EWADE
European Workshop on Aircraft Design Education

THE IMPORTANCE OF NON-LINEARITIES IN AIRCRAFT PRELIMINARY DESIGN

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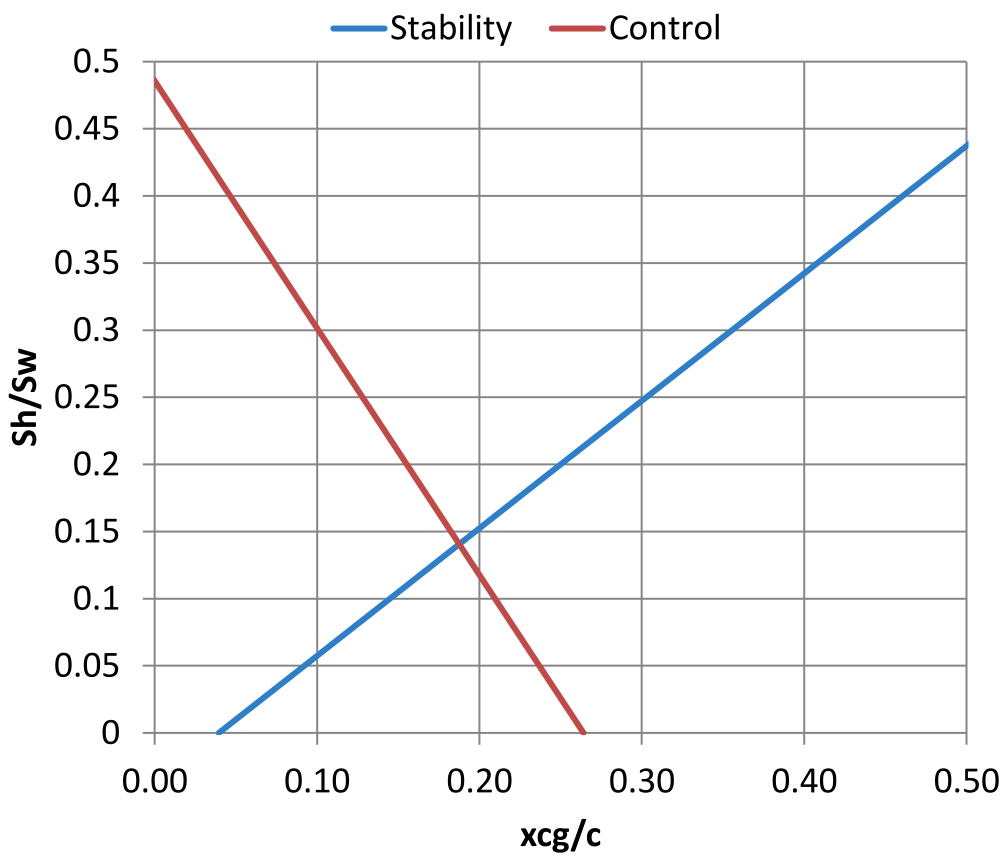
Layout of presentation

- Some remarks on longitudinal stability and control:
  - correct approach
  - non-linearities
- Influence on Horizontal Tail preliminary design
- Non-linearities in directional stability and control and vertical tail design
- Conclusions

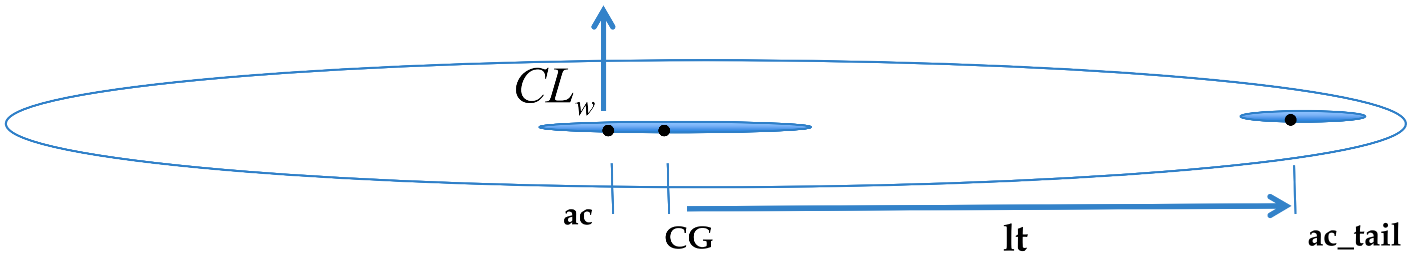
DIPARTIMENTO DI INGEGNERIA INDUSTRIALE
SEZIONE INGEGNERIA AEROSPAZIALE

Horizontal tail design – scissors plot

- Stability condition
- Control condition



Longitudinal stability and control



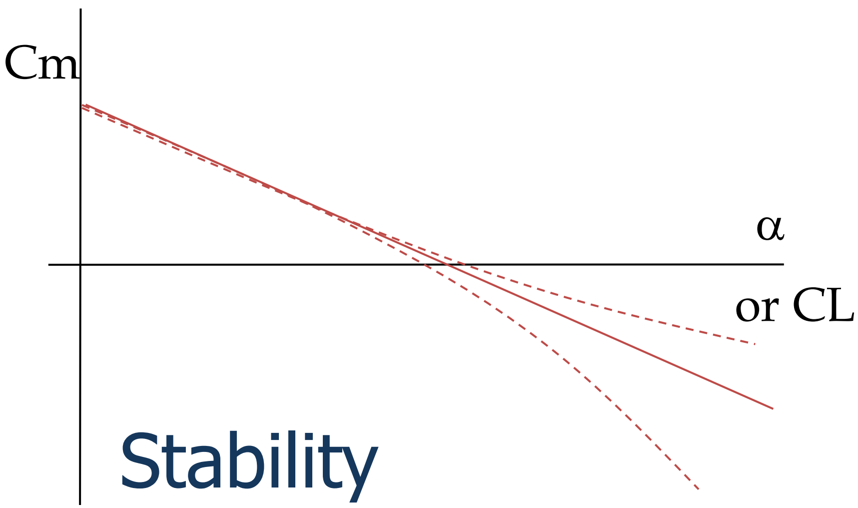
$$x_a = \frac{x_{cg} - x_{ac_{wb}}}{c}$$

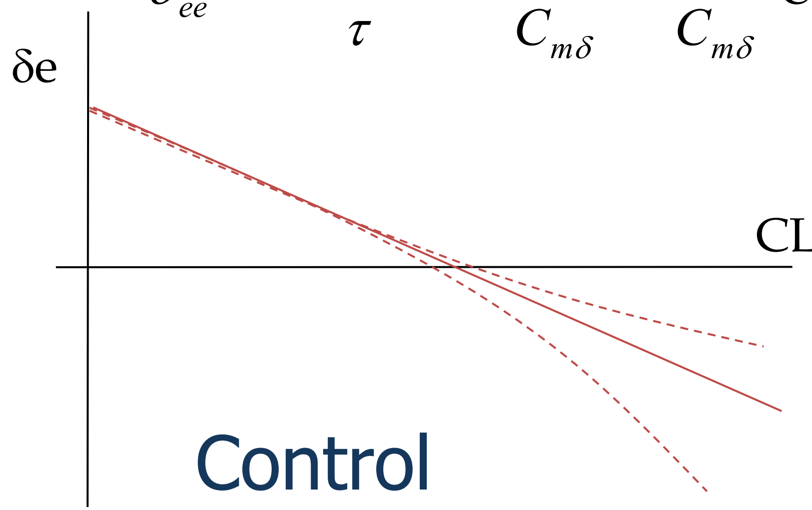
$$\bar{v}_t = \frac{S_t l_t}{S_w c_w}$$

$$C_m = C_{m_{ac}} + \left[x_a - \frac{a_t}{a_w}\bar{v}_t\left(1-\frac{d\varepsilon}{d\alpha}\right)\right]C_{Lw} - a_t \bar{v}_t \left(i_t - \alpha_{0w}\right)$$

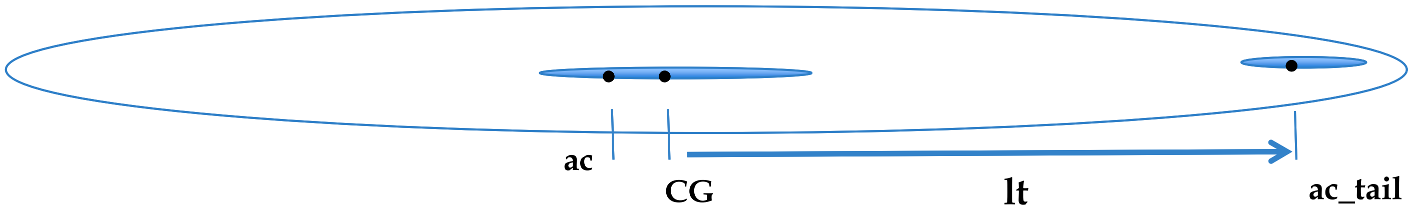
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a_w}\bar{v}_t\left(1-\frac{d\varepsilon}{d\alpha}\right)\eta_t$$

$$\delta_{ee} = \frac{\alpha_{0w} - i_{t0}}{\tau} - \frac{C_{m0w}}{C_{m\delta}} - \frac{C_{m_{CL}}}{C_{m\delta}} C_{Le}$$





Longitudinal stability and control



$$x_a = \frac{x_{cg} - x_{ac_{wb}}}{c}$$

$$\bar{v}_t = \frac{S_t l_t}{S_w c_w}$$

$$C_m = C_{m\,\mathrm{ac}} + \left[x_a - \frac{a_t}{a_w}\bar{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]C_{Lw} - a_t\bar{v}_t\left(i_t - \alpha_{0w}\right)$$

$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a_w}\bar{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

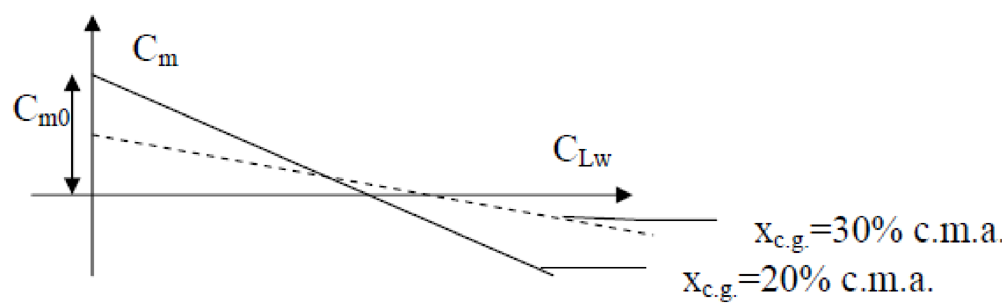
Main infl of CG

Sec. infl of CG

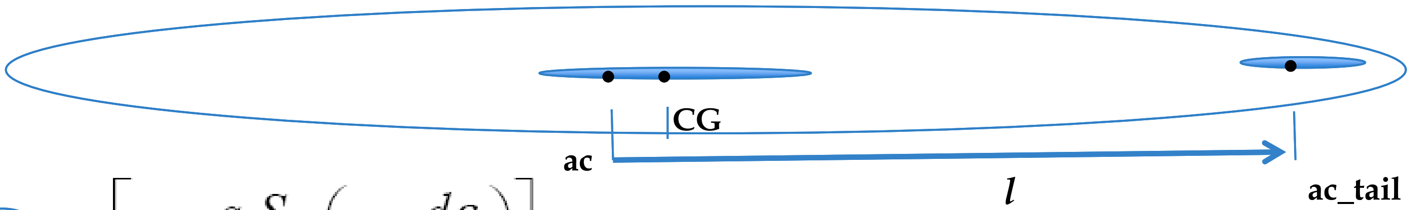
Influence of CG position on Volumetric coeff.
=> Cm @ C_{Lw} =0 depends on CG

Neutral point is function (weak) of the CG position

$$N_0 = x_{ac} + \frac{a_t}{a_w}\bar{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$



Longitudinal stability and control – correct approach



$$x_a = \frac{x_{cg} - x_{ac_{wb}}}{c}$$

$$a = a_w\left[1 + \frac{a_t S_t}{a_w S_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]$$

Complete aircraft lift curve slope

$$\overline{V}_t' = \left[\overline{V}_t + x_a \frac{S_t}{S_w}\right] = \left[\frac{S_t}{S_w}\left(\frac{l_t}{c} + x_a\right)\right] = \frac{S_t \cdot l}{S_w \cdot c}$$

$$C_m = C_{m_{ac}} + \left[x_a - \frac{a_t}{a_w}\overline{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] C_{Lw} - a_t \overline{v}_t\left(i_t - \alpha_{0w}\right)$$

$$C_m = C_{m_{ac}} + \left[x_a - \frac{a_t}{a}\overline{v}_t'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] C_L - a_t \overline{v}_t' \alpha_{t0}$$

$$\alpha_{t0} = \frac{1}{\left[1 + \frac{a_t S_t}{a S_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]}\left(i_t - \alpha_{0w}\right)$$

$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a_w}\overline{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

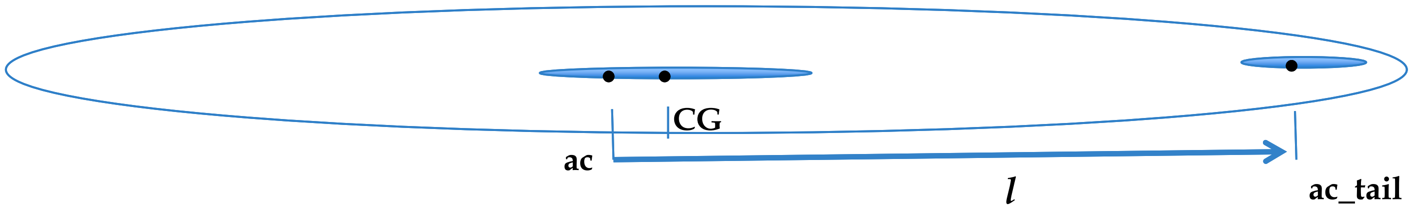
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a}\overline{v}_t'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

$$\frac{S_H}{S}$$

$$N_0 = x_{ac} + \frac{a_t}{a}\overline{v}_t'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

$$\frac{S_H}{S}$$

Longitudinal stability and control – correct approach



$$x_a = \frac{x_{cg} - x_{ac_{wb}}}{c}$$

- The equations include the contribution of the horizontal tail on lift
- The lift and moment equation are linked

