Reference Trajectory Optimization Using the Particle Swarm Optimization

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ABSTRACT

Aircraft requires fossil fuel to power engines. This fact brings as a consequence releasing polluting particles to the atmosphere. Among these particles, carbon dioxide has gained special importance due to its contribution to global warming. Aware of this problem, the aeronautical community has been improving over time to reduce fuel consumption. One way of reducing fuel requirements is by improving the aircraft's reference trajectory. Finding the altitudes and speeds that minimize the fuel consumption needed to fly a given mission brings as consequence reducing the pollution released to the atmosphere. There is also the advantage of reducing the flight cost. This paper proposes an algorithm able to find those speeds and altitudes while fulfilling the RTA constraint by using the particle swarm optimization algorithm. This paper has two main objectives. The first objective is to analyze if the algorithm is able to find more economical solutions than the shortest path trajectory by taking weather into consideration. The second objective is to observe if the algorithm provides trajectories respecting current flight constraints. Fuel consumption was computed by interpolating data obtained from a numerical performance database. Weather information was obtained from data provided by Weather Canada. Results showed that the algorithm was able to find economical trajectories respecting their current traffic constraints.

KEYWORDS: **TRAJECTORY OPTIMIZATION, PARTICLE SWARM ALGORITHM, FUEL CONSUMPTION, OPERATIONS, AIRCRAFT**.

NOMENCLATURE

B_x –Leading particles position
c_x – Particle's position fixed influence
B_x – Leading particles position
Rx – Leading particles random influence
ff_l – Fuel flow lower limit
ff_H– Fuel flow higher limit
S_d–Distance segment
W_l– Aircraft weight lower limit
W_H– Aircraft weight higher limit

Greek
φ– Aircraft heading
ε– Flight time tolerance
ω–Inertia value

Subscripts
i - Dimension
k – Current iteration

1 INTRODUCTION

Human development has brought the undesirable consequence of pollution. This is the case of burning fossil fuel to power engines for industries such as aviation, automotive, and transportation in general. Among the polluting particles released to the atmosphere, we can find Carbon Dioxide (CO_2), Nitrogen Oxides (NOx), Sulphur, Hydrocarbons, and etcetera. In the last years, CO_2 has been given special importance due to its contribution to global warming and wind patterns changes. For this reason, the aeronautical industry has set itself the goal of reducing CO_2 emissions to 50% of those recorded in 2005 by the year 2050[1]. This ambitious goal requires different improvements in manyengineering areas, especially as it is expected that aircraft number will increase in the next years[2]. New generation of aircraft will not be able to meet these goals by itself as a result of aerospace industry[3]. For these reasons, improvements in operations should as well be conducted to meet pollution goals.

As a matter of fact, there are studies analyzing continental United States flights, where it was concluded that airline politics and airspace constraints do not allow aircraft to fly at their optimal speeds and altitudes[4, 5]. Another study in Turkish airspace concluded that aircraftsmay load morefuel than needed, resulting in more expensive flights [6].

Before taking off, there are specialized airline teams which compute the most economical trajectory by taking into account different aspects such as weather, loads, flight distance, and air traffic restrictions. These trajectories are also known as flight plans. Air Traffic Management (ATM) approves, or provides modification to the flight plan before it can be followed. Just before taking-off, the crew loads the authorized flight plan in a device called Flight Management System. The flight plan (or reference trajectory) can only be changed prior vocal Air Traffic Control (ATC) authorization. The change on the trajectory can be performed for either, ATC or crew request.

Different algorithms in the literature have addressed the aircraft reference trajectory optimization.One of the first reference trajectories studies consisted in the constant descent cruise concept [7]. Lidenwas a pioneer in this research where he computed the optimal aircraft altitude [8], the influence of winds on the optimal trajectory [9]; he developed an algorithm able to optimize trajectories while fulfilling the Required Time of Arrival (RTA) constraint [10]. The RTA is, as its names suggests, a constraint which requires the aircraft to arrive at a given waypoint at a given time. As pointed out by Liden, winds are important to reduce fuel burn. Headwinds reduce the ground speed, and tailwinds increment it. Staying more time airborne logically requires more fuel. Important wind currents, called jet streams, were later analyzed,and it was shown that by successfully following them (or avoiding them if they were headwinds) could lead to important fuel and flight time savings[11].

