Effects of MR Damper on Flutter of a Wing/Store Configuration

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ABSTRACT

In this paper, the flutter analysis of an aircraft wing carrying an external store by means of a Magneto Rheological (MR) damper is studied. The wing performs as a thin beam with the structural model, incorporating bending-torsion flexibility and transverse shear deformation; rotary inertia and warping restraint are neglected. Thus, the wing structural model is strictly along the lines of classical beam theory and valid for long, straight, homogeneous, isotropic wings. Modified Bouk-Wen model is used to simulate the MR damper. The aeroelastic partial governing equations are determined via Hamilton's variational principle. Also, modified Peter's finite-state aerodynamic model is employed. The resulting partial differential equations are transformed into a set of ordinary differential equations through the assumed mode method. The numerical results for a wing are compared with published results and good agreement is observed. Then, simulation results for the wing with an elastically attached external store via MR damper are presented to show the effects of MR damper voltage and wing design parameters on the wing/store flutter. Results show that increasing the MR damper voltage increases the wing flutter speed and also decreases the flutter frequency.

KEYWORDS: *wing/store, flutter, MR damper*

NOMENCLATURE

A - Wing cross area
b - Semi-chord length
C - Damper Coefficient
E - Elastic modulus
G - Shear modulus
h – Mass displacement in z' direction
I - Wing cross-sectional moment of inertia
K - Elastic coefficient
l - Wing length
L - Aerodynamic lift
m- Wing mass per unit length
M - Aerodynamic moment
M_e - Store Mass
T_e - Mass kinetic energy
T_w - Wing kinetic energy
U – Strain energy
U_∞ - Air stream velocity
V- Voltage
V_f – Dimensionless flutter speed
W - Work done by non-conservative forces
w - Wing vertical deflection
X_e, Y_e, Z_e - Dimensionless mass location
x_e, y_e, z_e - Mass location
δ - Variational operator
φ - Wing torsional deflection
θ - Store torsional deflection
ρ_∞ – Air density
ω_f – Flutter frequency

1 INTRODUCTION

Mounting external stores on aircraft wings is a common configuration of modern aircrafts. External fuel tanks and missiles in the case of military aircraft and engine nacelles for transport aircrafts are some examples of these external stores which their elastic and inertial coupling effects with the wing may influence the aeroelastic behavior of aircraft wings, significantly.
The large quantity of papers on wing/store flutter analysis makes it almost impossible to consider them all. Here, only some important articles in this area are included to give an idea of the published research. One of the first papers to study the effects of external stores on wing flutter was performed by Goland and Luke in 1948 [1]. They solved the governing equations for the aeroelasticity of a uniform wing with a tip weight, analytically determining the flutter speed. They mentioned that results presented by Goland [2] were in error and they presented the corrected values. The aeroelastic stability of an unrestrained vehicle, with swept composite wings and tip weights, was considered by Lottati [3]. It was observed that flutter occurs at a lower speed relative to the same vehicle with a clean wing. Librescu and his research group made some valuable efforts to study the effects of external stores on the wing divergence and flutter boundaries [4, 5]. They developed a wing model in the form of a solid beam or thin/thick-walled beam and incorporated a number of non-classical effects such as warping restraint, transverse shear, material anisotropy, the wing rotary inertia and its span-wise non-uniformity. Na and Librescu [6] studied the effects of time-dependent external excitations on the dynamic behavior of adaptive cantilever beams with stores. Furthermore, the free vibration and dynamic response of aircraft wings carrying heavy stores was investigated by Librescu and Song [7].
More recently, Fazelzadeh et al. conducted several works to study the wing/store flutter [8-10]. They used classic and shear deformable wings and studied effects of store mass, store mounting location in chord-wise, span-wise and vertical directions relative to the wing, and store arrangement and deployment sequences on the wing flutter speed. They showed that the store influence on the wing flutter is strongly affected by the kind of store and the aircraft flight condition.
Governing equations for aeroelastic behavior of a wing carrying external stores are usually derived based on the assumption that the connection between the wing and store is rigid. Since the 1980s, some researchers have used a decoupler pylon system to model passive flutter suppression and treat the effects of pylon stiffness on wing flutter characteristics [11]. Pourshamsi et al. analyzed the flutter of an aircraft wing carrying an elastically attached external store [12]. In this work, the wing was considered as a uniform cantilevered beam, and the external mass was connected to the wing by a spring and damper.
According to the best of the authors' knowledge the aeroelastic modeling and flutter analysis of aircraft wings containing an arbitrarily placed mass by means of MR damper have not yet been addressed and will be presented in this study. Furthermore, discussions about the effects of MR damper voltage and wing design parameters on the wing/store flutter speed and frequency are presented.

