

Bézier-curve Navigation Guidance for Impact Time and Angle Control

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ABSTRACT

This paper addresses a novel impact time and angle control guidance law using a Bézier curve. The 2nd order Bézier curve consists of one control point and two boundary points; initial point P0, middle point P1 and end point P2. The curve is tangent to the line P0-P1 and the line P1-P2, respectively, and always exists in the convex hull of the control points. Proposed Bézier curve navigation guidance, makes the missile follow the Bézier curve with the feedback form of guidance command so that the missile hits the target on the desired time in the desired direction. We conducted numerical simulations on several terminal conditions to demonstrate the performance of proposed method.

KEYWORDS : *Bézier-curve navigation guidance, impact time, impact angle, anti-ship missiles*

NOMENCLATURE

Latin

x – x coordinates variable

y – y coordinates variable

a - coefficient for x

b - coefficient for y

c - coefficient

T_{go} - desired time-to-go

V - velocity

R_{go} - range-to-go

Greek

γ - flight path angle

λ - line of sight angle

τ - Bézier curve independent variable

Subscripts

f - terminal value

0 - initial value

go - terminal value – initial value

1 INTRODUCTION

In modern warfare, as high valuable targets can protect themselves effectively against the guided missile, the guided missile has evolved to use some smart strategies. For example, recent anti-tank missiles have top-attack mode; the turret on the tank is vulnerable in vertical direction. The anti-ship missiles attack the vessel simultaneously, SALVO attack, to saturate the target's defense resources. These strategies are achievable with special guidance laws which controls impact time or angle on the terminal phase.

The impact angle control (IAC) guidance law steers the missile in a vulnerable direction so that the target get fatal damage. Ryoo proposed an IAC guidance law minimizing time-to-go weighted energy through the optimal control theory in [1]. Manchester developed an IAC guidance that follows the

circular path in the homing phase in [2]. In the space launch vehicle area, the explicit guidance is often used for trajectory shaping. Ohlmeyer created the generalized vector explicit guidance (GENEX) which is available for the 3D IAC guidance [3]. A mixture guidance method employing the PN guidance for IAC has been proposed by Park in [4].

The impact time control (ITC) guidance makes the missile hit the target on a desired time. Through the ITC guidance, the SAVLO attack can be accomplished in an implicit way. A biased proportional navigation scheme [5][6], and some nonlinear control theory based method [7] have been developed for ITC.

It is more successful if one can hit the target simultaneously in the vulnerable direction of the target. Impact time and angle control (ITAC) guidance can do this. There were some attempts to control the impact time and angle together. J.-I. Lee, and et al., used jerk command scheme for ITAC guidance in [8]. T.-H. Kim proposed a polynomial guidance method for ITAC in [9].

In this paper, we propose a novel Bézier-curve navigation guidance impact time and angle control guidance law that uses a Bézier curve as a reference trajectory. BNG/ITAC consists of two phases; deployment phase and terminal BNG phase. On the deployment phase, guidance law makes the missile adjust the flight path to decrease the impact time error that is expected for the terminal BNG phase. In the terminal BNG phase, a feedback form of Bézier-curve navigation guidance law is employed to meet the desired impact angle.

This paper composed with following items; chapter 2 is about preliminary background, chapter 3 is BNG/ITAC guidance law, and chapter 4 is numerical simulation and the conclusion.

2 PRELIMINARY BACKGROUND

2.1 Problem Description

Consider the following two dimensional anti-ship missile engagement kinematics with a static target in Fig. 1. The vehicle can accelerate perpendicular to the flight path direction.

$$\dot{x} = V \cos \gamma, \quad \dot{y} = V \sin \gamma, \quad \dot{\gamma} = a / V \quad (1)$$

In this paper, we assume that the vehicle has a constant speed through the speed controller. Anti-ship missiles have a designed cruise speed so that they nominally fly with the designed speeds. By satisfying the following boundary condition, the missile can hit the target in the desired γ_f direction. The terminal time is specified as t_f .

$$\begin{aligned} x(t_0) &= x_0, \quad y(t_0) = y_0, \quad \gamma(t_0) = \gamma_0 \\ x(t_f) &= x_f, \quad y(t_f) = y_f, \quad \gamma(t_f) = \gamma_f \end{aligned} \quad (2)$$

