Enhanced Kinematics Calculation for an Online Trajectory Generation Module

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ABSTRACT

This paper presents an extension to a previously published method by the authors, which implemented clothoids within an integrated flight guidance and control system with independent speed control. This method was then used in real flight tests of a highly-automated CS-23 aircraft. The method provided a steady entry and exit manoeuvre to a turn. Now, the aim of this paper is to show an enhancement of the previously published algorithm in the context of the trajectory reference point kinematics calculation for the clothoid manoeuvre. Nonetheless, the proposed methodology remains applicable for a variety of other trajectory curves (e.g. splines). The improved reference point calculation yields a smoother command for the trajectory controller, which uses 2nd order error dynamics. To illustrate the enhancements by the proposed algorithm, exhibits from a high-fidelity simulation framework of the CS-23 aircraft are depicted. Additionally, results of a flight test with the CS-23 aircraft are shown, discussed, and related to the previously published results. Furthermore, a worst-case approximation, in a Monte-Carlo like approach, of the maximal projection error for the proposed method is presented.

KEYWORDS: *trajectory generation, flight path following, clothoids, orthogonal projection, waypoint flying*

NOMENCLATURE

Within this paper, the following nomenclature is generally applied: vectors are in bold font and Cartesian vectors are additionally depicted by an arrow. Scalars and indices are in normal font. The different indices, abbreviations, and symbols are illustrated in the following listing:

Vectors:
$(\vec{\boldsymbol{r}}^{a})_c$: location vector from the origin of the inertial coordinate frame to the point a, with coordinates given in the frame c
$(\vec{\boldsymbol{r}}^{ab})_c$: location vector from the point a to the point b, with coordinates given in the frame c
Abbreviations:
T: trajectory frame, i.e. frame for which x-axis is tangential to the current trajectory, the y-axis points right of the trajectory, and the z-axis down (right-handed frame)
ref: reference point
ac: aircraft position
tang: tangential vector
cur: current quantity
prev: previous quantity
init: initial quantity
cc: circle centre
Symbols:
τ: clothoid reference parameter (non-dimensional time)
s: clothoid arc length
κ: curvature
r_{circle}: circle radius
μ_{cmd}: commanded / desired bank angle
α_T: angle between two flight plan legs
χ_T: trajectory course angle
A: clothoid shape parameter
φ: clothoid manoeuvre angle
r: radius
Δx: increment by projection / projection error

1 INTRODUCTION

Just recently, the authors published a paper concerned with the online trajectory generation using clothoids for the entry and exit manoeuvre of a turn [1]. The method itself described an enhancement to another published methodology, which simply combined straight lines with arcs [2]. Here, the main issues were curvature steps at the transition points between the straight lines and the arcs, which led to a poor performance of the trajectory controller, which uses 2nd order error dynamics and therefore, requires smooth higher order derivatives of the trajectory [3].
Thus, [1] implemented clothoids within the trajectory generation submodule, which enforce a linear curvature change that is more suitable for the trajectory controller. For more information on clothoids the interested reader is referred to [4–6]. Note that for the previously published method [1], the kinematic reference point calculation was still not satisfying in the sense of accuracy of the approximated orthogonality relation of the projection algorithm. This will be overcome by this paper.
The kinematic reference point calculation by projection is necessary, because the trajectory generation module is part of an integrated mission management system with independent speed control on-board a CS-23 aircraft [7] (p. 146ff.). Thus, the trajectory, and therefore, the trajectory reference point, cannot be defined with respect to the global flight time, but must be calculated based on the current aircraft position. The structure of the integrated mission management system on-board the CS-23 aircraft is depicted by Fig. 1. Here, the different modules of the mission management system are displayed and their relations and interactions are visible.
This paper aims at the enhancement of the previously proposed model for the reference point calculation. Therefore, the differences between the two approaches will be stated at first: the previous approach relied on the projection on the connection line between the clothoid start and end point. Now, the new approach is based on a local linearization of the clothoid. The two approaches are then compared in a simulative assessment of a high-fidelity simulation of the CS-23 aircraft. Additionally, we look at the projection error, for different aircraft positions during the clothoid manoeuvre, in a Monte-Carlo like approach. This will give an insight into the maximal and minimal errors of the proposed method. Finally, the presented strategy is tested in flight tests conducted within the CS-23 aircraft.
It should be noted that the proposed method is depicted for clothoids only in this paper, but is also more generally applicable, e.g. to splines [5, 8].