2 AEROELASTIC MODEL

The cantilever wing containing a mass subjected to a lateral follower force as shown in Fig.1 is considered. The wing performs as a thin beam and the structural model, which incorporates bending-torsion flexibility, is used. Also, the external mass inertia are accounted for in deriving the governing equations.

Figure 1: Schematic of a wing carrying an external store via MR damper

The wing typical section is represented in Fig.2, where y_e and z_e are the distances between the center of gravity of the external mass and the elastic axis of the wing at the static stability position. Also, points AE, AC and cg_e refer to the wing elastic axes, aerodynamic center of the wing and store center of gravity, respectively.

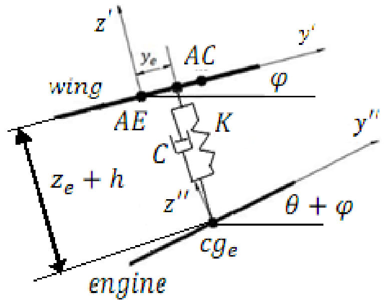


Figure 2: Schematic of a wing carrying an external store via MR damper

Because of the wing flexibility and also elastic connection between the wing and store, three coordinate systems have been used here. As shown in Fig. 1, the orthogonal axes x, y, z are fixed on the wing root in which the x axis lies in the wing spanwise direction. The other coordinate system, $x'y'z'$, has been fixed on a deformed wing. After the wing deformation, the shear center of the cross-section located at x is displaced by an amount of w in z direction. Additionally, the angle of twist of the cross-section changes to φ about the x axis. Furthermore, the coordinate system, $x''y''z''$, has been located on the center of gravity of the store. Because of the elastic connection, the store center of gravity is displaced by an amount of h in z' direction and the angle of twist of the store changes to $\varphi + \theta$ about the x' axis.

The equations of motion and boundary conditions are derived using Hamilton's variational principle that may be expressed as:

$$\int_{t_1}^{t_2} [\delta U - \delta T_w - \delta T_s - \delta W] dt = 0 \qquad \delta w = \delta\theta = 0 \quad \text{at} \quad t = t_1 = t_2 \tag{1}$$

where U and T are strain energy and kinetic energy, and W is the work done by non- conservative forces. The indices w and s identify the wing and externally mounted mass, respectively. The first variations of the wing kinetic energy and strain energy are simply [8]:

$$\delta T_W = \int_0^l \left\{ \left(-m\ddot{w} - me\ddot{\varphi}\right)\delta w + \left(-mk_m{}^2\ddot{\varphi} - me\ddot{w}\right)\delta\varphi \right\} dx \tag{2}$$
$$\delta U = \int_0^L (EI_y w^{(4)}\delta w - GJ\varphi''\delta\varphi) dx$$