The missile position is described in the inertial coordinate system. The guidance command is calculated on a guidance frame, where the x-axis is aligned in the flight path direction. Then the x-go and y-go are calculated on the G-frame as follows,

$$\begin{aligned} R_{go} &= \sqrt{(x_f - x)^2 + (y_f - y)^2} \\ \lambda_{LOS} &= \text{atan2}(y_f - y, x_f - x) \end{aligned} \quad (3)$$

$$\begin{aligned} x_{go} &= R_{go} \cos(\lambda_{LOS} - \gamma) \\ y_{go} &= R_{go} \sin(\lambda_{LOS} - \gamma) \end{aligned} \quad (4)$$

$$\gamma_{go} = \gamma_f - \gamma \quad (5)$$

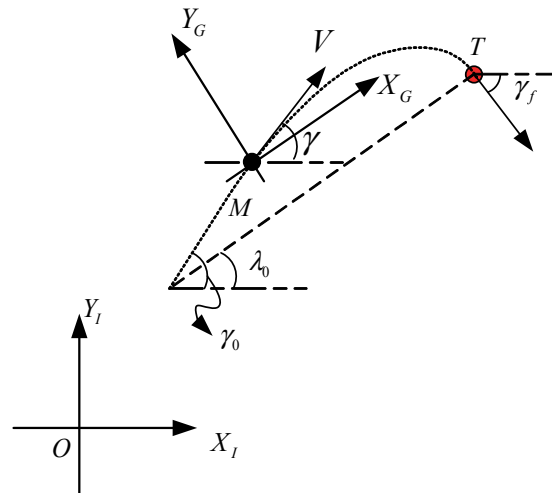


Figure 1: Planar Engagement Geometry

2.2 Bézier Curve

A Bézier curve is described with control points and two boundary points. This curve penetrates both boundary points in the tangential direction between the boundary point and the consecutive control point. Bézier curves always lie in the convex hull that is sequentially made with the boundary points and control points.

The 2nd order Bézier curve, quadratic Bézier curve, has two boundary points, and one control point so that the curve lies in a triangle. In general, this curve is expressed with the Bernstein polynomial form as follows,

$$\begin{aligned}\tilde{x} &= (1-\tau)^2 x_0 + 2(1-\tau)\tau x_c + \tau^2 x_f \\ \tilde{y} &= (1-\tau)^2 y_0 + 2(1-\tau)\tau y_c + \tau^2 y_f \quad (0 \leq \tau \leq 1)\end{aligned}\tag{6}$$

The Bézier curve independent variable τ varies from zero to one. This equation can be reformulated as following 2nd order polynomials for each coordinate variable.

$$\begin{aligned}\tilde{x} &= a_0 + a_1\tau + a_2\tau^2 \\ \tilde{y} &= b_0 + b_1\tau + b_2\tau^2 \quad (0 \leq \tau \leq 1)\end{aligned}\tag{7}$$

The quadratic Bézier curve is deterministic, if the two boundary points and two slopes one the points are given. Especially, the length of the quadratic Bézier curve is obtainable analytically as follows,

$$L_B = \int_0^1 \sqrt{\tilde{x}'^2 + \tilde{y}'^2} d\tau = \int_0^1 \sqrt{c_2 \tau^2 + c_1 \tau + c_0} d\tau \quad (8)$$

where

$$\begin{aligned} c_2 &= 4(a_2^2 + b_2^2) \\ c_1 &= 4(a_2 a_1 + b_2 b_1) \\ c_0 &= a_1^2 + b_1^2 \end{aligned} \tag{9}$$

