Flutter Uncertainty Analysis of an Aircraft Wing Subjected to a Thrust Force Using Fuzzy Method

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ABSTRACT

Flutter uncertainty analysis of an aircraft wing subjected to a thrust force using fuzzy method is investigated. The wing model contains bending and torsional flexibility and the aeroelastic governing equations are derived based on Hamilton's Principle. The aerodynamic loading is simulated based on finite state unsteady thin airfoil theory. Partial differential equations of motion are converted to a set of ordinary differential equations using Galerkin method. The wing bending and torsional rigidity, aerodynamic lift curve slope and air density are modeled as fuzzy uncertain parameters with triangle membership function. The eigenvalue problem with fuzzy input parameters is solved using fuzzy Taylor expansion method and a sensitivity analysis is performed. Flutter boundary is extracted as a membership function. Furthermore the upper and lower bounds of Flutter region in different α-cuts are extracted. Results show that this method is a low-cost method with reasonable accuracy to estimate the flutter speed and frequency.

KEYWORDS: *Uncertainty; Flutter; Aircraft wing; Thrust Force; Fuzzy Method*

NOMENCLATURE

A, B - Eigenvalue problem matrixes
$\underset{\sim}{A}$ - Finite state pressure loading coefficient
$C_{l\theta}$ – Lift Curve Slope Coefficient
E - Elastic modulus
G - Shear modulus
H - Heaviside function
I - Wing cross-sectional moment of inertia
J - Wing cross-sectional polar moment of inertia
K_e - Engine mass radius of gyration
L - Aerodynamic lift
M - Aerodynamic moment
M_e - Engine Mass
P - Dimensionless trust force
P_e - Engine thrust force
T_e - Engine kinetic energy
T_w - Wing kinetic energy
U - Airstream velocity
U_s - Strain energy of the wing
W_a - Work done by aerodynamic forces
W_f - Work done by follower forces
X_e, Y_e, Z_e - Dimensionless engine location
b - Wing semichord
$\underset{\sim}{c}$ - Finite state pressure loading coefficient
$m_{(x)}$ - Wing mass per unit length
n - Number of modes
n_w - Number of bending modes
n_θ - Number of torsional modes
n_λ - Number of induced flow states
q_j - j^{th} eigenvector corresponding to λ_j
v_f – Dimensionless flutter speed
w - Wing bending deflection
x_e, y_e, z_e - engine location
λ_j - j^{th} eigenvalue
θ – Wing torsion deflection
ρ – Air density
$\tilde{\xi}$ - Fuzzy uncertain parameters
ℓ - Wing length