Using the kinematical procedure, the first variation of the kinetic energy of the external mass can be derived as:

$$\begin{aligned}
\delta T_e &= -\int_0^L \iint_{A_e} \rho_e (\ddot{\boldsymbol{R}}_e . \delta\boldsymbol{R}_e)\delta_D(x - x_e) d\eta d\xi dx = \\
&= -\int_0^L \rho_e (-\ddot{h}A\delta w - \ddot{h}A\delta h + y_e A sin\Lambda \ddot{w}'\delta h - A\ddot{w}\delta w - y_e A e_A cos\Lambda \ddot{\theta}\delta\varphi - \\
&-2y_e A e_A cos\Lambda \ddot{\varphi}\delta\varphi - y_e A e_A cos\Lambda \ddot{\varphi}\delta\theta - cos\Lambda sin\Lambda A y_e{}^2 \ddot{\varphi}'\delta w + \\
&+cos\Lambda sin\Lambda A y_e{}^2 \ddot{w}'\delta\varphi + y_e{}^2 A (sin\Lambda)^2 \ddot{w}''\delta w + 2z_e A e'_A \ddot{w}''\delta w + \\
&+z_e{}^2 A\ddot{w}''\delta w + I_y \ddot{w}''\delta w - z_e A e'_A \ddot{\varphi}\delta\varphi - I_y \ddot{\varphi}\delta\varphi - I_y \ddot{\varphi}\delta\theta - \\
&-cos\Lambda y_e A\ddot{\varphi}\delta w - z_e A e'_A \ddot{\theta}\delta\varphi - I_y \ddot{\theta}\delta\varphi - cos\Lambda y_e A\ddot{w}\delta\varphi - \\
&-z_e A e'_A \ddot{\varphi}\delta\theta - y_e{}^2 A (cos\Lambda)^2 \ddot{\varphi}\delta\varphi - A e_A \ddot{\varphi}\delta h - I_y \ddot{\theta}\delta\theta - \\
&-z_e{}^2 A\ddot{\varphi}\delta\varphi - cos\Lambda y_e A\ddot{\varphi}\delta h - \ddot{w}A\delta h - I_z \ddot{\theta}\delta\varphi - I_z \ddot{\theta}\delta\theta - \\
&-I_z \ddot{\varphi}\delta\varphi - I_z \ddot{\varphi}\delta\theta - A e_A \ddot{\theta}\delta h)\delta_D(x - x_e) dx
\end{aligned} \tag{3}$$

The virtual work of non-conservative forces acting on the wing may be expressed as:

$$\delta W_A = \int_0^l (L\delta w + M\delta\theta) dx \tag{4}$$

where L and M are aerodynamic lift and moment, respectively which are derived from the finite-state aerodynamic model of Peters et al.[13]. Because of the wing sweep angle some corrections in aerodynamic model are implemented [10].

$$\begin{aligned}
L &= \pi\rho_\infty b^2[-\ddot{w} + U_\infty \cos\Lambda\dot{\theta} - U_\infty \sin\Lambda\dot{w}' - ba(\ddot{\theta} + U_\infty \sin\Lambda\dot{\theta}')] + \\
&\quad 2\pi\rho_\infty U_\infty b\cos\Lambda[-\dot{w} + U_\infty \cos\Lambda\theta - U_\infty \sin\Lambda w' + \\
&\quad b(\frac{1}{2} - a)(\dot{\theta} + U_\infty \sin\Lambda\theta') - \lambda_0] \\
M &= b(\frac{1}{2} + a)L - \pi\rho_\infty b^3[-\frac{1}{2}\ddot{w} + U_\infty \cos\Lambda\dot{\theta} - U_\infty \sin\Lambda\dot{w}' + \\
&\quad b(\frac{1}{8} - \frac{a}{2})(\ddot{\theta} + U_\infty \sin\Lambda\dot{\theta}')]
\end{aligned} \tag{5}$$

λ_0 is the induced-flow velocity and needs to be expressed in terms of the airfoil motion. The induced-flow theory of Peters et al. does just that, representing the average induced flow λ_0 in terms of *N* induced-flow states $\lambda_1, \lambda_2, \ldots, \lambda_N$ as:

$$\lambda_0 \approx \frac{1}{2}\sum_{n=1}^{N} b_n \lambda_n \tag{6}$$

where b_n's are found by the least-squares method. The induced-flow states λ_n are obtained by solving a set of *N* first-order differential equations approximating the unsteady flow over the airfoil.

