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Modeling method of strength analysis for thin-walled structure

Functions of this work

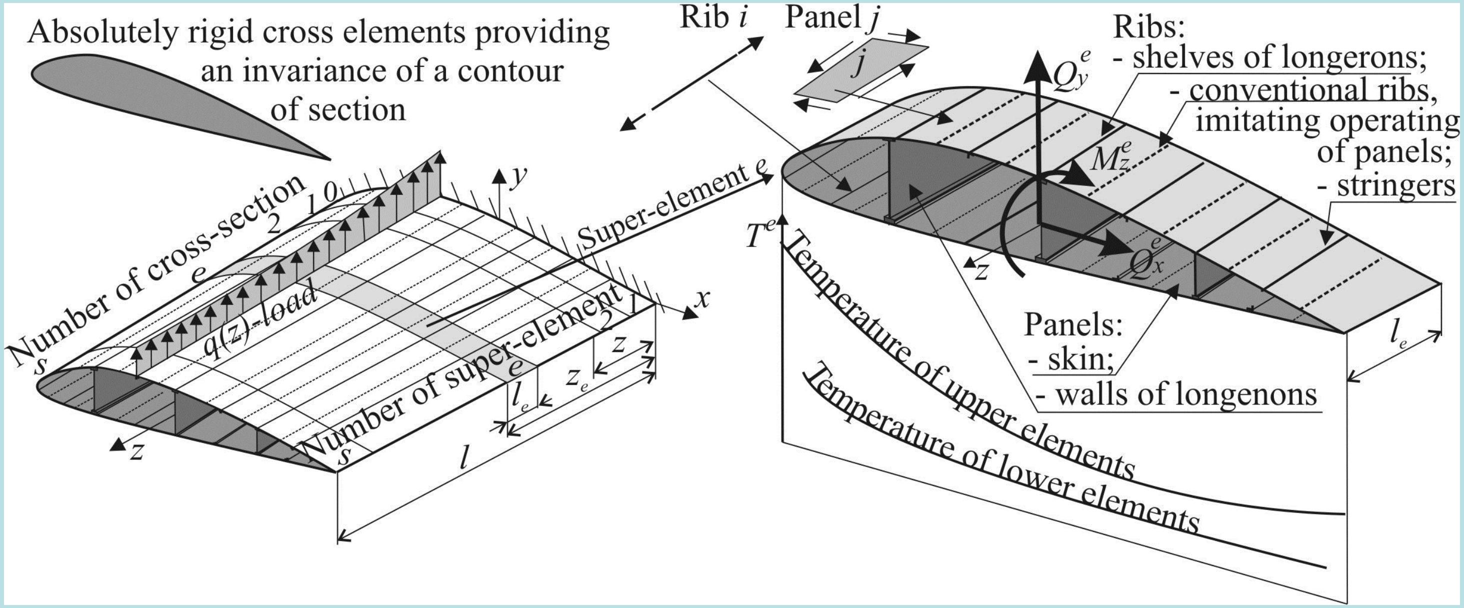
Real Design:

Conceptual Design
Preliminary Design

Educational process:

Stady of desing hypoteses and FEM

About considered structure



Let's consider a typical construction of type of an aircraft's wing or fuselage, by the right-hand orthogonal coordinate system, *xy* plane falls in the plane of the mounting and the *z*-axis is directing along the length of the structure (Fig.1).

The considered structure consists from *n* one-dimensional elements ("ribs"), corresponding to the real carried-load elements work in tension-compression,

and *m* two-dimensional elements ("panel") – skin, walls of spars.

About the proposed model

The model is based on the variational principle of Lagrange

$$\delta U = \delta A, \tag{1}$$

where δU and δA are variation of potential energy of deformation and the work of the external forces on the possible movements;

$$\delta U = \int \delta \boldsymbol{\varepsilon}^{\mathrm{T}} \boldsymbol{\sigma} d\tau; \ \delta A = \int \delta \mathbf{u}^{\mathrm{T}} \mathbf{R} d\tau + \int \delta \mathbf{u}^{\mathrm{T}} \mathbf{p} d\omega \tag{2}$$

$\delta\boldsymbol{\varepsilon}^{\mathrm{T}}$ – the transposed matrix-column of deformations on possible movements; $\boldsymbol{\sigma}$ – the matrix-column stresses; τ – the volume of the structure; $\delta\boldsymbol{u}^{\mathrm{T}}$ – transposed matrix-column of the possible movements; $\boldsymbol{R}$ and $\boldsymbol{p}$ – matrix-columns of volume forces and surface forces which act on the surface ω.

