-2-1-1 doublets and a step are used to perturb the MAV at both trim conditions.

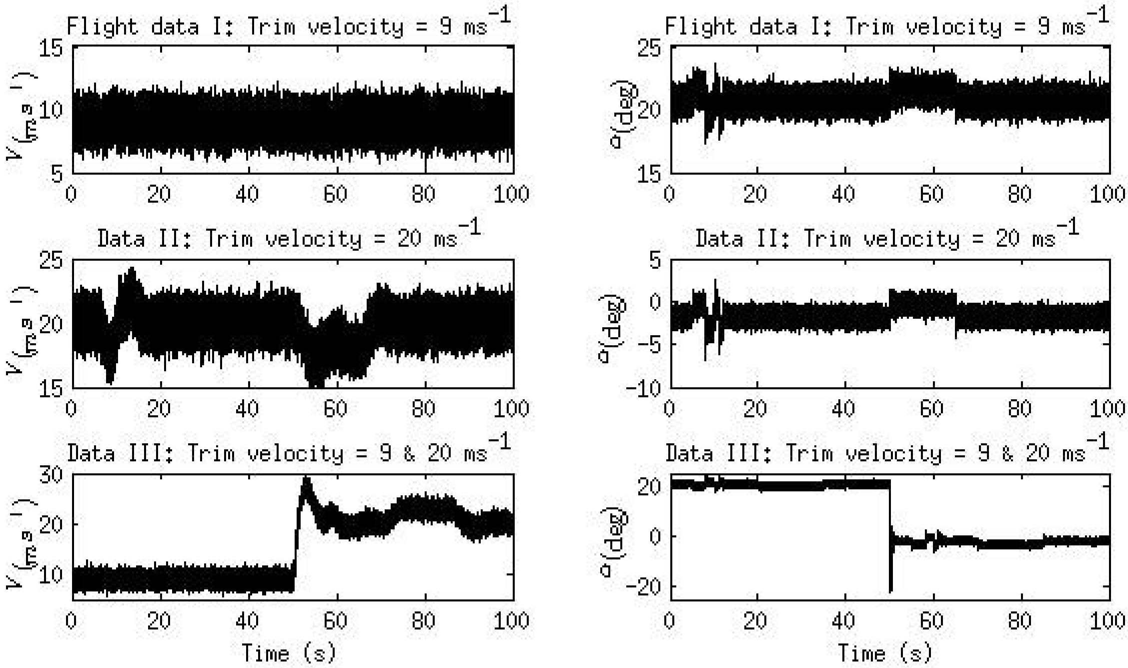

Figure 2: Angle-of-attack α and velocity V vs time t for three flight data sets

4. PARAMETER ESTIMATION

4.1. Data processing

Acceleration components in x-axis and z-axis of body fixed frame are measured along with pitch rate q and airspeed$V$. As these measurements are corrupted by noise, Kalman filter is used to get less noisy measurements. Time histories of lift coefficientC_L, drag coefficient C_D and pitching moment coefficient C_M are extracted from these measurements as follows.

$$C_X = \frac{a_x m - T}{\frac{1}{2}\rho V^2 S} \tag{17}$$

$$C_Z = \frac{a_z m}{\frac{1}{2}\rho V^2 S} \tag{18}$$

$$C_D = -C_X \cos\alpha - C_Z \sin\alpha \tag{19}$$

$$C_L = C_X \sin\alpha - C_Z \cos\alpha \tag{20}$$

$$C_M = \frac{\dot{q}}{\frac{1}{2}\rho V^2 \bar{S}\bar{c} I_{yy}} \tag{21}$$

4.2 Least square estimation

As the aerodynamic model is known, lift coefficientC_L, drag coefficient C_D and pitching moment coefficient C_M are expressed as Eq. 22, Eq. 23 and Eq. 24 respectively.