$$[\mathbf{A}]\{\dot{\boldsymbol{\lambda}}_{\mathbf{n}}\}+\frac{U_\infty}{b}\{\boldsymbol{\lambda}_{\mathbf{n}}\}=\{\mathbf{C}\}[-\ddot{w}+U_\infty \cos\Lambda\dot{\theta}-U_\infty \sin\Lambda\dot{w}'+b(\frac{1}{2}-a)(\ddot{\theta}+U_\infty \sin\Lambda\dot{\theta}')] \tag{7}$$

where $[\mathbf{A}]$ and $[\mathbf{C}]$ are constant matrices given by Hodges and Pierce [14].

For modeling the MR damper, modified Bouc-Wen hysteresis model as shown in Fig.3 is used.

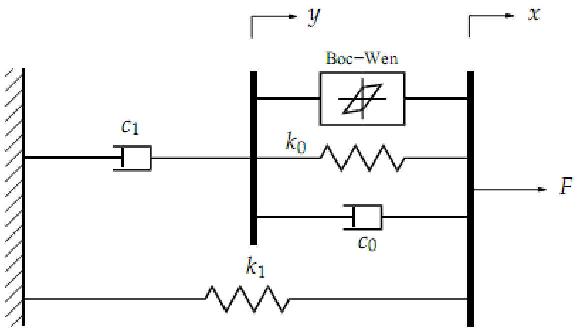


Figure 3: Modified Bouc-Wen hysteresis model for MR damper

The damper force F is obtained as:

$$F=c_1\dot{y}+k_1(x-x_0) \tag{8}$$

where

$$\dot{y}=\frac{1}{(c_0+c_1)}[\alpha z+c_0\dot{x}+k_0(x-y)] \tag{9}$$

$$\dot{z}=-\gamma|\dot{x}-\dot{y}|z|z|^{n-1}-\beta(\dot{x}-\dot{y})|z|^{n}+A(\dot{x}-\dot{y}) \tag{10}$$

In these equations, α, c_0 and c_1 are voltage constants which can be written as:

$$\begin{aligned}&\alpha=\alpha(v)=\alpha_a+\alpha_b v\\&c_1=c_1(v)=c_{1a}+c_{1b}v\\&c_0=c_0(v)=c_{0a}+c_{0b}v\end{aligned} \tag{11}$$

Substituting Eqs. (2-5) and (8-11) Eq. (1), and noticing that for every admissible variation ($\delta w, \delta\theta, \delta h, \delta\varphi$) the coefficient of these variations must be zero, the aeroelastic governing equations are obtained as:

$$\begin{aligned}&EI_y w^{(4)}+[\rho_w(-Ae_A\ddot{\varphi}+I_y\ddot{w}''-A\ddot{w})-\rho_e(-\ddot{h}A-A\ddot{w}-cos\Lambda sin\Lambda y_e{}^2A\ddot{\varphi}'\\&+y_e{}^2A(sin\Lambda)^2\ddot{w}''+2z_eAe_A'\ddot{w}''+z_e{}^2A\ddot{w}''+I_y\ddot{w}''-cos\Lambda y_eA\ddot{\varphi})+\\&+\left(\frac{c_1\alpha A}{c_0+c_1}\right)(h-w+y_e{}^2(sin\Lambda)^2w''-y_ecos\Lambda\varphi-y_e{}^2cos\Lambda\sin\Lambda\varphi'-y)+\\&+\left(\frac{c_0c_1}{c_0+c_1}\right)(\dot{h}-\dot{w}+y_e{}^2(sin\Lambda)^2\dot{w}''-y_ecos\Lambda\dot{\varphi}-y_e{}^2cos\Lambda\sin\Lambda\dot{\varphi}')+\\&+\left(\frac{k_0c_1}{c_0+c_1}\right)(h-w+y_e{}^2(sin\Lambda)^2w''-y_ecos\Lambda\varphi-y_e{}^2cos\Lambda\sin\Lambda\varphi'-y)+\\&+k_1(h-w+y_e{}^2(sin\Lambda)^2w''-y_ecos\Lambda\varphi-y_e{}^2cos\Lambda\sin\Lambda\varphi'-x_0)]\delta_D(x-x_e)-Li=0\end{aligned} \tag{12}$$