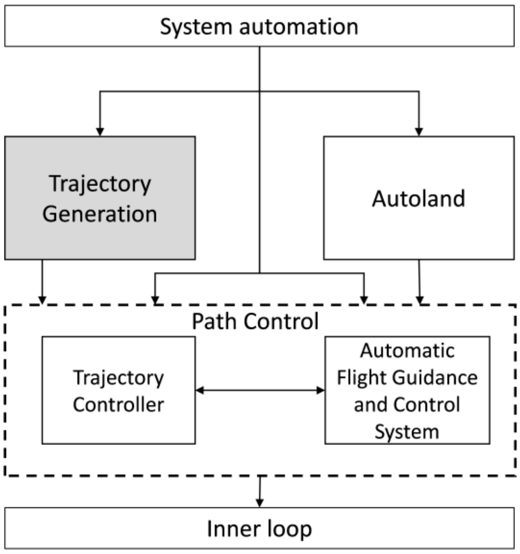

Figure 1: Structure of integrated mission management system of the CS-23 aircraft [2].

The paper is organized as follows: section 2 gives an overview on the theoretical aspects of the implementation: at first, the previously published method will be briefly recapitulated and after that, the proposed method is introduced. Then, section 3 depicts the simulative comparison of the two proposed methodologies in the high-fidelity simulation framework. Section 4 looks at the Monte-Carlo-like assessment of the maximal projection error. The flight tests will be considered in section 5. These will show the applicability of the developed algorithms in a real test flight environment. Some conclusive remarks and further development opportunities are given in section 6.

2 THEORETICAL ASPECTS

Just recently, the authors proposed a method to incorporate clothoids in an integrated flight guidance and control system with a second order error controller and independent speed control. By this a steady entry and exit manoeuvre to a turn was modelled (given by radius r_{circle} and center $\vec{r}^{cc}$; Fig. 2) [1]. This approach is said to be the *current* method in the following. A clothoid itself is defined by [1, 4, 5]:

$$\vec{x}_{cl}(\tau) = \begin{bmatrix} A \cdot \int_0^\tau \cos(k^2)\,dk \\ A \cdot \int_0^\tau \sin(k^2)\,dk \end{bmatrix} \tag{1}$$

Here, τ is a curve parameter, interpreted to be a non-dimensional time, and A is the shape parameter, which basically relates to the speed of the curvature increase.
The independent speed control creates the necessity to calculate the reference point on the trajectory, and thus, also on the clothoid of Eq. 1, based on the current aircraft position and not based on a global time. On the other hand, the second order error controller [3] requires the trajectory command values to be smooth up until second order. Additionally, as the applied methods should be developed under certification aspects and hence, as simple as possible, the published methodology [1] was designed to be only based on trigonometric relations. These demands yield, especially for the reference point calculation on the clothoid, which is based on the current aircraft position, to only an approximate solution of the exact projected position. The exact projected position is given by the orthogonality relation in Eq. 2:

$$(\vec{r}^{refac})_{Tref} \circ \left(\vec{r}_{tang}^{ref}\right)_{Tref} \equiv 0 \tag{2}$$