⇒The neutral point does not depend anymore on CG

⇒ The Cm @ CL=0 does not depend on the CG (it is a pure moment)

$$C_m = C_{m_{\text{ac}}} + \left[x_a - \frac{a_t}{a}\overline{v_t}'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]C_L - a_t\overline{v_t}'\alpha_{t0}$$

$$\overline{v_t}' = \frac{S_t \cdot l}{S_w \cdot c}$$

$$\alpha_{t0} = \frac{1}{\left[1 + \frac{a_t S_t}{a S_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]}(i_t - \alpha_{0w})$$

$$\frac{S_H}{S}$$

$$N_0 = x_{ac} + \frac{a_t}{a}\overline{v_t}'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

$$\frac{S_H}{S}$$

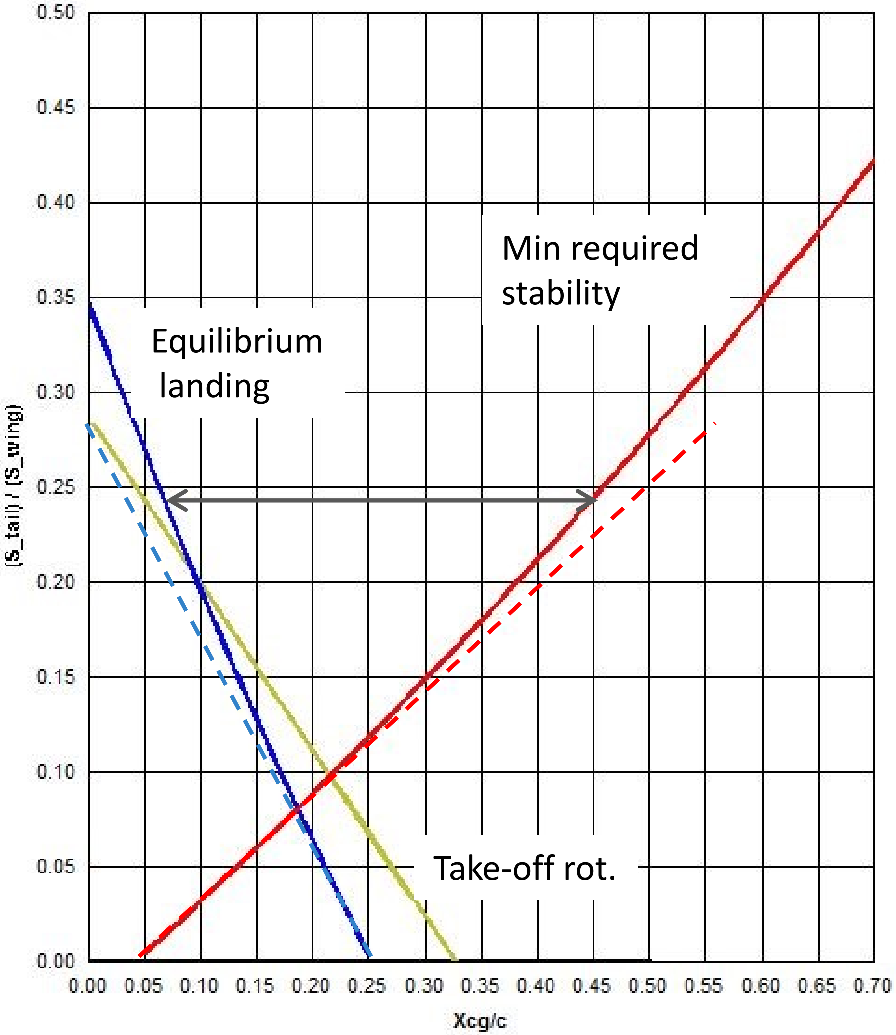
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a}\overline{v_t}'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

Effect of new correct approach on scissors plot

The influence of tail area ratio on neutral point and maximum allowable FWD CG position for equilibrium in landing
is not more linear.

Allowable CG range
It is becoming MORE CRITICAL !

(even -10% xcg travel with Sh/S=0.25



Non-linear effects

$$a = a_w \left[1 + \frac{a_t S_t}{a_w S_w} \left(1 - \frac{d\varepsilon}{d\alpha} \right) \right]$$

Non linearity in h tail lift curve slope

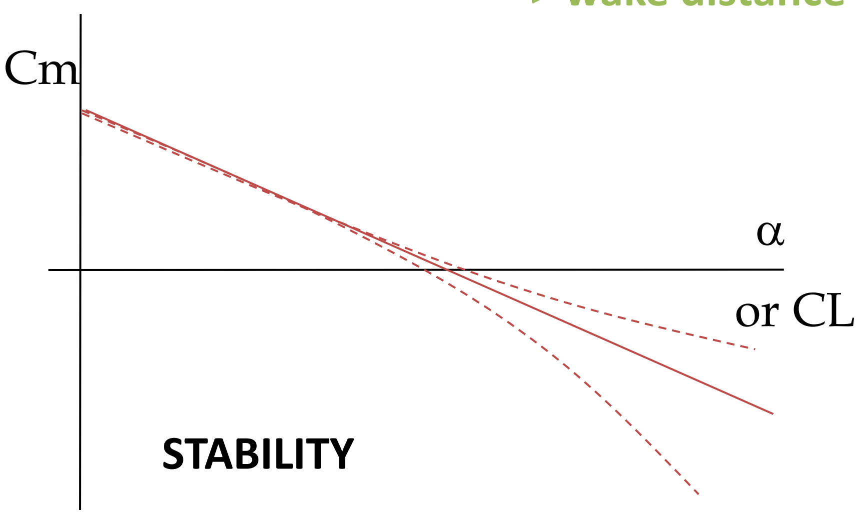
Dynamic pressure @ H tail (influenced by relative wake position)

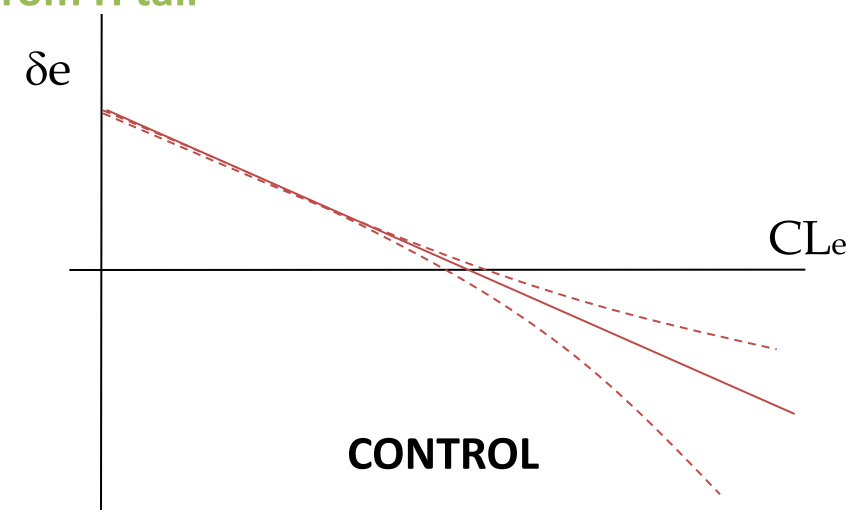
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a} \bar{v}_t' \left(1 - \frac{d\varepsilon}{d\alpha} \right) \eta_t + \underbrace{C_L \left(\frac{2}{\pi A R_e} - \frac{2}{a_w} \right) \cdot \frac{z_a}{c}}$$

Non linearity in wing lift curve slope at high AofA

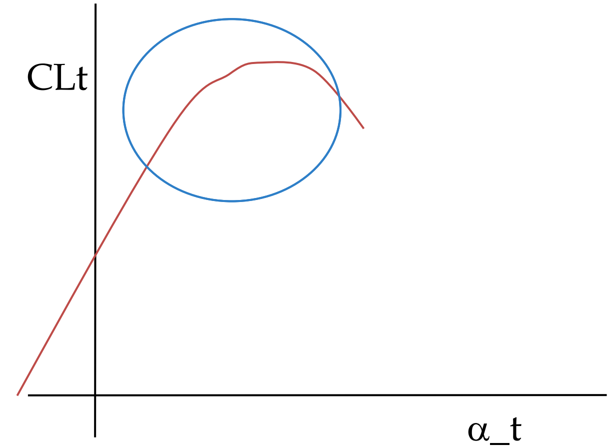
Non linearity in downwash:
-> wing lift curve slope
-> wake distance from H tail

Pendular stability





Stability and control ; effect of a_t



- a_t reduces at high angles of attack of for the H tail, high C_{Lt} and C_{Lw}

$$C_m = C_{m0w} + \left[x_a - \frac{a_t}{a} \bar{v}_t' \left(1 - \frac{d\varepsilon}{d\alpha} \right) \right] C_L - a_t \bar{v}_t' \alpha_{t0}$$

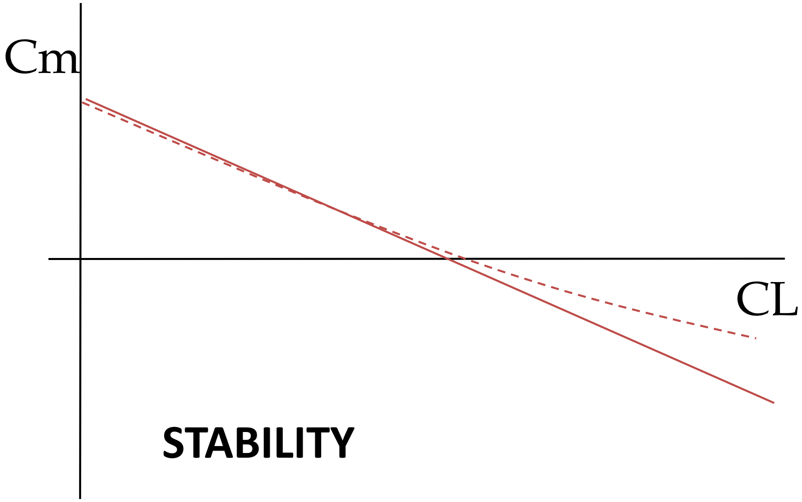
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a} \bar{v}_t' \left(1 - \frac{d\varepsilon}{d\alpha} \right) \eta_t$$

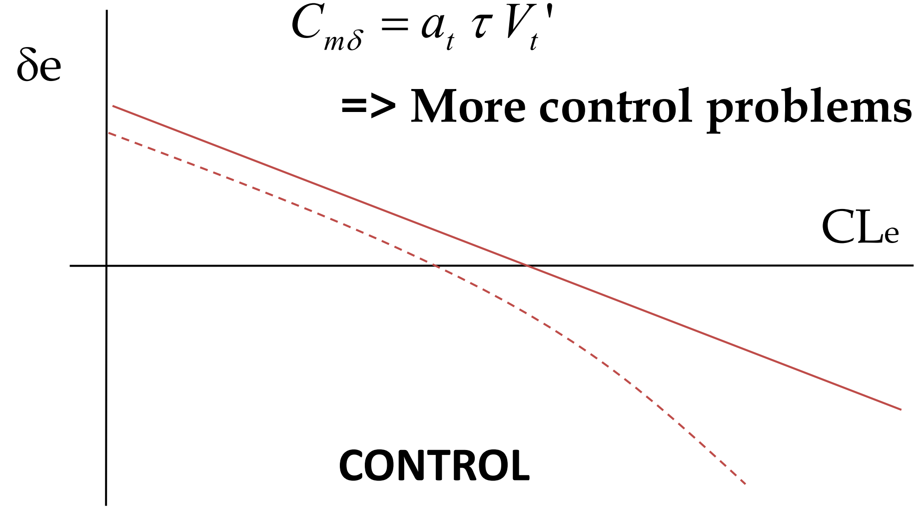
Reduced a_t => **Lower stability margin,**
but also lower control power

$$\delta_{ee} = \frac{\alpha_{0w} - i_{t0}}{\tau} - \frac{C_{m0w}}{C_{m\delta}} - \frac{C_{m_{C_L}}}{C_{m\delta}} C_{Le}$$

$$C_{m\delta} = a_t \, \tau \, V_t'$$

=> More control problems !

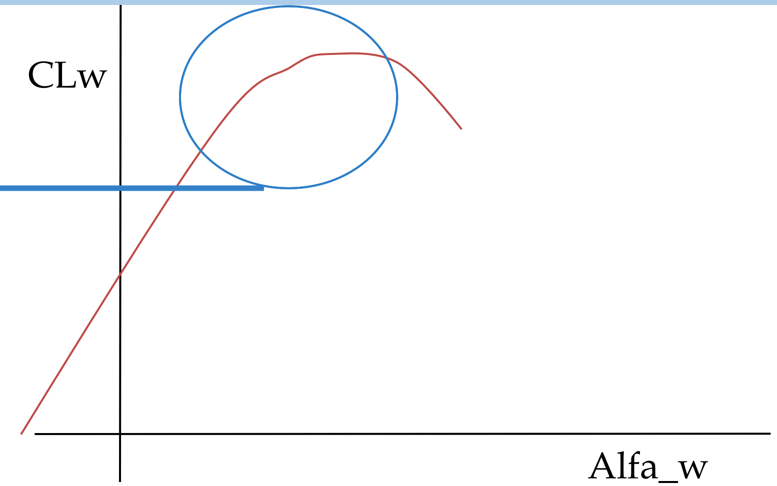




Stability and control; effect of a_w

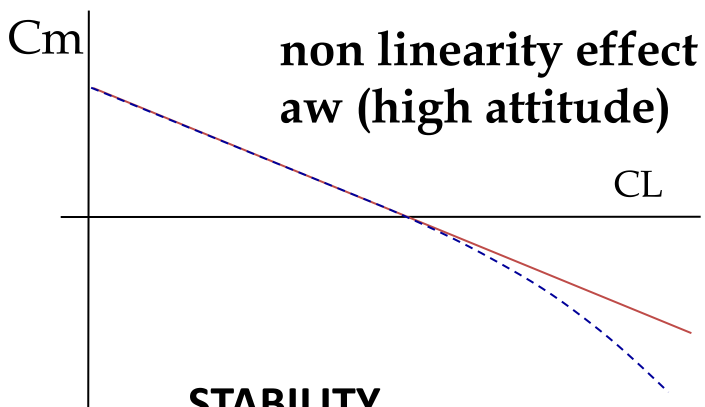
a_w reduces at high attitude, and
static margin of stability increases !