The indefinite integral of above equation gives the length,

$$L(\tau) = \int \sqrt{c_2 \tau^2 + c_1 \tau + c_0} d\tau$$

$$= \frac{1}{8c_2^{3/2}} \left[2\sqrt{c_2} (2c_2 \tau + c_1) \sqrt{c_2 \tau^2 + c_1 \tau + c_0} + (4c_2 c_0 - c_1^2) \ln \left(\sqrt{c_2 \tau^2 + c_1 \tau + c_0} \right) + \frac{2c_2 \tau + c_1}{2\sqrt{c_2}} \right] \quad (10)$$

$$L_B = L(1) - L(0)$$

2.3 Acceleration Command on the Bézier Curve

Consider that missiles fly along the path which is function of τ , and the path can be expressed as,

$$x = \tilde{x}(\tau), \quad y = \tilde{y}(\tau) \quad (11)$$

The velocity of the missile is

$$V = \sqrt{\dot{\tilde{x}}^2 + \dot{\tilde{y}}^2}. \quad (12)$$

By the rule of chain, we obtain

$$V^2 = (\tilde{x}'^2 + \tilde{y}'^2) \dot{\tau}^2. \quad (13)$$

And the flight path angle and the slope of the path are related as follows,

$$\tan \gamma = \frac{\tilde{y}'}{\tilde{x}'} \quad (14)$$

Differentiating above equation gives,

$$\dot{\gamma} = \frac{1}{\sec^2 \gamma} \frac{\tilde{x}'' \tilde{y}' - \tilde{x}' \tilde{y}''}{\tilde{x}'^2} \dot{\tau}. \quad (15)$$

From Eq.(14), we have

$$\sec^2 \gamma = 1 + \tan^2 \gamma = \frac{\tilde{y}'^2 + \tilde{x}'^2}{\tilde{x}'^2}. \quad (16)$$

Substituting Eq.(13), (15), and (16) to $\dot{\gamma} = a/V$ gives the acceleration on the curve,

$$a_c = \frac{V^2 (\tilde{x}'' \tilde{y}' - \tilde{x}' \tilde{y}'')}{(\tilde{x}'^2 + \tilde{y}'^2)^{3/2}}. \quad (17)$$

If the curve is a quadratic Bézier curve, the command becomes,

$$a_{BNG}(\tau) = \frac{V^2 (2a_2(2b_2\tau + b_1) - (2a_2\tau + a_1)2b_2)}{(c_2\tau^2 + c_1\tau + c_0)^{3/2}} \quad (18)$$

If $\tau = 0$,

$$a_{BNG}(0) = \frac{2V^2(a_2b_1 - a_1b_2)}{(b_1^2 + a_1^2)^{3/2}} \quad (19)$$

3 BÉZIER-CURVE NAVIGATION GUIDANCE

In this paper, we propose a composite guidance scheme impact time control guidance law. It consists of two phases of guidance law. The 'deployment phase' adjusts the flight path so that the remaining time to fly in the terminal phase is to be the predicted time-to-go. The terminal phase employs the Bézier-curve navigation guidance so that the missile hit the target in desired direction.

3.1 Bézier-Curve Navigation Guidance

Bézier-curve navigation guidance control the missile to follow a quadratic Bézier curve, which begins at the current point and ends at the target point. Then the Bézier curve should satisfy following conditions.

$$\begin{aligned}\tilde{x}(0) &= a_0 = -x_{go}, \quad \tilde{y}(0) = b_0 = -y_{go} \\ \tilde{x}(1) &= a_0 + a_1 + a_2 = 0, \quad \tilde{y}(1) = b_0 + b_1 + b_2 = 0\end{aligned}\quad (20)$$

Let the slope of the curve at the starting boundary point be same with the current flight path angle, then,

$$\begin{aligned}\tilde{x}'(0) &= a_1, \quad \tilde{y}'(0) = b_1 \\ \tilde{y}'(0) / \tilde{x}'(0) &= \tan \gamma\end{aligned}\quad (21)$$

In the same manner, let the slope of the curve at the terminal boundary point be same with the desired impact angle.

$$\begin{aligned}\tilde{x}'(1) &= a_1 + 2a_2, \quad \tilde{y}'(1) = b_1 + 2b_2 \\ \tilde{y}'(1) / \tilde{x}'(1) &= \tan \gamma_f\end{aligned}\quad (22)$$