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\alpha^2}}\alpha^2 + C_{L_{\alpha^3}}\alpha^3 + C_{L_{\alpha^4}}\alpha^4 + C_{L_{\delta_e}}\delta_e + C_{L_{{\delta_e}^2}}{\delta_e}^2 \tag{22}$$

$$C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\alpha^2}}\alpha^2 + C_{D_{\delta_e}}\delta_e + C_{D_{{\delta_e}^2}}{\delta_e}^2 \tag{23}$$

$$C_M = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\alpha^2}}\alpha^2 + C_{M_{\delta_e}}\delta_e + C_{M_{{\delta_e}^2}}{\delta_e}^2 \tag{24}$$

where C_L, C_D and C_M are dependent variables. αandδ_e are independent variables and C_{L_0}, C_{L_α}, $C_{L_{\alpha^2}}$, $C_{L_{\alpha^3}}$, $C_{L_{\alpha^4}}$, $C_{L_{\delta_e}}$, $C_{L_{{\delta_e}^2}}$, C_{D_0}, C_{D_α}, $C_{D_{\alpha^2}}$, $C_{D_{\delta_e}}$, $C_{D_{{\delta_e}^2}}$, C_{M_0}, C_{M_α}, $C_{M_{\alpha^2}}$, $C_{M_{\delta_e}}$, $C_{M_{{\delta_e}^2}}$ are unknown parameters.

As measurements are corrupted by noise, Eq. 22, Eq. 23 and Eq. 24 can be expressed as Eq. 25.

$$Z = X\Theta + \epsilon \tag{25}$$

where$Z$ is an array of dependent variables, X is a matrix of independent variables, Θ is an array of unknown parameters and ϵ is an array of errors due to noise in measurements.

$$Z = [Z(1) \quad Z(2) \quad \cdots \quad Z(N)]^T \tag{26}$$

$$X = \begin{bmatrix} 1 & X_1(1) & \cdots & X_n(1) \\ 1 & X_1(2) & \cdots & X_n(2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & X_1(N) & \cdots & X_n(N) \end{bmatrix} \tag{27}$$

$$\Theta = [\Theta(1) \quad \Theta(2) \quad \cdots \quad \Theta(N)]^T \tag{28}$$

$$\epsilon = [\epsilon(1) \quad \epsilon(2) \quad \cdots \quad \epsilon(N)]^T \tag{29}$$

The unknown parameters Θ are estimated by minimizing sum of squares of errors. The least square cost function $J(\Theta)$ is defined as Eq. 30.

$$J(\Theta) = \epsilon^T \epsilon = (Z - X\Theta)^T (Z - X\Theta) \tag{30}$$

Differentiating Eq. 30 with respect to Θ gives,

$$\frac{dJ(\Theta)}{d\Theta} = -Z^T X + \Theta^T X^T X \tag{31}$$

The unknown parameters are estimated using Eq. 32.

$$\Theta = (X^T X)^{-1} X^T Z \tag{32}$$

Estimated parameters are compared with true parameters in Table 4. The error in terms of percentage is mentioned below the parameters in brackets.