This just means that the dot product between the connection line from the reference point on the clothoid to the current aircraft position, $(\vec{r}^{refac})_{Tref}$, given in the trajectory reference frame, $Tref$, i.e. the trajectory frame at the reference point, must be orthogonal to the tangent vector at the current reference point, $\left(\vec{r}_{tang}^{ref}\right)_{Tref}$. Note that Eq. 2 cannot be solved analytically, for e.g. the non-dimensional time, as the definition of the clothoid is nonlinear and given by a transcendent integral (Eq. 1). Thus, iterative solutions would be required that have no deterministic convergence times and are, therefore, not suitable for a certifiable algorithm.
To overcome this, the method proposed in [1] was a simple projection on the connection line between the starting *(clp1)* and the ending point of the clothoid *(clp2)*, which itself was determined by analytic relations (using auxiliary point, $\vec{r}^{ap}$, and transient manoeuvre turn angle φ_{trans}, which is analogous to the transient non-dimensional time τ_{trans}). The procedure is depicted in Fig. 2. The aircraft *(ac)* position is projected on the connection line *(ref*)* to get the trajectory reference point *(ref)*. Here, the distance of the projected point to the starting point is interpreted as the arc length *(s)* of the clothoid, which directly yields the reference point position on the clothoid [1].
This method exhibited quite a large error in the orthogonality condition (Eq. 2) that is normally present for the projection of the current aircraft position on the trajectory (e.g. for straight line and arc) [2].
Thus, this paper proposes a method that uses a local linearization approach around the previously *(prev)* calculated trajectory reference point, which is called the *proposed* method in the following. It is evident from Fig. 3 that the method uses the tangent of the last trajectory reference point for the projection. Thus, the projection line is adapted while the aircraft flies the clothoid manoeuvre rather than in the previously published approach, where it is fixed during the complete manoeuvre.
The distance between the last point and the projected *(proj)* point, $\Delta x_T = (x^{refac})_T$, is interpreted as a change in the arc length on the clothoid (Δs). This directly yields an approximation of the new trajectory reference point via the non-dimensional time.
The proposed method provides a significant decrease in the orthogonality error as it will be examined in sections 3 and 4, which increases the overall performance of e.g. the trajectory error controller.

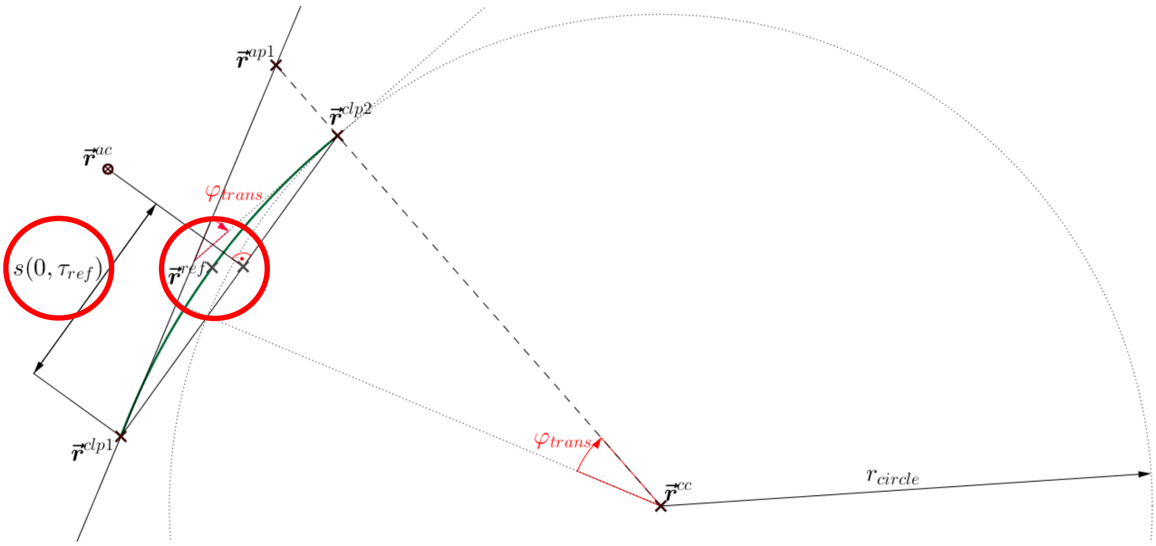

Figure 2: Current procedure for reference point calculation by projection on connection line between clothoid starting and ending point [1].

