THE RELATIVE MOTION OF A SPACECRAFT NEAR A GEOSTATIONARY POSITION

Roxana Alexandra Petre
University "Politehnica" of Bucharest
PhD student
313 Splaiul Independenţei, Bucharest 060042, Romania
petre.roxana.alexandra@gmail.com

Ion Stroe
University "Politehnica" of Bucharest
Professor
ion.stroe@gmail.com

Andrei Craifaleanu
University "Politehnica" of Bucharest
Professor
ycraif@yahoo.com

Thien Van Nguyen
University "Politehnica" of Bucharest
PhD student
bangden33468@gmail.com

ABSTRACT

In this paper the relative motion of a vehicle in the neighborhood of a geostationary orbit is studied. The vehicle is a satellite equipped with a propulsion system capable of providing the necessary force for pulse orbital transfer, so that the satellite remains in the desired region. Different numbers of necessary maneuvers applied to the satellite were considered and compared, in order to determine the most advantageous situation. Also considering that the fuel consumption is directly proportional to the developed pulse, it results that a lower number of maneuvers is more advantageous for the fuel consumption.

KEYWORDS: *relative motion, satellite, latitude, longitude, pulse.*

NOMENCLATURE

A - fixed point in space
B - point on the equatorial line above which the satellite is at a certain moment of time
i - inclination of orbit
M_o- satellite
n - number of maneuvers
R - radius of the Earth
θ - position angle of the satellite on the inclined orbit
γ - longitude difference between the satellite and the station
φ - latitude of the satellite
λ - longitude
α - angle between the velocity direction and the local parallel
v - total speed
Δp - pulse
Δp_t - total pulse
m - mass of satellite
n- number of maneuvers

1 INTRODUCTION

Numerous technical applications, such as radio communications and TV programs broadcasting, are facilitated by the use of geostationary satellites, since such bodies orbiting in the equatorial plane are in apparent rest with respect to the ground. But the increasing use of this type of space objects has led to the congestion of the circumterrestrial zone in which the geostationary motion is possible.

A solution to this problem is the flight in geostationary orbits parallel to the equatorial plane. However, a satellite in free motion in a non-equatorial plane is moving with respect to the Earth [1]. As a result, maintaining the geostationary motion at non-zero latitudes requires the application of a continuous traction in meridian plane [2], which involves continuous fuel consumption. In reference [5] the possibility of using space sails in order to produce this traction is considered.

In this paper another strategy is proposed, which consists in maintaining a satellite near a small latitude geostationary orbit by following a path composed of arcs on which the motion takes place freely, the transition from one arc to the other being carried out with pulse traction [2], [3] (Fig. 1). The study is performed for a number n of maneuvers. By pulse orbital transfer, the shape and orientation of the satellite orbit is changed.

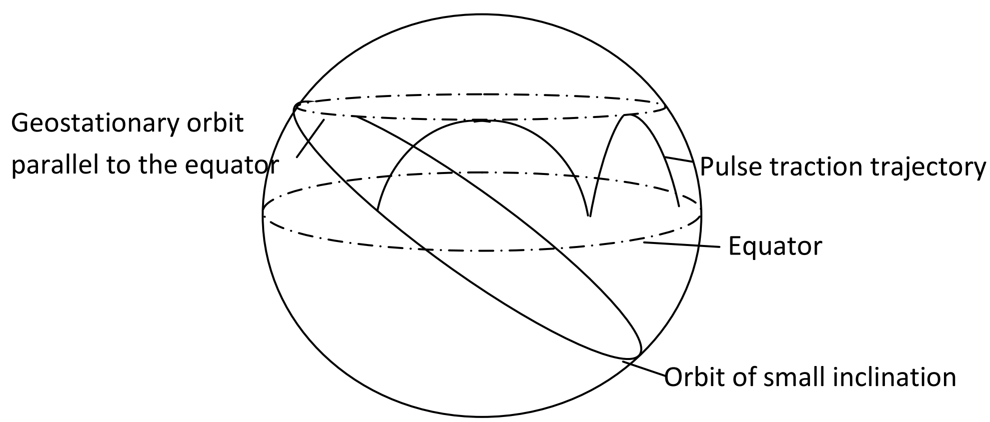

Figure 1: The pulse traction trajectory of the satellite

2 APPARENT MOTION OF THE SYNCHRONOUS SATELLITE ON AN INCLINED ORBIT

It is considered that the satellite orbit is sincronous and inclined with a certain angle. The motion takes palce in a central gravitational field, with the asumption that the satellite is a material point upon which there are no influences from aerodynamic forces. The radiation pressure is neglected.
The trajectory of the sattelite is described in Fig. 2.