About features of model

1. Along the longitudinal axis *z* we make *c+1* cross-sections which divide the structure on *c* substructures.

2. In each calculation cross section has been added to the coordinate axis *s* directed along the tangent to the contour of the cross section.

3. The movements in ribs and panels are expressed through generalized one's and defined approximated functions. Consequently displacement are written

$$u = \sum_{k=1}^{M} \vartheta_k \varphi_k, \quad v_i = \sum_{\eta=1}^{N} r_{\eta i} \psi_\eta \qquad (3)$$

where ϑ_k and $r_{\eta i}$ are functions, approximated respectively displacement of cross-section in the transverse and longitudinal directions.

4. In according with the traditional finite element method the unknown generalized displacement at any point is expressed through the generalized movement of nodes:

$$\varphi = \alpha\, \varphi, \quad \psi = \alpha\, \psi, \qquad (4)$$

in wich α is shape function.

5. We accept that one-dimensional elements (“ribs”), corresponding to the real carried-load elements work in tension-compression, and two-dimensional elements ("panel") – in shear. Also we can insert an additional longitudinal ribs to account the carrying capacity of the panels in the longitudinal direction of the normal stress, its area corresponding to the area of the panels in cross section with a reduction factor, or this area is distributed between the ribs, limiting panels. Normal stress in the direction of the axis *s* are taken into account similarly – related to carrying capacity of the edges of the transverse load-bearing elements.

As a result of this schematization, in the longitudinal direction in the computational model following relations are used:

– for *n* longitudinal ribs: $\sigma_{zi}=E_{zi}\varepsilon_{zi}$; $\varepsilon_{zi}=dv_i/dz$;

– for *m* panels: $\tau_{zs}=G_{zs}\gamma_{zs}$; $\gamma_{zs}=dv/ds+du/dz$,

where *v* and *u* – respectively the displacement of an arbitrary point along the axes *z* and *s*.

The similar structure of ratio is for c flat transverse load-bearing elements, bounding of c compartments

Consequently displacement $\boldsymbol{f}$

$$u = \sum_{k=1}^{M} \vartheta_k \varphi_k \, , \; v_i = \sum_{\eta=1}^{N} r_{\eta i} \, \psi_\eta \qquad (5)$$

where ϑ_k and $r_{\eta i}$ are given functions, approximate by there are respectively displacement of cross-section in the transverse and longitudinal directions.