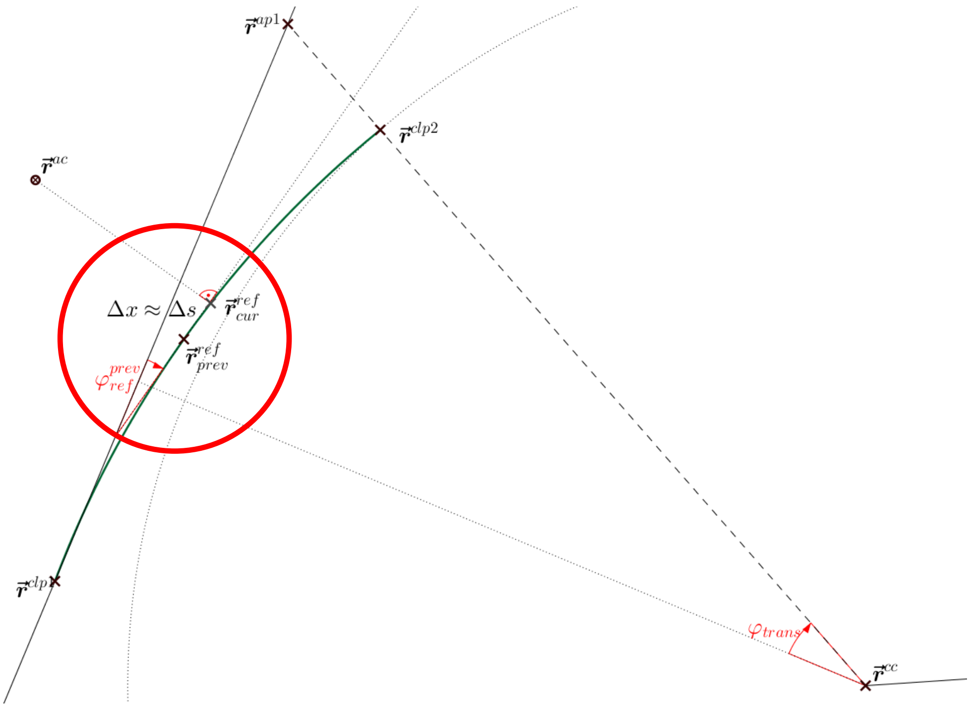

Figure 3: Proposed procedure for reference point calculation by projection on tangent of the previous clothoid reference point.

Now, the following equations, Eqs. 3-6, give a brief review of the mathematical formulation of the proposed approach: at first, we start with the definition of the non-dimensional time increment, $\Delta\tau$, from the projected increment in the arc length, Δs, which are directly proportional [1]:

$$\Delta\tau = \frac{\Delta s}{A} \approx \frac{\Delta x_T}{A} \tag{3}$$

The constant of proportionality is the inverse of the previously introduced clothoid shape parameter, A. We can use this increment to approximate the current position on the clothoid by our last known position. Therefore, we use the non-dimensional time as the propagation variable (the initial value is zero as we are starting the clothoid manoeuvre close to the clothoid starting point):

$$\tau_{ref} = \tau_{prev} + \Delta\tau; \quad \tau_{init} = 0 \tag{4}$$

As the non-dimensional time is directly linked to the clothoid position (Eq. 1), we consequently get the approximation of the current aircraft reference point based on the known clothoid starting point [1], $clp1$, and the clothoid definition of Eq. 1:

$$(\vec{r}^{ref})_{Tref} = (\vec{r}^{clp1})_{Tref} + \begin{bmatrix} A \cdot \int_0^{\tau_{ref}} \cos(k^2)\, dk \\ A \cdot \int_0^{\tau_{ref}} \sin(k^2)\, dk \end{bmatrix}_{Tref} \tag{5}$$

Note that the trajectory reference frame, $Tref$, at the reference point, is known, because the non-dimensional time is directly linked to an angle increment by the square of itself [1]. Furthermore, the computational burden to evaluate the integral in Eq. 5 (or Eq. 1) can be relaxed by expanding the integral into a power series [1, 4].

The projection distance of Eq. 3 is calculated by:

$$\Delta x_T = (x^{refac})_{Tprev} = (x^{ac})_{Tprev} - (x^{ref}_{prev})_{Tprev} \tag{6}$$

Eq. 6 just states that we simply must consider the aircraft movement along the tangent of the previous trajectory point to update the previous reference position. The reference frame here is the trajectory frame at the previous point, $Tprev$.

Overall, Eqs. 3-6 provide us with a simple analytic framework, just as with the method of [1], which can be easily incorporated in the current mission management system [9]. This will be shown for the high-fidelity simulation framework in section 3 and for the real flight test in section 5.