1 INTRODUCTION

The evaluation of flutter for an aircraft wing has been a major challenge for aeronautical engineering for many years [1]. Hodges et al. [2] investigated the effect of thrust on the flutter of a high-aspect-ratio wing. Fazelzadeh et al. [3-4] presented a deterministic model for bending torsional flutter characteristic of a wing under follower force. They have studied the flutter of an aircraft wing carrying a powered engine and indicated the importance of follower force on flutter speed and frequency.
The aircraft wing is a system with many structural and non-structural uncertainties. That's why the study of various phenomena on aircraft wings in the presence of uncertain parameters is more real and significant. In this regards Rao and Berke [5] investigated the modelling of uncertain structural systems using interval analysis. They represented each uncertain input parameter as an interval number. Muhanna and Mullen [6] presented a non-traditional uncertainty treatment for mechanics problems. In their work uncertainties are introduced as bounded possible values (intervals). Qiu and Wang [7] presented the non-probabilistic interval analysis method for the dynamical response of structures with uncertain-but-bounded parameters. Qiu [8] used convex models and interval analysis method to predict the effect of uncertain-but-bounded parameters on the buckling of composite structures. Muhanna et al. [9] presented an interval approach for the treatment of parameter uncertainty for linear static problems of mechanics. They combined interval analysis and finite element methods to analyse the system response due to uncertain stiffness and loading. Xiaojun and Zhiping [10] studied the influences of uncertainty parameters on the flutter speed of a wing. The uncertain parameters was described by interval numbers. They found the upper and lower flutter bound of speed using first order Taylor series expansion. They have only studied the structural parameters and other parameters such as geometrical, aerodynamic and loading have not been mentioned in their work. Khodaparast et al. [11] investigated the problem of linear flutter analysis in the presence of structural uncertainty. Badcock et al. [12] reviewed the use of eigenvalue stability analysis of very large dimension aeroelastic numerical models arising from the exploitation of computational fluid dynamics. Sofi et al. [13] evaluated the lower and upper bound of the natural frequencies of structures with uncertain but bounded parameters. They applied the improved internal analysis via extra unitary interval (EUI).
Some researchers used fuzzy approach to analyse uncertainty problems. De Gersem et al. [14] examined the interval and fuzzy finite element method for the eigenvalue and frequency response function analysis of structures with uncertain parameters. They combined non-probabilistic methods with the component mode synthesis technique in order to reduce the calculation time. Massa et al. [15] presented a fuzzy methodology to calculate the eigenvector and eigenvalue of a mechanical structure defined by imprecise parameters. They described material and geometric parameters as imprecise fuzzy numbers. Damping and other non-conservative parameters were not considered in their work. Tartaruga et al. [16] used probabilistic and non-probabilistic approaches to predict the flutter dynamic pressure of a semi-span super-sonic wind-tunnel model.
According to the best of the authors knowledge, in the pertinent literature, aeroelastic analysis of wings subjected to thrust force under all type of uncertainties containing bending and torsional rigidity, lift curve slope and air density using Fuzzy approach have not yet been presented. This study intends to fill the gap in knowledge associated with this problem. In this paper, parameter sensitivity with various order of magnitudes is done for different airspeeds. Furthermore, modal damping vs airspeed diagrams, in different α-cuts, are presented.

2 PROBLEM STATEMENT

The aircraft wing subjected to a powered engine as shown in Fig.1 is considered [3]. The undeformed shape of the wing is shown in Fig.1 (a) and the typical section of the wing is shown in Fig.1 (b). *AE*, *AC*, cg_w and cg_s are the wing elastic axis, the wing aerodynamic center, the wing center of gravity and the engine center of gravity, respectively. The structural model of the wing contains bending and torsional flexibility. Aerodynamic pressure loading based on Finite State unsteady thin airfoil theory is also applied on this model. Torsional and bending rigidity, lift curve slope and air density are considered as fuzzy uncertain parameters, in the model. These uncertain parameters are modelled as fuzzy membership functions.

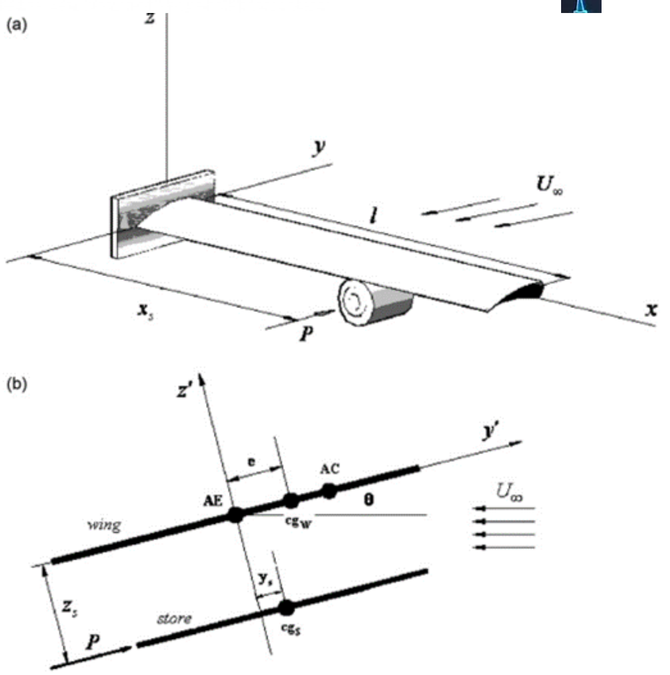

Figure 1: (a) Aircraft wing subjected to a power engine, (b) the wing engine typical section [5]

3 GOVERNING EQUATION

The equations of motion and boundary conditions are developed by Hamiltonian variational principle as

$$\int_{t_1}^{t_2}\left(\delta U_s - \delta T_e - \delta T_w - \delta W_a - \delta W_f\right)dt = 0 \qquad \delta w = \delta\theta = 0 \ \ at\ \ t = t_1 = t_2 \tag{1}$$

