Convexification in Energy Optimization of a Hybrid Electric Propulsion System for Unmanned Aerial Vehicles

Ye Xie
Cranfield University
PhD Student
Bedford, UK, MK43 0AL
ye.xie@cranfield.ac.uk

Al Savvaris
Cranfield University
Reader

Antonios Tsourdos
Cranfield University
Professor

ABSTRACT

This paper presents the energy management of a hybrid electric propulsion system for unmanned aerial vehicles, using convex optimization. The primary energy optimization problem is formulated and then converted to a convex problem. The introduction of variable–battery internal energy is firstly proposed to convexify the state equality function. This convexification method can yield to a more straightforward and clear form of convex problem than previous studies. The relaxation of equality constraints is also employed without loss of equality. The numerical examples and forward simulations are carried out to validate the convexified problem. The result of flight scenario infers that the convex relaxation does not prejudice the optimality of the solution. By comparing with the benchmark optimization–dynamic programming, the convex optimization performs gentler optimal control and minimal optimal cost results, with much less optimization time. The most significant is that the convexification reduces the optimization computation to a level compatible with the practical application.

KEYWORDS: *UAV, Hybrid Electric Propulsion System, Fuel Optimization, Convex Optimization, Convexification, Lossless Relaxation*

NOMENCLATURE

x	- state variable of system
u	- control variable of system
J	- cost function
$\dot{m}_f$	- fuel rate of engine
I	- current of battery
Q_{max}	- the maximum capacity of battery
V_{oc}	- open circuit voltage of battery
R_{int}	- internal resistance of battery
G	- gear ratio of gear box
P	- power
T	- torque
ω	- rotational speed
E	- battery internal energy
Subscripts	
ICE	- engine
EM	- electric motor
$loss$	- power loss

1 INTRODUCTION

Nowadays, a growing number of Unmanned Aerial Vehicles (UAVs) are powered by electric motors (EMs), because of lower emissions and noise, better overall efficiency, and lower maintenance requirements. However, the specific energy of electric energy storage sources, e.g. batteries, is much lower than that of fossil fuel [1]. As a result, the internal combustion engine (ICE) is preferred for relatively large or long-endurance UAVs, due to its high power and energy density [2]. The hybrid electric propulsion system (HEPS) combines an electric powertrain with a conventional combustion engine to provide propulsion, in other words, being able to have the energy efficiency of an electric propulsion system with the extended range of an ICE. The aforementioned benefits make HEPS an attractive option to explore for powering UAVs.

The Air Force Institute of Technology (AFIT) has conducted several studies on aircraft hybrid electric technologies. Harmon et al. began the project with the theoretical design of neural network control for a parallel HEPS [3]. Continuing on from this work, Ausserer [4] implemented the physical integration and validation of a prototype. Queensland University of Technology (QUT) also conducted some studies [5]. Glassock et al. successfully downsized the engine and improved the overall propulsive efficiency compared to the non-hybrid powered aircraft.

HEPS can provide better fuel economy and lower emissions without compromising performance. In addition, it can provide on-board electrical regeneration for powering different systems, but this flexibility and diversity comes at a cost of increase complexity in hybrid energy management.

The energy optimization is one of the most popular topics regarding hybrid energy management. Research work by scholars investigated various types of optimizations in order to achieve the best control trajectory for a given mission. The Dynamic Programming (DP) is one of the most studied theories because it can guarantee the global optimality of the solution [6]. However, large computational cost exists in DP due to nonconvex characteristics of the energy optimization problem. Consequently, the optimization result from DP is usually used as a benchmark for other strategies.

Convex optimization problem is widely favoured because it can be solved, very reliably and efficiently [7]. Using interior-point methods, the problem can be guaranteed convergence to the global optimum with a deterministic upper bound on the number of iterations, without requirement of pre-supplied initial guess. In other words, the global optimality, lower complexity and no request of use-specified initial value make the convex programming very promising for real application.

The convex optimization for energy management of hybrid ground vehicles was firstly proposed by researchers of Chalmers University of Technology [8], [9]. The studies are concentrated on convexifying the nonconvex primary problem. The model of energy buffer (capacitor or battery) was convexified by introducing the variable–pack energy [10]. Furthermore, a lossless relaxation and its detailed proof were also provided in [10], [11]. The works indicate that the global minimum of original problem can be obtained by solving the relaxed problem without the loss of equality. This paper proposes a new convexification method. It can retain the feature of lossless relaxation from previous studies, but also lead to a more straightforward and clear form of convex problem.

