RESPONSE SURFACE ANALYSIS and DESIRABILITY FUNCTION OPTIMIZATION TO OBTAIN AERODYNAMICALLY OPTIMIZED STORE with LOW ASPECT RATIO WINGS

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ABSTRACT

In the current work multi objective response surface analysis and desirability function optimization are performed to determine optimum size and shape of the lifting surfaces on a store with strake-fin configuration. In order to cover various flow conditions, CFD simulations are conducted at Mach numbers of 0.8, 1.5 and 2.5 and for angle of attack values 4°, 7°, 12°. Through the results of CFD analysis surrogate models are constructed with least squares estimation and desirability function optimization is performed to find geometry which satisfy externally defined requirements.

KEYWORDS: *Response Surface, CFD, Optimization, Strake*

NOMENCLATURE

CFD – Computational Fluid Dynamics
CPU – Central Processing Unit
D – Distance
k – Ratio of Strake Span to Fin Span
FRC – Fin Root Chord
FS – Fin Span
SRC – Strake Root Chord
SS – Strake Sweep Angle
TR – Taper Ratio

1 INTRODUCTION

Traditionally, the amount of time which is separated for the preliminary design stages are limited such that it is not practically possible to investigate all design options with great detail. However, it is generally expected from the results of the preliminary design studies to cover all possible ways that design can develop through the dynamically changing mission requirements. Therefore, fast aero-prediction methods play crucial role especially in the preliminary design stages. Unfortunately, these fast aero-prediction methods which are generally rely on linear theories and/or experimental correlations may not be so accurate depending on the investigated flow scenarios.

Aerodynamic investigation of stores with very low aspect ratio wings (i.e strakes) are one of the most challenging case for fast-aero prediction codes. In such a case, CFD analyses provide more accurate results compared to the semi-empirical methods. Recent work conducted by Christopher et al [1] compares prediction capabilities of semi empirical aero prediction codes and CFD methods in capturing aerodynamic behaviors of the missile configurations with strakes. Their study shows that for most of the investigated flight conditions and missile geometries, CFD analyses provide more accurate results compared to the semi-empirical methods. However, CFD computations are time consuming and therefore costly such that investigating large number of aerodynamic shapes via CFD analyses is not practical in the early stages of the design process. To reduce number of geometries that are investigated and to decrease the amount of CPU time spend in CFD analyses, response surface methodology can be used as a very efficient tool. Response surface analysis can provide an analysis environment which can answer all possible mission requirements even in the early stages of the design stages.

In this study response surface analysis and desirability function optimization is performed to obtain aerodynamically optimized lifting surfaces on a cylindrical store geometry. For the investigations, effect of 7 different design parameters are examined. Experimental points are determined with Face Centered Composited Design and each of the geometry obtained with design of experiment are computationally analysed at various flow conditions. Results of the computational analyses are used to obtain surrogate models which basically explain effect of each parameter on static stability, lift to drag ratio and normal force of the store.

2 COMPUTATIONAL APPROACH

Steady RANS solutions are carried out with the commercially available flow solver ANSYS Fluent 14.0.7 by using one equation turbulence model Spalart Allmaras. Computations are obtained with pressure-based coupled algorithm and gradients are calculated with Green Gauss Node Based Method with 2nd order accuracy. In order to capture flow near the wall 12 layers of boundary layer elements are employed and first layer thickness is determined such that y+ value kept below 1 for each investigated flow condition.