The final equations of motion are derived by extending the above equation [3].

$$\begin{aligned}\delta w:\ & m_{(x)}\ddot{w} + m_{(x)}y_\theta\ddot{\theta} + EIw'''' + M_e\left(-z_e^2\ddot{w}'' + y_e\ddot{\theta} + \ddot{w}\right)\delta_D\left(x - x_e\right)\\ & + P_e\left(x_e - x\right)H\left(x_e - x\right)\theta'' - 2P\theta' = L\left(x,t\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\delta\theta:\ & m_{(x)}k_{EA}^2\ddot{\theta} + m_{(x)}y_\theta\ddot{w} - GJ\theta'' + M_e\left(\left(z_e^2 + y_e^2 + K_e^2\right)\ddot{\theta} + y_e\ddot{w}\right)\delta_D\left(x - x_e\right)\\ & + P_e\left(x_e - x\right)H\left(x_e - x\right)w'' = M\left(x,t\right)\end{aligned} \tag{3}$$

Peters et al. finite state unsteady aerodynamic model is used to simulate aerodynamic forces [17]:

$$L\left(x,t\right) = -\pi\rho b^2\left[\ddot{w} - U\dot{\theta} + ba\ddot{\theta}\right] + C_{L\theta}\rho Ub\left[-\dot{w} + U\theta - ba\dot{\theta} + \frac{b}{2}\left(\frac{C_{L\theta}}{\pi} - 1\right)\dot{\theta} - \lambda_0\left(t\right)\right] \tag{4}$$

$$\begin{aligned}M\left(x,t\right) == & -\pi\rho b^3\left[\frac{1}{2}\left(\frac{C_{L\theta}}{\pi} - 1\right)U\dot{\theta} - Ua\dot{\theta} + a\ddot{w} + b(\frac{1}{8} + a^2)\ddot{\theta}\right]\\ & + C_{L\theta}\rho Ub^2\left(a + \frac{1}{2}\right)\left[U\theta - \dot{w} - ba\dot{\theta} + \frac{b}{2}\left(\frac{C_{L\theta}}{\pi} - 1\right)\dot{\theta} - \lambda_0\left(t\right)\right]\end{aligned} \tag{5}$$

where $\lambda_0 = \sum_{n=1}^{\infty} b_n\lambda_n$ is the induced flow velocity, calculate through a system of N first order coupled differential equations [18].

4 SOLUTION APPROACH FOR DETERMINISTIC MODEL

Due to the complexity of the governing equations an approximate solution methodology should be used to solve them. Galerkin method is a simple and accurate choice for solving these equations. In this method, the wing bending and torsion are expressed as the following series:

$$w(y,t)=\sum_{j=1}^{n_w} W_j(y)\varphi_j(t) \quad , \quad \theta(y,t)=\sum_{n=1}^{n_\theta} \Theta_j(y)\psi_j(t) \tag{6}$$

where $\varphi_j(t)$ and $\psi_j(t)$ are time dependent modal coordinates and $W_j(y)$ and $\Theta_j(y)$ are bending and torsional trial functions. n_w and n_θ are the number of trial functions used for representation of w and φ, respectively.
By using suitable family of orthogonal functions for w and φ [18], substituting Eq.7 in Eqs.2 and 3, and applying the Galerkin procedure discrete equations of motion are considered in general following form:

$$[M]\{\ddot{q}\}+([C]+U[G])\{\dot{q}\}+([K]+U[L]+U^2[H])\{q\}=0 \tag{7}$$

where [M] is mass matrix, [C] is damping matrix, U [G] is damping matrix due to aero elastic terms,[K] is structural stiffness and $U[L]+U^2[H]$ is aeroelastic stiffness matrix due to circularity force. The final state space form of discrete governing equations can be developed as:

$$A\{\dot{q}\}=B\{q\} \tag{8}$$

After solving above eigenvalue problem the modal damping and frequency in different airspeeds are obtained.