The results presented in this paper are part of the ongoing work performed as part of the AIRSTART project. AIRSTART is a £3.2 million collaborative Research and Development project developing key technologies to support long-endurance UAVs. Cranfield is working on the hybrid propulsion system, converting a Rotron UAV engine into a hybrid combustion-electric system. The platform for testing the hybrid propulsion system is a remotely piloted multi-purpose UAV—the Aegis. First, the primary energy management is formulated, in which the minimization of the total fuel use is selected as the objective function. Later, three various techniques are implemented to convexify the nonconvex original problem. The cost is fitted with the piecewise linear function, while the battery voltage and electric motor losses are approximated with quadratic and exponential equation, respectively. A new variable, called the battery internal power, is firstly introduced here to convexify transition equation of system state. The same techniques applied on [10] is then employed to relax the equality to inequality constraint. The last numerical examples and forward simulations are carried out to verify the optimal results of convex optimization.

2 ORIGINAL PROBLEM FORMULATION

Initially, the formulation of original problem has been presented. In this study, the objective of the energy optimization is to minimize the fuel consumption of engine during the complete flight mission. The cost function is expressed by:

$$J = \int \dot{m}_f(t)\, dt, \tag{1a}$$

where $\dot{m}_f$ denotes the fuel consumption rate of engine, which is dependent on time t.

The control variable of formulated system is the engine output power P_{ICE}, i.e. $u(t) = P_{ICE}$; whereas the state variable is the battery's State-of-Charge (SoC), denoted by $x(t)$. According to the definition of SoC, the system state transition equation can be written as:

$$\dot{x}(t) = -\frac{I}{Q_{max}}, \quad (1b)$$

where I is the current flowing through the battery and Q_{max} is the battery maximum capacity.
The battery model is commonly described by an ideal open-circuit voltage source in a series with an internal resistance [12] in hybrid vehicle analysis. It uses the open circuit voltage V_{oc} and internal resistance R_{int} to obtain the battery output power P_{batt}:

$$P_{batt} = I * V_{oc}(x) - I^2 R_{int}, \quad (1c)$$

in which the resistance R_{int} is assumed to be constant.
In addition to cost function and system dynamics, the optimization is also subject to limitations of each component and the powertrain capability. Put differently, the power demand P_{req}, in addition to the motor/generator losses $P_{EM,loss}$, appears as the sum of the engine and battery power contributions (see equation (1d)). Also, other constraints (power, state and current constraints) can be transformed and addressed by the equations (1e-1g).

$$P_{ICE} + P_{batt} = P_{req} + P_{EM,loss}, \quad (1d)$$

$$P_{ICE,min}(\omega_{ICE}) \leq P_{ICE} \leq P_{ICE,max}(\omega_{ICE}), \quad (1e)$$

$$x_{min} \leq x \leq x_{max}, \quad (1f)$$

$$I_{min} \leq I \leq I_{max}. \quad (1g)$$

3 CONVEXIFICATION

The original Problem (1) is not convex. The inequality constraints (1e-1g) are affine, but equality equations and cost function do not satisfy requirement of convex programming [7]. In the following text, three techniques–approximation, change of variables and relaxation are carried out to convert the Problem (1) into a convex Problem **(10)**.

3.1 Approximation

The approximation is normally employed to reveal the inherent correlation between different variables with an algebraic expression, instead of the original numerical data from experiments. During the approximation, the convexity of original problem can also be investigated thoroughly.

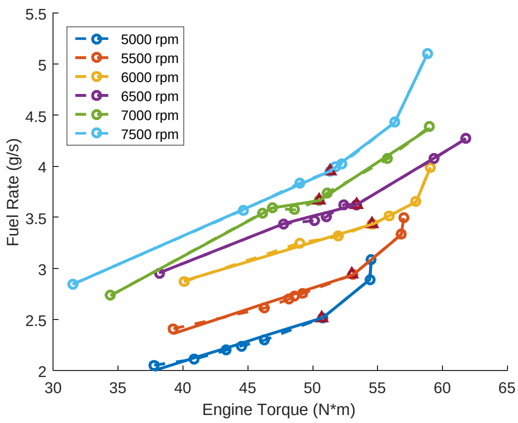

Figure 1. Piecewise linear approximation of the fuel rate at different speeds. The best efficiency points are represented by triangle marks and fitted data are denoted by the solid curve.