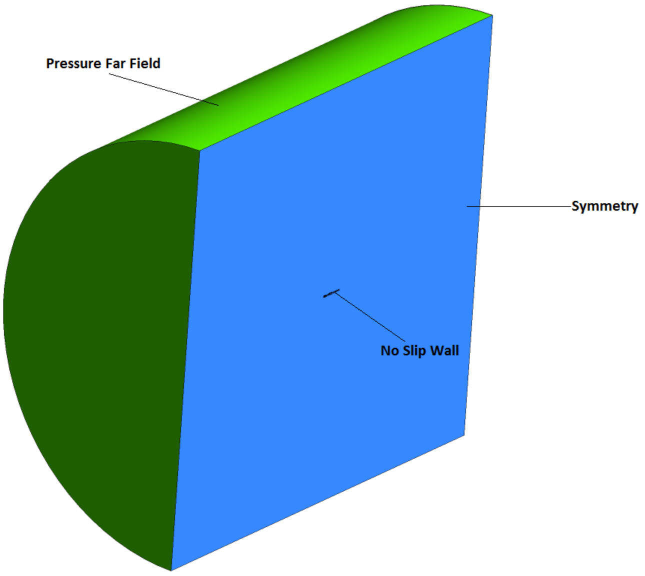

Figure 1. Computational Domain and Boundary Conditions

Computational domain and defined boundary conditions for flow solver is shown in Figure 1. External boundaries except symmetry plane are defined as pressure far field and no slip wall boundary condition is applied to the store surface. Symmetry boundary condition is also used to reduce computational cost. Although total number of mesh changes with the change in geometrical parameters, it is kept on the order of 2.5 million for each case. Surface mesh on the store and symmetry plane is shown in the Figure 2 for one of the geometric alternatives.

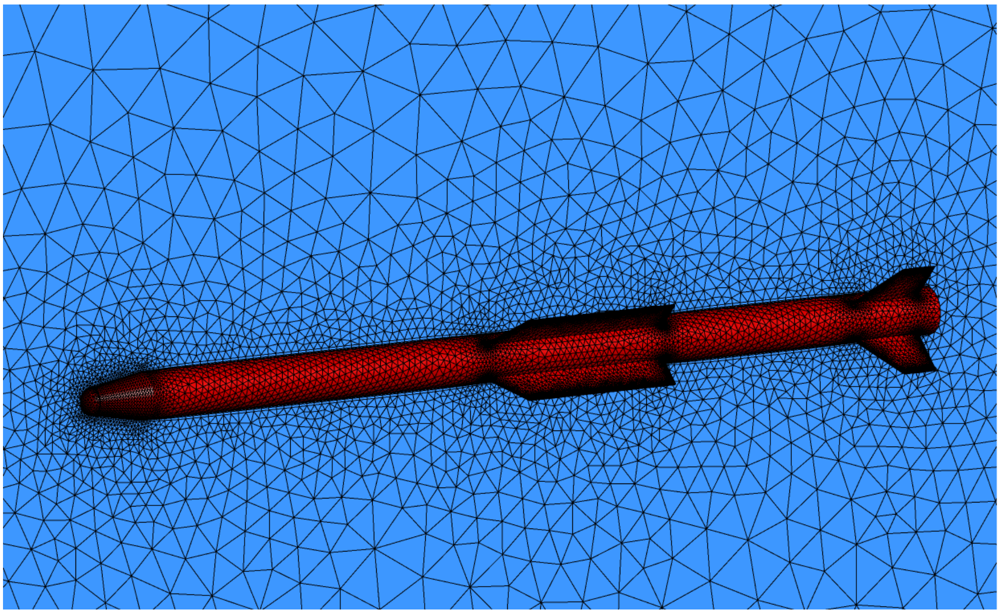

Figure 2. Surface Mesh on Store and Symmetry Plane

3 PERFORMANCE MEASURES

To construct aerodynamic design environment, lift to drag ratio, stability margin and normal force coefficient are taken as primary performance parameters. Lift to drag ratio is directly related to aerodynamic efficiency of the air vehicle. For agile store, higher lift to drag ratios leads to less energy lost during the maneuver. Furthermore, maximum range of the air vehicle heavily depends on the lift to drag ratios. At the same time, stability margin determines the response of the air vehicle to the disturbances and its tendency to restore its position. For the purposes of the study, only static stability is taken into account. Static stability of any air vehicle can be explained with the stability margin which is the distance between the aerodynamic center, where resultant aerodynamic forces are acting, and center of gravity. At the same time, Normal force is a measure of maneuvering capability of the air vehicle. Depending on the requirements it is applicable to limit the normal force or maximize it.