$$
\begin{aligned}
&-GJ\varphi'' + [\rho_w(-I_{y'}\ddot{\varphi} - Ae_A\ddot{w} - I_{z'}\ddot{\varphi}) - \rho_e(-y_e Ae_A cos\Lambda\,\theta - 2y_e Ae_A cos\Lambda\ddot{\varphi} + \\
&+cos\Lambda sin\Lambda A y_e{}^2\ddot{w}' - z_e Ae'_A\ddot{\varphi} - I_{y'}\ddot{\varphi} - z_e Ae'_A\ddot{\theta} - I_{y'}\ddot{\theta} - cos\Lambda y_e A\ddot{w} - \\
&-y_e{}^2 A(cos\Lambda)^2\ddot{\varphi} + \left(\frac{c_1\alpha A}{c_0+c_1}\right)(hy_e cos\Lambda - wy_e cos\Lambda + y_e{}^2 cos\Lambda \sin\Lambda w' - \\
&-y_e{}^2(cos\Lambda)^2\varphi - y) + \left(\frac{c_0 c_1}{c_0+c_1}\right)(\dot{h}y_e cos\Lambda - \dot{w}y_e cos\Lambda + y_e{}^2 cos\Lambda \sin\Lambda \dot{w}' - \\
&-y_e{}^2(cos\Lambda)^2\dot{\varphi}) + \left(\frac{k_0 c_1}{c_0+c_1}\right)(hy_e cos\Lambda - wy_e cos\Lambda + y_e{}^2 cos\Lambda \sin\Lambda w' - \\
&-y_e{}^2(cos\Lambda)^2\varphi - y) + k_1(hy_e cos\Lambda - wy_e cos\Lambda + y_e{}^2 cos\Lambda \sin\Lambda w' - \\
&-y_e{}^2(cos\Lambda)^2 \times \varphi - x_0)]\delta_D(x - x_e) - Mo = 0
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&[\rho_e(-\ddot{h}A + y_e A sin\Lambda\ddot{w}' - Ae_A\ddot{\varphi} - cos\Lambda y_e A\ddot{\varphi} - \ddot{w}A - Ae_A\ddot{\theta}) + \\
&+\left(\frac{c_1\alpha A}{c_0+c_1}\right)(-h + w - y_e w' sin\Lambda + y_e\varphi cos\Lambda - y) + \left(\frac{c_0 c_1}{c_0+c_1}\right)(-\dot{h} + \\
&+\dot{w} - y_e\dot{w}' sin\Lambda + y_e\dot{\varphi} cos\Lambda) + \left(\frac{k_0 c_1}{c_0+c_1}\right)(-h + w - y_e w' sin\Lambda + \\
&+y_e\varphi cos\Lambda - y) + k_1\left(-h + w - y_e w' sin\Lambda + y_e\varphi cos\Lambda - x_0\right)]\delta_D(x - x_e) = 0
\end{aligned}
\tag{14}
$$

$$
[\rho_e(-y_e Ae_A cos\Lambda\ddot{\varphi} - I_{y'}\ddot{\varphi} - z_e Ae'_A\ddot{\varphi} - I_{y'}\ddot{\theta} - I_{z'}\ddot{\theta} - I_{z'}\ddot{\varphi}))]\delta_D(x - x_e) = 0 \tag{15}
$$

3 SOLUTION METHODOLOGY

Due to intricacy of governing equations, it is difficult to get the exact solution. Therefore, in order to solve the above equations in a general way, the Galerkin's method is used. To this end, w, φ are represented by means of series of trial functions, φ_i that should satisfy the boundary conditions, multiplied by time dependent generalized coordinates, $\mathbf{q_i}$.

