A method for calculus of Internal Forces

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ABSTRACT

A new method to compute internal forces in a multi-bodies system is presented in the paper. Lagrange equations are used to study the motion of a system under the action of known external forces. If an internal force has to be found, a supplementary mobility is considered in the system and the corresponding internal force for the new mobility is found for null value of the mobility, as well as its first and second orders of derivatives. The method is a general one, but a particular case of mechanism used in the dynamics of the airplane elevator is analyzed to verify the validity of the proposed method.

KEYWORDS: *airplane, multi-bodies system, constraint, dynamics.*

NOMENCLATURE

E - Total kinetic energy expressed with the base frame
Q_k - Generalized force
U_Φ - Analytical function
$\{J_{Ck}\}$ - Matrix of inertia of link k about link frame $C_k x_k y_k z_k$
$\{r_{Ck}\}$ - Position vector of mass center of link k with respect to the base frame
$\{\dot{r}_{Ck}\}$ - Derivative of position vector of mass center of link k with respect to the base frame
$\Re_{h+1}$ - Internal force
$\{\omega_k\}$ - Angular velocity vector of link k with respect to the base frame
δW_k - Virtual work produced by forces acting upon the system corresponding to δq_k
δq_k - Virtual displacement
λ_j - Lagrange multiplier

1 INTRODUCTION

Determining internal forces and constraint force are important steps in dynamic analysis, which is the base for structure design of a mechanism. As known, calculus of internal forces for a static system of rigid bodies is common in the field known as strength of materials. According to the standard procedure, first, external reactions at external supports need to be computed and next, by using sections perpendicular to the rigid body axis, the internal forces such as shear force, or bending moment at specified points along the rigid body are calculated based on the principle of equilibrium. However, for a complex mechanism with a large number of degrees of freedom, the analysis of the constraint forces in dynamic state are extremely difficult. Consequently, the calculus of the internal forces will encounter a lot of difficulties.
In recent years, the problems related to the dynamic analysis of rigid bodies systems have attracted attention of researchers and some of them have got valuable results in their works: based on the observation method and the theory of the reciprocal screw system, Zhi and Wang [4] have solved and expressed the constraint forces of the kinematic pair of a slider-crank mechanism and a single loop spatial RUSR mechanism by introducing the solution coefficient of the constraint wrench of the kinematic pair; Y. Zhao, J. F. Liu and Z. Huang [5] used the screw theory to determine all the

reactions, as well as the active forces, for the spatial 3-RPS parallel manipulator (consisting of a mobile platform connected to a fixed base by three identical supporting limbs with symmetrical kinematic structure) without over-constraint; using Newton-Euler method and d'Alembert's principle, Y. Jiang, T. M. Li and L.P. Wang [6] established the force analysis equations and also put forward the dynamic analysis model of a parallel mechanism based on the deformation compatibility method; A. Rotaru, L. Dudici in [7] calculated the reaction wrench components of all the kinematic pairs of the linkages of the Stewart platform, by applying the Denavit-Hartenberg transformation matrices and the principle of virtual work; by using Lagrange equations and the principle of virtual work, I. Stroe and S. Staicu [1] calculated the joint forces in the double pendulum; I. Stroe, S. Staicu and A. Craifaleanu determined the bending moment in a compass robotic arm based on Lagrange equations [2].
As known, by using Lagrange equations, the differential equations of motion of a rigid bodies system can be obtained easily without considering constraint forces. In addition, if an internal force has to be found, a supplementary mobility related to it is considered in the system and the corresponding internal force for the new mobility is calculated for null values of mobility as well as its first and second derivatives. The paper presents a new method for determining internal forces. Not only this method can calculate the internal forces in a rigid body but it can also calculate them in a group of links having translational movement one with respect to the other. A system for controlling the aircraft elevator is considered to illustrate the proposed method.