By studying triangle ABM_o in Fig. 2, the following relations can be obtained [1]:

$$\begin{cases} \sin\theta = \dfrac{\sin\varphi}{\sin i} \\ \cos\theta = \cos\varphi\cos\left(\theta - \gamma\right) \end{cases} \qquad (1)$$

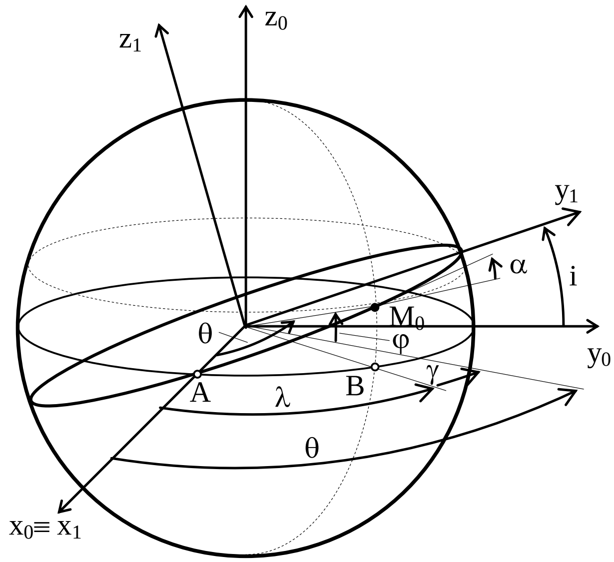

Figure 2: Apparent motion of satellite

The necessary calculations can be performed by studying only the first quadrant represented in Fig. 2, due to symmetry reasons. Thus, by eliminating θ from Eq.1, γ is obtained:

$$\gamma = \theta - \arccos\frac{\cos\theta}{\cos\varphi} = \theta - \arccos\frac{\sqrt{1-\frac{\sin^2\varphi}{\sin^2 i}}}{\cos\varphi} \tag{2}$$

or

$$\gamma = \arccos\sqrt{1-\frac{\sin^2\varphi}{\sin^2 i}} - \arccos\frac{\sqrt{1-\frac{\sin^2\varphi}{\sin^2 i}}}{\cos\varphi} \quad . \tag{3}$$

The above equation can be rewritten as

$$\gamma = \arccos\frac{1}{\cos\varphi}\left(1-\frac{1-\cos i}{\sin^2 i}\sin^2\varphi\right) \tag{4}$$

and noting

$$K = \frac{1-\cos i}{\sin^2 i} = \frac{1}{2\cos^2\frac{i}{2}} \quad , \tag{5}$$

the following relation is obtained [1]:

$$\cos\gamma = \frac{1-K\sin^2\varphi}{\cos\varphi} \quad . \tag{6}$$

The above relation describes the dependency between φ and γ.

3 PULSE ORBITAL TRANSFER

In order to apply a number n of pulse orbital transfer maneuvers, so that the satellite remains in the geostationary region, the angle α must be calculated (Fig. 2). This angle determines the orientation of the pulse that has to be applied to the satellite.

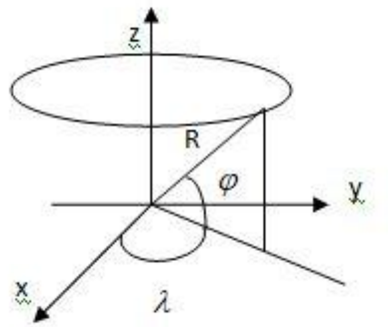

Figure 3: Coordinates of the satellite

The following relations were determined using the notations in Fig. 2 and Fig. 3:

$$\begin{cases} x = R\cos\varphi\cos\lambda \\ y = R\cos\varphi\sin\lambda \\ z = R\sin\varphi \end{cases}, \tag{7}$$

where $\varphi = const$.