$$
w = \varphi_1{}^{\mathbf{T}}\mathbf{q_1} \qquad , \varphi = \varphi_2{}^{\mathbf{T}}\mathbf{q_2} \tag{16}
$$

Substituting Eq.(16) in Eqs.(12-15) and applying the Galerkin procedure on these governing equations the following set of ordinary differential equations are obtained.

$$
\dot{\mathbf{Z}} = [\mathbf{A}]\,\mathbf{Z} \tag{17}
$$

where

$$
\begin{aligned}
&\mathbf{Z} = \left\{\mathbf{q}^{\mathbf{T}}\ \dot{\mathbf{q}}^{\mathbf{T}}\right\}^{\mathbf{T}} \\
&\{\mathbf{q}\} = \left\{\mathbf{q_1}^{\mathbf{T}}\quad \mathbf{q_2}^{\mathbf{T}}\right\}^{T}
\end{aligned}
\tag{18}
$$

The problem is now reduced to that of finding out the eigen-values of matrix $[\mathbf{A}]$ for a given values of the air speed parameter U_∞. The eigen-value ω is a continuous function of the air speed U_∞. For

$U_\infty \neq 0$, ω is in general complex, $\omega = \mathrm{Re}(\omega) + i\,\mathrm{Im}(\omega)$. When $\mathrm{Re}(\omega) = 0$ and $\mathrm{Im}(\omega) \neq 0$ the wing is said to be in critical flutter condition. At some point, as U_∞ increases, $\mathrm{Re}(\omega)$ turns from negative to positive so that the motion turns from asymptotically stable to unstable.

4 NUMERICAL RESULTS

Pertinent data for the particular wing-weight combination used here are the same as those utilized in Ref. [9] and the relevant data for the MR damper are considered in Table 1. Also, dimensionless parameters used in the numerical simulation are:

Table 1. Characteristics of the MR damper

Parameters	Value	Parameters	Value
c_{0a}	20.2 N.s/cm.V	α_a	44.9
c_{0b}	2.68 N.s/cm.V	α_b	638 V^{-1}
K_0	15 N/cm	γ	39.3 cm^{-2}
c_{1a}	15 N.s/cm	β	39.3 cm^{-2}
c_{1b}	70.7 N.s/cm	A	47.2
K_1	5.37 N/cm	n	2
x_0	18.4 cm		

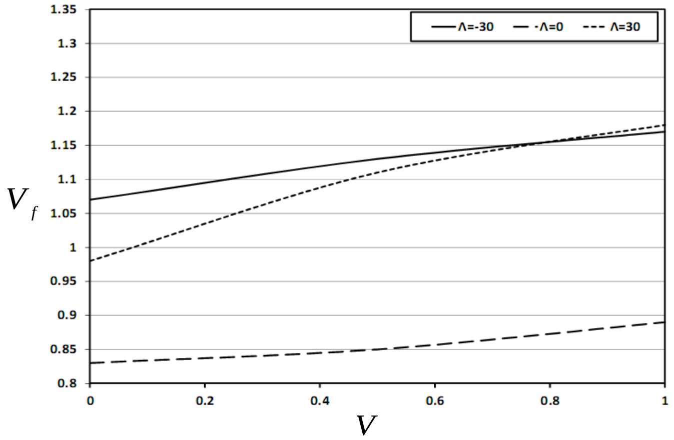
$$V_f = \frac{U_F}{b\omega_\varphi}, \quad X_e = \frac{x_e}{L}, \quad Y_e = \frac{y_e}{b}, \quad Z_e = \frac{z_e}{b}, \quad \eta_e = \frac{M_e}{mL} \tag{19}$$

In Table 2, for the purpose of validating the results for the rigid connection between the wing and external mass, is compared with Ref. [15] for different spanwise locations of the external mass, and good agreement with the theoretical and experimental results is observed.