2 METHOD FOR CALCULATING INTERNAL FORCES BY USING LAGRANGE EQUATIONS

2.1 Equations of motion of a rigid bodies system

When constraints are expressed by functions of coordinates, the motion of the systems can be studied with Lagrange equations for holonomic systems with dependent variables, while if the constraints are expressed by velocities, the motion is described with Lagrange equations for non-holonomic systems.
For a non-holonomic system, the Lagrange equations corresponding to a system of h generalized coordinates

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right)-\frac{\partial E}{\partial q_k}=Q_k+\sum_{i=1}^{p}\lambda_i a_{ik} \ , \ (k=1,\,2,...,h) \ , \tag{1}$$

are completed with the constraints

$$\sum_{k=1}^{h} a_{ik}\dot{q}_k+b_i=0 \ , \ (i=1,\,2,...,\,p) \ , \tag{2}$$

while

$$Q_k=\frac{\delta W_k}{\delta q_k} \ . \tag{3}$$

By solving a system of h equations in Eq. (1), and p equations in Eq. (2), the coordinates q_k and the Lagrange multipliers λ_i will be found.
From Eq. (1), the equations for the holonomic system can be obtained by replacing the functions a_{ik}.
In the case of a holonomic system, the constraints are of the form

$$\Phi_i\left(q_1,...,q_h,t\right)=0 \ , \ (i=1,\,2,...,\,p) \ , \tag{4}$$

From the above formula, the following differential form is obtained:

$$\sum_{k=1}^{h}\frac{\partial \Phi_i}{\partial q_k}\dot{q}_k+b_i=0 \, , \ (i=1,\,2,...,\,p) \ . \tag{5}$$

By comparing Eq. (5) and Eq. (2), it follows

$$a_{ik}=\frac{\partial \Phi_i}{\partial q_k} \ . \tag{6}$$

Then, Eq. (1) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right)-\frac{\partial E}{\partial q_k}=Q_k+\frac{\partial}{\partial q_k}\sum_{i=1}^{p}\lambda_i\Phi_i\ ,\ (k=1,\ 2,\ldots,\ h)\,. \tag{7}$$

By defining the analytical function

$$U_\Phi=\sum_{i=1}^{p}\lambda_i\Phi_i\ . \tag{8}$$

Eq. (7) can be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right)-\frac{\partial E}{\partial q_k}=Q_k+\frac{\partial U_\Phi}{\partial q_k}\ ,\ (k=1,\ 2,\ldots,\ h)\ . \tag{9}$$

Starting from these h differential equations and using p relations of constraints, the generalized coordinates q_k and the Lagrange multipliers λ_i are determined.

2.2 Calculus of Internal Forces

For a mechanical system with h degrees of freedom represented by the independent generalized coordinates q_k $(k=1,\ 2,\ldots,\ h)$, the Lagrange equations are expressed as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right)-\frac{\partial E}{\partial q_k}=\frac{\partial U}{\partial q_k}+Q_k^*\ ,\ (k=1,\ 2,\ldots,\ h)\ . \tag{10}$$

An internal force Q_{h+1}, as the new generalized force, can be found if a new fictitious mobility according to the force is considered. Then the mechanical system becomes one with $h+1$ degrees of freedom. The equation for the new mobility is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_{h+1}}\right)-\frac{\partial E}{\partial q_{h+1}}=\frac{\partial U}{\partial q_{h+1}}+Q_{h+1}\ . \tag{11}$$

Considering again the mechanism, the internal force $\Re_{h+1}$ is easily obtained from Eq. (11) in the form

$$\Re_{h+1}=\left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_{h+1}}\right)-\frac{\partial E}{\partial q_{h+1}}-\frac{\partial U}{\partial q_{h+1}}\right]_{\substack{q_{h+1}=0\\ \dot{q}_{h+1}=0\\ \ddot{q}_{h+1}=0}}\ . \tag{12}$$