3 SIMULATION RESULTS

Within this section, the authors present exhibits from a high-fidelity simulation framework for standard waypoint flight. The track consists of twelve waypoints close to *LOAN (Wiener Neustadt East Airport)*, in the area where also the real flight tests were conducted [10]. The results show an exhibit of fly-by manoeuvres conducted during the flight plan, which is itself a specifically tailored flight plan for testing the trajectory generation module rather than for real flight tests. In the following, the "current" method (solid blue line) always relates to the method published in [1], while the "proposed" method (dashed black line) is the method introduced in section 2 of this work.
Fig. 4 depicts the horizontal position deviation in x-direction (left plot) and y-direction (right plot) for the entry and exit manoeuvre of a fly-by. The x-direction is equivalent to the orthogonality relation, which should be enforced (Eq. 2). Thus, a value of zero must be the goal to have an exact orthogonal projection. It is evident that the proposed method has a much smaller projection error than the current method. The current method has projection errors up to $1m$, while the proposed method has errors, which are far below $0.05m$. This error is based on the linear, i.e. first order approximation and naturally also increases with a larger curvature, i.e. at the end of the entry manoeuvre. This can also be seen in the zoomed-in portion of the left plot (here, the end of the fly-by entry is shown). Nonetheless, the proposed method shows a significant increase in case of the accuracy of the reference point calculation. Additionally, it can be observed in the right plot, where the y-direction, i.e. the distance to the design trajectory is depicted: By only using a better orthogonal projection and therefore, smoother trajectory, the lateral directional error can be reduced (zoomed-in portion). This smoother trajectory command now yields the possibility to use larger controller gains and effectively follow the trajectory even better in the real flight test of section 5.
After the simulative assessment, we now concentrate on the worst-case calculation in a Monte-Carlo-like assessment.

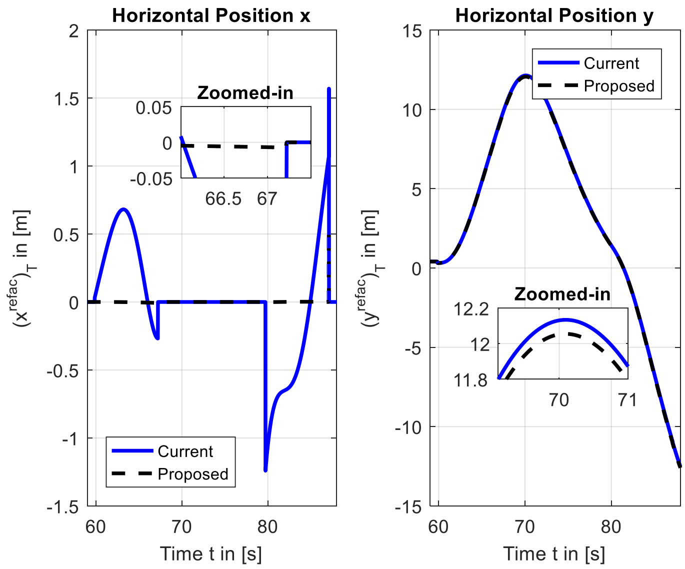

Figure 4: Comparison of horizontal position deviations of current and proposed method.

4 WORST-CASE CALCULATION – MAXIMAL PROJECTION ERROR

This section gives an overview on the test results to obtain the maximal projection error of the two methods and compare these. These results are obtained for a reference clothoid manoeuvre given by the shape parameter $A = 717.7m$ and the dimensionless time $\tau_{trans} = 0.59$. The results are depicted by Fig. 5 and Fig. 6. Here, Fig. 5 illustrates at the errors obtained from the previous, auxiliary connection line, approach, while Fig. 6 presents the results for the approach proposed in this work. The error is always calculated with respect to the exact solution of Eq. 2, which was derived using a Newton iteration scheme. Thus, the error manifests itself in the arc length, which is analogous to an error in the non-dimensional time.

At first, the maximal projection error is calculated for three scenarios, which are special cases of the worst-case flight paths the aircraft will fly following the flight plan. These are: The aircraft is flying on a straight line (dashed-dotted blue), i.e. it is staying on the leg. Additionally, the aircraft is perfectly following the clothoid (solid red), and finally, the aircraft is following the auxiliary connection line (dashed black). The auxiliary connection line, i.e. the current approach, always has a larger error than the local linearization, i.e. the proposed approach. This can be especially seen, when the aircraft follows the clothoid exactly (solid line): While the local linearization retains an error in the order of $10^{-5}m$, the auxiliary connection line creates an error of up to $2m$. This undesired behaviour can also be observed for the other two test cases that show the superiority of the proposed approach compared to the previous approach. Additionally, we can observe from Fig. 5 that the calculated error is changing sign, i.e. the reference point is changing from a lagging position behind the real reference point into a leading position. This behaviour is also highly undesired when following the reference point.