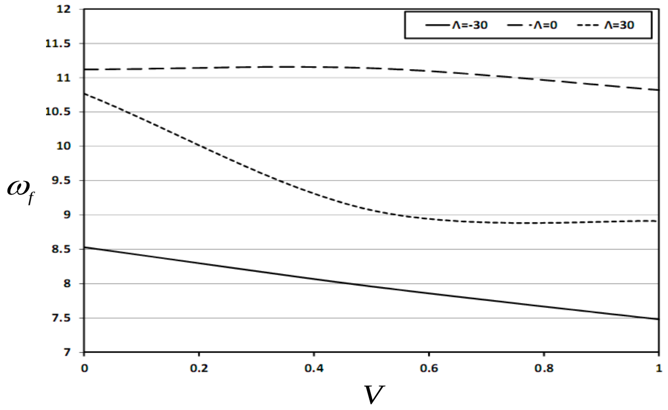
Table 2: Validation of flutter speed and frequency for wing/store with rigid connection

	Ref. [15]		Present			
Location of the store	V_f	ω_f	V_f	Error (%)	ω_f	Error (%)
0	1.88	11.85	1.88	0	11.85	0
0.2	1.82	11.65	1.83	0.55	11.69	0.34
0.4	1.7	10.9	1.69	0.59	10.88	0.18
0.6	1.64	10.1	1.58	3.65	10.62	5.1

Figure 4 shows the variation of the flutter speed and frequency of the wing/store for selected values of the sweep angle due to variations in the MR damper voltage. It can be seen from this figure that flutter speed increases by increasing the MR damper voltage. On the other hand, increasing the voltage, decreases the flutter speed as shown in Fig.4 (b). Furthermore, both backward and forward sweep angles improve the stability domain of the wing.



(a)



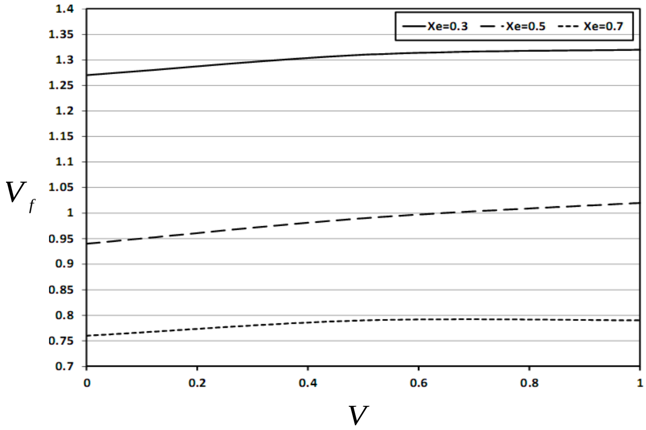
(b)

Fig. 4. Effects of the MR damper voltage on the wing critical behaviour for $X_e = 0.5,\ Y_e = -0.25,\ Z_e = -1,\ \eta_e = 0.5$**: (a) Flutter speed, (b) flutter frequencies.**

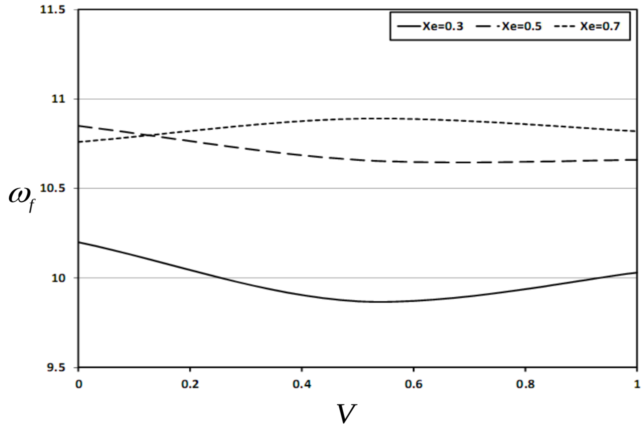
The influence of the MR damper voltage on the flutter speed and frequency of the wing for selected values of the external mass spanwise location is shown in Fig. 5. In this case $Y_e = -0.25,\ Z_e = -1,\ \eta_e = 0.5,\ \Lambda = 30°$. It can be seen from this figure that increasing the voltage will expand the aeroelastic stability region of the wing/store. Also, increasing the distance of the external mass from the wing root will decrease the flutter speed. Figure 5 also reveals that the flutter frequency generally increases by moving the external mass towards the wing tip.
Figures 6 shows the effects of the external store mass on the flutter speed and frequency of the swept wing for different MR damper voltage values. The store is located at the middle of the wing span and $Y_e = -0.25, Z_e = -1$. Also in this case, the wing sweep angle is $\Lambda = 30$. Results show that the store mass decreases the flutter speed and plays a destabilizing role in the dynamic stability of the wing. Furthermore, the effects of the MR damper voltage on the wing flutter speed and frequency can also be observed in this figure. As expected, by increasing the voltage the flutter speed of the wing/store configuration increases.