In this case, the fuel rate (the integrand of objective function) is not a convex function of control variable, but it is piecewise linear (affine) dependent on engine torque at a given speed (see Figure 1). In other words, the fuel rate at each speed can be addressed by $\dot{m}_f = k(\omega_{ICE})\frac{P_{ICE}}{\omega_{ICE}} + d(\omega_{ICE})$. The first turning point of different pieces indicates the best efficiency point for a given speed, if the fuel rate curve is piecewise convex [13]. This condition is not fulfilled at 6500 rpm and 7000 rpm, so their best efficiency points move to the second turning point due to the effect of nonconvex segment. To avoid large distortion and shift of the best efficiency region, the best efficiency points are needed to be kept at its original value. The results of piecewise linear approximation is also revealed in Figure 1 with the solid curve.

In general, the curve of voltage as a function of SoC can be divided into three segments by two turning points: the end of the exponential zone and the end of the nominal zone (as shown in Figure 2). It is clear that the function V_{oc} is nonlinear and also not convex. Fortunately, the SoC is typically limited between (0.2, 0.8), i.e. the nominal zone, to extend the lifetime of battery. As a result, the V_{oc} can be fitted with a quadratic function: $V_{oc} = a_v x^2 + b_v x + c_v$. The convex approximation result is also drawn in Figure 2 and the coefficients (a_v, b_v, c_v) are (24.95, 9.319, 291).

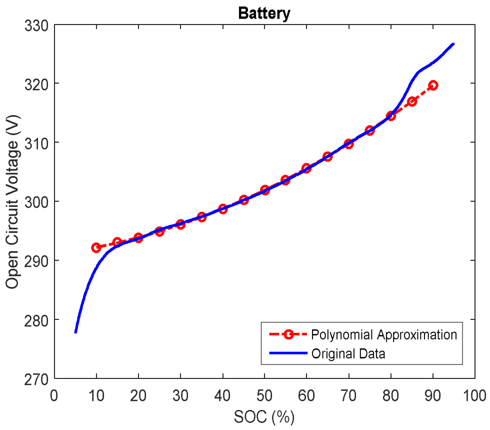

Figure 2. Approximation of open circuit voltage.

When a second order polynomial is applied on fitting, the coefficient of determination is 0.9993.

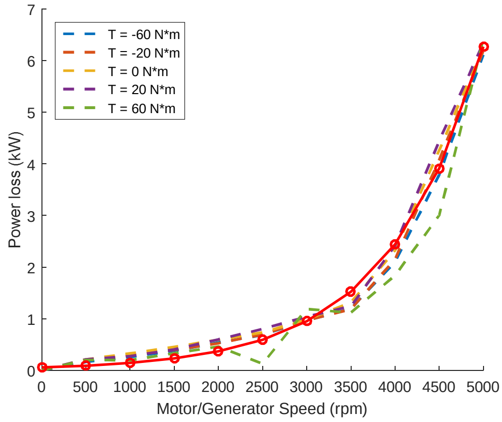

Figure 3. Approximation of the motor/generator power losses.

The fitted data are denoted by the solid curve marked with circles and its coefficient of determination is 0.9919.

The power loss of motor/generator is dependent on the output speed and torque. Figure 3 displays the relationship between power loss and speed under difference torques. The distance correlation between power loss and torque is 0.0541. This means that the power loss is weakly dependent on torque. As a result, the power loss is approximated as a formula only involving speed, using convex function $P_{EM,loss} = a_m * \exp(b_m \omega_{MG})$. The fitting results are also plotted in the Figure 3, with the red solid line marked by circles. The fitted coefficient is $a_m = 0.0563$, $b_m = 9.4248 * 10^{-4}$, when the unit of speed is revolutions per minute. It is notable that the original data is the expectation of power loss data at different torques.

3.2 Change of Variables

Afterwards, the change of variables is implemented to prepare for the formulation of new convex problem. Firstly, a battery internal power P_b which does not include battery loss is introduced, then equality constraints (1c-1d) are converted to:

$$P_b = I * V_{oc}(x), \tag{2}$$

$$P_{ICE} + P_b = P_{req} + P_{EM,loss} + P_{b,loss}. \tag{3}$$

The power loss of battery $P_{b,loss}$ defined here is mainly the dissipative power of resistance, which is dependent on the internal battery power and battery SoC (see Figure 4). Similar to $P_{EM,loss}$, the power loss is mainly correlated with battery power, rather than SoC. Meanwhile, it can be fitted with a convex quadratic function: $P_{b,loss} = a_l P_b^2$. The fitted coefficient is $a_l = 3.24 * 10^{-6}$. Likewise, the original data is the expectation of power loss data at difference charge-states.