$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a}\bar{v}_t'\left(1-\frac{d\varepsilon}{d\alpha}\right)\eta_t$$



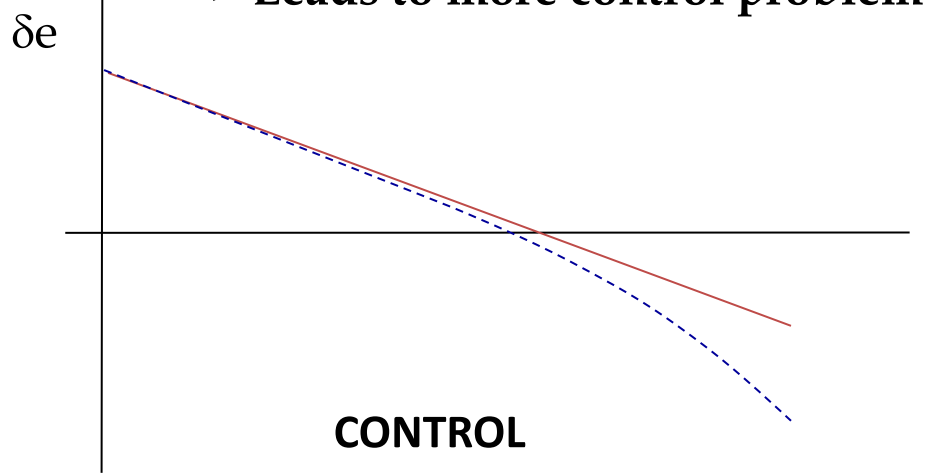
$$\delta_{ee} = \frac{\alpha_{0w} - i_{t0}}{\tau} - \frac{C_{m0w}}{C_{m\delta}} - \frac{C_{m_{C_L}}}{C_{m\delta}} C_{Le}$$

non linearity effect of aw (high attitude)



STABILITY

=> Leads to more control problem



CONTROL

Stability and control; effect of downwash non-linearities

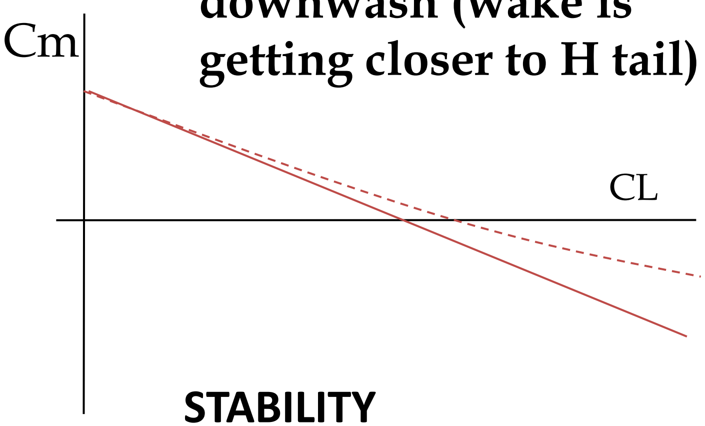
$$\frac{dC_m}{dC_L} = x_a - \frac{a_t}{a}\bar{v}_t'\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

Downwash influences SSM $\frac{d\varepsilon}{d\alpha}$ $\frac{dC_m}{dC_L}$

$$\delta_{ee} = \frac{\alpha_{0w} - i_{t0}}{\tau} - \frac{C_{m0w}}{C_{m\delta}} - \frac{C_{mC_L}}{C_{m\delta}} C_{Le}$$

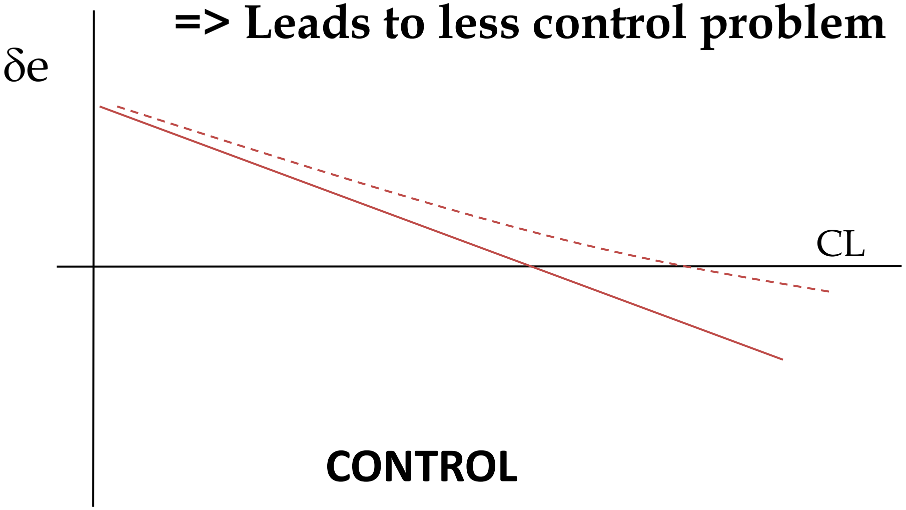
Downwash influences control $\frac{d\delta_{ee}}{dC_L}$

non linearity effect of downwash (wake is getting closer to H tail)



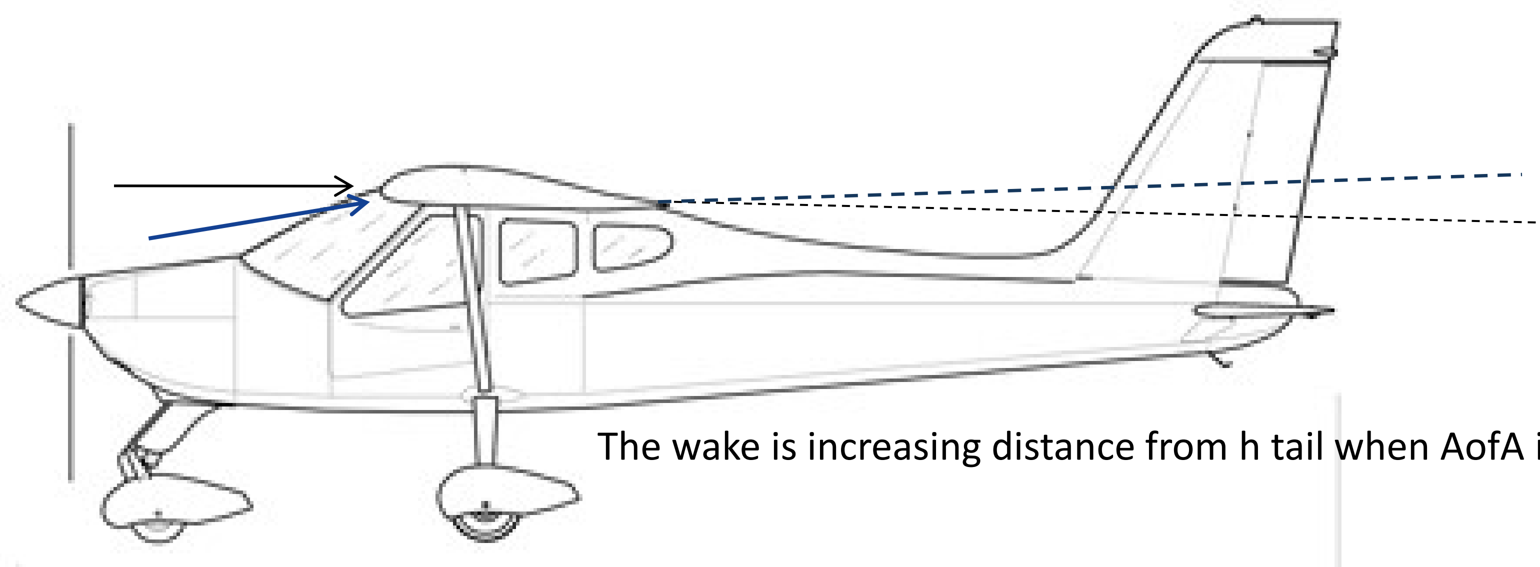
STABILITY

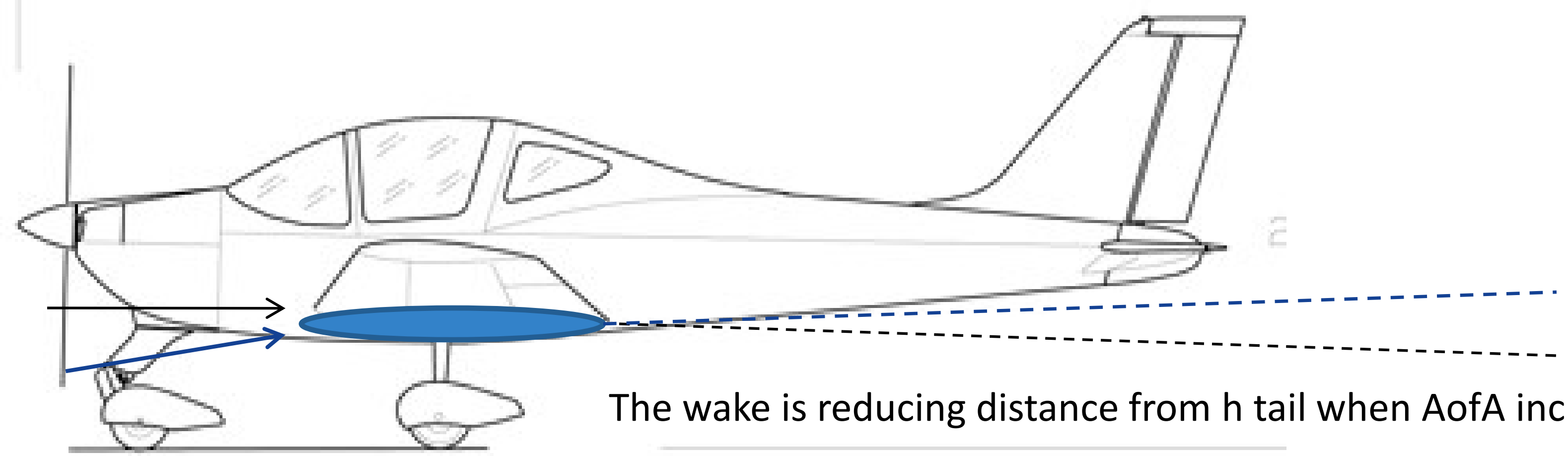
=> Leads to less control problem



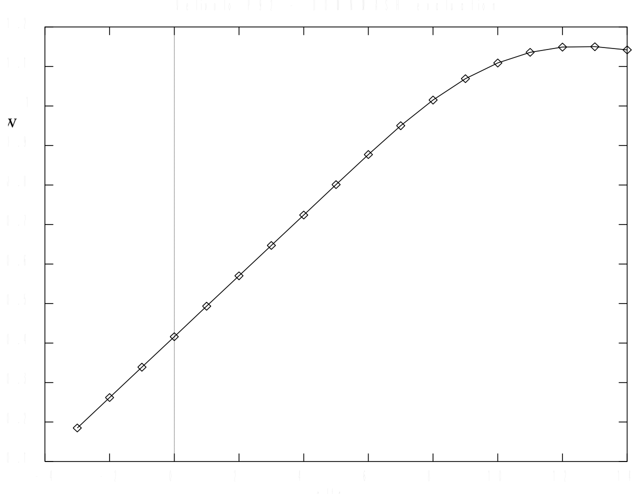
CONTROL

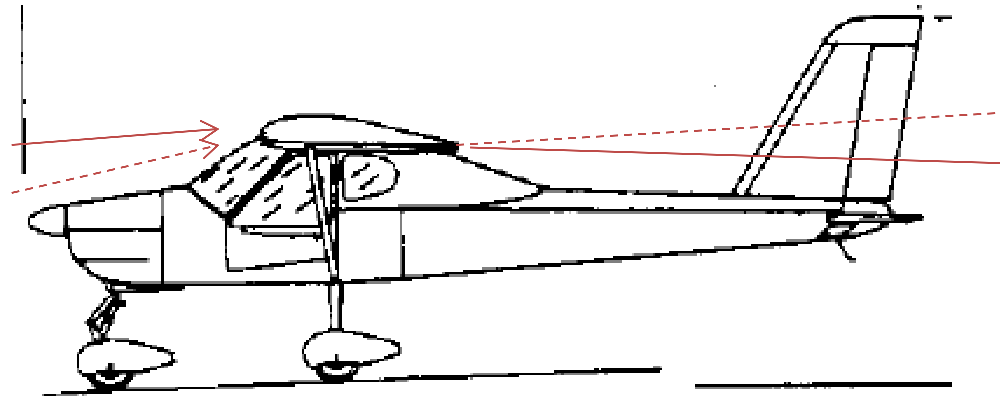
Effect of downwash non-linearities (high wing vs low-wing)