5 CONCLUSION

Effects of using MR damper for connecting an arbitrarily placed external mass to the wing on the aeroelastic modelling and flutter analysis of the wing/store is considered in this paper. The governing equations include effects of both MR damper and lift and aerodynamic moment induced forces.



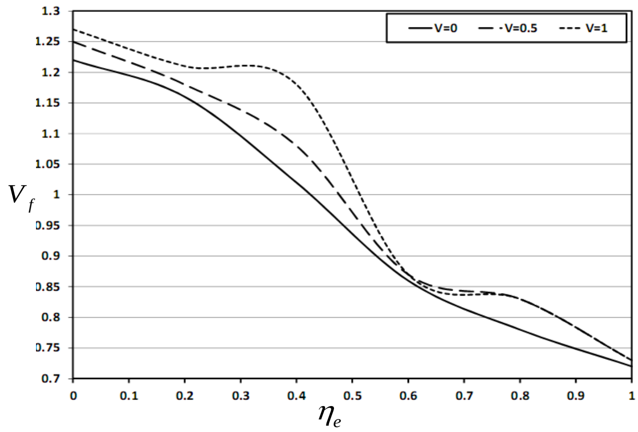
(a)



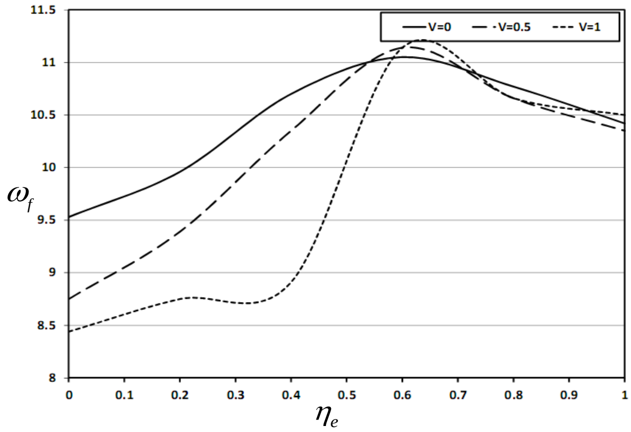
(b)

Fig. 5. Effects of the MR damper voltage on the wing flutter boundary for $Y_e = -0.25,\ Z_e = -1,\ \eta_e = 0.5,\ \Lambda = 30°$ **and for selected values of the mass spanwise mounting position: (a) Flutter speed, (b) flutter frequencies.**

A parametric study of the MR damper voltage and the external mass magnitude and location on aeroelastic flutter is performed. Results are indicative of the important influence of the MR damper voltage on flutter speed and frequency of the wing/store. The results show that stability region can be expanded by increasing the MR damper voltage.



(a)



(b)

Fig. 6. Effects of the store mass on the wing flutter boundary for $X_e = 0.5,\ Y_e = -0.25,\ Z_e = -1,\ \Lambda = 30°$ **and for selected values of the MR damper voltage: (a) Flutter speed, (b) flutter frequencies.**

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