Solving the system of equations; Eq.(20), (21) and (22), determines the coefficients of the Bézier curve as,

$$\begin{aligned}\Delta &= \tan \gamma - \tan \gamma_f \\ a_1 &= -2(b_0 - a_0 \tan \gamma_f) / \Delta \\ a_2 &= -(a_0 \tan \gamma - 2b_0 + a_0 \tan \gamma_f) / \Delta \\ b_1 &= -2 \tan \gamma (b_0 - a_0 \tan \gamma_f) / \Delta \\ b_2 &= (b_0 \tan \gamma + b_0 \tan \gamma_f - 2a_0 \tan \gamma \tan \gamma_f) / \Delta\end{aligned}\quad (23)$$

If the curve is described on a guidance frame which is aligned to the missile velocity direction, the current flight path angle is zero, and the desired terminal angle becomes γ_{go} .

$$\gamma = 0, \quad \gamma_f = \gamma_{go}\quad (24)$$

again,

$$\begin{aligned}a_1 &= 2(-z_{go} + x_{go} \tan \gamma_{go}) / \tan \gamma_{go} \\ a_2 &= (2z_{go} - x_{go} \tan \gamma_{go}) / \tan \gamma_{go} \\ b_1 &= 0 \\ b_2 &= z_{go}\end{aligned}\quad (25)$$

Substituting these coefficients to Eq. (19) gives the acceleration command to follow the Bézier curve as,

$$a_{BNG} = \frac{V^2 z_{go} \tan^2 \gamma_{go}}{(-z_{go} + x_{go} \tan \gamma_{go})^2} = \frac{V^2 z_{go}}{(-z_{go} \cot \gamma_{go} + x_{go})^2}\quad (26)$$

Here, we define this guidance law as Bézier-curve navigation guidance (BNG). Fig. 2 shows the feasible impact angle region in the guidance frame. If the terminal flight path exists out of this region, there is no feasible Bézier curves that satisfy the given boundary condition. This region can be represented by following inequalities.

$$0 < \lambda_{LOS} - \gamma \leq \gamma_f - \gamma < \pi \quad (0 < y_{go}) \quad (27)$$

$$-\pi \leq \gamma_f - \gamma < \lambda_{LOS} - \gamma < 0 \quad (y_{go} < 0) \quad (28)$$

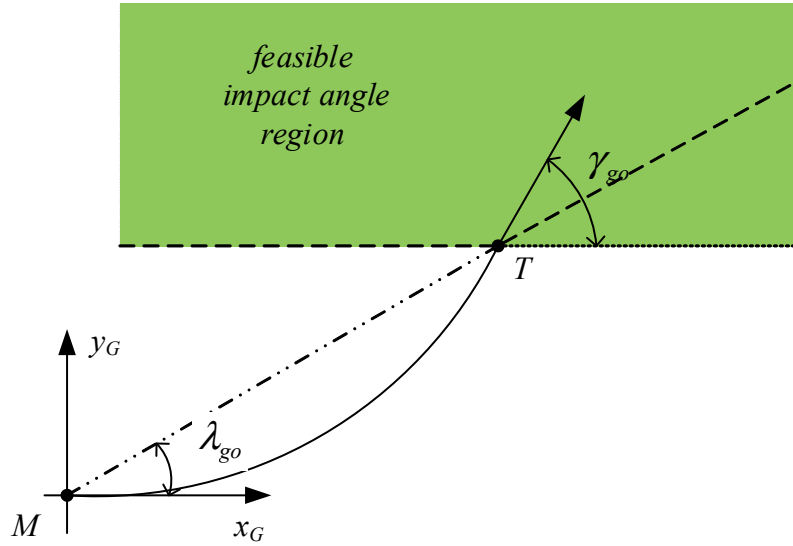


Figure 2: Terminal guidance phase, Bézier-curve navigation guidance feasible impact angle region schematic

3.2 Deployment Phase for Impact Time and Angle Control

The deployment phase control the flight path of the missile to reduce the time-to-go error of the terminal phase. The inner loop controls the flight path angle, and the outer loop controls the time to go error. The FPA error is defined as follows, and it is attenuated in proportional control scheme as,

$$\begin{aligned} \gamma_{err} &= \gamma_{cmd} - \gamma \\ a_{AP} &= \tau_\gamma^{-1} V \gamma_{err} = N_\gamma V \gamma_{err} \end{aligned} \quad (29)$$

The flight path angle time constant, τ_γ , is reciprocal to the navigation constant, N_γ . If γ_{cmd} is constant, the flight path angle will converges to the command similar to a first order system.