After the assessment of the three reference cases to compare the current and the proposed approach (Fig. 5 and Fig. 6), Fig. 7 shows a Monte-Carlo like assessment of the proposed method. Here, the aircraft follows a grid of straight line trajectories and on each of the grid points the error in the reference point approximation is calculated. The solid black line is the reference clothoid trajectory, which is defined as before (Fig. 5 and Fig. 6). We can observe the behaviour that is to be expected from the assessment of the previous paragraphs: Within a close perimeter around the reference trajectory, the projection error is small. If the aircraft deviates significantly from the reference clothoid manoeuvre the error grows rather rapidly. As these flight paths are normally not present within a controlled aircraft, these large errors are not present in real flight situations. Therefore, also the Monte-Carlo-like assessment shows the validity of the proposed approach in the sense of an enhanced reference point approximation.

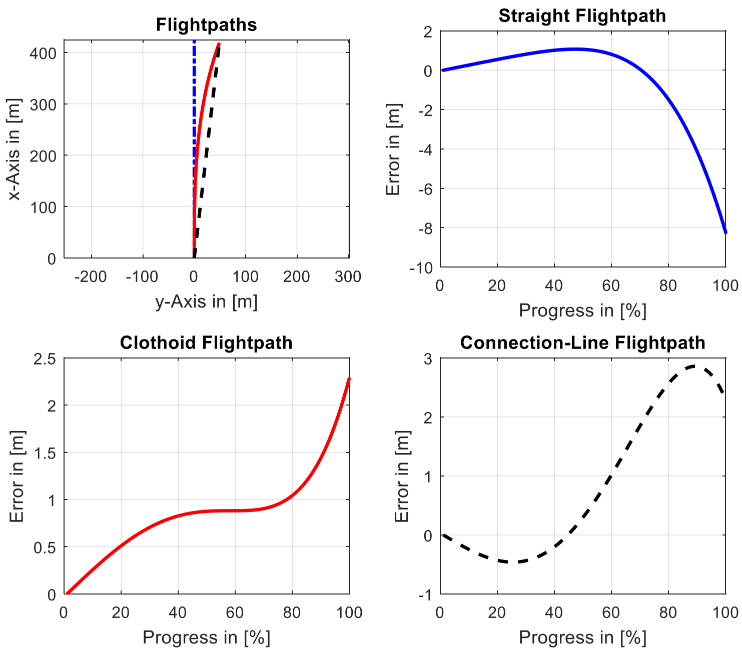

Figure 5: Error in reference point projection of connection line for straight line (dashed-dot blue), clothoid (solid red), and connection line (dashed black) flight of the aircraft.

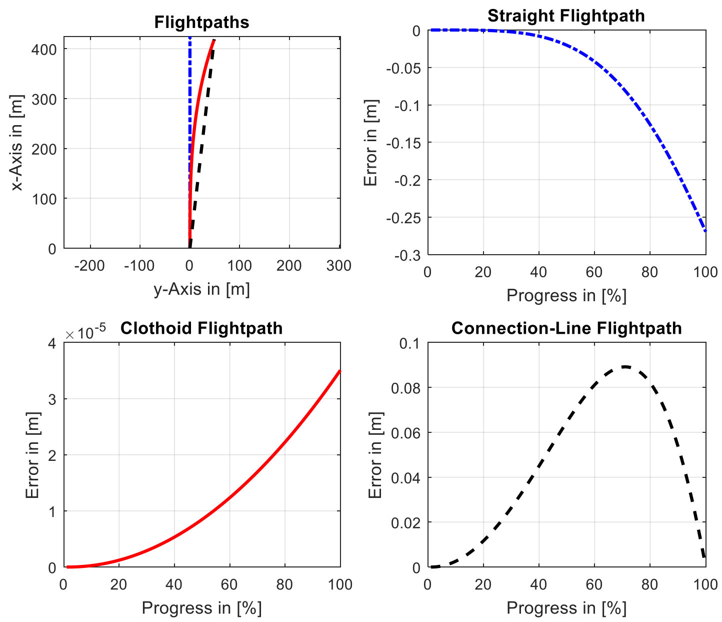

Figure 6: Error in reference point projection of local tangent for straight line (dashed-dot blue), clothoid (solid red), and connection line (dashed black) flight of the aircraft.