5 MODELING UNCERTAINTY WITH FUZZY APPROACH

In this section the modelling of parameter uncertainty using fuzzy expansion approach [15] is investigated. The eigenvalue problem of Eq.8 can be described as:

$$(B-\lambda_j A)q_j=0 \qquad j=1,2,\ldots,n \;\;\&\;\; n=2n_w+2n_\theta+n_\lambda \tag{9}$$

where λ_j is the jth eigenvalue, q_j is the jth eigenvector, n_w is the number of bending modes, n_θ is the number of torsional modes and n_λ *is* the number of induced flow states. It's assumed that bending and torsional rigidity, lift curve slope and air density are not deterministic parameters. Because these parameters are imprecise they are modelled by fuzzy numbers. Each fuzzy value $\tilde{\zeta}$ is represented as a fuzzy membership function showing in figure 2 and as:

$$\tilde{\zeta}=\xi_c+\Delta\tilde{\zeta} \tag{10}$$

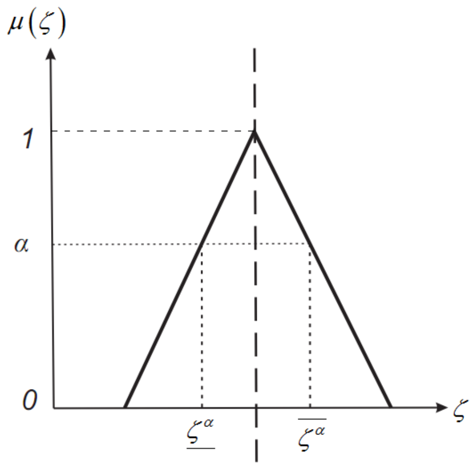

Figure 2: Fuzzy triangle membership function

ζ_n is a nominal or crisp value and $\Delta\zeta^{\alpha}$ is the variation associated to each α-cut. For each α-cut

$$\tilde{\zeta} = \xi_c + \left[\underline{\Delta\zeta^{\alpha}};\overline{\Delta\zeta^{\alpha}}\right] \tag{11}$$

In which $\underline{\zeta^{\alpha}}$ and $\overline{\zeta^{\alpha}}$ are minimum and maximum values of fuzzy parameter $\tilde{\zeta}$ for a given α-cut, respectively. The membership function are discretized by different intervals which are linked to an α-cut ranging from 1 to 0 [19].

In the presence of m fuzzy parameters the eigenvalue problem can be rewritten as:

$$B\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_m\right)\tilde{q}_j = \tilde{\lambda}_j A\left(\tilde{\zeta}_1,\tilde{\zeta}_2,...,\tilde{\zeta}_m\right)\tilde{q}_j \tag{12}$$

A-cut method is an approach for solving this types of eigenvalue problems [19]. In this method the fuzzy membership function is discretize to different intervals using α-level cut concept. For each α-level cut the eigenvalue problem is solved with the Neumann series of first order perturbation method.

In this paper, to solve the flutter uncertain problem the Taylor series expansion is used to determine the crisp value (TSEC). TSEC is a method that evaluates the derivative of crisp values to calculate the eigenvalue and eigenvector of fuzzy parameters. In this method the fuzzy eigenvalue and eigenvector is determined as:

$$\begin{aligned}\tilde{\lambda}_j^{\alpha} &= \lambda_{j_c} + \sum_{j=1}^{n}\frac{\partial\lambda_j}{\partial\zeta_i}\Delta\tilde{\zeta}_i^{\alpha}\\ \tilde{q}_j^{\alpha} &= q_{j_c} + \sum_{j=1}^{n}\frac{\partial q_j}{\partial\zeta_i}\Delta\tilde{\zeta}_i^{\alpha}\end{aligned} \tag{13}$$

where $\Delta\tilde{\zeta}_i^{\alpha} = \left[\underline{\Delta\zeta^{\alpha}};\overline{\Delta\zeta^{\alpha}}\right]$. The value of $\frac{\partial\lambda_j}{\partial\zeta_i}$ can be determined by following Equation [19].

$$\frac{\partial\lambda_j}{\partial\zeta_i} = q_{j_c}^T\left(\frac{\partial B}{\partial\zeta_i} - \lambda_j\frac{\partial A}{\partial\zeta_i}\right)q_{j_c} \tag{14}$$