Each performance parameters are obtained for 3 different Mach numbers (0.8, 1.5, 2.5) and for 3 angle of attacks (4˚, 7˚, 12˚). Performance parameters obtained at each flight condition contributes equally to the overall performance of the air vehicle. In other words, overall performance is calculated as the average of the performance measures obtained at each flight condition. Similar approach is also used in earlier study [2] such that overall performance calculated with 1 where λ and n represent performance parameter and the number of flight conditions respectively.

$$\lambda_{overall} = \frac{1}{n}\sum_{k=1}^{n}\lambda_k \qquad \text{Equation 1}$$

4 GEOMETRICAL VARIABLES

In order to construct relevant response surfaces to capture the aerodynamic problem, it is first mandatory to define geometrical design variables that may affect the aerodynamic behaviour of the store. In this study, fin span, fin root chord, fin taper ratio which is defined as ratio of fin tip chord to the fin root chord, strake root chord, strake sweep angle, distance between strake trailing edge and fin leading edge are taken as design variables with ratio of strake span to the fin span. For the purposes of this study, trailing edges for both strakes and tail fins are kept perpendicular to the store surface. General view of store with variable geometric parameters is shown in Figure 3.

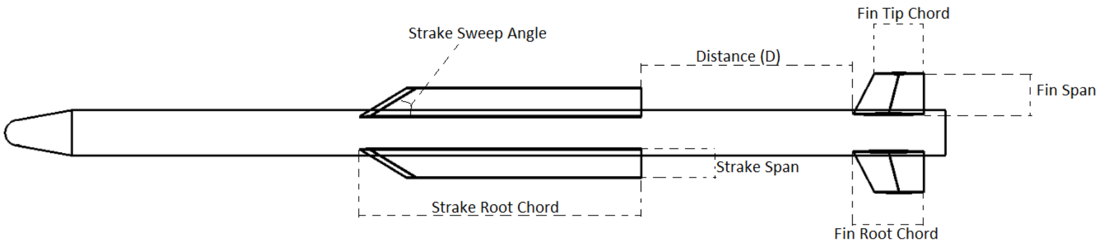

Figure 3. General View of Strake-Fin Configuration Store with Variable Geometric Parameters.

5 RESPONSE SURFACE METHODOLOGY

Response surface methodology is used to construct empirical models, which represent dependency of the responses on input variables. In order to construct response surfaces experimental runs are carried out with systematically selected input variables. Determining experimental data points plays crucial role in accuracy and validity of the response surface model. Experimental points must be properly placed in the design space such that maximum amount of information can be gained from limited number of experimental runs. It is possible to find many methods for constructing experimental designs in the literature. BOX-Behnken, Space Filling and Central Composite Designs are only few of the most widely used experimental design methods in the industry. In this study, Face Centered Composite Design(FCCD) with seven variables is employed to get maximum amount of information with limited number of experimental points. Originally central composite design is a first order experimental design which is enriched by additional points such that it allows second order estimations [3] with high efficiency. FCCD introduces $2^{(k-f)}$ + 2k + Cp number of experimental points where k is the number of geometrical variables, Cp is the central point and f is the factorial number. Factorial number is used to limit the number of design points by excluding some diagonal points. In this work, by taking factorial number equal to 1, number for design points is reduced to 79.

After collection of experimental data, responses of these experimental runs are used to develop empirical formulations. The degree of the response surface equations may change depending on the complexity of the process and required accuracy. Generally, first order or second order models produce sufficiently good results. Since, it was expected to have some non-linear behaviour of the responses in the current study, second order models are constructed with standard least squares method [4]. Second order models have following form;

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{1 \le i \le j}^{k} \beta_{ij} x_i x_j + \varepsilon \qquad \text{Equation 2}$$

Where y is response, x is regressor, β is regression coefficient and ε is the residual. For least square regression modelling it is automatically assumed that residuals are randomly and independently distributed. To check whether any of these assumptions are violated or not it is customary to investigate normal probability plot and plot of residuals versus predicted response values [5].

In this study, normal probability plots and plots of residuals versus predicted response values are constructed for lift to drag ratio, dimensionless static margin and normal force coefficient. Straight line in the normal probability plot indicates normal distribution of the residuals, whereas S-shape curve indicates normality assumption is violated. Expected tendency in the plot of residuals versus predicted values is random distribution of the residuals which represents homogeneity of the residuals.

Normal probability plots are shared in Figure 4, Figure 5 and Figure 6. As can be seen from these figures residuals are placed around straight line such that normality satisfied with 95% confidence interval. Furthermore, plots of residuals versus predicted values are shown in Figure 7, Figure 8 and Figure 9. From these figures it is seen that residuals have sufficiently homogeneous distribution through the investigated design space. Therefore, constructed regression models for lift to drag ratio, dimensionless static margin and normal force coefficient do not violate normality and independent distribution assumption.