Table 4: Estimated parameters using least square estimation technique

Parameter	True value	Estimated parameters		
		Flight data I	Flight data II	Flight data III
C_{L_0}	0.1784	-0.6676	0.3368	0.1707
		(481.90)	(92.69)	(2.36)
C_{L_α}	2.4530	6.9386	1.3391	2.5619
		(-182.86)	(45.41)	(-4.44)
$C_{L_{\alpha^2}}$	-1.6910	-3.7067	33.0846	-2.1298
		(-119.20)	(2056.51)	(-25.95)
$C_{L_{\alpha^3}}$	29.9860	-6.7453	423.2488	27.3422
		(122.50)	(-1311.77)	(8.80)
$C_{L_{\alpha^4}}$	-49.2450	-5.5334	-60.4454	-53.0795
		(88.76)	(22.74)	(-7.79)
$C_{L_{\delta_e}}$	0.7405	0.5739	-0.9698	0.7470
		(22.50)	(230.97)	(-0.88)
$C_{L_{\delta_e^2}}$	-0.3638	0.4988	1.2140	0.1499
		(237.12)	(433.69)	(141.21)
C_{D_0}	0.0871	-0.3464	0.0647	0.0992
		(497.61)	(25.74)	(-13.92)
C_{D_α}	-0.0559	2.5371	0.2768	-0.0494
		(4636.23)	(594.82)	(-11.63)
$C_{D_{\alpha^2}}$	3.4825	-1.8942	4.3937	3.2420
		(154.39)	(-26.16)	(6.91)
$C_{D_{\delta_e}}$	0.1471	0.0853	0.4037	0.0439
		(41.98)	(-174.42)	(70.16)
$C_{D_{\delta_e^2}}$	0.2258	0.6083	0.0074	0.2778
		(-169.38)	(96.71)	(-23.02)
C_{M_0}	0.0385	-0.7006	0.0286	0.0380
		(1919.85)	(25.63)	(1.24)
C_{M_α}	-0.6000	3.7705	-0.4418	-0.5732
		(728.66)	(26.33)	(4.43)
$C_{M_{\alpha^2}}$	-1.2740	-7.3574	-0.9275	-1.2261
		(-477.49)	(27.20)	(3.76)
$C_{M_{\delta_e}}$	-0.4106	-0.2423	-0.3021	-0.3985
		(40.99)	(26.43)	(2.94)
$C_{M_{\delta_e^2}}$	0.1587	0.3088	0.1016	0.1404
		(-94.55)	(36.01)	(11.56)

Table 4 shows that the errors gradually reduce if flight data set III is used for parameter estimation. Linear least square estimation technique has provided good estimates with flight data set III. $C_{L_{\delta_e^2}}$ and $C_{D_{\delta_e}}$ have large errors in terms of percentage. Measurements are regenerated for flight data set III using these estimated coefficients.

A cost function J is defined to know the performance of least square estimation technique. It is defined such that it shows mean absolute error for any measured quantity. It is defined as Eq. 33.

$$J = \frac{\sqrt{\sum_{i=1}^{N}\left(Z_{m.i}-Y_{m,i}\right)^2}}{N} \tag{33}$$

where $Z_{m,i}$ is measured quantity at discrete time i, $Y_{m,i}$ is estimated quantity at discrete time i and N is number of total samples of measurements. The values of cost function for measurements are shown in Table 5.

Table 5: Cost functions for different flight data sets

Quantity	Data set I	Data set II	Data set III
V (ms^{-1})	10.44	0.29	0.04
α (deg)	1.18	0.08	0.02
θ (deg)	8.98×10^5	6.25×10^4	1.02
q ($rads^{-1}$)	644.49	14.58	0.60×10^{-5}
a_x (ms^{-1})	3.71	47.25	0.16
a_z (ms^{-1})	4.94	129.28	0.05

Table 5 shows that the estimated parameters from flight data set III using least square are accurate. Fig. 3 shows that errors between regenerated states and measurements using estimated parameters have very less errors.