The above equation also demonstrates the sensitivity of eigenvalues to parameter ζ_i. For modelling the uncertainty in the flutter problem the fuzzy parameter should be determined, primarily. $EI, GJ, \tilde{\rho}, C_{l\theta}, \tilde{P}$ are considered as uncertain parameters of the wing. The bending and torsional rigidity EI and GJ are structural uncertainty parameters, the air density $\tilde{\rho}$ is an aerodynamic uncertain parameter which varies with the aircraft flight height. Also, the wing lift curve slope $C_{l\theta}$ and follower force are other uncertain parameters. These parameters are modelled using triangle fuzzy membership function as shown in Fig.2. After modelling the uncertain parameters, the final equation for fuzzy eigenvalue problem is determined as:

$$\tilde{\lambda}_j^{\alpha} = \lambda_{j_c} + \frac{\partial\lambda_j}{\partial(EI)}\Delta EI^{\alpha} + \frac{\partial\lambda_j}{\partial(GJ)}\Delta GJ^{\alpha} + \frac{\partial\lambda_j}{\partial\rho}\Delta\rho^{\alpha} + \frac{\partial\lambda_j}{\partial C_{l\theta}}\Delta C_{l\theta}{}^{\alpha} + \frac{\partial\lambda_j}{\partial P}\Delta P^{\alpha} \tag{15}$$

6 NUMERICAL RESULTS

6.1 Validation of Deterministic problem

Related data for the particular wing which is used here is given in table 1. By considering two bending modes in w direction, two torsion modes and two aerodynamic states, the Eqs.2 and 3 convert to a set of first order coupled ordinary differential equations. The following dimensionless parameters are used in this study:

$$P = \frac{P_e\ell^2}{\sqrt{GJ\,EI}}\ ,V_f = \frac{U_f}{b\,\omega_\theta}, X_e = \frac{x_e}{\ell}, Y_e = \frac{y_e}{b}, Z_e = \frac{z_e}{b} \tag{16}$$

As shown in Fig. 3, flutter boundary results are compared with previous published studies, such as Fazelzadeh et al. [3] and Hodges et al. [2] and good agreement is observed. This validation is performed to determine the accuracy of the current aeroelastic governing equations and the solution methodology in the presence of engine thrust.

Table 1: The wing model characteristics [2]

Parameters	**Value**
Wing Length	16 m
Semi-chord	0.5 m
Bending rigidity	2e4 $N.m^2$
Torsional rigidity	2e3 $N.m^2$
Mass per unit length	0.75 Kg/m
Wing moment of inertia	0.1 Kg.m
Engine moment of inertia	20 Kg.m

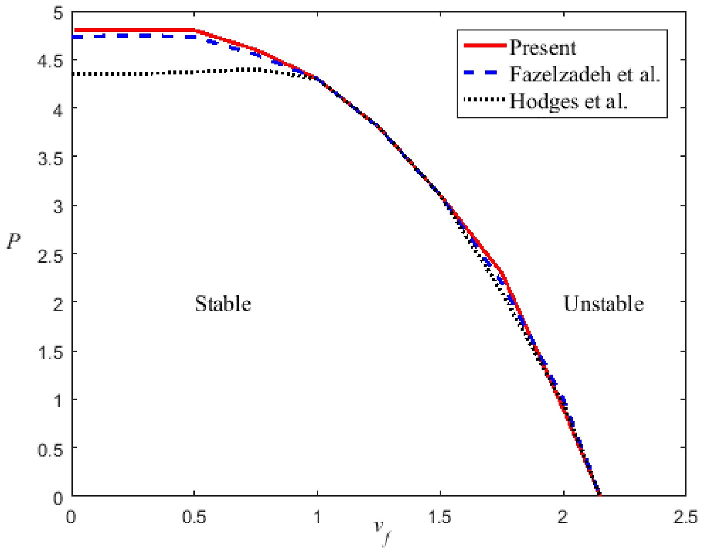

Figure 3 : Flutter Boundary of a clean wing subjected to trust force

6.2 Investigating Flutter under Uncertainty

In this section the flutter analysis with uncertain parameters is investigated. The values of uncertain parameters are specified in table 2.