Now, the inequality (1g) is substituted by:

$$I_{min} \le \frac{P_b}{V_{oc}(x)} \le I_{max}, \tag{4}$$

which is affine in the battery internal power P_b, but not guarantees convexity in the battery voltage $V_{oc}(x)$.

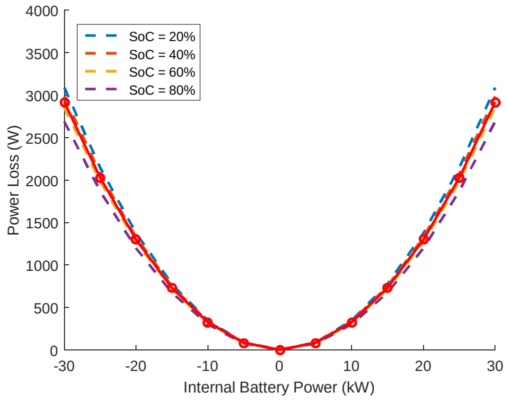

Figure 4. Approximation of the battery power losses.
The fitted data are denoted by the solid curve marked with circles and its coefficient of determination is 0.9998.

Consider approximating $V_{oc}(x)$ with two affine functions: $V_{lb}(x)$ and $V_{ub}(x)$. The $V_{lb}(x)$ is the Taylor expansion of $V_{oc}(x)$ around $x = 0.5$, while the curve defined by $V_{ub}(x)$ goes through two points at $x = 0.2$ and $x = 0.8$. Since $V_{oc}(x)$ is a convex function, the following constraints is satisfied:

$$I_{min}V_{oc}(x) \leq I_{min}V_{ub}(x) \leq P_b \leq I_{max}V_{lb}(x) \leq I_{max}V_{oc}(x). \quad (5)$$

Therefore, by solving problem with constraints **(5)**, one never obtains solutions which violate constraints of the physical problem. The bounds on errors (e_{lb}, e_{ub}) introduced by the second approximation is computed and displayed in Figure 5. The figure clearly shows that the inequalities **(4)** are not compromised for all practical purposes when replaced by **(5)**.

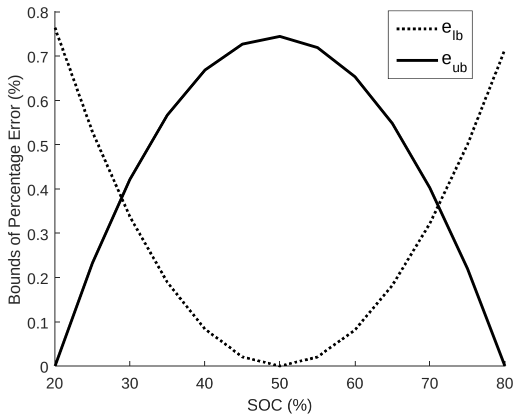

Figure 5. Error bounds of the second approximation.
The percents of errors of upper and lower bounds are both smaller than 1%.

From equation (1b) and (2), the state equality function can be deduced:

$$\dot{x}(t) = -\frac{P_b}{V_{oc}(x)Q_{max}}, \quad (6)$$

which still does not meet the requirement of convex programming. Therefore, a new variable called battery internal energy E is introduced to replace the original state variable (SoC). The new state transition equality is formulated:

$$\dot{E}(t) := Q_{max}V_{oc}(x)\dot{x}(t) = -P_b, \quad (7)$$

while x still denotes the battery SoC. Simple integration manipulation leads to:

$$E(x) = Q_{max}\int V_{oc}(x)dx = Q_{max}\left(\frac{1}{3}a_v x^3 + \frac{1}{2}b_v x^2 + c_v x + d_v\right), \quad (8)$$

where d_v is set to zero without loss of generality. Meanwhile, the inequality (1f) is transformed into:

$$E_{min} \leq E \leq E_{max}, \quad (9)$$

since the $E(x)$ is monotonically increased with x in the domain of definition. The E_{min}, E_{max} are calculated by $E(x_{min})$ and $E(x_{max})$, respectively.