The tangent vector to the local parallel and its components with respect to reference system $x_0y_0z_0$ are:

$$\bar{u}: \begin{cases} \dot{x} = -R\dot{\lambda}\cos\varphi\sin\lambda \\ \dot{y} = R\dot{\lambda}\cos\varphi\cos\lambda \\ \dot{z} = 0 \end{cases}. \tag{8}$$

The parametric equations of the inclined orbit with respect to reference system $x_1y_1z_1$ are:

$$\begin{cases} x_1 = R\cos\theta \\ y_1 = R\sin\theta \\ z_1 = 0 \end{cases}. \tag{9}$$

The tangent vector to the inclined orbit and its components with respect to reference system $x_1y_1z_1$ are:

$$\bar{v}: \begin{cases} \dot{x}_1 = -R\dot{\theta}\sin\theta \\ \dot{y}_1 = R\dot{\theta}\cos\theta \\ \dot{z}_1 = 0 \end{cases}. \tag{10}$$

The transformation formula of the components of an arbitrary vector $\bar{x}$ with respect to the reference system $x_1y_1z_1$ to reference system $x_0y_0z_0$ is:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_0 = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{Bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_1. \tag{11}$$

By combining Eq.10 and Eq.11, the following expression is obtained for the components of vector $\bar{v}$ with respect to reference system $x_0y_0z_0$:

$$\bar{v}:\begin{Bmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{z}_1\end{Bmatrix}_0=\begin{Bmatrix}-R\dot{\theta}\sin\theta\\ R\dot{\theta}\cos\theta\cos i\\ -R\dot{\theta}\cos\theta\sin i\end{Bmatrix} . \tag{12}$$

The angle α is obtained:

$$\cos\alpha=\frac{\bar{u}\cdot\bar{v}}{|\bar{u}|\cdot|\bar{v}|}=\sin\theta\sin\lambda+\cos\theta\cos\lambda\cos i \quad . \tag{13}$$

By replacing Eq.1 in Eq.13, the following form is obtained

$$\cos\alpha=\frac{\sin(\theta-\gamma)\sin\varphi}{\sin i}+\cos^2(\theta-\gamma)\cos\gamma\cos i \quad . \tag{14}$$

The unitary pulse, i.e. the pulse divided by the mass of the satellite, is

$$\frac{\Delta p_t}{m}=2v\sin\alpha \quad . \tag{15}$$

The total pulse is

$$\Delta p_t=n\Delta p \quad . \tag{16}$$

4 NUMERICAL APPLICATION

A Matlab program was used in order to obtain the needed numerical results, and the dependency of one date upon the other, in order to deduce the optimum solution.

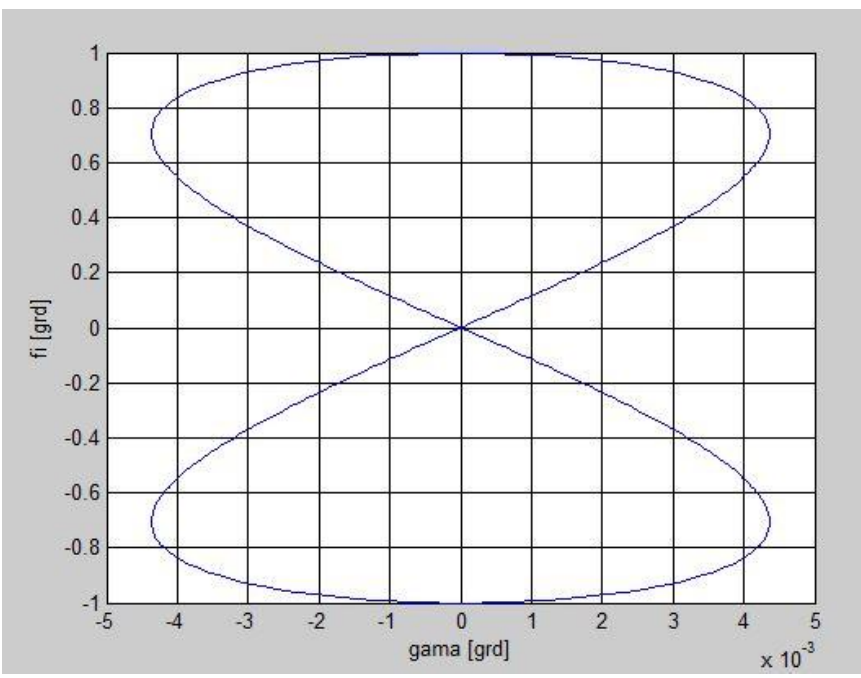

Figure 4: The apparent motion of the satellite at incidence 1º

Fig. 4 shows the dependence between φ and γ when the orbit has a small inclination of 1º. Since the distances covered by the satellite are proportional with φ and γ, the satellite performs an 8-shaped apparent motion on the celestial sphere.