In the outer loop, FPA command is updated by the t-go error. The t-go prediction is obtained as,

$$\hat{t}_{go} = L_B(x, y, \gamma_{cmd}) / V \quad (30)$$

Here the predicted t-go is calculated with FPA command which will be meet when the terminal guidance phase starts. Then the time-to-go error is calculated as,

$$\varepsilon_{go} = T_{go} - \hat{t}_{go} \quad (31)$$

The flight path angle command is controlled as follows,

$$\gamma_{cmd} = \gamma_{cmd} + V \left(\frac{dL_B}{d\gamma} \right)^{-1} \frac{\varepsilon_{go}}{T_{go}} \quad (32)$$

The Bézier curve derivative respect to the FPA is calculated numerically, so it leads the FPA to converge a proper initiative FPA of the terminal guidance phase. When the FPA error decrease less than the tolerance values, it initializes the terminal guidance phase. At the phase, BNG is used to meet both the impact angle and the impact time. Fig. 3 the shows block diagram of the deployment phase. In this scheme the t-go Loop should be faster than the FPA loop.

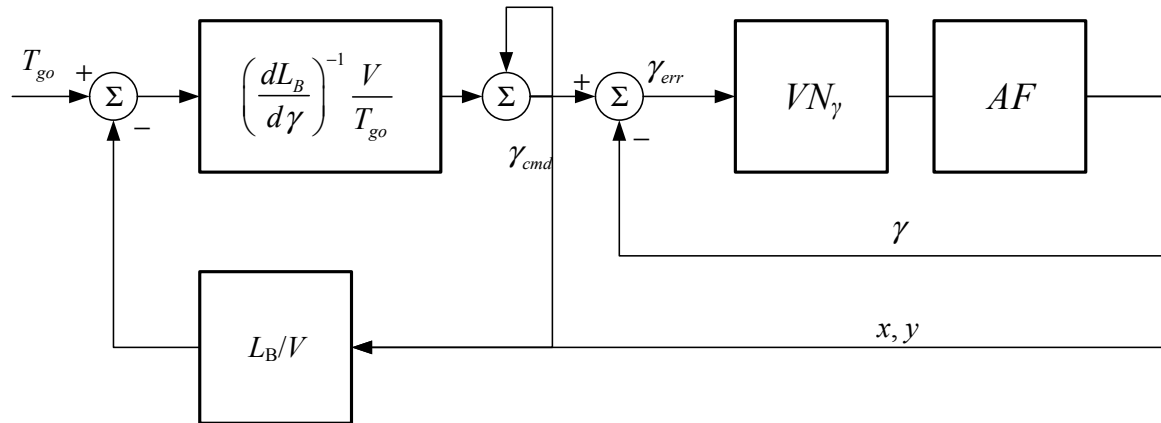


Figure 3: Deployment Phase Controller (Inner loop-FPA control, Outer loop-tgo error compensation)

4 SIMULATIONS

4.1 Simulation Scenarios

The proposed guidance law is simulated to demonstrate its performance. Table 1 shows the simulation scenarios. For each scenario, it is performed with the different impact time constraints.

Table 1: Table of simulation scenarios

	Scenario # 1	Scenario # 2
X Pos (m) (initial/terminal)	0 / 10000	
Y Pos (m) (initial/terminal)	0 / 0	
FPA (deg) (initial/terminal)	0 / -30	0 / -60
N_γ	1	1
t_f (sec)	70 80 90 100	70 80 90 100

4.2 Simulation Results

Fig. 4 shows the trajectories of scenario 1. As shown in the figure, BNG guided the missile toward the desired target along the proper Bézier curve. Fig. 5 shows the FPA of scenario 1. The FPAs at the final moment are almost -30 deg for each case, and each case satisfies desired impact time closely.

Fig. 6 shows the acceleration command of scenario 1. At the beginning large guidance command is applied at the deployment phase. After then, a bell shaped acceleration command is exerted by BNG. At the end point, the sharp jump is related to the cross over between the missile and target.