The Continuous Descent Approach (CDA) corresponding to a constant path instead of the typical step-descent approach has proved to give important fuel savings, and to reduce noise [12, 13]. An algorithm based on soft dynamics programming with neural network was developed while fulfilling the RTA constraint [14]. An algorithm able to optimize the cruise trajectory for an aircraft while avoiding obstacles was developed using the Dijkstra's algorithm [15].Reducing the Mach number can help reducing fuel burn for long flights, however flight time will be increased [16]. Another study optimized the Mach number for a constant altitude cruise, similarly, fuel burn reductions were reported due to aircraft speed reduction [17]. Not only speed, but also altitude is helpful to reduce fuel burn. In [18], an algorithm able to find the optimal constant cruise altitude and mach number was developed. The beam search algorithm was implemented to execute step climbs (changes in altitude during cruise) to a pre-defined route in order to save fuel [19]. The golden search section was used to find the optimal altitudes for short flights [20]. The Dijksta's algorithm was also used to optimize the lateral reference trajectory by taking advantages of winds[21].

There are algorithms such as those exposed in [14] which optimize the flight trajectory by changing altitude (vertical reference trajectory), and the lateral reference trajectory (these are also called 3D trajectories). As shown in [22] where the optimal lateral reference trajectory is computed, then its corresponding optimal vertical reference trajectory is computed. There are also algorithms that optimize the trajectory for constant altitudes while modifying the aircraft's heading, direction, and altitude (3D) in order to provide trajectories avoiding contrails (clouds generated by aircraft) formation zones [23, 24]. Dynamic programming was also used to optimize flights in 3D, and to meet the RTA constraint (4D flight) [25].Multiphase Mixed-Integer Optimal Control was also been implemented to optimize a 4D trajectory by taking into account contrails [26].

Most of the algorithms above mentioned are deterministic algorithms. There is another type of algorithms called metaheuristic algorithms. These algorithms are normally based on nature findings. These algorithms are functioning by exploring the search space, and by adding random values for its exploration when local optimal solutions are obtained. Genetic algorithms were implemented in order to optimize the lateral reference trajectory [27], the 3D reference trajectory with constant Mach number [28], and with varying Mach number[29].The artificial bee's colony was implemented to optimize the vertical reference trajectory while fulfilling the RTA constraint [30]. The ants' artificial colony optimization algorithms were used to optimize the Mach number for a constant altitude flight [31] and to optimize the lateral reference trajectory [32].

This paper has two main objectives. The first objective is to analyse if the algorithm is able to find more economical solutions than the shortest path trajectory while fulfilling the RTA constraint. The second objective is to observe if the algorithm provides trajectories respecting current flight constraints such as trajectories that do not change altitudes or headings too often by creating zig-zag like trajectories. It is desirable to have trajectories that keep their heading and their altitude constant for certain flight segments.

This paper is divided in different sections. Firstly, the Aircraft Fuel Burn model is described. Secondly, the optimization problem is defined. Thirdly, the search space, the fuel burn methodology, and the Particle Swarm Optimization algorithm used to find the optimal trajectory are explained. Finally, results and conclusions are given.

2 AIRCRAFT FUEL BURN MODEL

The fuel burn model is given under the form of database. As this algorithm will focus on the cruise phase, only the Mach cruise phase, the Mach climb phase, and the Mach descent phase will be the ones used.

This database is also called *numerical performance model*, and it was developed from experimental flight data. This database can be represented as a black box as shown in Fig. 1.

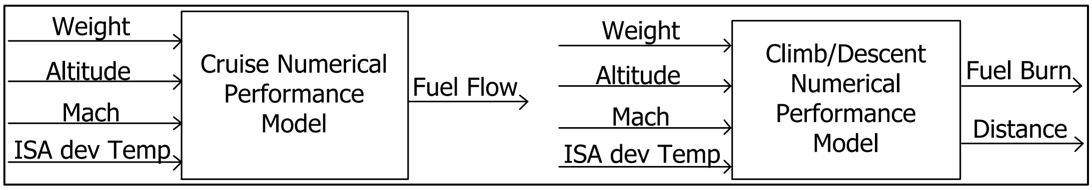

Figure 1: Numerical performance model cruise and climb/descent phases

All inputs must be provided in order to obtain the desired outputs. These inputs define the current aircraft's flight envelope and consist of altitudes, Mach numbers, gross weights, and the ISA standard temperature deviations (ISA dev). The outputs are thefuel flow for cruise phase, the fuel burn, and the horizontal traveled distance to climb from the reference altitude (normally 0 ft.) to the targeted altitude.