3 CALCULUS OF THE INTERNAL FORCES IN THE AIRPLANE ELEVATOR SYSTEM

In order to verify the validity of the method presented above, a system for controlling the airplane elevator as shown in Fig. 1 is considered as an example to calculate internal forces. For simplicity and without generality, the link (1) is supposed as a bar of length l_1 and mass m_1; the piston rod (2) is a bar characterized by the length l_2 and the mass m_2; the piston body is a plate characterized by the radius R_2 and the mass m_3; the cylinder (4) is represented by radii R_1 and R_2, the length l_4 and the mass m_4.
As shown, the mechanism has one degree of freedom, so θ is chosen as generalized coordinate. Then, by using Lagrange equations, the equation governing the motion of the system can be obtained easily. However, in order to find the linear and the angular velocities of the links, necessary for computing the total kinetic energy, some kinematic relations of the mechanism are needed to be determined first. Based on the geometric relation, as shown in Fig. 1, the kinematic relations between some important terms are written below:

$$\varphi_4 = \operatorname{atan}\left(\frac{l_1 (1-\cos\theta)}{l_2 + \frac{l_4}{2} + l_1 \sin\theta} \right), \tag{13}$$

$$\dot{\varphi}_4 = \frac{(1-\cos\theta) + \frac{\left(l_2 + \frac{l_4}{2}\right)}{l_1}\sin\theta}{\left(\frac{l_2 + \frac{l_4}{2}}{l_1} + \sin\theta\right)^2 \left(1+\tan^2\varphi_4\right)} \dot{\theta} \ , \tag{14}$$

$$x = \left(l_2 + \frac{l_4}{2}\right)\left(\frac{l_1}{\left(l_2 + \frac{l_4}{2}\right)} \sin(\theta - \varphi_4) + \sqrt{\left(\frac{l_1}{l_2 + \frac{l_4}{2}}\right)^2 \sin^2(\theta - \varphi_4) + 1} - 1 \right) , \tag{15}$$

$$\dot{x} = l_1 . \left(1 + \frac{l_1}{l_2 + \frac{l_4}{2}} \frac{\sin(\theta - \varphi_4)}{\sqrt{\left(\frac{l_1}{l_2 + \frac{l_4}{2}}\right)^2 \sin^2(\theta - \varphi_4) + 1}} \right) . \left(\dot{\theta} - \dot{\varphi}_4\right)\cos(\theta - \varphi_4) \ , \tag{16}$$

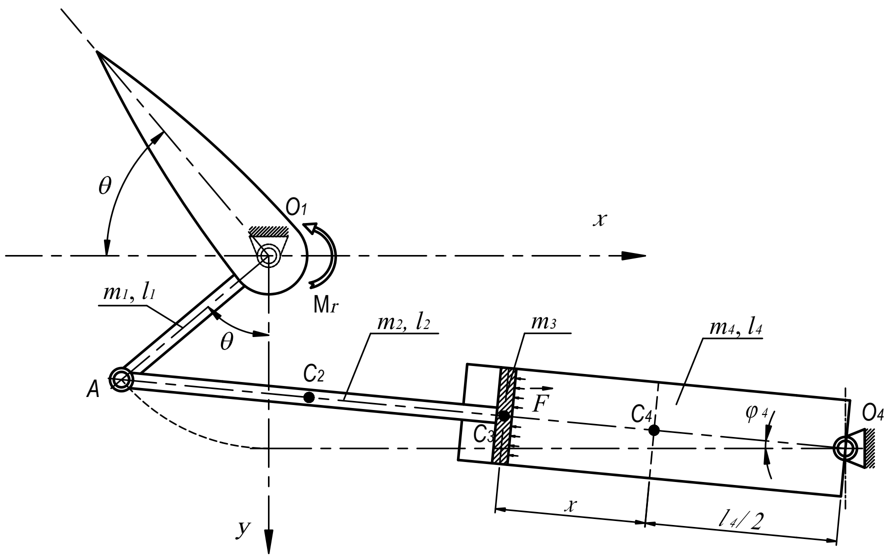


Figure 1: The system for controlling the aircraft elevator

After computing all the necessary terms, such as kinetic energy *E*, force function *U*, generalized forces Q_k, and taking all their partial derivatives as well as total derivatives with respect to time and replacing into Eq. (9), the equation of motion is achieved:

$$\mathrm{A}\ddot{\theta} - \mathrm{B} = -\mathrm{M_r} + \mathrm{F}_{cy} . l_1 \cos(\theta - \varphi_4) , \tag{17}$$