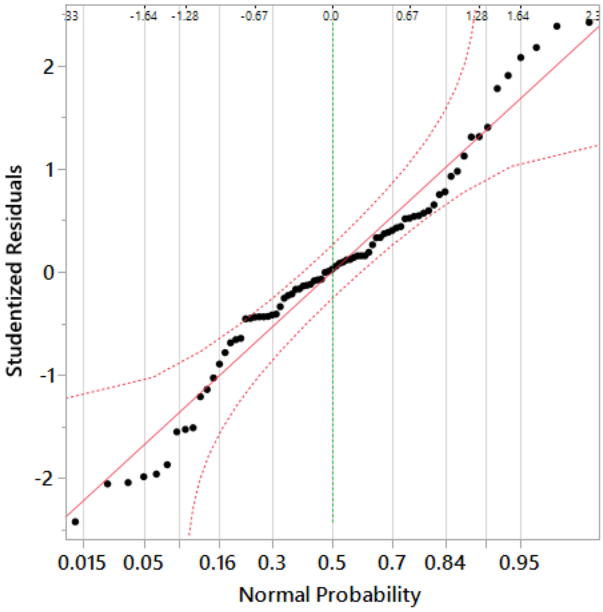

Figure 4. Normal Probability Plot with 95% Confidence Interval for Lift to Drag Ratio (L\D)

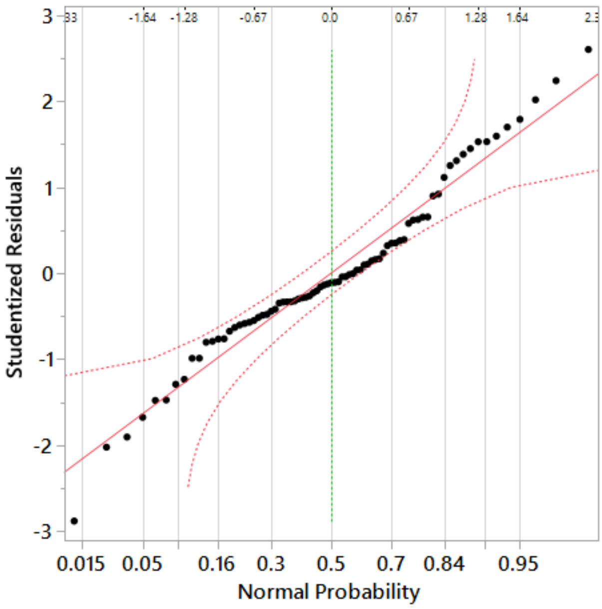

Figure 5. Normal Probability Plot with 95% Confidence Interval for Dimensionless Static Margin (SM)

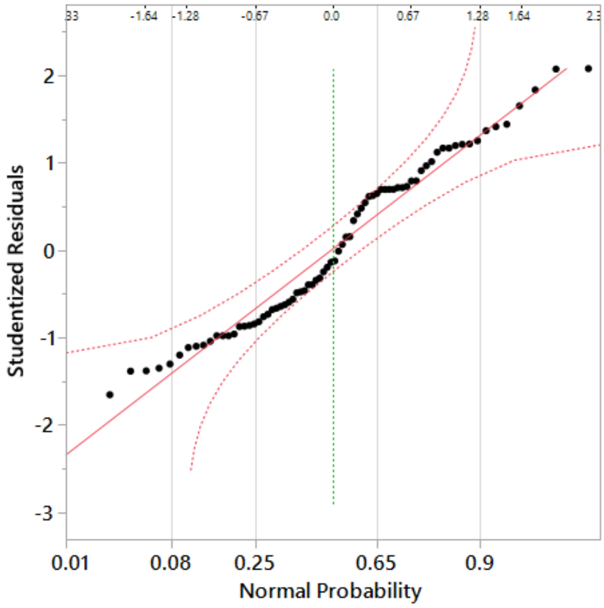

Figure 6. Normal Probability Plot with 95% Confidence Interval for Normal Force Coefficient (CN)