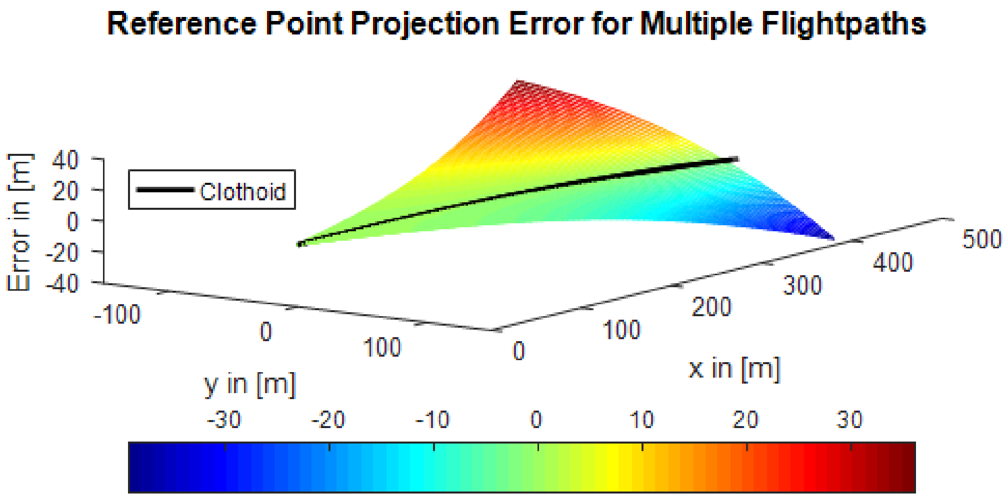

Figure 7: Error in reference point projection for multiple straight line flight paths from the clothoid start around a reference clothoid.

5 FLIGHT TEST RESULTS

This section gives an overview of the flight test results of the CS-23 aircraft. Note that the flight plan of the flight tests is different from the simulation test in section 3 and therefore, the results cannot be compared directly. The aircraft flies the flight path as depicted by Fig. 8. Here, the dashed red part is a Radius To Fix manoeuvre, which uses the reference point calculation for the clothoid of this paper for turning in and out. The projection error for this manoeuvre is illustrated in Fig. 9. It is evident that we can reproduce the results of Fig. 4 in the sense that we have a maximal error, which is approximately $1mm$. Additionally, we can see that the development of the error is very similar. Thus, the flight tests show that the proposed method also achieves good tracking that was seen in simulation within the real flight test even with disturbances. Overall, the flight tests show the applicability of the proposed algorithm to real applications.

6 CONCLUSIONS AND PERSPECTIVE

This paper presented an enhancement of a previously published approach by the authors on the reference point calculation for a clothoid manoeuvre within an integrated mission management system with independent speed control of a CS-23 aircraft. The extension of the previously published method was based on a further enhancement of the reference point calculation, which is based on the reference aircraft position during the clothoid manoeuvre. This was necessary to enhance the performance of the trajectory controller with second order error dynamics.
Here, the proposed projection algorithm must approximate the exact solution, given by an orthogonal projection, as accurate as possible. The proposed methodology was based on a local linearization around the previous reference point. Thus, the method does always project the current aircraft position on the local tangent, i.e. local first order expansion, of the clothoid.
The method presented in this paper showed significantly better results than the previously published method, in the sense of reproducing an accurate orthogonal projection by simple analytic relations. We could observe a decrease in the orthogonality error of about maximal $1m$ for the old method to below $0.05m$ for the new method. This originates from the fact that the previously published method just relied on the projection onto a constant line during the manoeuvre, while the proposed approach, as stated before, is based on a local linearization and therefore, a varying line over the manoeuvre. This naturally increases the accuracy of the method.