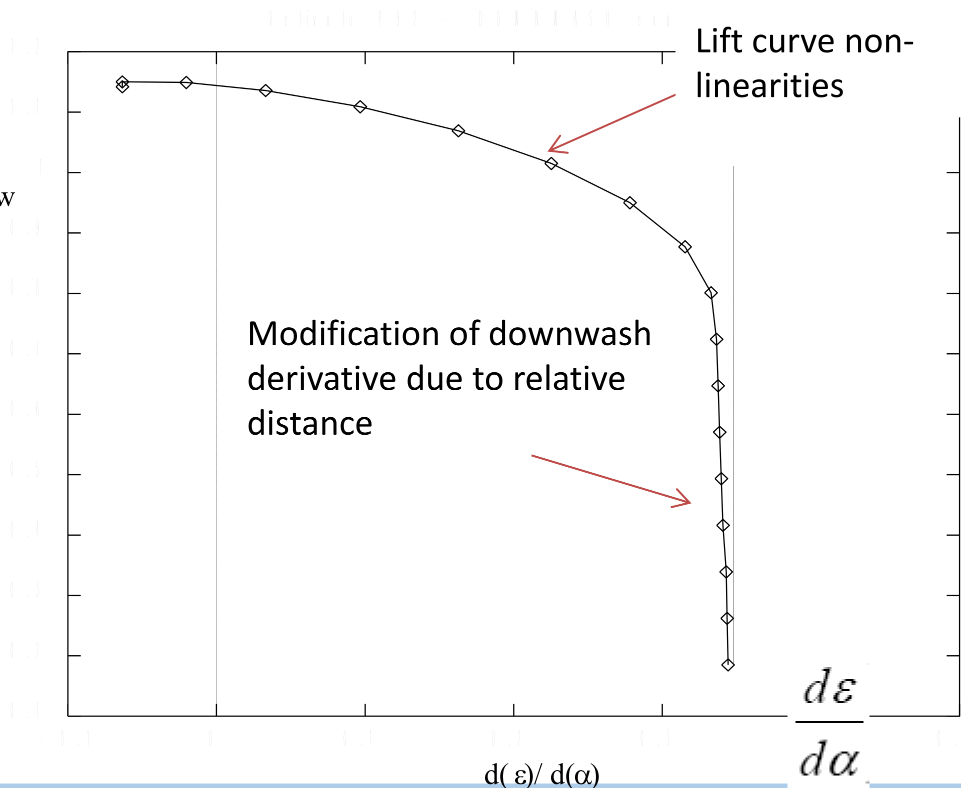




Effect of downwash non-linearities



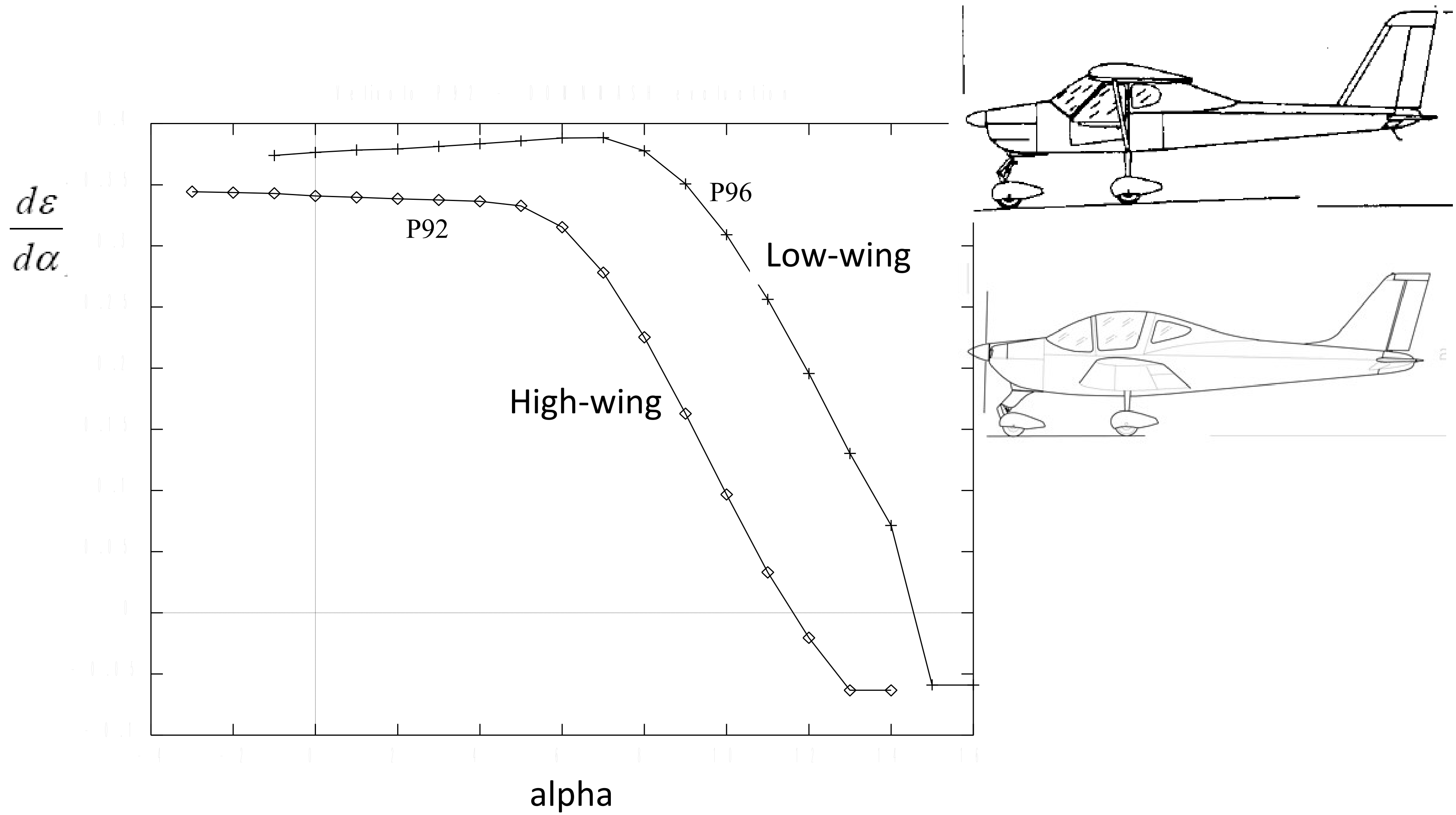




In the linear wing lift curve range (< 10 deg) we can have differences in downwash derivative of about 5-7 % (due to different wake distance).

=> Which flight condition is more critical for stability and which one for control ?

Effect of downwash non-linearities (high wing vs low-wing)



Pendular Stability

$$C_m = \underbrace{C_{macwb} + x_a \cdot C_{Lw} + C_C \cdot \frac{z_a}{c}}_{\text{Wing-body contribution}} - C_{Lt}\,\overline{v_t}\,\eta_t$$

$$C_C = C_D \cdot \cos(\alpha) - C_{Lw} \cdot \sin(\alpha)$$

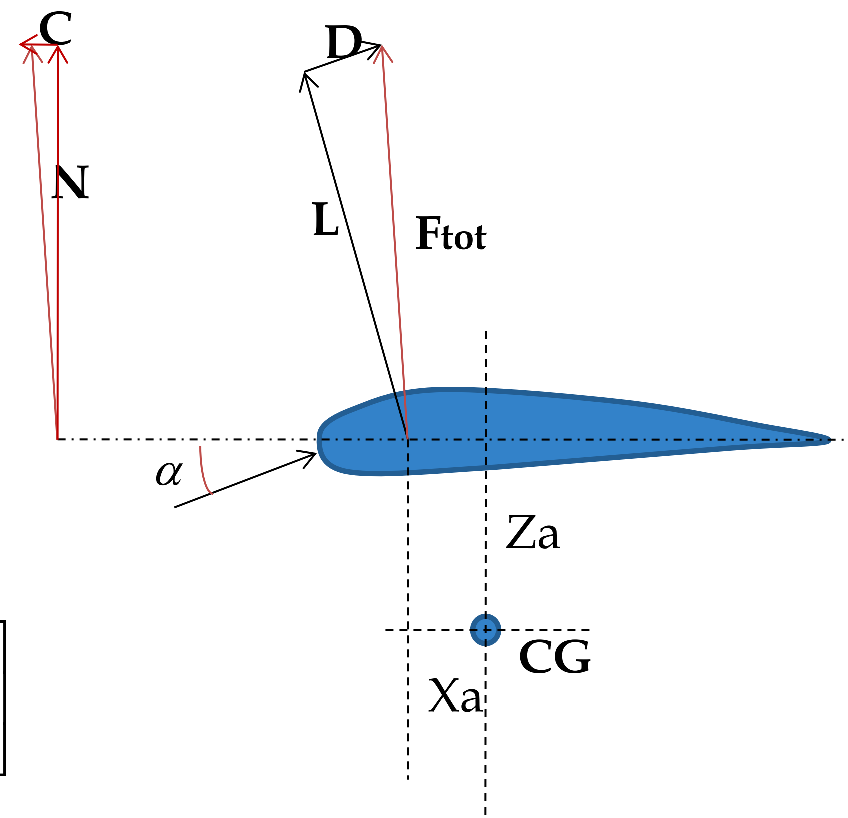
$$C_C = C_D - C_{Lw} \cdot (\alpha_w - i_w)$$

$$C_C = \left[C_{D_o} + C_L^{\,2}\left(\frac{1}{\pi AR_e} - \frac{1}{a_w}\right) - C_L \cdot (-\alpha_{ow} - i_w) \right]$$

about 1/20=0.05 about 1/(5 [1/rad])=0.20

$$\frac{dC_C}{dC_L} = 2 \cdot C_L \underbrace{\left(\frac{1}{\pi AR_e} - \frac{1}{a_w}\right)}_{\textbf{About -0.15} \quad <0} + (\alpha_{ow} + i_w)$$

about 3-5 deg => 0.07 rad



Ex. → $\dfrac{dC_C}{dC_L} \approx -0.30 \cdot C_L + 0.07$

Always < 0 for CL >0.25

Pendular Stability

$$\frac{dC_m}{dC_L} = \underbrace{x_a + C_{Lw}\left(\frac{2}{\pi AR_e} - \frac{2}{a_w}\right)\cdot\frac{z_a}{c}}_{\text{Wing contribution to the longitudinal stability}} - \frac{a_t}{a}\bar{v}_t'\left(1-\frac{d\varepsilon}{d\alpha}\right)\eta_t$$

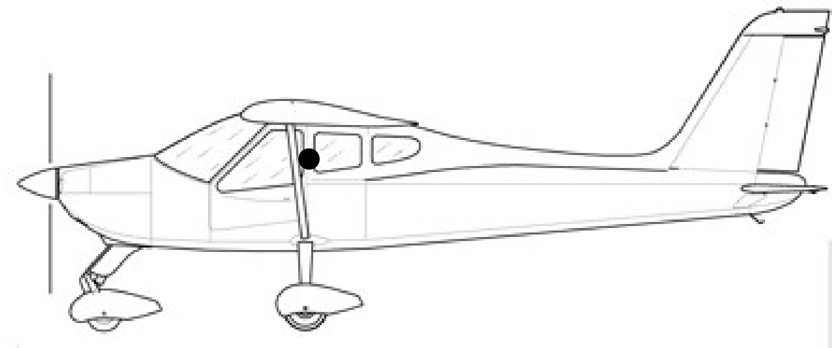
$$\left.\frac{dC_m}{dC_L}\right|_w = x_a + C_{Lw}\left(\frac{2}{\pi AR_e} - \frac{2}{a_w}\right)\cdot\frac{z_a}{c} \approx x_a - 0.30\cdot C_{Lw}\cdot\frac{z_a}{c}$$

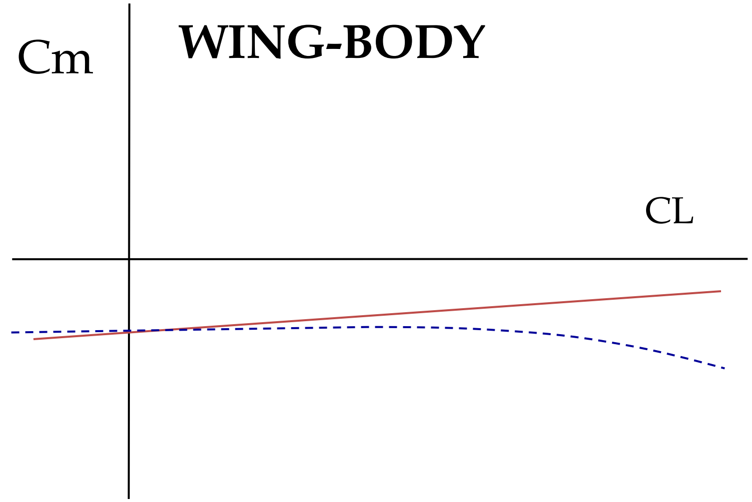
CG at al 10% of MAC below the wing at CL=1 leads to 3% higher SSM

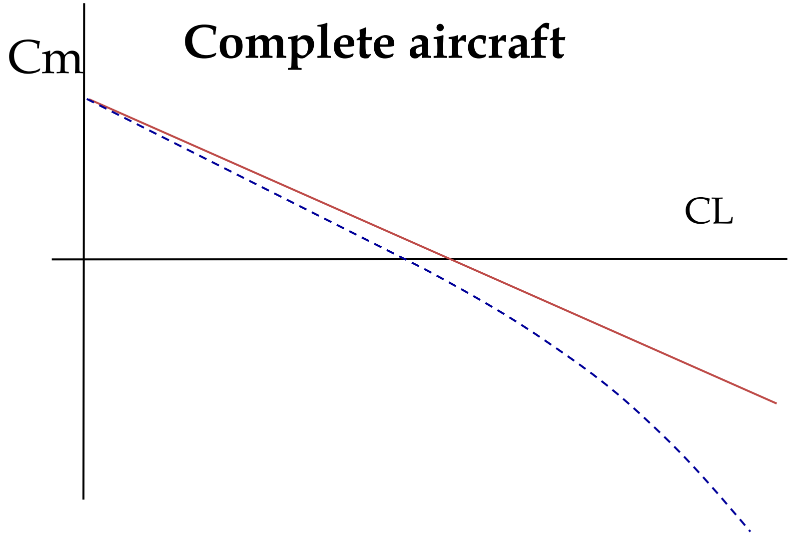
CG at al 30% of MAC below the wing at CL=1 leads to 9% higher SSM

Pendular Stability

$$\frac{dC_m}{dC_L} = x_a + C_{Lw}\left(\frac{2}{\pi AR_e} - \frac{2}{a_w}\right)\cdot\frac{z_a}{c} - \frac{a_t}{a}\overline{v_t}'\left(1-\frac{d\varepsilon}{d\alpha}\right)\eta_t$$







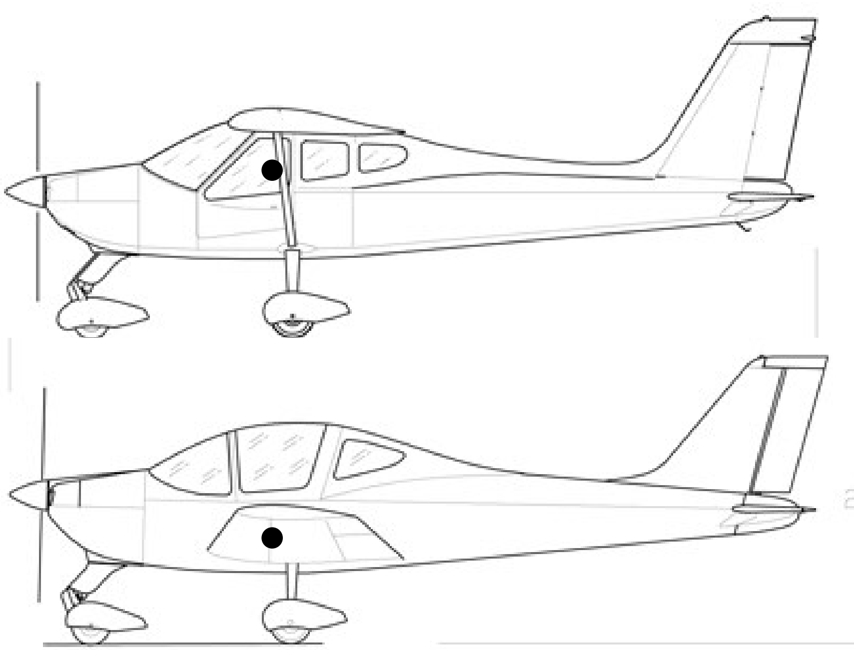
High wing aircraft usually has a 25-30% ZCG below the wing

As it has been highlighted this can lead to about +10% of SSM at CL=1 (climb)

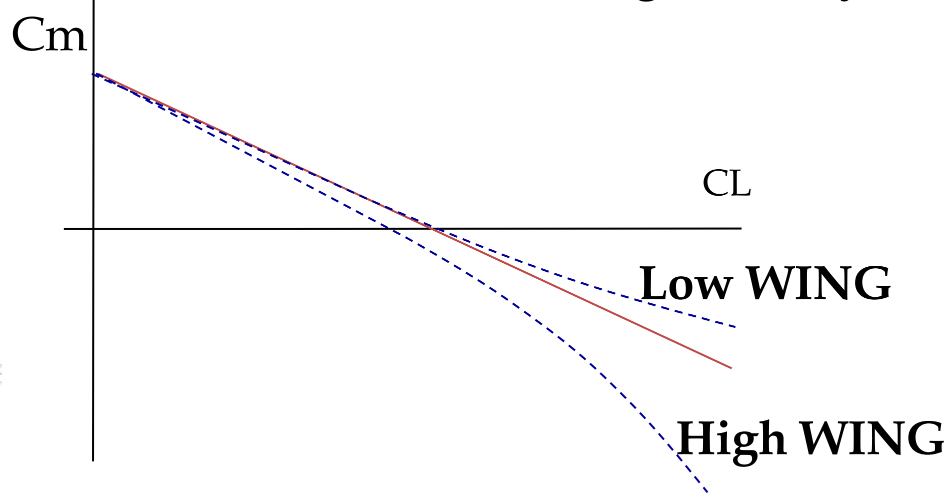
In full flap condition (approach in landing) , with CL=2 => + 20%