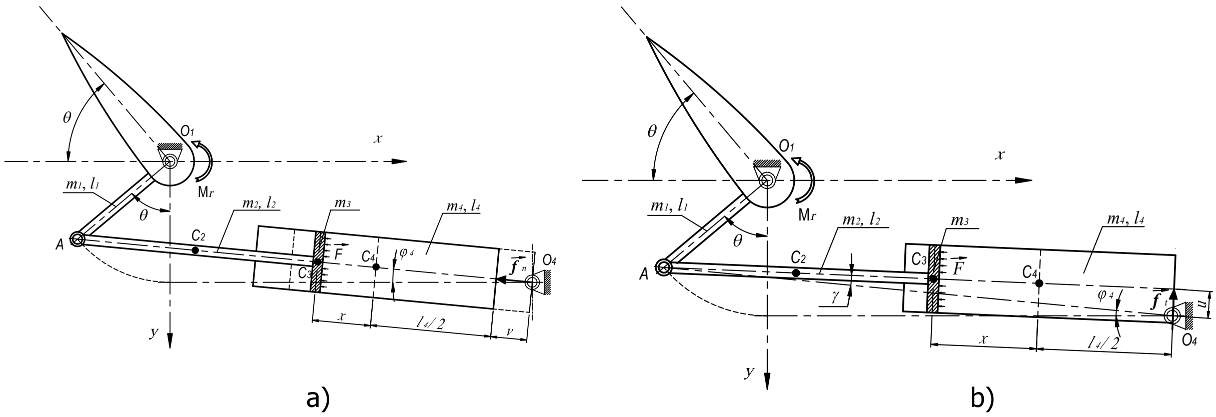
where $\mathrm{A} = \mathrm{f}_1(\theta)$ stands for the inertia component,

$\mathrm{B} = \mathrm{f}_2(\theta, \dot{\theta})$ stands for the Coriolis/centrifugal and gravity components,

F_{cy} is the external force produced by the pressure of the hydraulic cylinder,

M_r stands for the moment created by the resistance effect of the air.

As mentioned above, one of the most important applications in using Lagrange equations is the possibility to calculate directly the internal forces as well as constraint forces in a rigid bodies system. For this case, the constraint force at point O_4 can be considered as the sum of two components: the normal constraint force $\vec{f}_n$ with the direction along the centerline of the cylinder, and the tangent constraint force $\vec{f}_t$, with the direction perpendicular to $\vec{f}_n$. Both these two forces lie in the vertical plane.



a) b)

Figure 2: Diagrams for calculating the normal constraint force (a) and the tangent constraint force (b)

The supplementary mobility corresponding to the normal constraint force $\vec{f}_n$ is v. Thus, the generalized coordinates representing the considered mechanism are chosen as $q_1 = \theta,\ q_2 = v$ as shown in Fig. 2a. Subsequently, the normal constraint force can be determined by using Lagrange equation and is written in the form

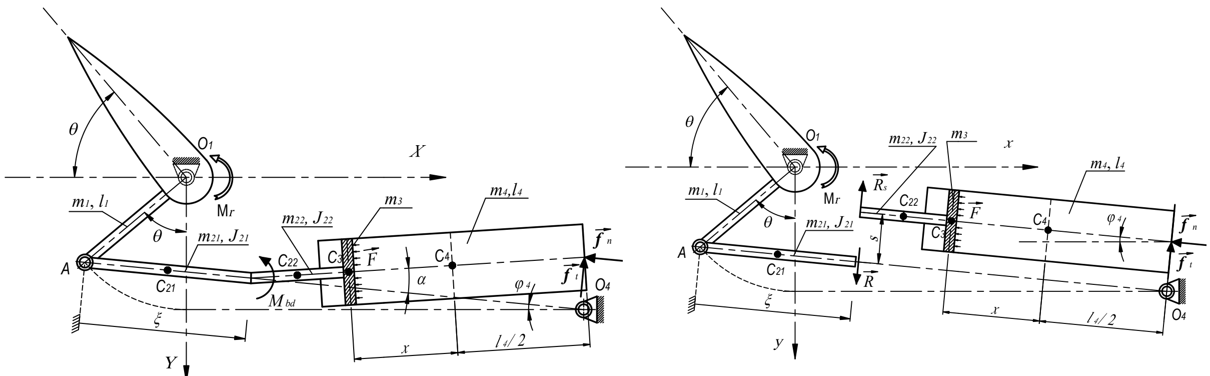
$$f_n = C\ddot{\theta} + D + F_{cy}\,, \tag{18}$$

where $C = f_3(\theta)$ stands for the inertia component,

$D = f_4(\theta, \dot{\theta})$ stands for the Coriolis/centrifugal and gravity components.