3 PROBLEM DEFINITION

As the goal of the algorithm is to find the combinations of waypoints that reduce the fuel burn, the objective function can be written as shown in next Eq. 1 – Eq. 5.

$$min(\, Fuel\ Burn) \tag{1}$$

Subject to:

$$ETA \ = RTA \pm \varepsilon \tag{2}$$

$$MACH \ \in MACH_{DB} \tag{3}$$

$$Altitude \ \in Altitude_{DB} \tag{4}$$

$$Waypoints \ \in Grid \tag{5}$$

wherethe Estimated Time of Arrival (*ETA*) should meet the Required Time of Arrival (*RTA*) within a given error (ε). The selected Mach numbers and altitudes should be available in the numerical performance model (*DB*), and the selected waypoints should be contained in the grid which will be described in Section 4.2.

4 METHODOLGY

This section aims to describe the way in which the fuel burn is computed from the aircraft model, the search space and the optimization algorithm.

4.1 Fuel burn computation

The numerical performance database described above gives information only if the inputs have exactly the expected values. For those values that are not the exact ones as those of the numerical performance model, linear interpolations are performed in order to obtain the required outputs. For example, if the fuel flow for a given segment is required for a weight W_r, by considering that altitude, Mach number and temperature are the same, the process in Fig. 2 is followed:

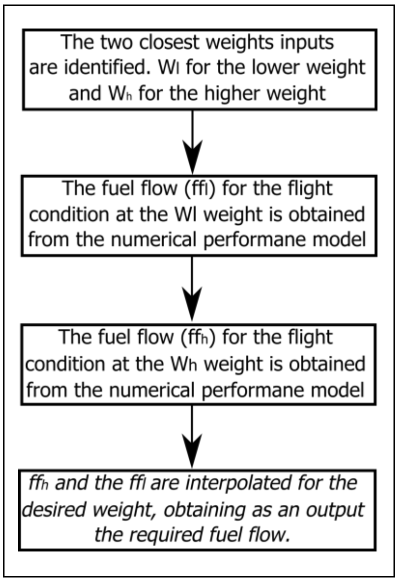

Figure 2. Fuel flow computation for a required weight

With the fuel flow knowledge, it is possible to determine the fuel burn to fly a segment (S_d) by computing the ground speed with Eq. 6.

$$GS = TAS \pm WS * (\varphi - WD) \tag{6}$$

Where *GS* stands for Ground Speed, *WS* for Wind Speed, *WD*for the wind angle, and φ is the aircraft's heading.

The fuel required to fly a constant altitude cruise segment (S_d) is then given by Eq. 7:

$$Fuel\ Burn = Fuel\ Flow * \left(\frac{S_d}{GS}\right) \tag{7}$$

Finally, the fuel burn sum of all segments provides the total fuel burn. A more detailed discussion about the way of computing fuel burn using a numerical performance model can be found in [33].

4.2 The Search Space

The search space is modeled under the form of a unidirectional weighted graph *G* (*V*,*E*) where *V* are the available nodes where the aircraft can fly to, and *E* are the links between every node. For the algorithm, every node is connected to its consecutive neighbors. The weights defined under the graph concept (not the aircraft weight) are defined as the flight cost to fly from a given waypoint to the other waypoint.

For this paper, this graph (search space) is created using a reference trajectory as it will be shown later. Two graphs can be identified, the vertical and the lateral. The vertical graph consists in flight's distance and altitude, while the lateral graph consists in the geographical coordinates. The vertical graph is shown in Fig. 3.

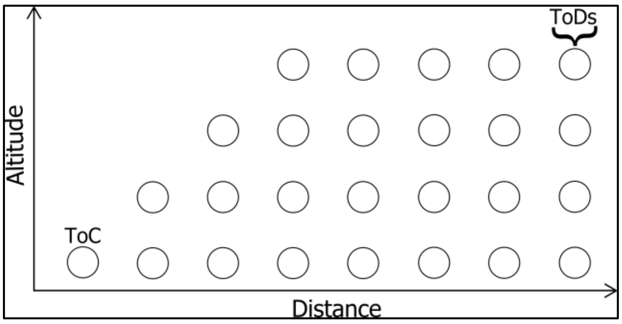

Figure 3: Vertical reference trajectory graph

The lateral graph is shown in Fig 4.