Table 2: Uncertain fuzzy parameters

Parameters	**Bending Rigidity**	**Torsional Rigidity**	**Air Density**	**Lift Curve Slope**
Crisp Value	20000	2000	0.0889	5.9027
Minimum Value	19000	1900	0.0845	5.6076
Maximum Value	21000	2100	0.0933	6.1979
Percentage of Variation	±5%	±5%	±5%	±5%

Figure 4 shows the sensitivity analysis of the above parameters, *EI*, *CJ*, ρ and Cl_θ in different air speeds. Because the magnitude orders of parameters sensitivity are very different, the y axis is shown in logarithmic scale. This figure shows that the sensitivity of air density and lift curve slope is much larger than the sensitivity of geometric and structured parameters. As expected, this result shows that the aerodynamic loading has significant impact on the wing flutter phenomenon.

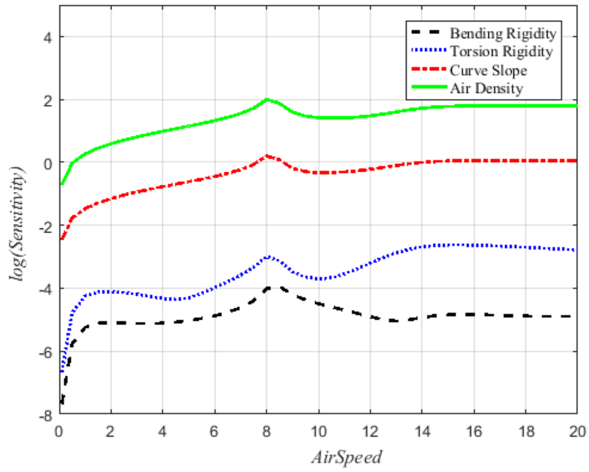

Figure 4. Parameter sensitivity vs airspeed

The modal damping versus air speed for uncertain fuzzy parameters at α-cut=0 (largest interval) and α-cut=0.5 for different dimensionless thrust forces P is shown in Fig.5. This figure relates to first modes of bending and torsion. In Fig.5 (a) and (b) the effect of thrust force in α-cut=0 is illustrated. It can be seen that increasing thrust force will decrease the flutter speed. Furthermore, increasing the thrust force tightens the flutter speed range due to uncertainties. These results are repeated for α-cut=0.5 that is shown in Fig .5 (c) and (d) and the same conclusion is also drawn in this case.