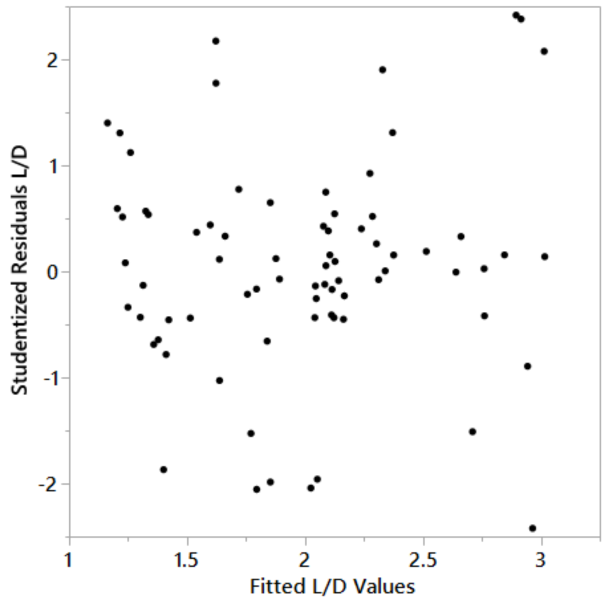

Figure 7. Plot of Residuals vs Fitted Values for Lift to Drag Ratio (L\D)

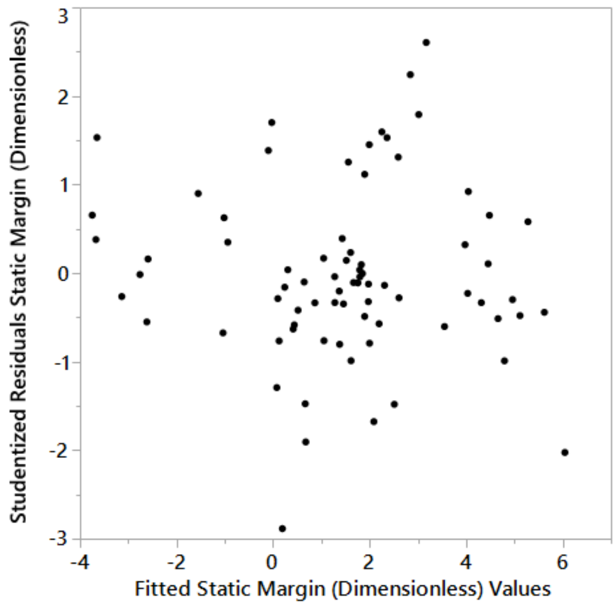

Figure 8. Plot of Residuals vs Fitted Values for Dimensionless Static Margin (SM)

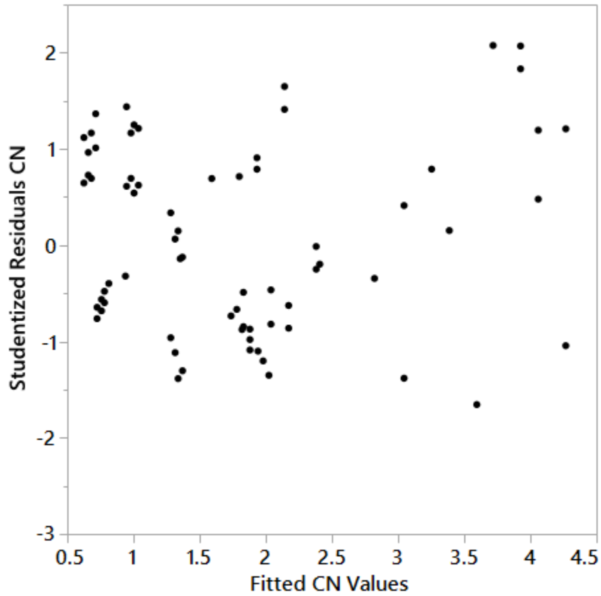

Figure 9. Plot of Residuals vs Fitted Values for Normal Force Coefficient

After obtaining statistically valid response surface models it is beneficial to check prediction capabilities of the regression models. In order to analyse performance of these constructed response surface equations for each of the responses, coefficient of determination(R^2), adjusted coefficient of determination (R_a^2) and percentage root mean square error (RMSE%) values are investigated. These values are presented in Table 1. As can be seen from this table all regression models have high performance values with acceptable RMSE values.