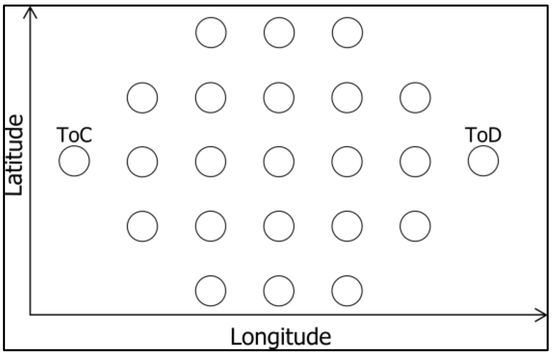

Figure 4: Lateral reference trajectory graph

Combining both graphs bring as a consequence a 3D search space described in Fig. 5.

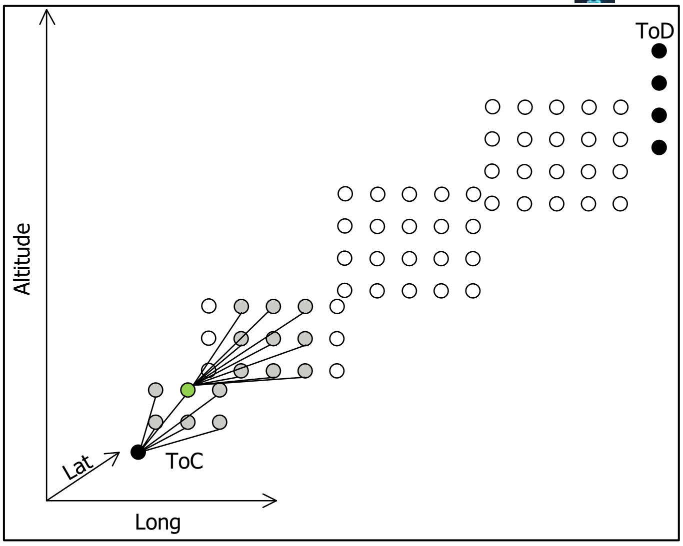

Figure 5: 3D reference trajectory graph

The aircraft begins the flight at the Top of Climb (ToC), and ends the flight at any of the available Top of Descents (ToDs).

4.3 The Optimization Algorithm

The optimization algorithm is based on the Particle Swarm Optimization (PSO) theory. The PSO algorithm reproduces the social behavior of animals, such as the fish school or the displacement of bird flock. The operationof this algorithm is different than the operation of other social algorithms.Contrary to ant colony or bee colony algorithms which are also based on the intelligence of a group, the PSO algorithm hasa leader. This leader can be replaced if another member of the group is better. In nature, this replacement can be seen as the bird flock leader rotates as he gets tired.

For the aircraft trajectory optimization, each particle represents a trajectory. There is also the local leader, which is defined as the most economical particle from a group of particles. Different groups form a set. Thus, the global leader is the most economical particle of the whole set of particles.

The PSO works using iterations. At every iteration, each particle moves within the search space from its current position towards the leaders. In other words, the particle's motion is influenced by its current position, and also by the leader positions. The algorithm mutates the trajectory for each dimension (i) using iterations. Dimensions are altitudes, geographical coordinates, and Mach numbers. This motion can be defined with Eq.8.

$$M_i(k+1) = \omega R_\omega M_i(k) + c_1 R_1\big(B_l - X_i(k)\big) + c_2 R_2\big(B_g - X_i(k)\big) \qquad (8)$$

where $i$is the *current*particle, and $k$is the current iteration. $M$is the computed motion,X is the particle's location,$B_g$is the global optimal particle position,$B_l$is the best local particle position,c_1 and c_2 are positive constants, which influence the particle'smotion with respect to the global and the local leaders. Parameter ω refers to the particle's inertia. Three random parameters calledR_ω, R_1 and R_2 (with values between 0 and 1)determine the influence of ω, B_l, and B_g respectively, on the particle's motion.The new particle position is then defined with Eq. 9:

$$X_i(k+1) = X_i(k) + M_i(k+1) \qquad (9)$$

The algorithm first *mutates* the trajectory in aircraft geographical coordinates and altitude, and thenmodifies the Mach numbers in order to meet the RTA constraint. The algorithm is defined in the next steps.