Similarly, when the tangent constraint force $\vec{f}_t$ is determined, the generalized coordinates are $q_1 = \theta,\ q_2 = u$, as shown in Fig. 2b. Then the tangent constraint force can be obtained and has the expression of the form

$$f_t = f_t(\theta, \dot{\theta})\ . \tag{19}$$



a) b)

Figure 3: Diagrams for calculating the bending moment (a) and shear force (b) in the piston rod-cylinder system

Based on the achieved results, the constraint at the point O_4 is released and replaced by the tangent and normal constraint forces $\vec{f}_t, \vec{f}_n$. At this moment, they are considered as external forces acting on the rigid bodies system.

For determining the internal forces, including the shear force and the bending moment in the piston rod-cylinder sub-mechanism, the generalized coordinates $q_1 = \theta$, $q_2 = \alpha$ as shown in Fig. 3a are chosen, corresponding to the case for calculating the bending moment M_{bd}.

The variable indicating the position along the piston rod-cylinder subsystem, where the bending moment is calculated, will be denoted with ξ. The expressions of the kinetic energy *E*, and of the force function *U* will be written as follows:

$$\begin{aligned} E = &\frac{1}{2}\{\omega_1\}^T\{J_{O1}\}\{\omega_1\} + \frac{1}{2}\{\omega_{21}\}^T\{J_{C21}\}\{\omega_{21}\} + \frac{1}{2}m_{21}\{\dot{r}_{C21}\}^T\{\dot{r}_{C21}\} + \\ &+\frac{1}{2}\{\omega_{22}\}^T\{J_{C22}\}\{\omega_{22}\} + \frac{1}{2}m_{22}\{\dot{r}_{C22}\}^T\{\dot{r}_{C22}\} + \frac{1}{2}\{\omega_3\}^T\{J_{C3}\}\{\omega_3\} + \\ &+\frac{1}{2}m_3\{\dot{r}_{C3}\}^T\{\dot{r}_{C3}\} + \frac{1}{2}\{\omega_{41}\}^T\{J_{C41}\}\{\omega_{41}\} + \frac{1}{2}m_{41}\{\dot{r}_{C41}\}^T\{\dot{r}_{C41}\} + \\ &+\frac{1}{2}\{\omega_{42}\}^T\{J_{C42}\}\{\omega_{42}\} + \frac{1}{2}m_{42}\{\dot{r}_{C42}\}^T\{\dot{r}_{C42}\} \end{aligned}, \tag{20}$$

$$U = m_1 g y_{C1} + m_{21} g y_{C21} + m_{22} g y_{C22} + m_3 g y_{C3} + m_{41} g y_{C41} + m_{42} g y_{C42}, \tag{21}$$

in which y_{Ck} is the vertical coordinate of the mass center of body *k* in the mechanism (*k* = 1, 21, 22, 3, 41, 42) with respect to the base frame.

Notice that the expressions of *E* and *U* depend on ξ, since the positions of the mass centers change as ξ varies along the length of piston rod-cylinder.

When ξ varies in the interval $\left[0 \div l_2 - \dfrac{l_4}{2} + x\right]$:

$$m_{21} = \frac{m_2 \xi}{l_2};\ m_{22} = \frac{m_2(l_2 - \xi)}{l_2};\ m_{41} = m_4;\ m_{42} = 0, \tag{22}$$

$$\{r_{C1}\} = \begin{bmatrix} -l_1 \sin\theta \\ l_1 \cos\theta \end{bmatrix}, \tag{23}$$

$$\{r_{C21}\} = \begin{bmatrix} -l_1 \sin\theta + \dfrac{\xi}{2}\cos\varphi_4 \\ l_1 \cos\theta + \dfrac{\xi}{2}\sin\varphi_4 \end{bmatrix}, \tag{24}$$