Table 1. Performance Parameters of the Constructed Regression Models

Parameter	Lift to Drag Ratio(L/D)	Dimensionless Static Margin (SM)	Normal Force Coefficient (CN)
R^2	0.9973	0.9963	0.9928
R_a^2	0.9972	0.9963	0.9927
RMSE%	1.3731	7.0090	4.8000

From the constructed regression models, it is possible to obtain important information about the impact levels of each factor to the regression model via p value approach [4]. Generally high p values indicate low impact of the corresponding variable on the regression model, whereas low p values indicate significant effects. Results of the p value approach for each of the regression model are given in Table 2, Table 3 and Table 4 for lift to drag ratio, normal force coefficient and dimensionless static margin respectively. From these tables it can be seen that interactions between geometrical variables have significant effect on the aerodynamic performance. Furthermore, information which is obtained from p value approach can be used for future aerodynamic design studies.

Table 2. Significance Levels of the Regressor Variables on Regression Model Constructed for Lift to Drag Ratio

Source	P Value
FS	0
k (SS/FS)	0
FRC	0
SRC	0
FS*k (SS/FS)	0
SRC*k (SS/FS)	0
FS*FS	0
FRC*k (SS/FS)	0
SSA	0
FRC*FS	0
D	0.00002
TR*k (SS/FS)	0.00134
FS*D	0.00171
TR*FS	0.00277
SSA*k (SS/FS)	0.02966

Table 3. Significance Levels of the Regressor Variables on Regression Model Constructed for Normal Force Coefficient

Source	PValue
FS	0
k (SS/FS)	0
FS*k (SS/FS)	0
SRC	0
SRC*k (SS/FS)	0
FRC	0
FS*SRC	0
FRC*FS	0
TR	0
TR*FS	0.00049

Table 4. Significance Levels of the Regressor Variables on Regression Model Constructed for Dimensionless Static Margin

Source	PValue
D	0
k (SS/FS)	0
FS	0
SRC	0
k (SS/FS)*D	0
SRC*k (SS/FS)	0
FS*k (SS/FS)	0
FS*D	0
FRC	0
FS*SRC	0
SRC*D	0
TR	0
FS*FS	0
k (SS/FS)*k (SS/FS)	0.00001
SSA	0.00004
SSA*k (SS/FS)	0.00009
FRC*k (SS/FS)	0.00018
FS*SSA	0.00305

6 DESIRABILITY FUNCTION APPROACH AND OPTIMIZATION

To perform multi objective aerodynamic optimization for a store with strake-fin configuration desirability function approach [6] is performed. In this approach it is possible to search and obtain optimal values for each of the responses depending on the optimization criterions and priorities. Basically, method of desirability function optimization involves creating desirability functions for each responses and a single composite response function. The composite response function represents global desirability and it is created by the geometric mean of the individual desirability values which takes values between 0 and 1 such that they indicate least and most desirable responses respectively.

It is also possible to arrange priorities of the responses with weighting factors such that high weighting factors indicate higher priority.

Different functions are employed depending on the optimization criteria. If certain response is to be maximized and U is the upper acceptable value for the response where L is the lower, desirability function (d) is defined by following equation;

$$d = \begin{cases} 0 & y < L \\ \left(\frac{y-L}{U-L}\right)^r & L \leq y \leq U \\ 1 & y > U \end{cases} \qquad \text{Equation 3}$$

Similarly, if response is desired to be minimized desirability function is defined with Equation 4;

$$d = \begin{cases} 1 & y < L \\ \left(\frac{U-y}{U-L}\right)^r & L \leq y \leq U \\ 0 & y > U \end{cases} \qquad \text{Equation 4}$$

In Equation 3 and Equation 4, *r* represents weighting factor which is basically measure of how important to be close to the target. High values of r should be employed when it is highly important to be close to the target value.

It is also possible to optimize response function when specific target value is the most desirable response. In this case if T is the target value, desirability function is given by;

$$d = \begin{cases} 0 & y < L \\ \left(\frac{y-L}{U-L}\right)^{r_1} & L < y < T \\ 1 & y = T \\ \left(\frac{U-y}{U-T}\right)^{r_2} & T < y < U \\ 0 & y > U \end{cases} \qquad \text{Equation 5}$$

After obtaining desirability values for each of the responses a single composite desirability function which indicates overall desirability value can be constructed by taking geometric mean of the desirability values as shown in the Equation 6.

$$D = (d_1 d_2 \ldots d_n)^{1/n} \qquad \text{Equation 6}$$

7 RESULTS OF THE DESIRABILITY OPTIMIZATION

Optimized store geometries, which are obtained with the desirability function optimization, are shown in this section. Although it is possible to increase number of design examples, for the purposes of the present work only two different optimization studies are carried out. For each case computed and predicted values are compared with each other to understand prediction capabilities of the desirability function approach.