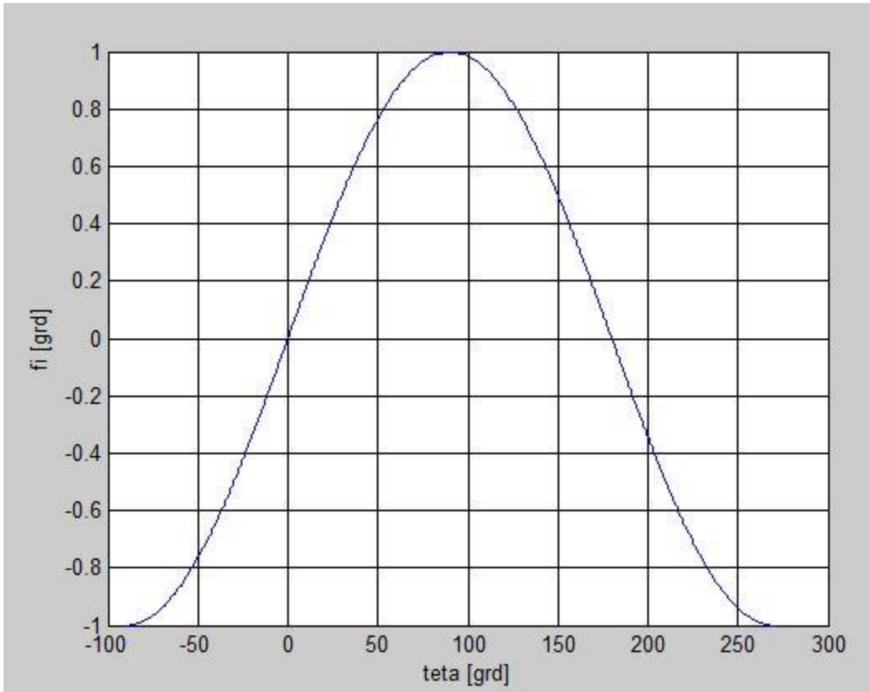

Figure 5: The variation of φ with respect to θ

Fig. 5 describes the variation of φ with respect to θ, i.e. the variation of the angular coordinates of the satellite with respect to the ground.