$$\{r_{C22}\} = \begin{bmatrix} -l_1 \sin\theta + \xi\cos\varphi_4 + \dfrac{(l_2 - \xi)}{2}\cos(\alpha - \varphi_4) \\ l_1 \cos\theta + \xi\sin\varphi_4 - \dfrac{(l_2 - \xi)}{2}\sin(\alpha - \varphi_4) \end{bmatrix}, \tag{25}$$

$$\{r_{C3}\} = \begin{bmatrix} -l_1 \sin\theta + \xi\cos\varphi_4 + (l_2 - \xi)\cos(\alpha - \varphi_4) \\ l_1 \cos\theta + \xi\sin\varphi_4 - (l_2 - \xi)\sin(\alpha - \varphi_4) \end{bmatrix}, \tag{26}$$

$$\{r_{C41}\} = \begin{bmatrix} -l_1 \sin\theta + \xi\cos\varphi_4 + (l_2 - \xi + x)\cos(\alpha - \varphi_4) \\ l_1 \cos\theta + \xi\sin\varphi_4 - (l_2 - \xi + x)\sin(\alpha - \varphi_4) \end{bmatrix}, \tag{27}$$

$$\{\omega_1\} = \left[0,\ 0,\ -\dot{\theta}\right]^T, \tag{28}$$

$$\{\omega_{21}\} = \left[0,\ 0,\ -\dot{\varphi}_4\right]^T, \tag{29}$$

$$\{\omega_{22}\} = \{\omega_3\} = \{\omega_{41}\} = [0,\ 0,\ -\dot{\varphi}_4 + \dot{\alpha}]^T . \tag{30}$$

When ξ varies in the interval $\left(l_2 - \frac{l_4}{2} + x \div l_2\right]$:

$$m_{41} = \frac{m_4\left(\frac{l_4}{2} + \xi - l_2 - x\right)}{l_4};\ m_{42} = \frac{m_4\left(\frac{l_4}{2} - \xi + l_2 + x\right)}{l_4}, \tag{31}$$

$$\{r_{C41}\} = \begin{bmatrix} -l_1 \sin\theta + \xi\cos\varphi_4 - \left(\frac{l_4}{2} + \xi - l_2 - x\right)\frac{\cos\varphi_4}{2} \\ l_1\cos\theta + \xi\sin\varphi_4 - \left(\frac{l_4}{2} + \xi - l_2 - x\right)\frac{\sin\varphi_4}{2} \end{bmatrix}, \tag{32}$$

$$\{r_{C42}\} = \begin{bmatrix} -l_1 \sin\theta + \xi\cos\varphi_4 + \left(\frac{l_4}{2} - \xi + l_2 + x\right)\frac{\cos(\alpha - \varphi_4)}{2} \\ l_1\cos\theta + \xi\sin\varphi_4 - \left(\frac{l_4}{2} - \xi + l_2 + x\right)\frac{\sin(\alpha - \varphi_4)}{2} \end{bmatrix}, \tag{33}$$

$$\{\omega_{41}\} = [0,\ 0,\ -\dot{\varphi}_4]^T , \tag{34}$$

$$\{\omega_{42}\} = [0,\ 0,\ -\dot{\varphi}_4 + \dot{\alpha}]^T . \tag{35}$$

Finally, when ξ varies in the interval $\left(l_2 \div l_2 + \frac{l_4}{2} + x\right]$:

$$m_{21} = m_2;\ m_{22} = 0 , \tag{36}$$

$$\{r_{C21}\} = \begin{bmatrix} -l_1 \sin\theta + \frac{l_2\cos\varphi_4}{2} \\ l_1\cos\theta + \frac{l_2\sin\varphi_4}{2} \end{bmatrix}, \tag{37}$$

$$\{r_{C3}\} = \begin{bmatrix} -l_1 \sin\theta + l_2\cos\varphi_4 \\ l_1\cos\theta + l_2\sin\varphi_4 \end{bmatrix}, \tag{38}$$