7.1 Case 1 – Maximum Lift to Drag Ratio, Maximum Normal Force Coefficient and Target Static Margin

Case 1 is to reach a design which has high aerodynamic efficiency, high normal force coefficient and zero static margin over the investigated flow conditions. According to these goals, constructed optimization setup and its results are given in Table 5.

Table 5. Optimization Summary for Case 1

Case 1: Total Desirability= 0.935							
Response	Lower Limit	Upper Limit	Goal	Relative Importance	Predicted Value	Computed Value	% Prediction Error
CN	0.5	4.5	Maximize	1	4.27	4.24	0.70
L/D	1	3.25	Maximize	1	3.02	2.98	1.32
S. Margin	-4	7	Target = 0	1	0.00	0.00	0.00

For the Case 1, total desirability value is 0.935 which indicates that all goals are highly satisfied. Furthermore, as can be seen from Table 5, predicted and computed values of three aerodynamic parameters have excellent agreement with each other.

Obtained geometry from the desirability optimization is shown in Figure 10 for the Case 1. As expected, total area of lifting surfaces is maximized in order to reach highest possible normal force coefficient. Moreover, distance between strake and fins are adjusted such that static margin meets the target value.

Figure 10. Desirable Geometry for Case 1

7.2 Case 2 – Maximum Lift to Drag Ratio, Target Normal Force Coefficient and Target Static Margin

Case 2 is designed to reach store geometry which has target normal force coefficient and target static margin with highest possible lift to drag ratio. This case represents a situation in which normal force and therefore maneuvering capability is limited possibly due to structural considerations. Furthermore, in this case predetermined static margin is to be obtained for ensuring level of static stability over the investigated flow conditions. Summary of the desirability function optimization for the case 2 is given in Table 6.

Table 6. Optimization Summary for Case 2

Case 2: Total Desirability= 0.906							
Response	Lower Limit	Upper Limit	Goal	Relative Importance	Predicted Value	Computed Value	% Prediction Error
CN	0.5	4.5	Target= 3	1	3.05	2.98	2.30
L/D	1	3.25	Maximize	1	2.71	2.63	2.95
S. Margin	-4	7	Target= 3	1	2.99	3.15	5.35

For the case 2, total desirability value is 0.906 which indicates that goals are again highly satisfied. Although, discrepancies between the predicted and computed values are more significant compared to the case 1 prediction errors are still in the acceptable range.

Desirable geometry, which is obtained from optimization to meet the goals of the case 2, is shown in Figure 11. For this case total area of the lifting surfaces are limited to meet the target value of normal force coefficient. Furthermore, it is seen that strakes are placed further downstream compared to the geometry obtained in Case 1 to get high level of static margin.

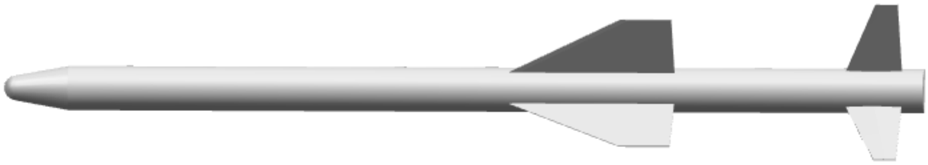

Figure 11. Desirable Geometry for Case 2

8 CONCLUSION

In this study, desirability function optimization is carried out to determine aerodynamic shape of a store with strake-fin configuration. Through the results of the study, it is clear that desirability function optimization is capable of determining aerodynamically optimized configurations. It is also shown that response surface methodology is very efficient way of obtaining and characterizing the effect of the design variables on the overall performance. To reduce computational cost and time spend in the process of analyzing effect of each design variables, response surface methodology can be used especially for the cases in which fast aero prediction methods are not sufficient. This study can be extended by adding more flight conditions and more specified considerations of flight mechanics. It can be also beneficial to perform optimization with more sophisticated methods, such as genetic algorithms, to compare relative advantageous of the different ways of optimization.

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