1. Initial trajectories are generated.
    a. 25 random trajectories are generated.
    b. Those trajectories are assigned to a group.
    c. If the maximal number of groups are reached, the algorithms goes to step 2, otherwise, step 1a is executed.
2. The trajectories' flight costs are computed.
3. The most economical trajectory for each group is selected as the *local optimal trajectory*.
4. The most economical trajectory from all trajectories is selected as the *optimal trajectory*.
5. The displacement (motion) for each trajectory in each dimension is computed with Eq. 8.
6. Each trajectory position is updated with Eq. 9.
7. The Mach number is modified with the aim to reach the RTA constraint.
8. The new trajectories costs are computed.
9. The most economical trajectory for each group is selected as the *local optimal trajectory*.
10. The most economical trajectory from all trajectories is selected as the *optimal trajectory*.
11. If the maximal number of iterations is reached, the algorithms deliver the most economical 4D trajectory, otherwise, Step 5 is executed.

The algorithm provides the most economical combination of Mach numbers that fulfill the RTA constraint within the given limits ε.

Since the algorithm goal is to fulfill the RTA constraint, it is desirable that the algorithm selects as leaders the economical trajectories having a realistic possibility to fulfill the RTA constraint. The trajectory cost is then re-defined as shown in Eq. 10.

$$Flight\ Cost = Fuel\ Burn + \ Penalty_{RTA} \tag{10}$$

wherethe parameter $Penalty_{RTA}$ can take three different values depending on the ETA and RTA relationship.

If the ETA is equal to the RTA, the $Penalty_{RTA}$ value is equal to 0.

If the ETA is within the RTA limits, then Eq.11 is used.

$$Penalty_{RTA} = abs(ETA - RTA) * Fuel\ Burn \tag{11}$$

wherethe ETA and RTA difference is given in *hours.*
If the ETA is over the RTA limits, Eq. 12 is used.

$$Penalty_{RTA} = 1.1 * Fuel\ Burn + \ abs(ETA - RTA) * Fuel\ Burn \tag{12}$$

It should be remarked that this $Penalty_{RTA}$ is a fictitious cost that is only taken into account to reject trajectories not meeting the RTA constraint.

5 RESULTS

The algorithm was tested using the numerical performance model of a long haul, 2 engines aircraft with a maximal takeoff weight and a ceiling altitude of 41,000 ft.
The analyzed flights are described in Table 1.

Table 1: Great Circle Trajectories Between Analyzed Flights

Number of flight	City Pairs		Distance (nm)
	Departure Airport	Arrival Airport	2981
1	Montreal (YUL)	Paris (CDG)	2981
2	Montreal (YUL)	Vancouver (YVR)	1990
3	Toronto (YYZ)	London (LHR)	3085
4	London (LHR)	Cancun (CUN)	1409
5	Boston (BOS)	Ponta Delgada (PDL)	2080
6	Calgary (YYC)	Gran Canaria (LPA)	4427
7	Montreal (YUL)	Amsterdam (AMS)	2975
8	Chicago (ORD	Amsterdam (AMS)	3574
9	Toronto (YYZ)	Innsbruck (INN)	3611
10	Amsterdam (AMS)	Cancun (CUN)	4477
11	Montreal (YUL)	Bucharest (OTP)	3943

5.1 A case study

In this results Section, it is intended to observe a solution in detail. In order to observe it, the 11thflight taking place from Montreal to Bucharest was selected. The reference flight reported a cost of 64,031 kg. The optimized cost was of 58,025 kg, meaning a 9.37 % of fuel burn savings. The RTA constraint was fulfilled within 14 seconds. As shown on Fig. 6, the reference trajectory is at the fixed altitude of 32,000 ft, whereas the optimal trajectory altitude begins at 38,000 ft. After having flown around 800 nm, a step climb is required for the aircraft to continue its flight at the altitude of 40,000 ft.