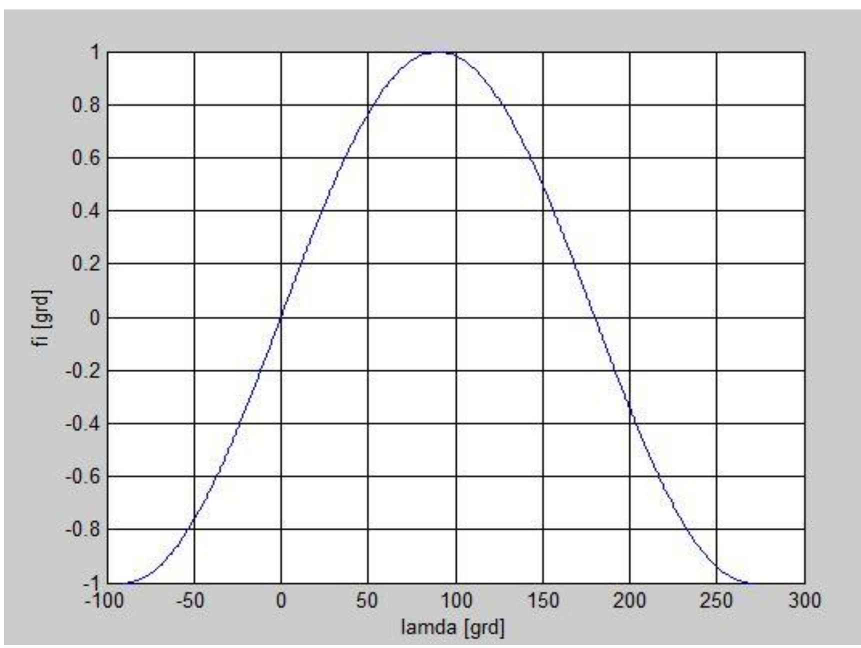

Figure 6: The variation of φ with respect to λ

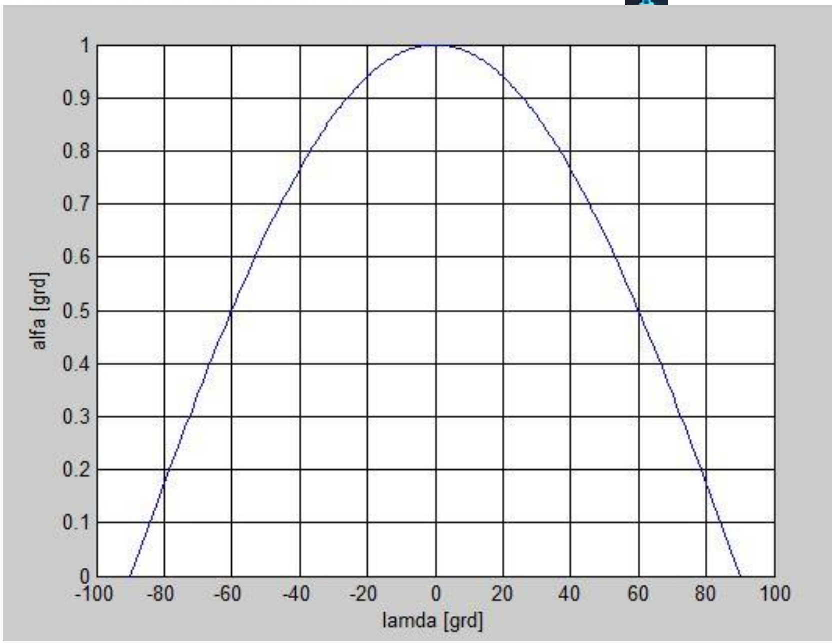

Figure 7: The variation of angle α with respect to λ

The numerical values obtained for various parameters defining the motion of the satellite are shown in Table 1.

Table 1: numerical values obtained in Matlab

n	φ (degree)	θ (degree)	λ (degree)	α (degree)	v (m/s)	$\frac{\Delta p_t}{m}$ (m/s)
3	-0.5	-29.9962	120	0.8660	92.8611	278.5833
4	0	0	90	1	107.2227	428.8907
6	0.5	30.0038	60	0.8660	92.8576	557.1453
12	0.8660	60.0038	30	0.5	53.6113	643.34
24	0.9659	75.002	15	0.2588	27.7513	666.03

The analyzed cases show that the total pulse necessary to maintain the satellite on the orbit increases with the number n of pulse traction maneuvers. It follows that among the analyzed cases the optimal situation is obtained when n=3 maneuvers, which means that the satellite will need to execute only three maneuvers during a whole rotation about the Earth. The fuel consumption is also optimal at n=3, since the fuel consumption is directly proportional to the pulse needed for the satellite to execute the maneuvers.

3 CONCLUSIONS

The paper deals with the relative motion of a satellite on an orbit of 1º inclination. The apparent free motion of the satellite is studied and, afterwards, a pulse is applied to the satellite so that it will move to the desired geostationary orbit. Various values were obtained, depending on the number of maneuvers applied to the satellite in order to bring it back to the orbit. The optimal value was obtained for only 3 maneuvers. This is also optimal from the point of view of fuel consumption, due to the fact that the fuel consumption is proportional to the value of the pulse needed to be applied.

REFERENCES

1. Mugur Savescu; 1976; "Intiere in radiocomunicatii prin sateliti"; Ed.Tehnica.
2. M.M. Niţă; 1973; "Teoria zborului spaţial"; Ed. Academiei.
3. I. Stroe; "Attitude control of the satellite for elliptic orbits"; IAF-96-A.5.07.
4. M.M. Niţă, I. Stroe, P. Pârvu, S. Cioc; 1998; "Rotaţia relativă a corpurilor în raport cu staţii orbitale de mari dimensiuni"; *Studii şi Cercetări de Mecanica Aplicată*; 57; (1);pp.18-28.
5. Jeannette Heiligers; 2010; "Displaced Geostationary orbits using Hybrid Low-Thrust Propulsion", *Advanced Space Concepts Laboratory*; University of Strathclyde; Glasgow, United Kingdom.