$$\{\omega_3\} = [0,\ 0,\ -\dot{\varphi}_4]^T . \tag{39}$$

After replacing the partial and the total derivatives with respect to time of the terms in the Lagrange equations, the bending moment can be obtained as

$$M_{bd} = -\left[\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\alpha}}\right) - \frac{\partial E}{\partial \alpha} - \frac{\partial U}{\partial \alpha} - \frac{\delta W_\alpha\left(\vec{f}_t + \vec{f}_n\right)}{\delta\alpha}\right]_{\substack{\alpha=0 \\ \dot{\alpha}=0 \\ \ddot{\alpha}=0}}, \tag{40}$$

where $\delta W_\alpha\left(\vec{f}_t + \vec{f}_n\right)$ is the virtual work produced by $\vec{f}_t$, $\vec{f}_n$ corresponding to the supplementary mobility α,

$$\delta W_\alpha\left(\vec{f}_t + \vec{f}_n\right) = \left(\frac{l_4}{2} + l_2 - \xi + x\right)\left(f_t\cos\varphi_4 + f_n\sin\varphi_4\right)\delta\alpha \tag{41}$$

and $\delta\alpha$ is virtual movement corresponding to supplementary mobility α.

For determining the shear force along the piston rod-cylinder subsystem, the generalized coordinates $q_1 = \theta$, $q_2 = s$ are chosen, as shown in Fig. 3b. By proceeding in a similar way, the shear force can be also determined directly.

In the inverse dynamics, the link (1) is imposed by the law of motion expressed as

$$\theta = \Omega_0 t + \frac{\varepsilon_0}{2} t^2 , (rad) , \tag{42}$$

and the moment M_r has the expression

$$M_r = \frac{9.10^4}{\pi} \theta , (Nm) , \tag{43}$$

where $\Omega_0 = \frac{\pi}{180} , (rad / s)$; $\varepsilon_0 = \frac{\pi}{90} , (rad / s^2)$.

For a simulation, a system for controlling the aircraft elevator with the following geometric and inertia characteristics is considered:

$l_1 = 0.5(m)$, $l_2 = 1(m)$, $l_4 = 1(m)$; $m_1 = 2(kg)$, $m_2 = 4(kg)$, $m_3 = 1(kg)$, $m_4 = 5(kg)$.

By using MATLAB software, the bending moment M_{bd}, and the shear force R with respect to " $\frac{\xi}{\left(l_2 + {}^{l_4}\!/_2 + x\right)}$ " along the length of piston rod-cylinder subsystem are released as shown in Fig. 4 and Fig. 5, respectively. Based on the proposed method, the internal forces can be calculated at any position of the mechanism, corresponding to the value of the rotation angle θ. However, the paper showed results for the special case when $\theta = 0$ (rad), with the aim to compare them with the results calculated for the static system mentioned below. The variations of the normal constraint force $\vec{f}_n$, and the tangent constraint force $\vec{f}_t$ with respect to the rotation angle θ are also calculated as shown in Fig. 6, Fig. 7, respectively.

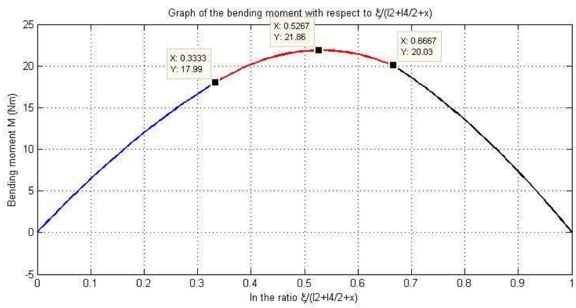


Figure 4: Variation of the bending moment along the piston rod-cylinder at *θ = 0* (rad)