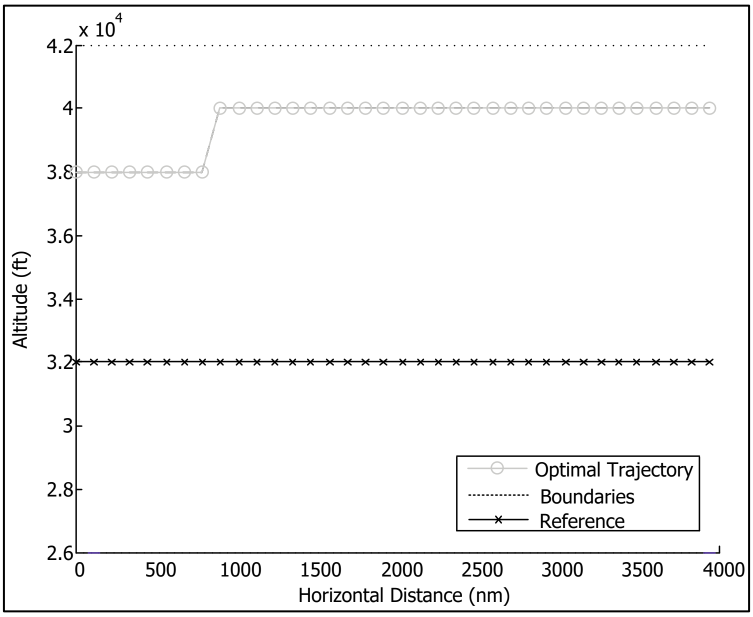

Figure 6: Vertical Reference Trajectory for the Great Circle and the Optimal Trajectory

The lateral reference trajectory can be seen in Fig. 7.

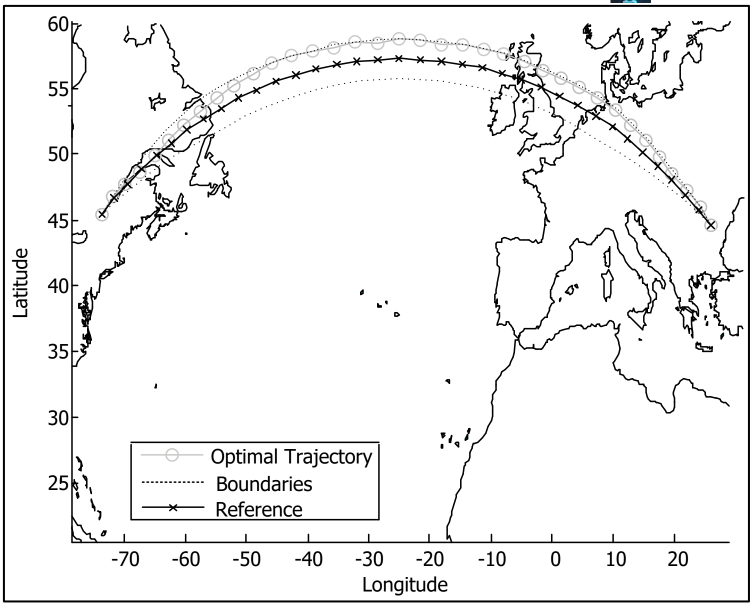

Figure 7: Lateral Reference Trajectory for the Great Circle and the Optimal Trajectory

Fuel savings can be explained as beginning the flight at a higher altitude and changing the flight level to altitudes where the engines are more efficient. The optimal lateral reference trajectory was computed by placing it near the upper search space limit. The reason is that winds are favorable in that region.

5.2 Fuel Burn Savings for Different Flights

This set of results presents the optimization algorithm potential on saving fuel burn. Different trajectories were selected, and their flight cost was computed following the geodesic route (shortest path) at 32,000 ft at a Mach number of 0.8. Fuel burn and savings for different flights are shown in Fig. 8.

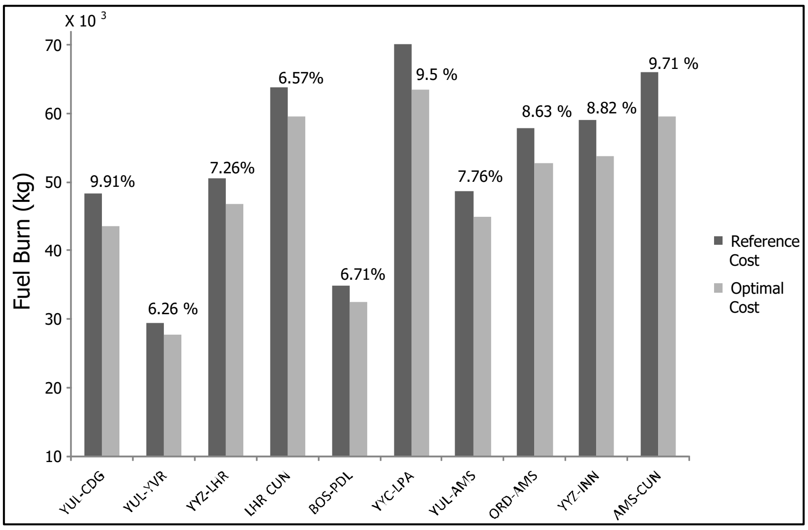

Figure 8: Fuel Burn Savings for Different Flights

All flights are optimized using this algorithm. The maximal fuel saving is of 9.91%, the minimal optimization is of 6.26%, thus the average optimization is of 8.11%. The optimization percentage (%) is normally dependent on the wind influence. The results shown in Figure 8 are optimistic as the reference trajectory does not execute step climbs.