3.3 Constraints Relaxation

With the substitution of variables, the equalities are converted to convex functions. Subsequently, the relaxation of equalities yields to a series of inequality constraints. As a result, the convex formulation of the original problem is constructed as follows:

$$J = \int_0^{t_n} \left(k(\omega_{ICE}) \frac{P_{ICE}}{\omega_{ICE}} + d(\omega_{ICE}) \right) dt, \quad (10a)$$

$$\dot{E}(t) = -P_b, \quad (10b)$$

$$P_{ICE} + P_b \geq P_{req} + P_{EM,loss} + P_{b,loss}, \quad (10d)$$

$$P_{ICE,min}(\omega_{ICE}) \leq P_{ICE} \leq P_{ICE,max}(\omega_{ICE}), \quad (10e)$$

$$E_{min} \leq E \leq E_{max}, \quad (10f)$$

$$I_{min} V_{ub}(x) \leq P_b \leq I_{max} V_{lb}(x). \quad (10g)$$

Note that the equality **(3)** is replaced by **(10d)**. As a consequence, the non-affine equality is successfully converted to a convex inequality. What is worth mentioning is that this relaxation does not prejudice the optimality of the solution. The claim has been logically reason in the [10]. Assume that the convex solver finds the optimal solution holding $P_{ICE} + P_b > P_{req} + P_{EM,loss} + P_{b,loss}$. This means that some energy supplied by the fuel and battery was wasted, thus a better solution can be found with **(10d)** holding with equality.

4 NUMERICAL EXAMPLE

The most commonly used hybrid powertrain configurations are series, parallel and series-parallel architecture. Compared with the series and series-parallel configuration, the parallel configuration is lighter and more reliable, whilst keeping the flexibility of hybrid. What's more, the parallel configuration is best suited for long-endurance UAVs, according to [14].
This paper applies the proposed convexification on the energy optimization of a parallel configured HEPS [15]. Hence, $P_{EM,loss}$ comes from the power loss of the single motor/generator. The power requirement is sum of power requested to drive the propeller and power demanded by the auxiliary devices of aircraft.
Two different hypothetical flight test scenarios are considered: the first test case is a complete mission that includes take-off, climbing, cruising and landing phases, where the battery charge-depleting and charge-sustaining strategy are both implemented on convex programming; second, the charge-sustaining based convex optimization is employed on a cruising flight mission and its performance is compared with the DP. Furthermore, the optimal control of second scenario is conducted and verified on a forward simulation model developed in the previous study [15].

4.1 Test Case 1

The test case used in this section simulates a complete 12 minutes flight mission that includes take-off, climbing, cruising and landing phases. The aircraft take-off run is in the first 1 minute and then continues to climb to the cruise height in the time interval 1-4.5 minute. Then, the aircraft will start its cruise phase at around 4.5 minute and finally the landing phase, after which the mission ends.
The power requirement of this flight mission is shown in Figure 6 with blue dotted line. The optimal control (ICE power) is also plotted with battery internal power. It is clear that the energy from two different sources combines together to power the aircraft during the climbing (between 1-4.5 minute). On the other hand, the battery is charged by the extra power from engine in take-off and landing phase.
Note that the power requirement is equal to the power consumption, which includes actual useful power and power losses. That means constraint **(10d)** holds with equality at the optimal trajectory. In other words, the relaxation presented in section 3.3 does not affect the results of optimization.

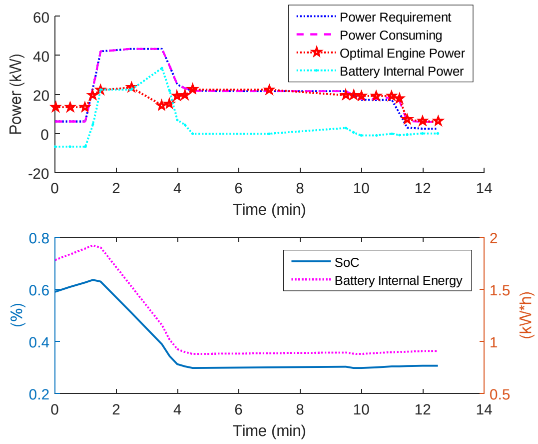

Figure 6. Optimization results of test case 1.

The initial value of SoC is set to 60%. The maximum and minimum value of SoC is 80% and 20%, respectively, considering the nominal operation range of battery. The SoC is allowed to be depleted to 30%, then the energy management is demanded to sustain the SoC around this value, to extend the lifetime of battery.