Pendular Stability

$$\frac{dC_m}{dC_L} = \underbrace{x_a + C_{Lw}\left(\frac{2}{\pi AR_e} - \frac{2}{a_w}\right)\cdot\frac{z_a}{c}} - \frac{a_t}{a_w}\overline{v_t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

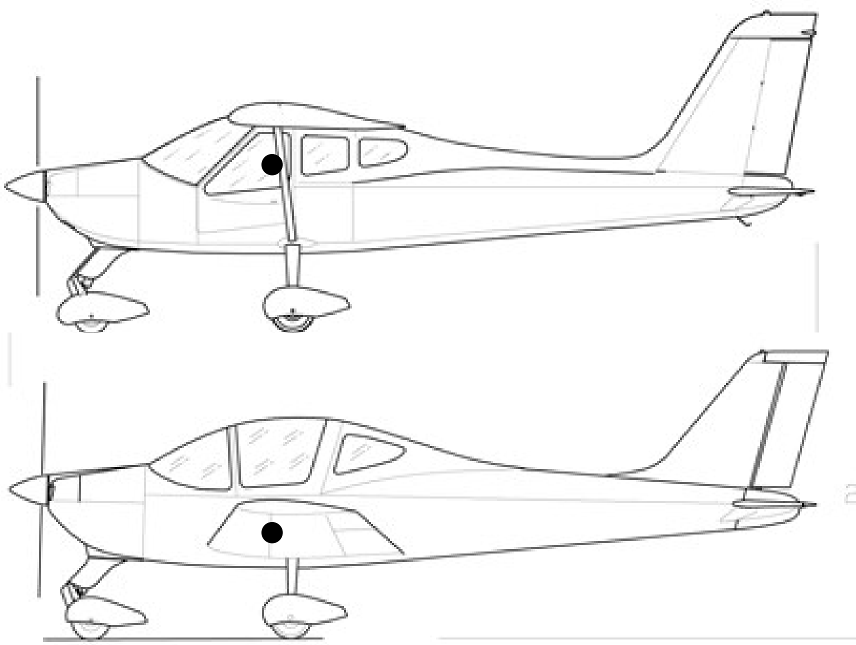


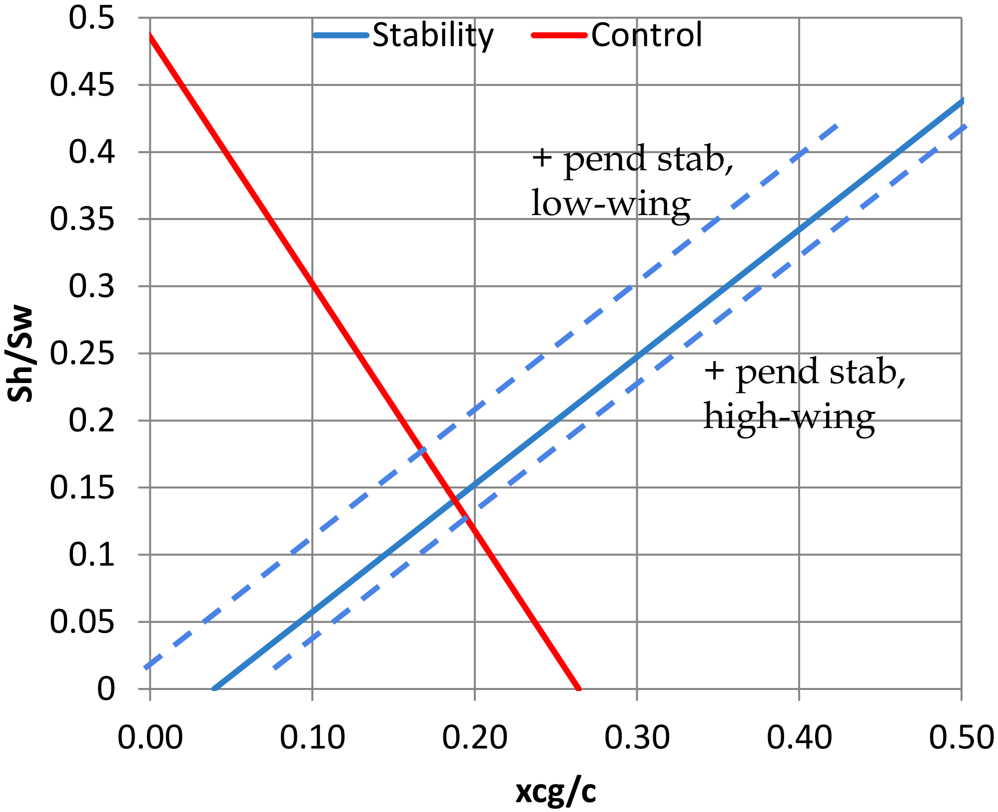
Different effect on long. stability



Pendular Stability; high wing vs low-wing, Design

- For high-wing, the worst case for stability is CL=0, in this case in preliminary design the effect on stability can be neglected. Maybe a value of the effect in high-speed cruise cond (CL=0.30) can be considered. => shift of about 3%
- For low-wing config., the worst case to be considered for design is stability at high CL. => shift left of 5 to 10 %

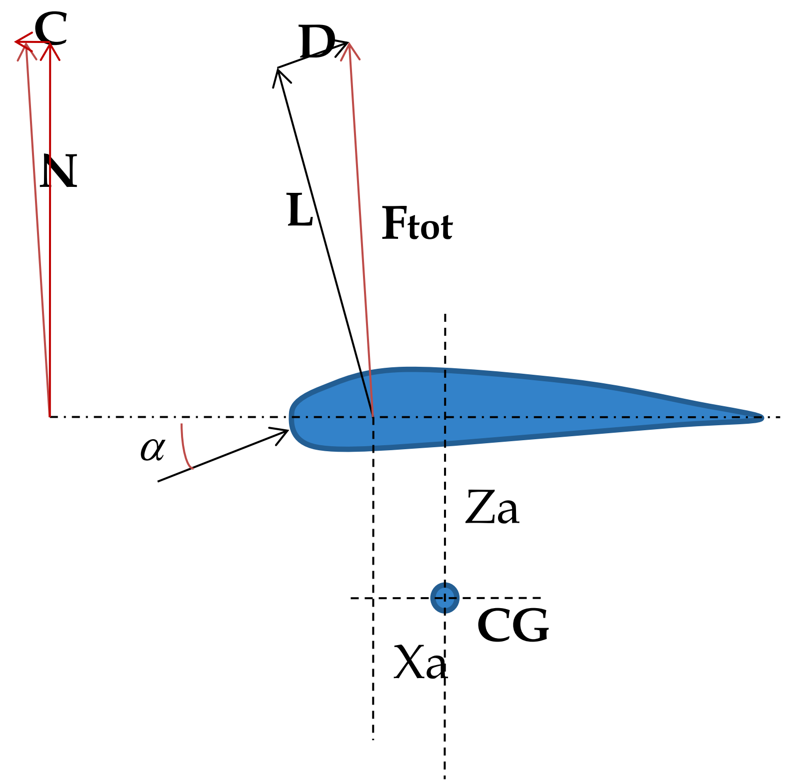




Longitudinal Control – Pendular effect

$$C_m = C_{macwb} + x_a \cdot C_{Lw} + C_C \cdot \frac{z_a}{c} - C_{Lt} \bar{v}_t \eta_t$$

$$C_C = \left[\underbrace{C_{D_o}}_{0.0070} + \underbrace{C_L^{\,2}\left(\frac{1}{\pi AR_e} - \frac{1}{a_w}\right)}_{-\,0.52} - \underbrace{C_L \cdot \left(-\alpha_{ow} - i_w\right)}_{-\,0.480} \right]$$



For a Regional Turboprop, LDG conditions:

- CD0w = 0.0070
- CLapproach = 2
- piARe = 27
- a_w = 6/rad
- α_{0w} = -12 deg.
- i_w = 2 deg.
- **z_a/c = 0.25**

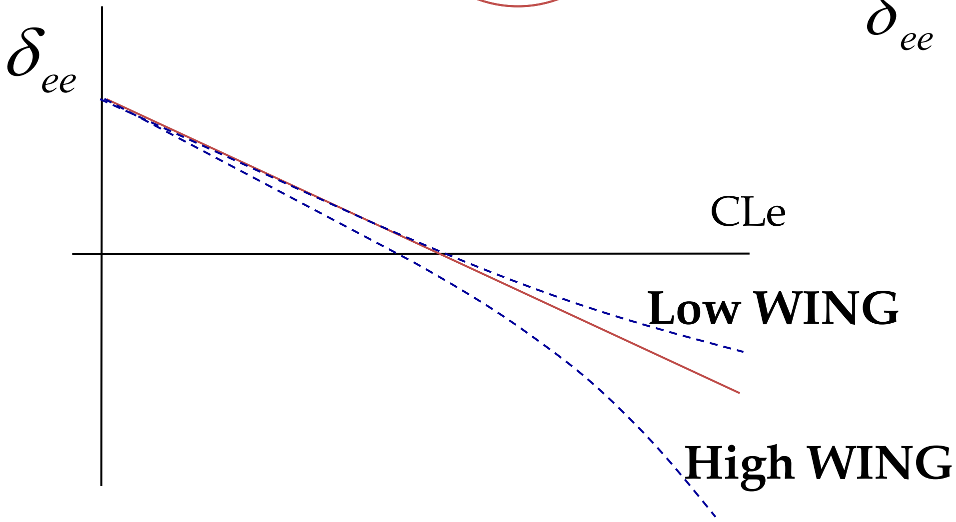
$$C_C \approx -0.90 \quad => \Delta C_m \approx -0.23$$

It is like comparable to increment due to flap

Longitudinal Control – Pendular effect

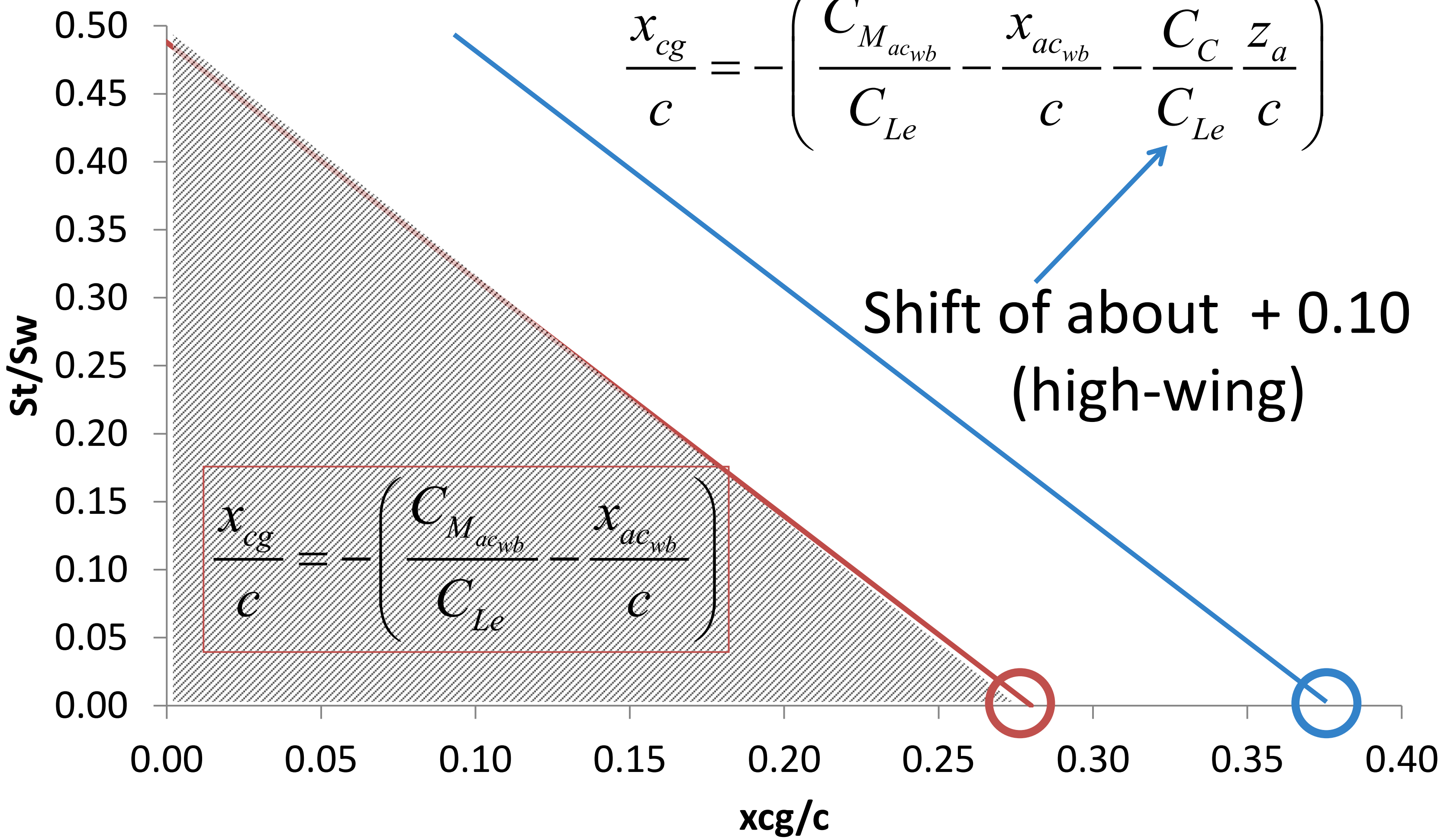
$$C_m = C_{macwb} + x_a \cdot C_{Lw} + C_C \cdot \frac{z_a}{c} - C_{Lt} \bar{v}_t \eta_t$$

$$\delta_{ee} = \frac{\alpha_{0w} - i_{t0}}{\tau} - \frac{C_{m0w}}{C_{m\delta}} - \frac{C_{m_{C_L}}}{C_{m\delta}} C_{Le}$$



For control in LDG, and for the high-wing configuration, pendular effect **must be considered**.