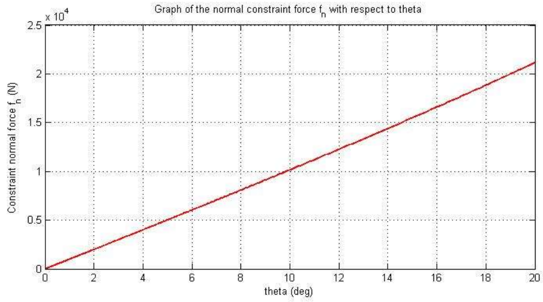


Figure 6: Variation of the normal constraint force versus rotation angle *θ*

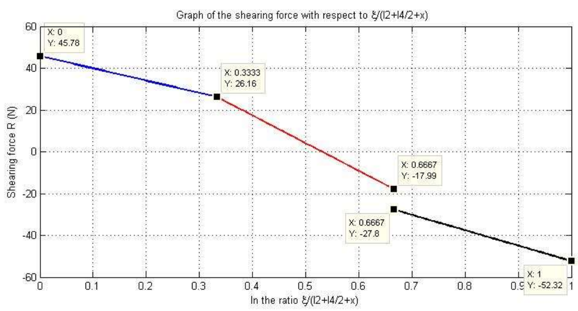


Figure 5: Variation of the shear force along the piston rod-cylinder at *θ = 0* (rad)

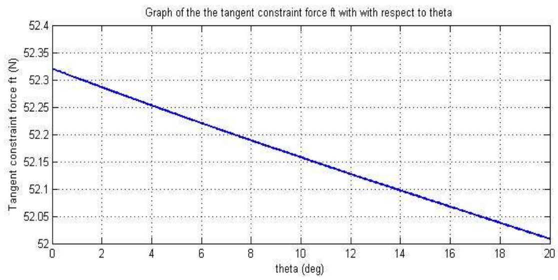


Figure 7: Variation of the tangent constraint force versus rotation angle *θ*

In order to verify the results above, the system for controlling the aircraft elevator at the position $\theta = 0$ (rad) is simplified as a static system, which is considered as a beam acted by distributed and concentrated forces. Then, by using the section method to compute manually the shear force and the bending moment, the results shown in Fig. 8 were obtained.

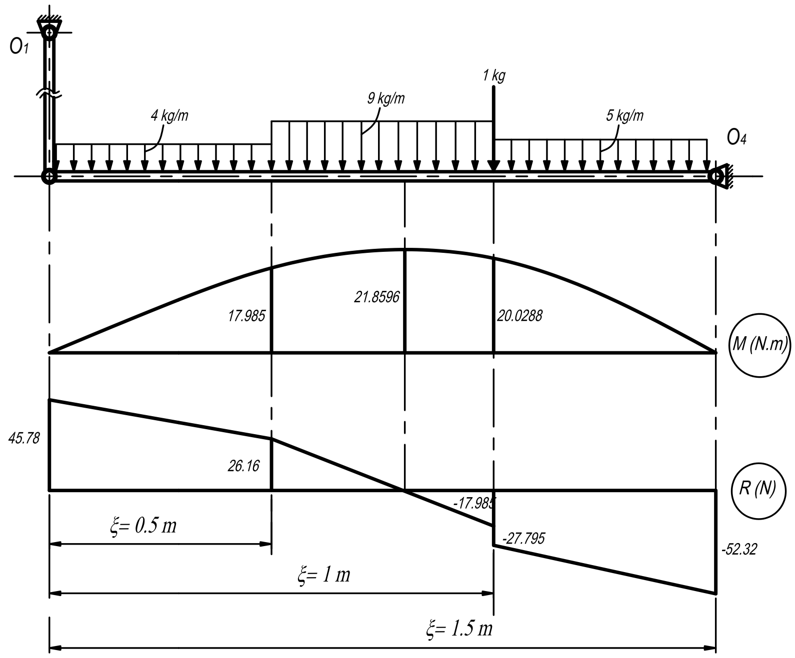


Figure 8: The internal force diagram of the simplified model

4 CONCLUSIONS

The paper presented a new method for determining the shear force and the bending moment in a mechanism, by using the Lagrange equations. Based on the method, internal forces in certain rigid bodies of a mechanism were calculated, in dynamic state, at any position, as well as at any moment of time, provided the supplementary mobilities are consistent with the constraints imposed to the mechanism. Besides, constraint forces at some positions can be calculated directly. Therefore, a closed mechanism can be transformed to an open one, which is more convenient for determining the kinematic relations.
The results obtained by using the proposed method are compared to the ones obtained by using a well-known method. So the validity of presented method is verified.

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