The Figure 6 also displays the virtual state variable E. The SoC and E both increase in the first 1 minute, due to battery charging during the take-off. Later, they drop by a large margin because the aircraft demands huge power for climbing. After reducing to 30%, the SoC is sustained around this value. The results demonstrate that the convex programming can achieve both depleting and sustaining strategies.

4.1 Test Case 2

The second instance exams the optimality of results from convex programming, by comparing with ones from global optimization–DP. Different from the test case 1, the forward simulation technique is applied on the optimization results to investigate their capabilities in practical application. The flight in this case considers cruising and excludes the take-off and landing phase. The power requirement of the flight mission is shown in Figure 7.

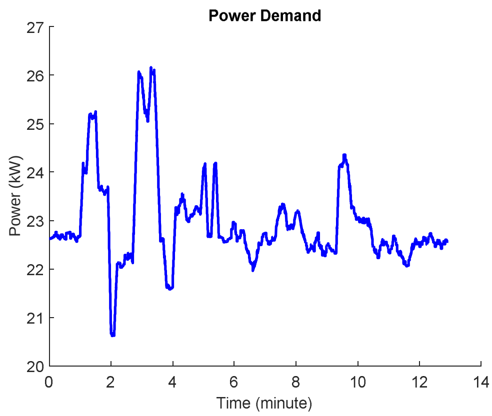

Figure 7. Power requirement of test case 2.

The Figure 8 compares the simulation results of battery SoC, engine power and fuel consumption between convex optimization and DP. The initial value of SoC is set to 50% and its value is supposed to be sustained in the 13-minute whole flight. The curves of SoC verify that two optimizations both can realize the charge-sustaining, but the DP obtains a more precise regulation of SoC. Moreover, the forward simulations demonstrate that optimal controls of two methods are both practical in real application. Yet the optimal control (engine power) derived from DP experiences more intense fluctuation than another one.

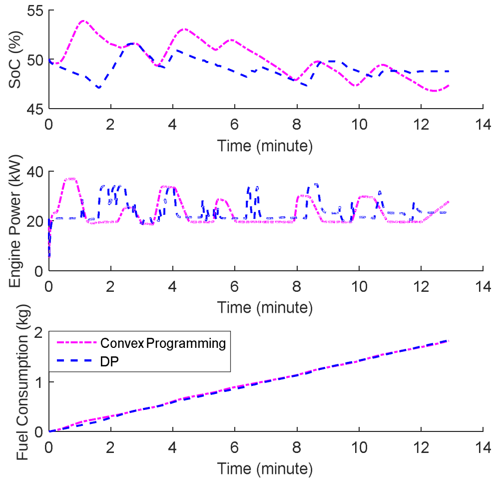

Figure 8. Comparison of convex optimization and DP in test case 2.

The most significant is that the convex programming can achieve better optimal cost (fuel use) than DP, as shown in Figure 8 and Table 1. To be fair, the final SoC of convex programming is lower than one of DP, which means the convex programming consumes more electric energy. In other words, the extra electric energy stored in the battery consumes the fuel spending in the DP. A real fuel consumption of two optimizations will be acquired by adjusting two final SoC values to the same one. Thus, the correction is introduced on the final of SoC.

Table 1: Comparison of consumptions between convex optimization and DP

	Convex Programming	DP
Final SoC (%)	47.37	48.80
Fuel Consumption (kg)	1.8235	1.8328
Corrected Final SoC (%)	50	50
Corrected Fuel Consumption (kg)	1.8304	1.8359
Energy Consumption (MJ)	84.85	85.13
Optimization Time (sec)	0.6708	7.9268

The corrected numbers are also displayed in the Table 1 and it shows that the convex optimization really has lower fuel usage compared with DP. Besides fuel using, another criterion—energy consumption also indicates that the convex optimization demands less energy to complete the flight mission.

Another significant advantage of convex optimization is the improved optimization time. It indicates that the convexification simplifies the original problem and reduces the computation to a level compatible with the practical application.

5 CONCLUSION

This paper presents the energy management of a hybrid electric propulsion system for unmanned aerial vehicles, using convex optimization. The primary energy management was formulated. The fuel minimization was selected as the cost and the battery SoC was chosen as the state variable.