Fig. 7 shows the trajectories of scenario 2. Fig. 8 shows FPAs of scenario 2. It shows that all desired impact angle and impact time is generally meet the reference command. Fig. 9 shows the guidance commands for scenario 2.

Table 2 shows the summary of the simulation results. It says that the given constraints are satisfied with the proposed method.

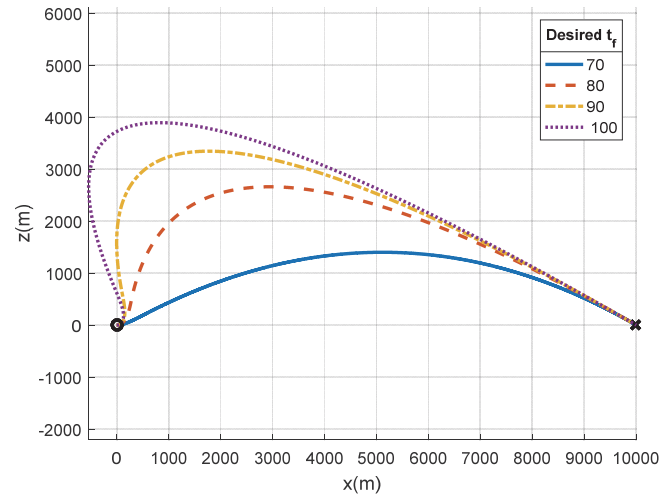


Figure 4: trajectories comparison to different impact time (Scenario 1)

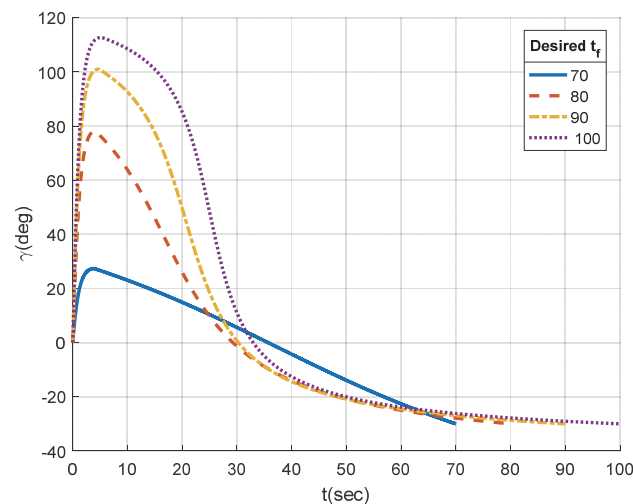


Figure 5: FPA comparison to different impact time (Scenario 1)

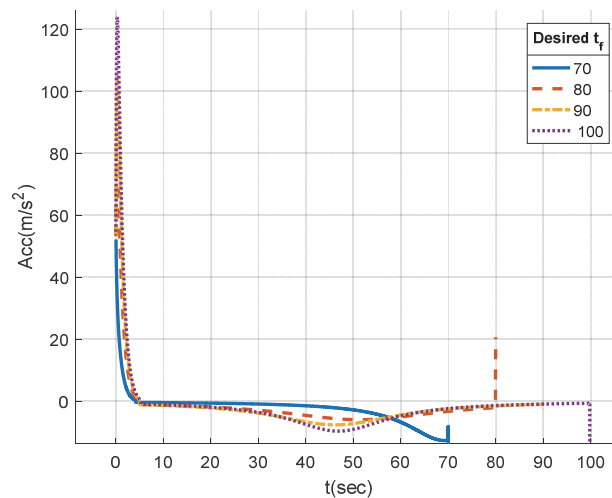


Figure 6: Acceleration command comparison to different impact time (Scenario 1)