All flights as well were able to reach the RTA constraint. For them, the tolerance was set to 15 seconds. The RTA constraint difference can be seen in Table 2.

Table 2: Required Time of Arrival Fulfillment for Different Flights

Flight	Reference Arrival Time (hh:mm:ss)	RTA hh:mm:ss	Optimal Arrival Time (hh:mm:ss)	RTA Difference (s)
YUL-CDG	16:50:43	16:54:00	16:53:45	-14
YUL-YVR	14:03:40	14:03:00	14:02:47	-13
YYZ-LHR	17:03:32	17:07:48	17:07:34	-13.
LHR-CUN	21:26:48	21:57:36	21:57:27	-9
BOS-PDL	17:17:27	17:16:12	17:15:57	-14
YYC-LPA	18:06:54	18:08:13	18:07:58	-14
YUL-AMS	15:00:06	15:01:48	15:01:33	-14
ORD-AMS	16:19:36	16:19:48	16:19:33	-14
YYZ-INN	15:37:45	15:36:36	15:36:21	-14
AMS-CUN	16:17:28	16:21:00	16:20:54	-6

For all flights, the RTA constraint was fulfilled. The optimal ETA was always reached before the RTA. This fact could be explained as the aircraft consumes less fuel as discussed in [16, 17].

5.3 Fuel Burn Savings for Real Flights.

The last set of tests were aim to evaluate the PSO flight optimization against real trajectories obtained from flightaware®. The flight information for three long-haul flights: Calgary (YYC) to Cancun (CUN), Edmonton (YEG) to Punta Cana (PUJ), and YYC to Varadero (VRA) is shown in Table 3. The RTA objective was the reported time of arrival for the selected as flown flights. For the algorithm a tolerance of 30-second was provided.

Table 3 4D trajectory optimization RTA fulfillment

Flight	Reference Flight Fuel Burn (kg)	Optimized Flight (kg)	Savings (kg)	RTA	4D ETA	RTA Difference (s)
YYC-CUN	29,123	28,719	404 (1.39%)	22h07m46s	22h07m56s	10
YEG-PUJ	42,170	41,594	576 (1.37%)	19h44m48s	19h44m33s	15
YYC-VRA	33,458	32,849	609 (1.82%)	15h40m54s	15h40m31s	23

As it can be observed in Table 3, the algorithm was able to reduce the fuel burn and meet the RTA constraint within the imposed 30-second limit. This performance was possible because the algorithm could give better altitudes and take advantage of winds.

Comparing these results against the ones shown in Fig. 8, it can be seen that evidently, the fuel saved diminished. This is normal as the trajectories in Table 3 flight at realistic altitudes and lateral trajectories. Nevertheless, results in Table 3 show that there exists the opportunity to improve those trajectories and save fuel.

The lateral and the vertical reference trajectories proposed by the 4D reference trajectory algorithm are compared to the as-flown trajectory for one flight in Fig.9 and Fig. 10, respectively. The optimal trajectory is represented in orange and the as-flown trajectory is shown in blue