Later, three techniques–approximation, variables and relaxation were implemented to convexify the nonconvex original problem. The nonconvex integrand of cost was fitted with the piecewise linear function. Under operation range, the battery voltage was able to be approximated with quadratic equation. The examination also shows that the power loss of motor is mainly dependent on motor speed with an exponential function. The convexification of state transition equality was successfully accomplished by proposed method, i.e. introducing battery internal voltage and energy. Subsequently, the lossless relaxation of equalities yielded to new inequality constraints.

The parallel hybrid configuration was nominated in the numerical test scenarios. The first test case verifies that the convex relaxation does not sacrifice the optimality of the solution. Also, the convex programming can reach optimal results under both charge depleting and sustaining strategies. By comparing with the DP in the test case 2, the convex optimization performs gentler optimal control and better optimal cost results. It even demands less total energy to accomplish the flight mission. The most significant advantage of convex optimization is that it can converge to the optimal solution with the time much less than DP.

On account of benefits of the convex programming as mentioned above, a more rigorous proof of equivalence between original problem and its convexified formation will be conducted in future studies.

REFERENCES

1. C. C. Chan; Apr. 2007; "The State of the Art of Electric, Hybrid, and Fuel Cell Vehicles," *Proc. IEEE*, vol. 95, no. 4, pp. 704–718.
2. M. Dudek, P. Tomczyk, P. Wygonik, M. Korkosz, P. Bogusz, and B. Lis; 2013; "Hybrid fuel cell-battery system as a main power unit for small Unmanned Aerial Vehicles (UAV)," *Int. J. Electrochem. Sci.*, vol. 8, no. 6, pp. 8442–8463.
3. F. G. Harmon, A. A. Frank, and S. S. Joshi; Jul. 2005; "The control of a parallel hybrid-electric propulsion system for a small unmanned aerial vehicle using a CMAC neural network," *Neural Networks*, vol. 18, no. 5–6, pp. 772–780.
4. J. K. Ausserer; 2012; "Integration, Testing and Validation of a Small Hybrid-Electric Remotely-Piloted Aircraft," Air Force Institute of Technology.
5. J. Y. Hung, and L. F. Gonzalez; 2012"On parallel hybrid-electric propulsion system for unmanned aerial vehicles," *Progress in Aerospace Sciences*, vol. 51. pp. 1–17.
6. P. Elbert, S. Ebbesen, and L. Guzzella; 2013; "Implementation of dynamic programming for n-dimensional optimal control problems with final state constraints," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 3, pp. 924–931.
7. S. Boyd, and L. Vandenberghe; 2010; *Convex Optimization*, vol. 25, no. 3. .
8. N. Murgovski, L. Johannesson, and J. Sjöberg; 2012; "Convex modeling of energy buffers in power control applications," *IFAC Proc. Vol.*, pp. 92–99.
9. X. Hu, N. Murgovski, L. Johannesson, and B. Egardt; 2013; "Energy efficiency analysis of a series plug-in hybrid electric bus with different energy management strategies and battery sizes," *Appl. Energy*, vol. 111, pp. 1001–1009.
10. P. Elbert, T. Nuesch, A. Ritter, N. Murgovski, and L. Guzzella; 2014; "Engine On/Off control for the energy management of a serial hybrid electric bus via convex optimization," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 3549–3559.
11. N. Murgovski, L. Johannesson, X. Hu, B. Egardt, and J. Sjoberg; 2015; "Convex relaxations in the optimal control of electrified vehicles," *Proc. Am. Control Conf.*, vol. 2015–July, no. 3, pp. 2292–2298.
12. O. Tremblay, L.-A. Dessaint, and A.-I. Dekkiche; 2007; "A Generic Battery Model for the Dynamic Simulation of Hybrid Electric Vehicles," in *2007 IEEE Vehicle Power and Propulsion Conference*, pp. 284–289.
13. C. Hou, M. Ouyang, L. Xu, and H. Wang; 2014; "Approximate Pontryagin's minimum principle applied to the energy management of plug-in hybrid electric vehicles," *Appl. Energy*, vol. 115, pp. 174–189.
14. R. M. Hiserote; 2010; "Analysis of hybrid-electric propulsion system designs for small unmanned aircraft systems," Air Force Institute of Technology.
15. Y. Xie, A. Savvaris, and A. Tsourdos; 2018; "Modelling and Control of a Hybrid Electric Propulsion System for Unmanned Aerial Vehicles," in *IEEE Aerospace Conference*.