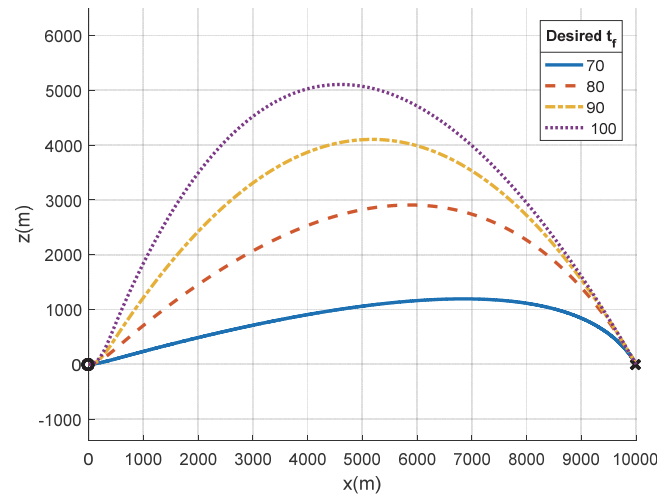


Figure 7: trajectories comparison to different impact time (Scenario 1)

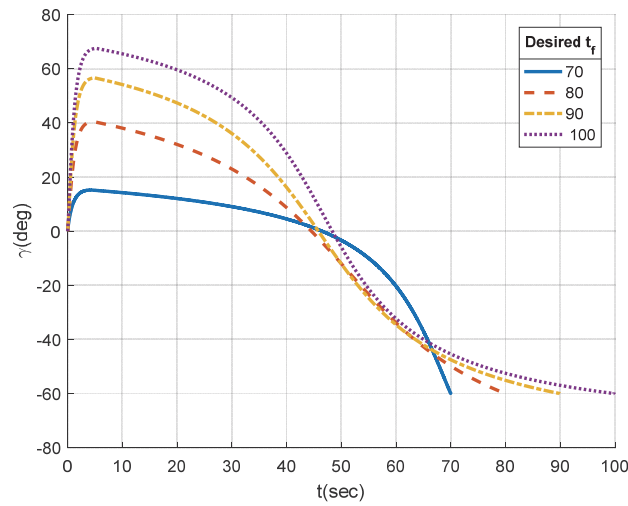


Figure 8: FPA comparison to different impact time (Scenario 2)

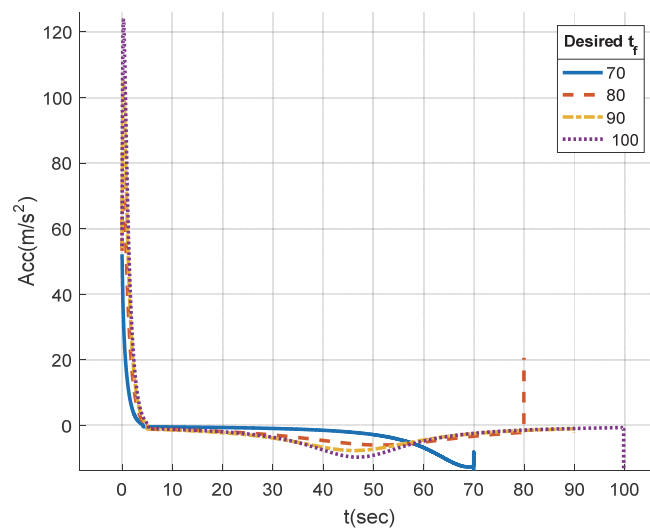


Figure 9: Acceleration command comparison to different impact time (Scenario 2)

Table 2: Summary of the simulation results

	Scenario # 1				Scenario # 2			
desired impact time (sec)	70	80	90	100	70	80	90	100
terminal FPA (deg)	-30	-30.07	-30	-30	-59.97	-59.98	-60.2	-60
t_f (sec)	69.99	80.02	89.98	99.88	69.96	79.94	89.88	99.74

5 CONCLUSION

This paper proposed a composite guidance style impact time and angle control guidance law, named as Bézier-curve navigation guidance impact time and angle control (BNG/ITAC). The BNG/ITAC consists of two phases; deployment phase and terminal BNG phase. On the deployment phase, the flight path is controlled to reduce the t-go error expected in the terminal BNG phase. At the terminal phase, the BNG guidance is applied to meet the desired impact angle. The performance of BNG/ITAC is demonstrated through the numerical simulation. The impact angle and impact time are all satisfied relatively with a little error, due to numerical reason.

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