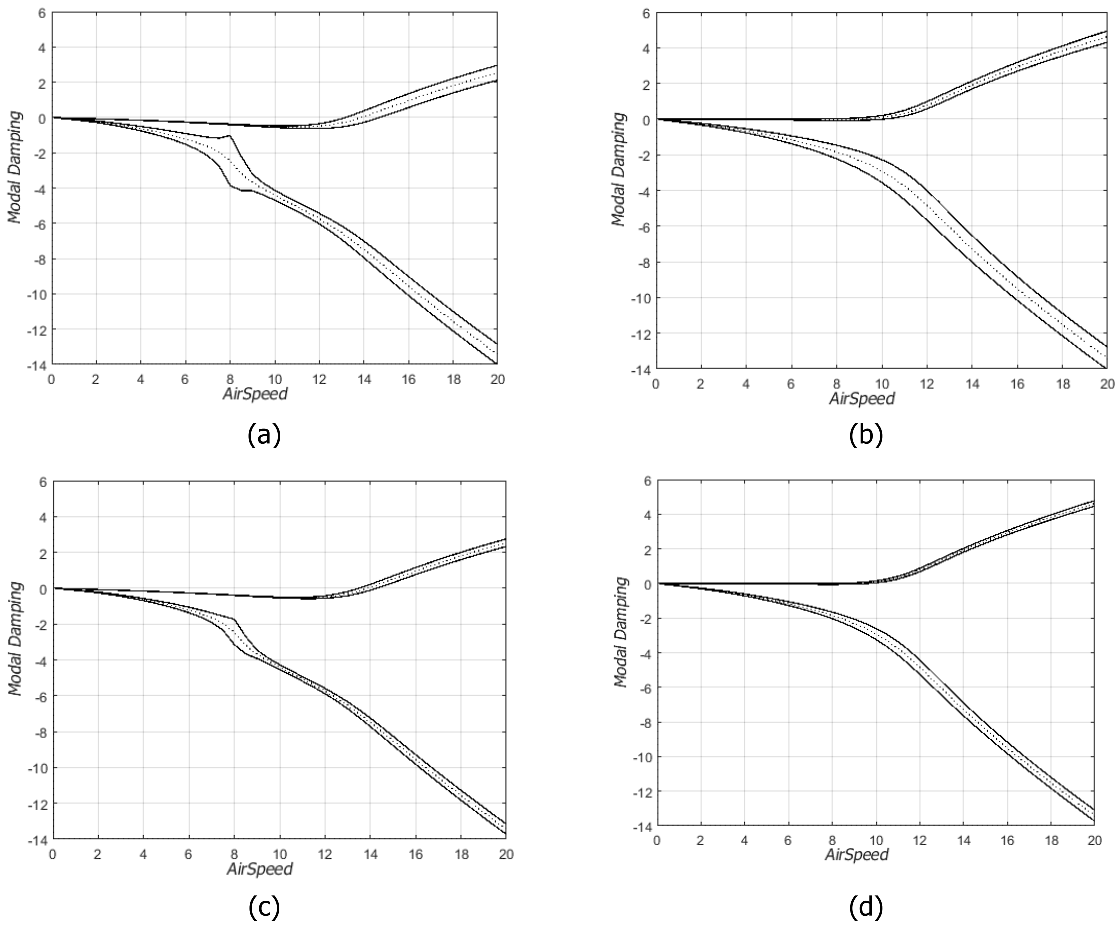

Figure 5 : modal damping vs airspeed and with follower force
(a) α-cut=0, P=0; (b) α-cut=0, P=4; (c)α-cut=0.5, P=0; (d)α-cut=0.5, P=4

Finally, the first bending mode modal damping vs airspeed in different α-cut and different dimensionless thrust forces is shown in Fig.6. In this figure, the flutter boundary range can be shown in a fuzzy mountain shape. For each value of the trust force and in every a-cat section, the upper and lower bounds of the flutter speed (where the modal damping equals to zero) can be extracted from this figure.

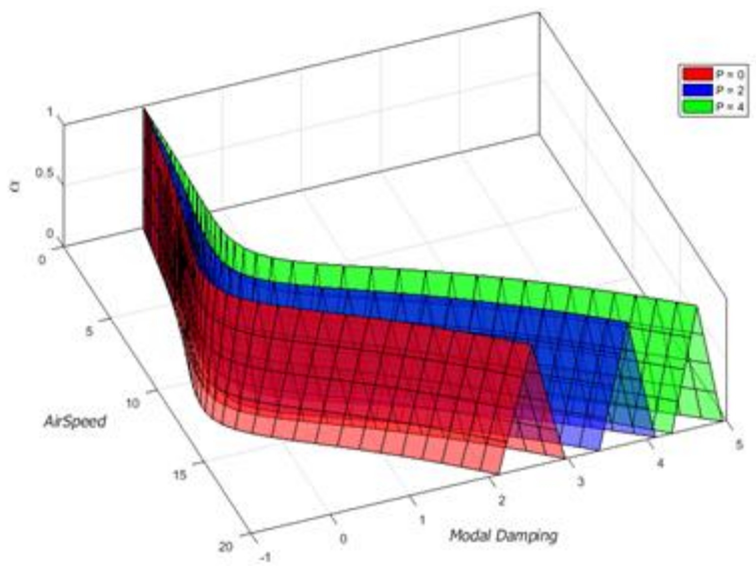

Figure 6.Modal Damping vs Airspeed in different α-cuts at P=0, 2, 4

7 CONCLUSION

In this paper the flutter of an aircraft wing subjected to engine thrust force is investigated. The wing model contains structural and geometrical uncertainties. Also, air density and lift curve slope are other uncertain parameters. These uncertain parameters are modelled as fuzzy membership functions and the α-cut method is used to solve this fuzzy eigenvalue problem. Sensitivity and flutter analysis is done with this methods. Results show that this method is useful to study the flutter phenomena in the presence of uncertainty. Also, simulation results indicate that increasing thrust force will generally decrease the flutter speed. Furthermore, increasing the thrust force tightens the flutter speed range due to uncertainties.

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