Longitudinal Control – Pendular effect Influence on Design (Scissors plot)



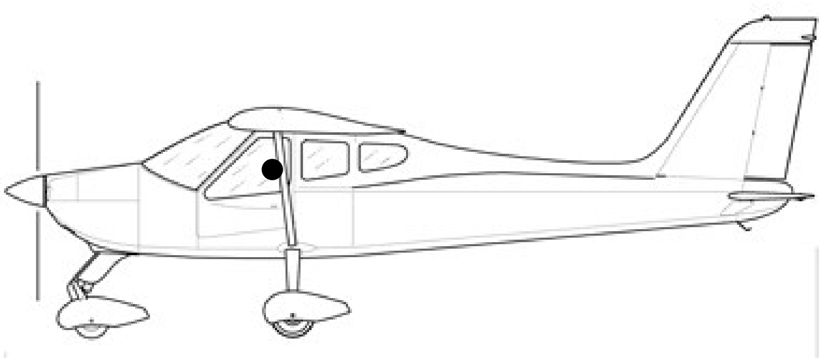
$$\frac{x_{cg}}{c} = -\left(\frac{C_{M_{ac_{wb}}}}{C_{Le}} - \frac{x_{ac_{wb}}}{c} - \frac{C_C}{C_{Le}}\frac{z_a}{c}\right)$$

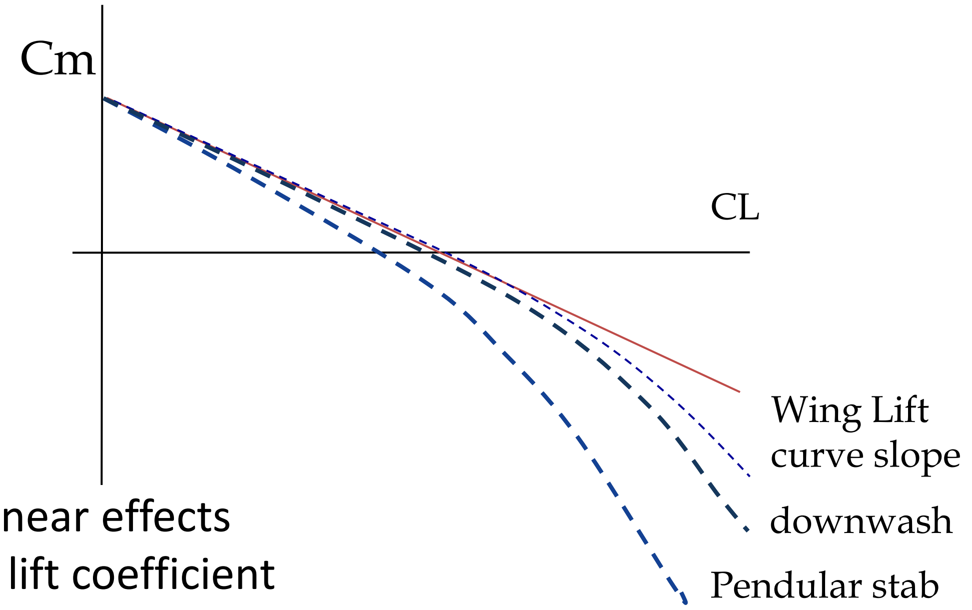
$$\frac{x_{cg}}{c} = -\left(\frac{C_{M_{ac_{wb}}}}{C_{Le}} - \frac{x_{ac_{wb}}}{c}\right)$$

Longitudinal Non-linear effects

$$\frac{dC_m}{dC_L} = x_a + C_{Lw}\left(\frac{2}{\pi AR_e} - \frac{2}{a_w}\right)\cdot\frac{z_a}{c} - \frac{a_t}{a_w}\overline{v}_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_t$$

High wing aircraft



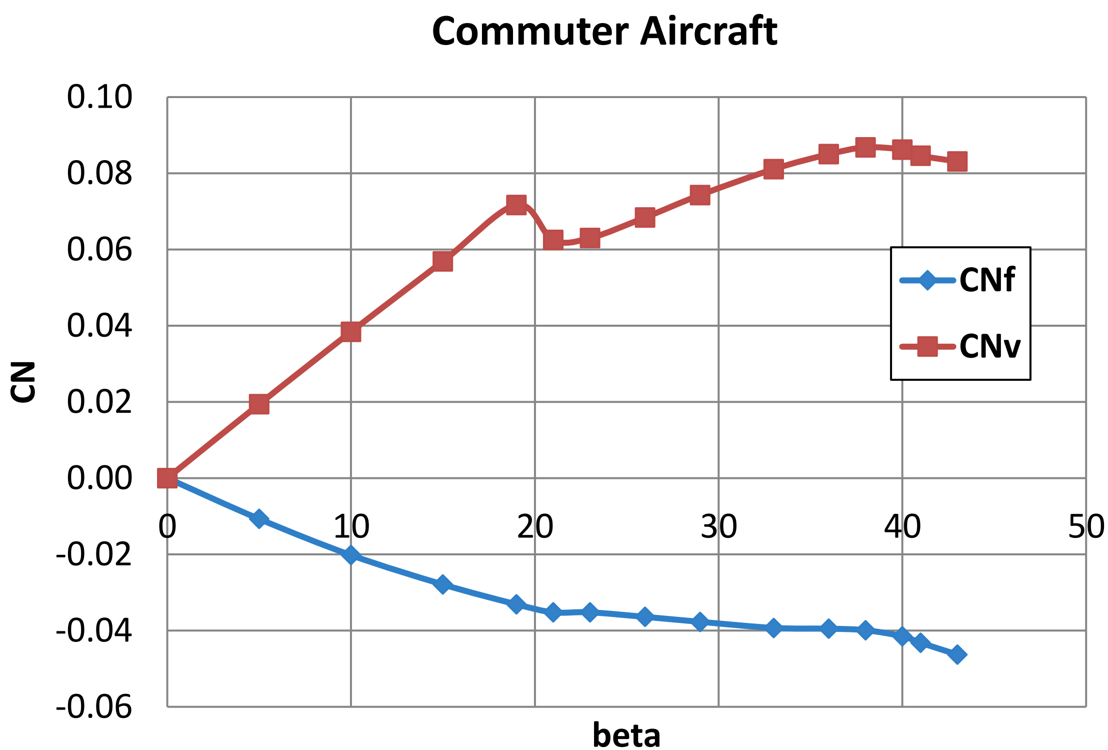


In case of an high-wing aircraft, all the non-linear effects **can lead to a big increase in stability** at high lift coefficient

=> Similar effect on control

Directional stability and control

- Vertical tail contribution
- Fuselage contribution (only stability)



Directional stability and control

- At high angle of sideslip both vertical and fuselage contribution to the directional stability can be non-linear
- Vertical tail can reach stall condition
- Fuselage instability is reduced
- Dorsal fin effect
- Rudder effectiveness is strongly dependent on rudder deflections

Directional Stability

- $C_N = C_{N_\beta}.\beta + C_{N_{\delta r}}.\delta r = 0$

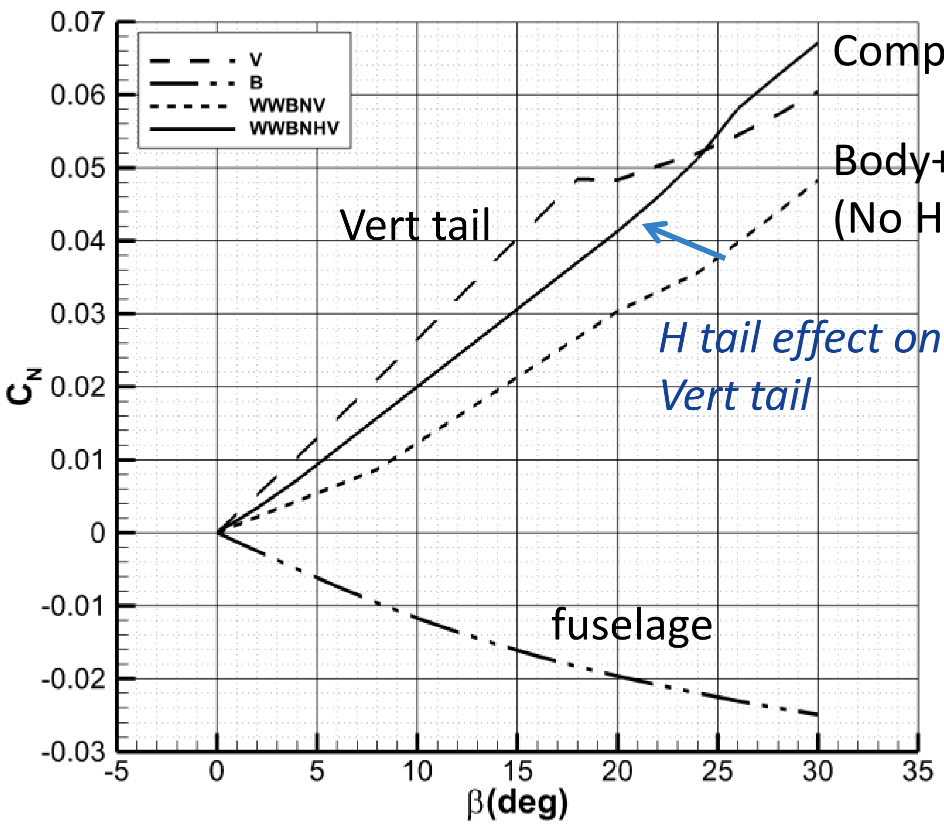
- $C_{N_\beta} = C_{N_{\beta_w}} + C_{N_{\beta_f}} + C_{N_{\beta_v}} + C_{N_{\beta_h}}$

Fuselage contribution

Vertical tail contribution

Directional Stability

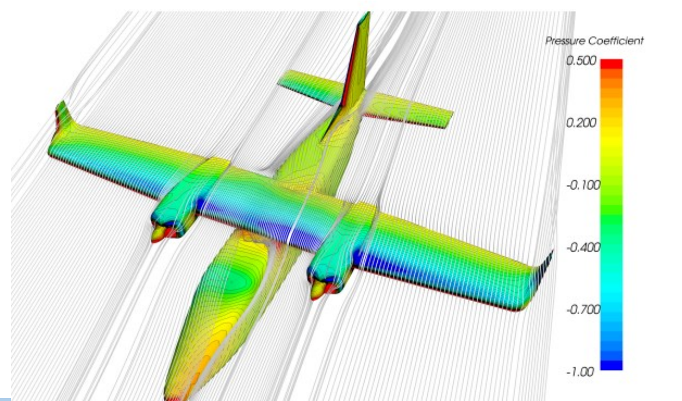
- In the linear range (0-10/15 deg.) mutual effects among aircraft components (fuselage, wing, horizontal tail)
- At high angle of sideslip Vertical tail stability contr reduces



Complete aircraft

Body+Wing+Nac+Vert tail (No HOR Tail)

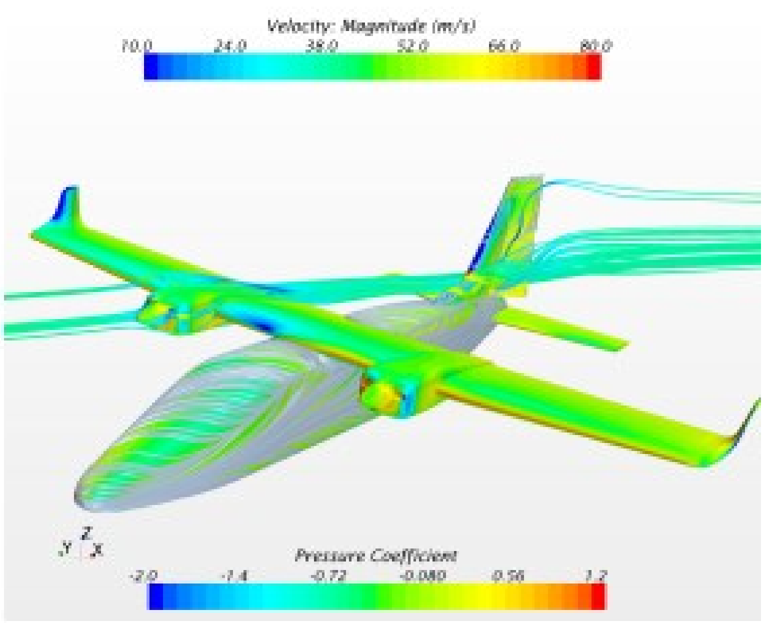
Commuter aircraft
M=0.15
Re=9.5 e6

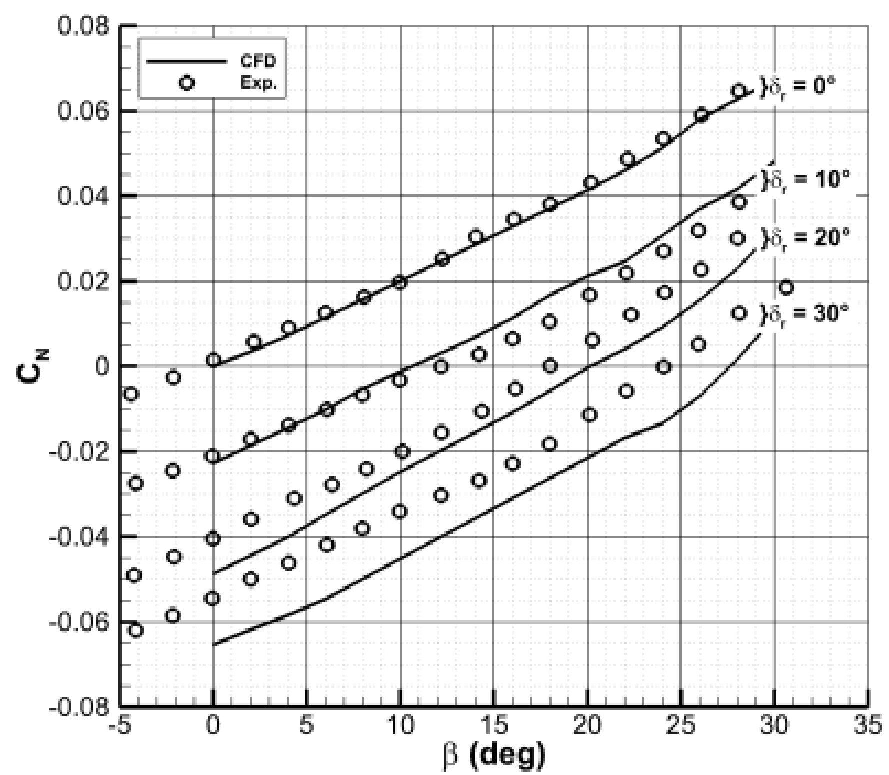


Directional control (Rudder)

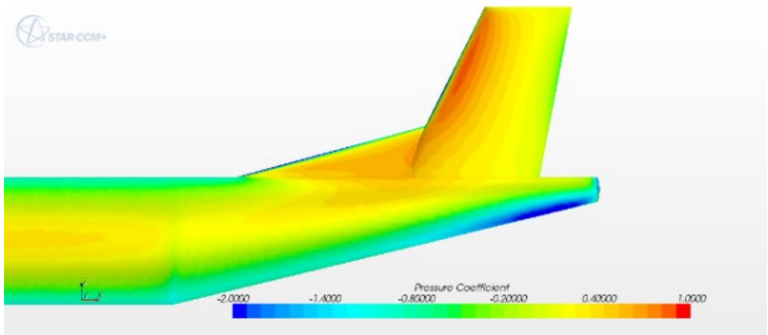
Critical conditions where non-linearity effects should be considered:

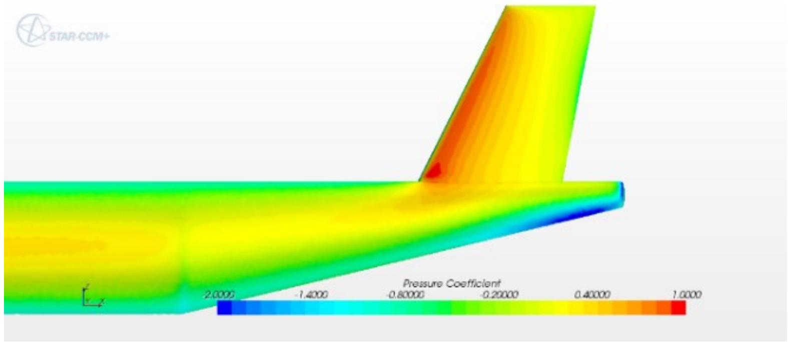
- VMC condition → rudder effectiveness non-linearity
- High sideslip angle in landing condition → Vertical, fuselage and dorsal fin non linearity

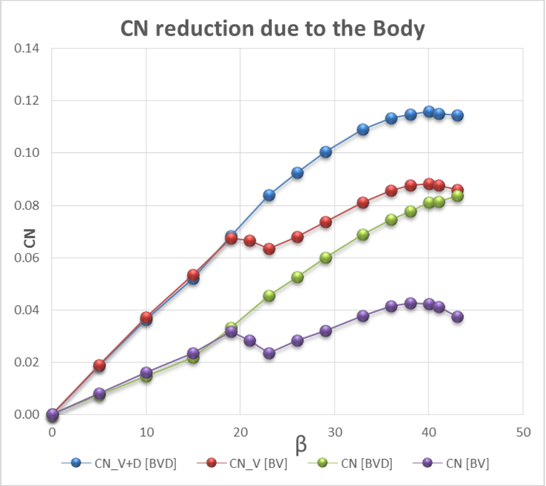


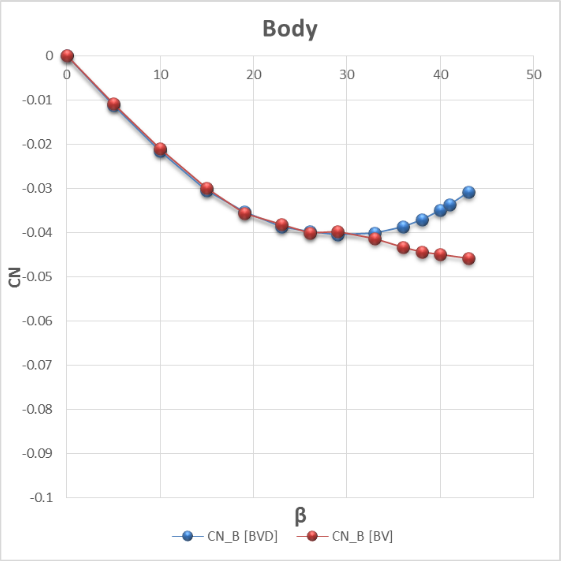


Directional Stability – Vertical tail+dorsal fin

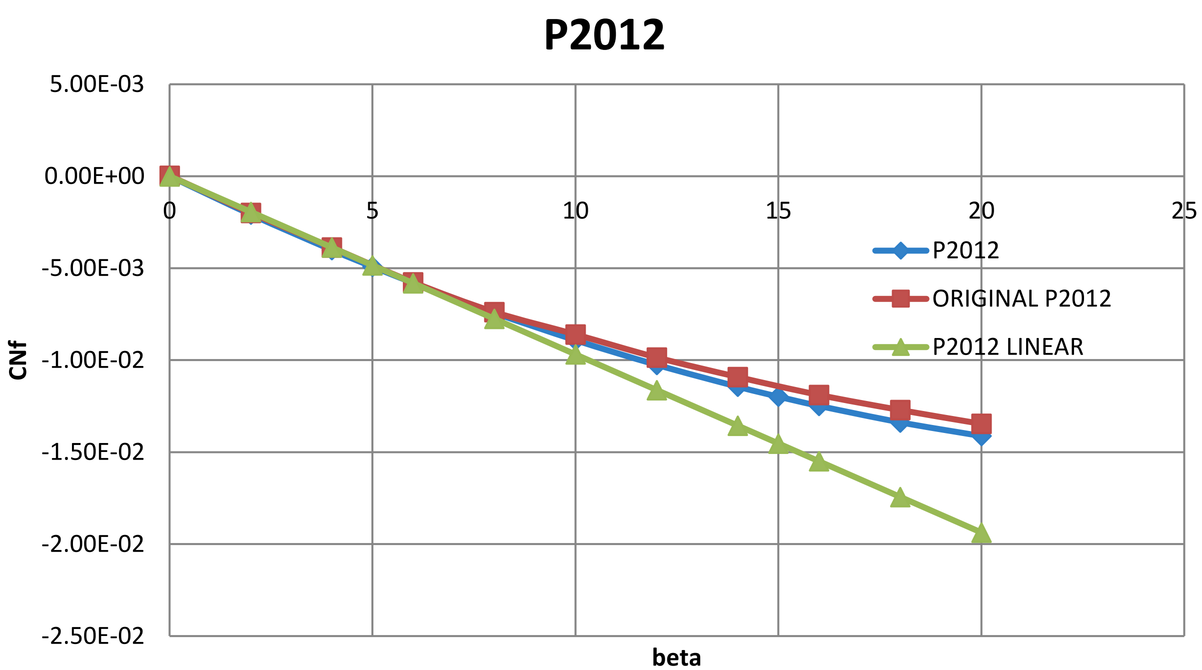








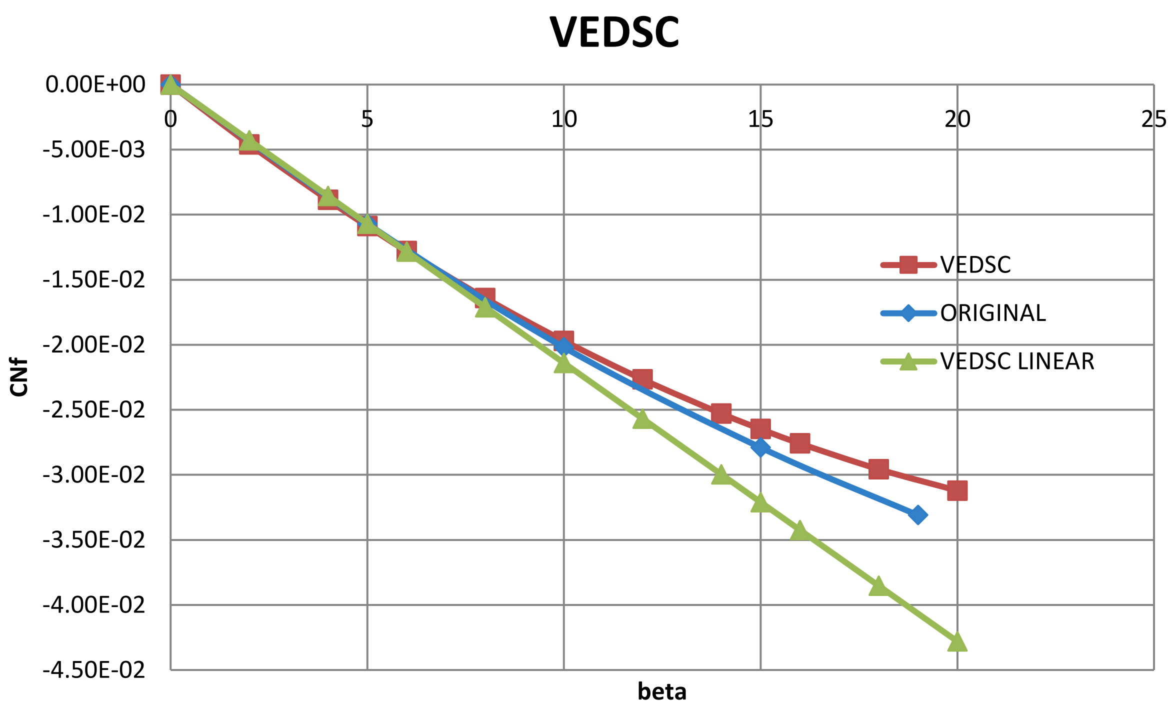
Directional Stability – Fuselage non linearities



$$C_{N_f} = K_f \cdot C_{N_\beta}$$

$$K_f = \left(\frac{C_{N_f}}{C_{N_\beta}}\right)_{\text{Poly}} = 1.11 \cdot \beta - 0.0191 \cdot \beta^2$$

Directional Stability – Fuselage

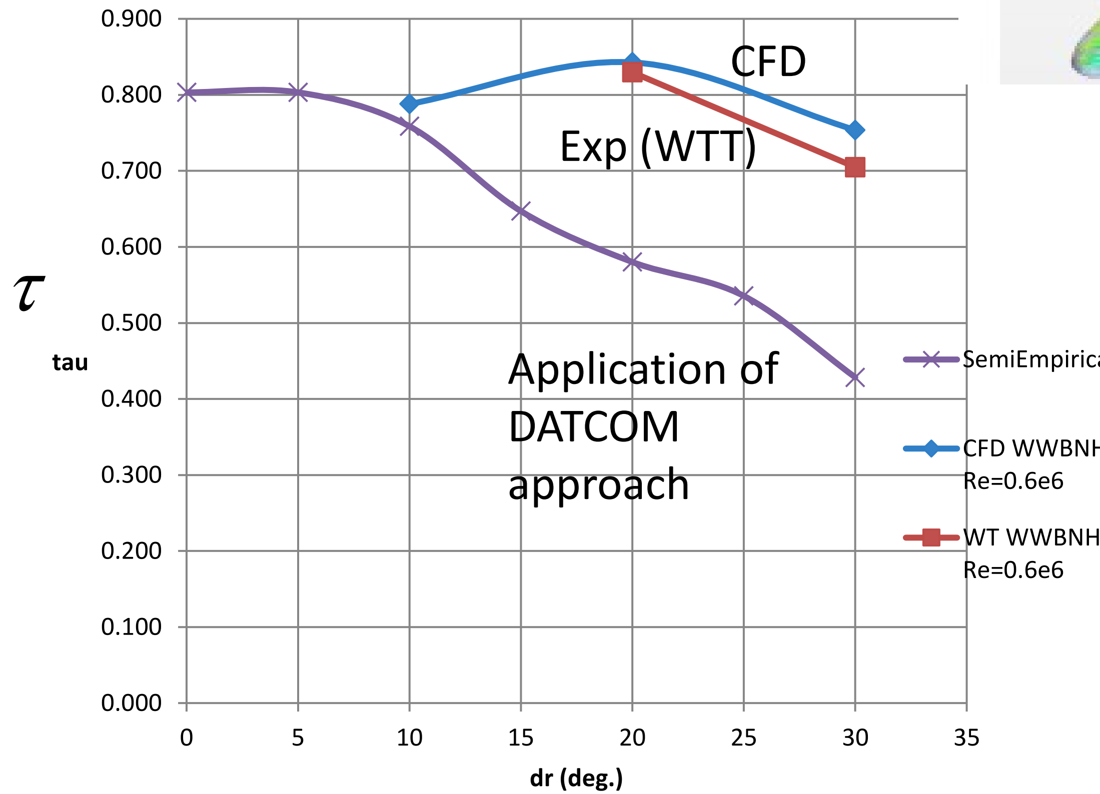


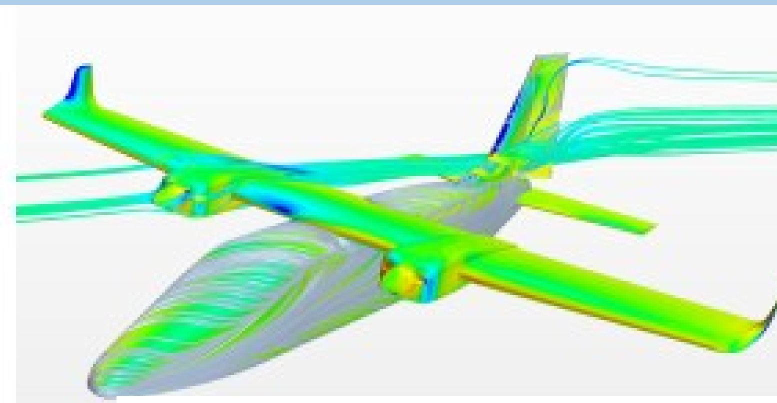
$$C_{N_f} = K_f \cdot C_{N_\beta}$$

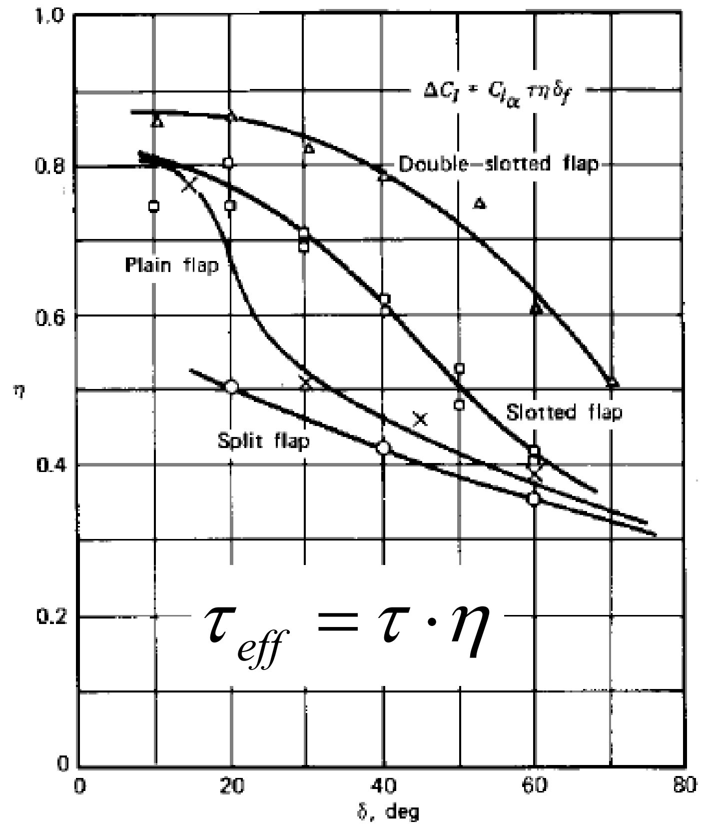
$$K_f = \left(\frac{C_{N_f}}{C_{N_\beta}}\right)_{\text{Poly}} = 1.11 \cdot \beta - 0.0191 \cdot \beta^2$$

Directional Control

P2012 Rudder control (average cr/cv=0.45)







Vertical Tail design

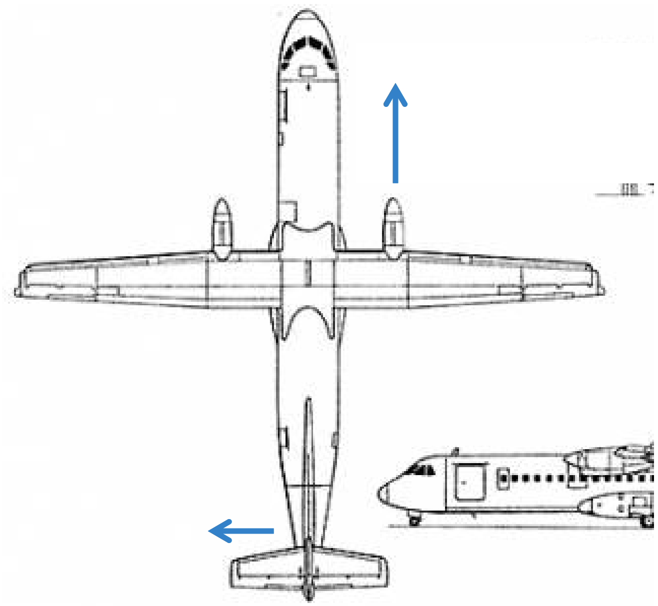
- **Vmc condition (non-linearities in rudder efficiency)**