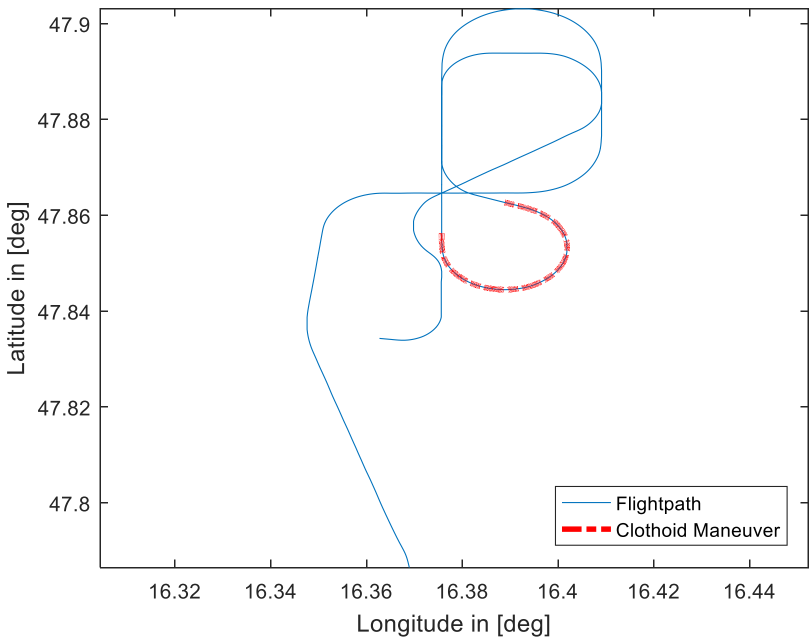

Figure 8: Aircraft flight path with marked clothoid manoeuvre.

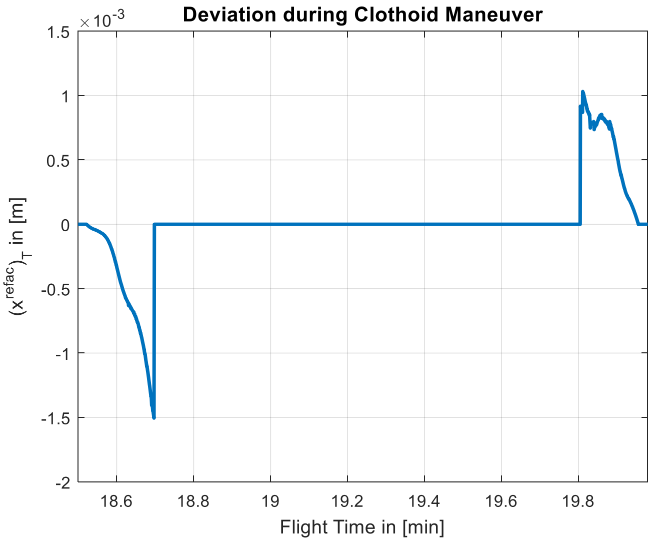

Figure 9: Horizontal position deviation during the radius–to–fix clothoid manoeuvre.

For further enhancement of the method, higher order trajectory functions, i.e. functions that are smoother up until a higher order than clothoids, should be considered. An example are splines. These will produce even smoother trajectories. Still, the methods for the reference point calculation, introduced in this paper, can be applied as they are mainly related to calculating a local linearization of the function, which is normally rather simple for smooth functions.
To get an even better approximation of the reference point kinematics, two further strategies could be tested: the first option would be to solve Eq. 2, i.e. the exact orthogonality relation, for a fixed number of times iteratively, e.g. by the *Newton-Raphson method*. Then, there would still be a deterministic convergence time, while we would get a significant improvement in the solution. Additionally, this approach would be a solution which secures convergence to the desired orthogonal solution. As we also always have a good initial guess for e.g. a *Newton-Raphson algorithm* from the previous reference point, we will reach a very fast convergence, i.e. a convergence within only few iterations.

Another option would be to use a foot-point propagation algorithm as proposed in [11]. Here, an error controller augments the foot-point propagation algorithm. By this, we again have a secured convergence in the sense of linear controller theory, while we still conserve the deterministic model structure. Again, these methods would be applicable to arbitrary trajectory functions.

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