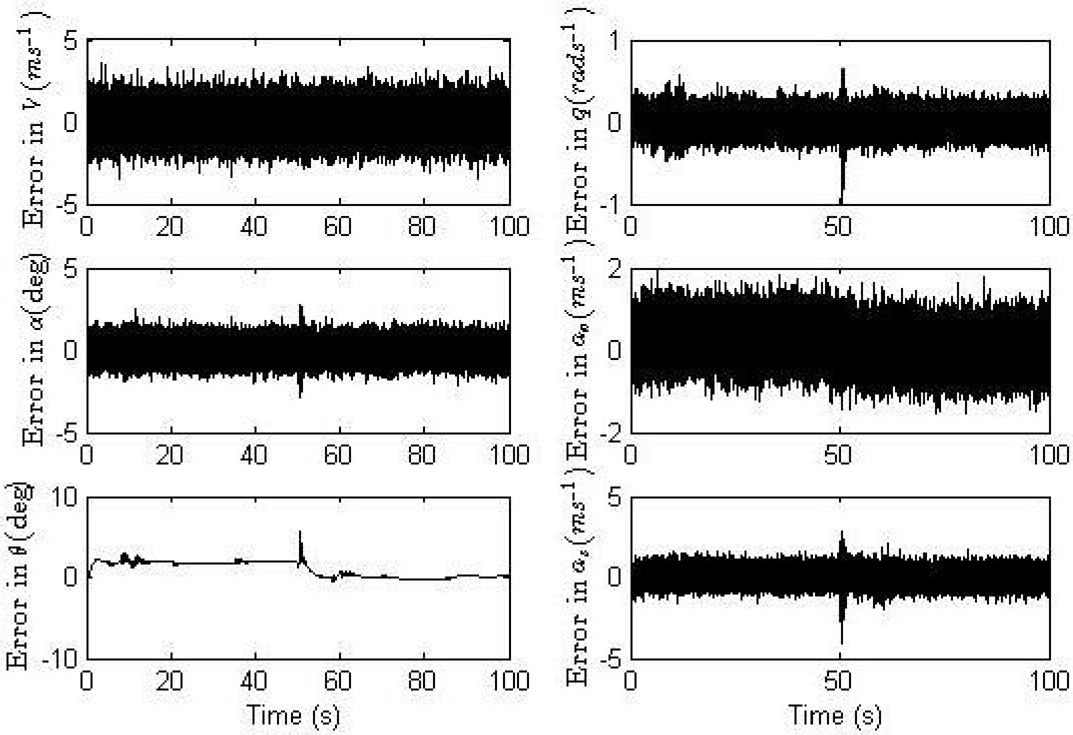

Figure 3: Errors between measured and estimated quantities

5 CONCLUSION

The nonlinear aerodynamic model of Black-Kite MAV is used to generate longitudinal flight data. Three different flight data sets are generated for different trim conditions. Artificial noise is added to make it similar to real measured flight data. Least square estimation technique is used to estimate unknown parameters.

It is observed that among three different flight data sets, the flight data set III which has multiple trim conditions provided best parameter estimates. Covering whole flight regime and perturbing aircraft such that more frequencies get excited are important aspects for flight tests. Least square estimation can estimate parameters accurately if multiple trim flight data is used. It can provide high accuracy initial guesses of parameters for maximum likelihood, Kalman filter based or heuristic methods.

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REFERENCES

[1] Wook-Je Park, Eung-Tai Kim, Kie-JeongSeong; 2006; "A study on the Parameter Estimation of DURUMI-II for the Fixed Right Elevator using Flight Test Data"; *Journal of Mechanical Science and Technology*; **20**(8); pp. 1224-1231

[2] Andrei Dorobantu, Austin M. Murch, Bernie Mettler, Gary J. Balas; 2011; "Frequency Domain System Identification for a Small, Low-Cost, Fixed-Wing UAV"; *AIAA Guidance, Navigation and Control Conference*; Portland, Oregon

[3] Willem H. Pietersen; 2010; "System Identification for Fault Tolerant Control of Unmanned Aerial Vehicles"; *M. S. Thesis*; Stellenbosch University, Stellenbosch, South Africa

[4] Russell J. Gates; 1995; "A comparison of flight input techniques for parameter estimation of highly augmented aircraft"; *M. S. Thesis*; Naval Postgraduate School, Monterey, California

[5] Eugene A. Morelli; 2012; "Flight Test Manoeuvres for Efficient Aerodynamic Modelling"; *Journal of Aircraft*; **49**(6);

[6] Stephen Carnduff; 2008; "System Identification of Unmanned Aerial Vehicles"; *Ph. D. Thesis*; Cranfield University, Cranfield, United Kingdom

[7] RavindraJategaonkar; 2006; *Flight Vehicle System Identification: A Time Domain Methodology*; **216**; AIAA Progress in Aeronautics and Astronautics, AIAA, Reston, Virginia

[8] Aero MAV; 2016; "Black-kite MAV figure";http://aeromav.free.fr/MAV08/team/NAL/black.jpg