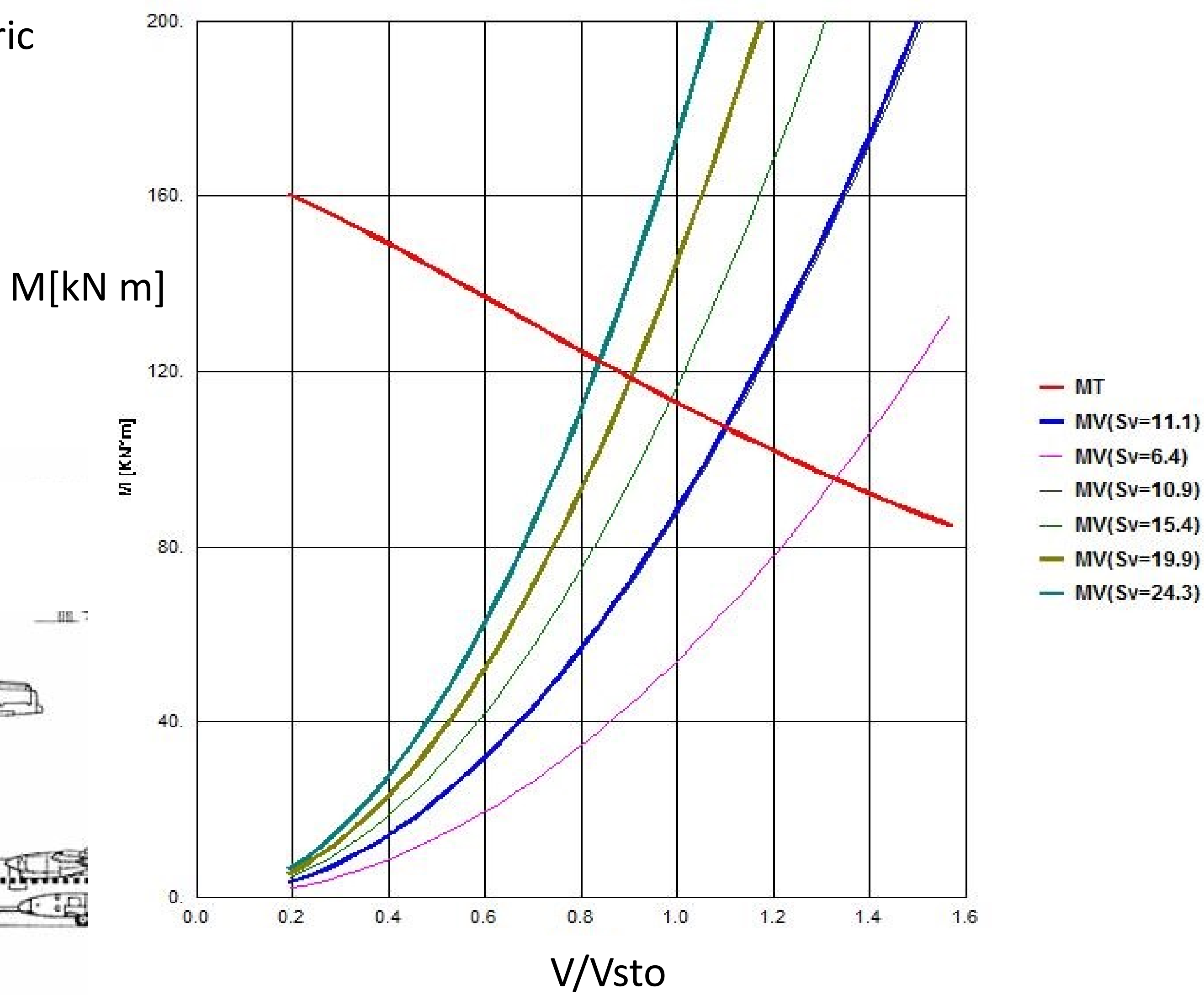
- **Equilibrium with cross-wind landing**
  high beta => non-lin effects on stability, dorsal fin effects
  high rudder deflections => non-lin in rudder efficiency

Vertical Tail design ; Vmc

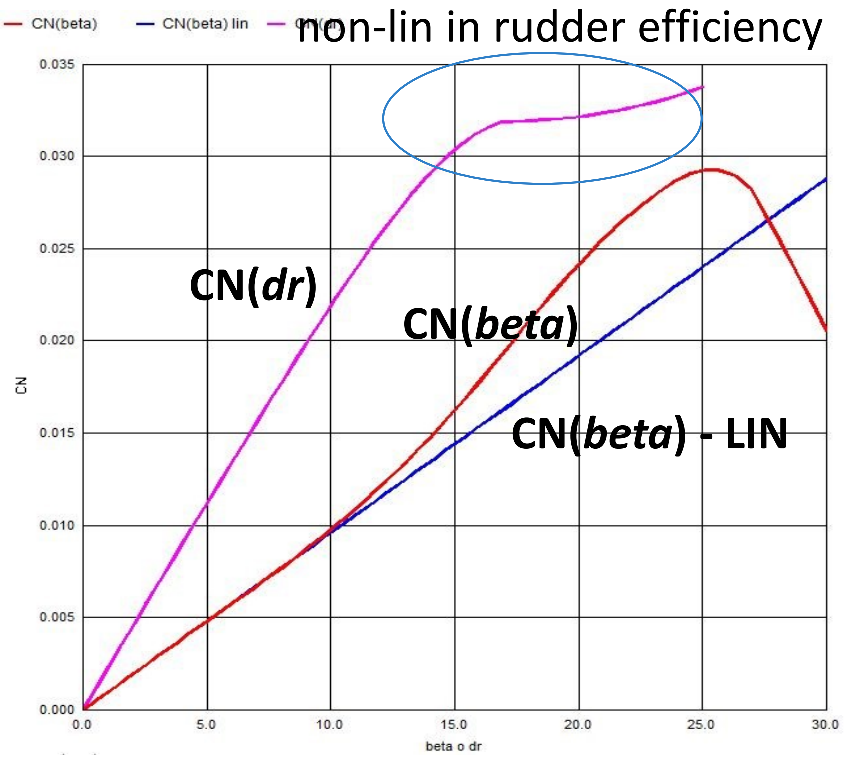
Example refers to a generic Regional Turboprop (like ATR72)

Vmc = 1.1 Vsto
leads to a certain Vertical Tail area
(**12 sq m**)



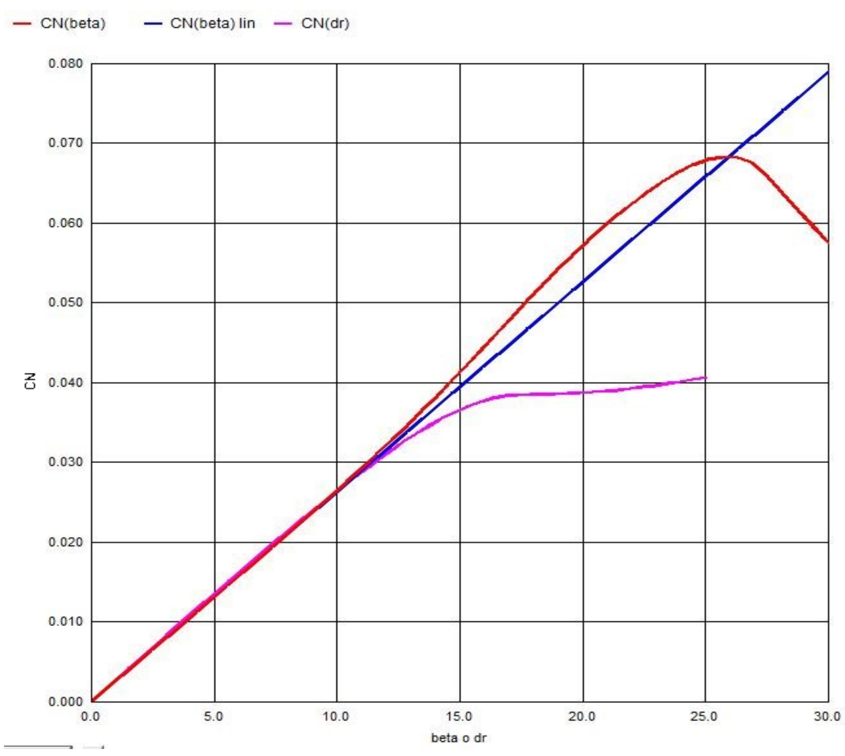


Vertical Tail design



Beta or *dr*

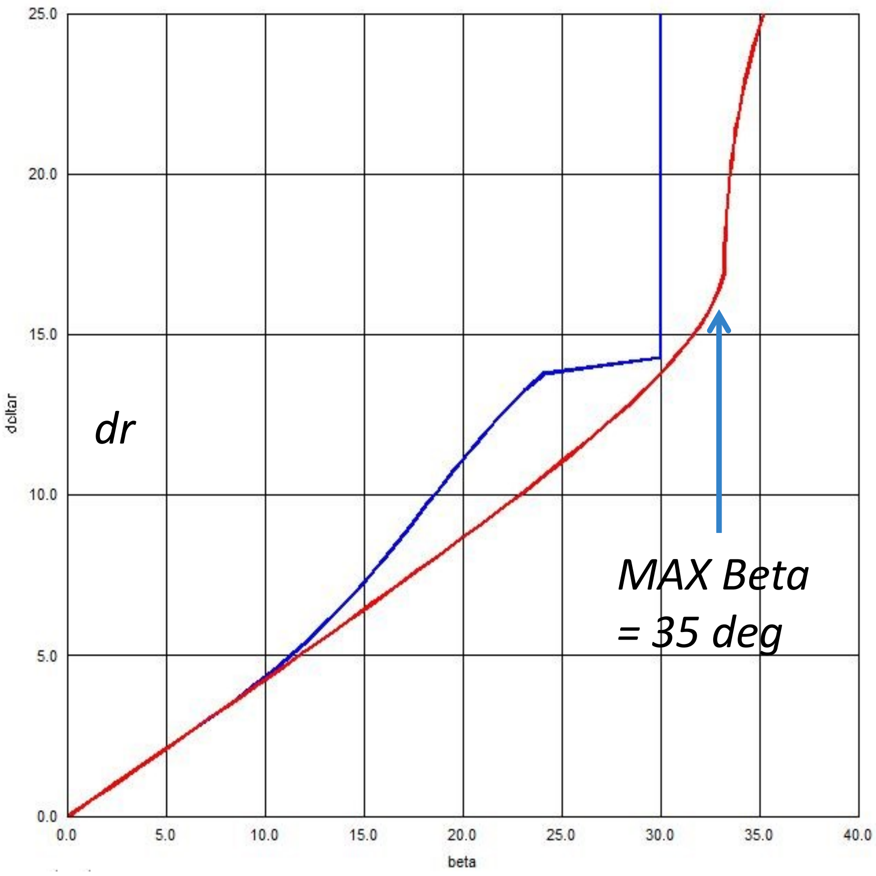
Sv=12 sq m



Beta or *dr*

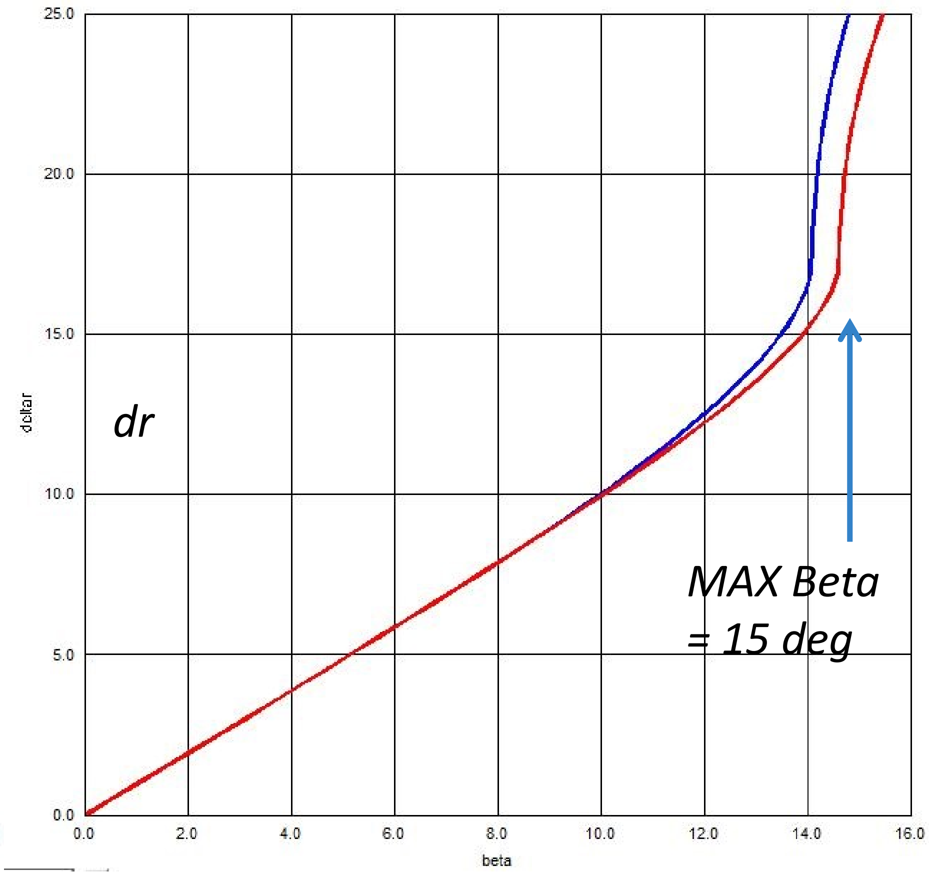
Sv=17 sq m

Vertical Tail design



Beta

Sv=12 sq m



Beta

Sv=17 sq m

Conclusions

- **Non linearities appears in longitudinal and directional stability and control (when indip. variable is alpha, beta or control surface deflection)**
- **Influence of wing-position (pendular stability)**
- **Carefully establish when it is worth to consider these effects**
- **Design of Horizontal tailplane must include these effects**

- **Design of Vertical tail (usually made for Vmc req) can be critical :**

$\Rightarrow$Vertical tail stall

$\Rightarrow$ Dorsal fin

$\Rightarrow$ Non-lin behavior of fuselage

$\Rightarrow$ Non linear rudder efficiency

Maximum achievable sideslip angle in LDG have to be checked and must include non-linear effects !