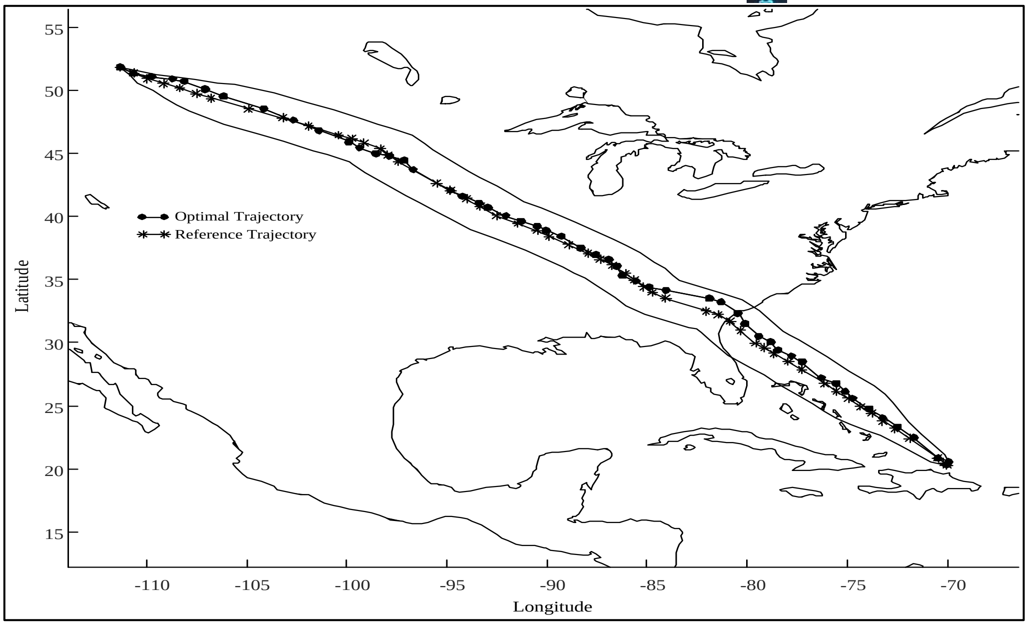

Figure 9.Lateral reference trajectory and the search space limit for the Calgary to Punta Cana flight

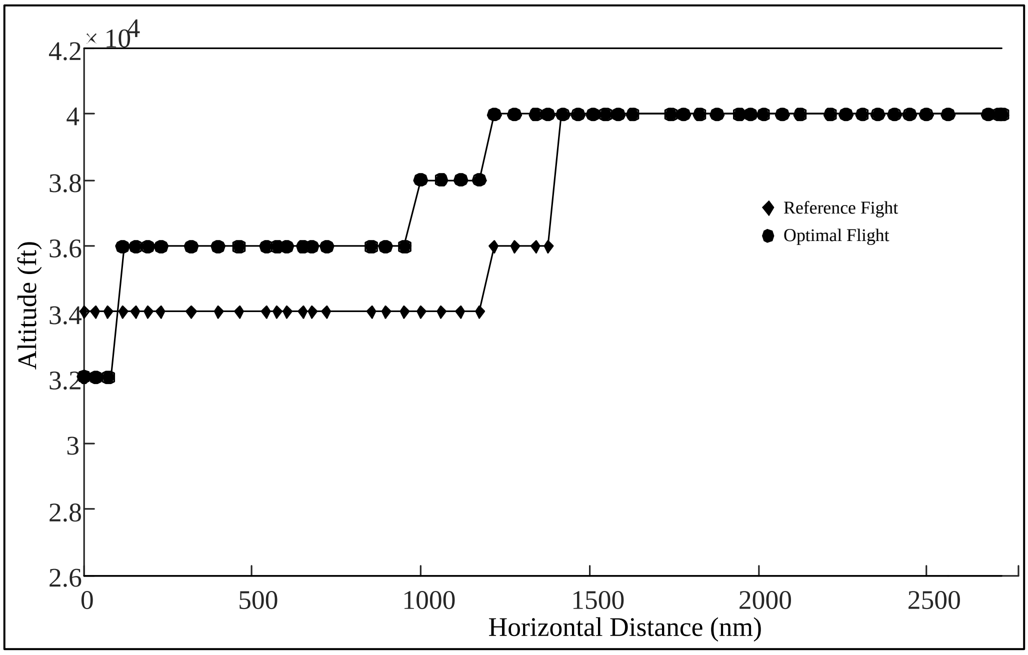

Figure 10.Vertical reference trajectory and the search space limits for the Calgary to Punta Cana flight

As shown in Fig.9, the optimal lateral trajectory diverges slightly from the reference trajectory; in this case, due to favorable winds. The vertical reference trajectory is quite similar to the real flight trajectory as shown in Fig. 10, with the main difference that the developed algorithm proposed beginning the cruise phase at 32,000 ft instead of 34,000 ft. The vertical trajectory delivered by the PSO algorithm proposed three 2,000 ft step climbs, and the *reference* algorithm executed only two step climbs: one 2,000 ft step climb, and one 4,000 ft step climb. Both flights ended at the same altitude (40,000 ft). These small differences provided the fuel savings reported in the last column of **Table 3**

6 CONCLUSIONS

This article gives three different conclusions. The first one is that the algorithm is able to provide economical trajectories. The second one is that the algorithm is able to fulfill the RTA constraint. The third and last conclusion is that the algorithm provides trajectories that respect ATC constraints such as constant altitude segments and constant Mach numbers. As future work, it is desirable to analyze as flown flights to quantify the algorithm potential to optimize real reference trajectories.

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