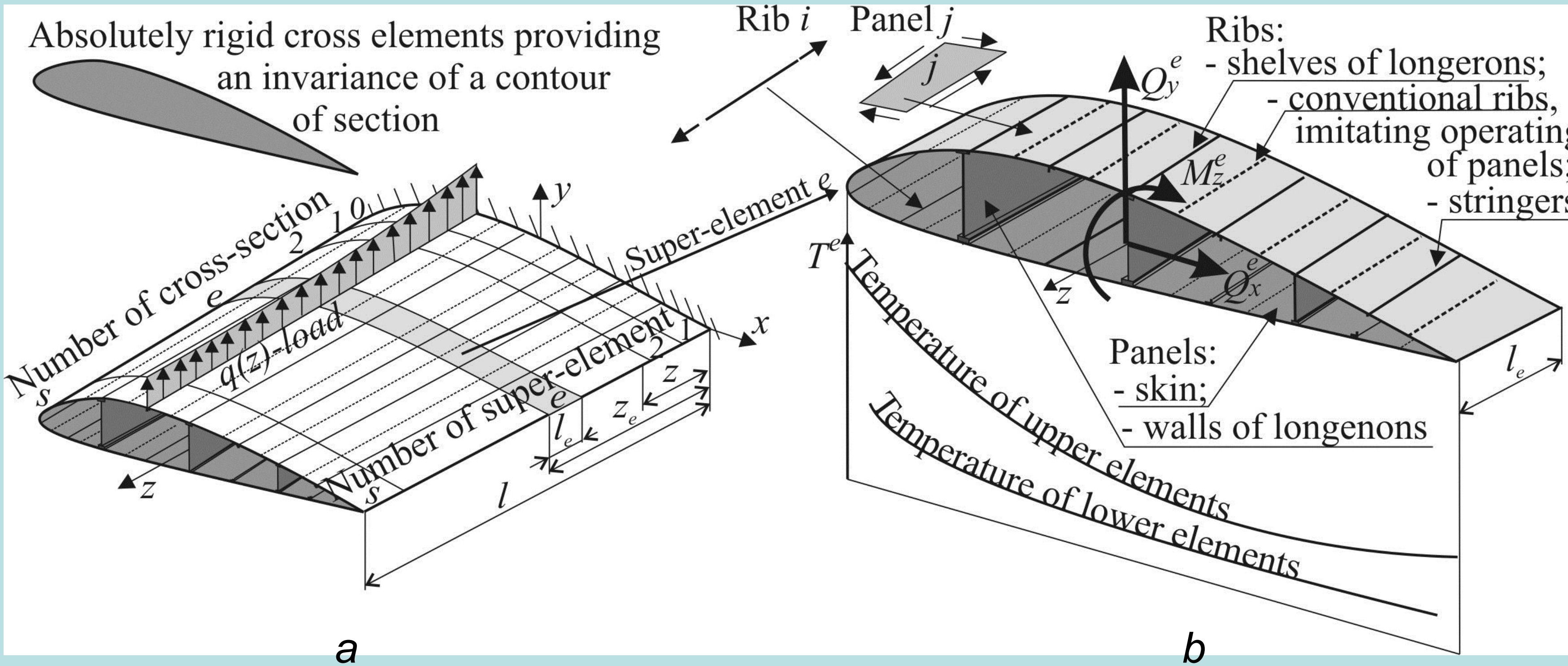


Fig.1. Design model: *a* - constructive-load circuit design layout and the breakup of the super-elements; *b* – scheme of super-element

• The solution for *M*=3 corresponds to the hypothesis of rigid contour cross-sections (RCC). In this approximation, ϑ is better to specify in this form: $\vartheta 1$, $\vartheta 2$ – inclination of axis *s* to the axes *x* and *y* in the considered point of the panel; the $\vartheta 3$ is radius vector drawn from the *z*-axis, normal to the coordinate *s*; $\varphi 1$, $\varphi 2$ – displacements the cross-section along the *x* and *y* axes, $\varphi 3$ – angle of rotation of the cross-section about the *z*-axis.

• If $3 < M \leq m$ – the solution takes into account of the deformation of the cross-sectional. If adding, for example, the fourth approximating function, it allow to take into account of the bending of cross-section in its plane. When $M = m$, the most accurate solution in this computational model will be obtained. If it is assumed that the displacement of a cross section, as rigid contour, by its attachment at 1, 2 and 3 panels (they have to form a plate, to prevent the bad conditioning of the stiffness matrix), then ϑk must be taken sequentially within each panel, starting with *k*=4, the value $\vartheta k = 1$ (*k*=4,..., *m*).

• When *N*=3 – the solution is for beam. The generalized displacements in the longitudinal direction will be: the $\psi 1$ is displacement of the entire cross-section in the direction of the *z* axis; $\psi 2$, $\psi 3$ are the rotation of a cross-section respect to the axes *x* and *y*.

• When we increase the number of functions $r\ \eta i$ ($3 < N \leq n$) it can consider warping of cross-section.

It is very convenient in the initial design stages in the conditions of lack of information about the created design using the option M=3 (hypothesis RCC). In this case, the third term in the right part of (7) is set to zero and no longer need the initial data of the transverse elements. Next we stop only on this version.

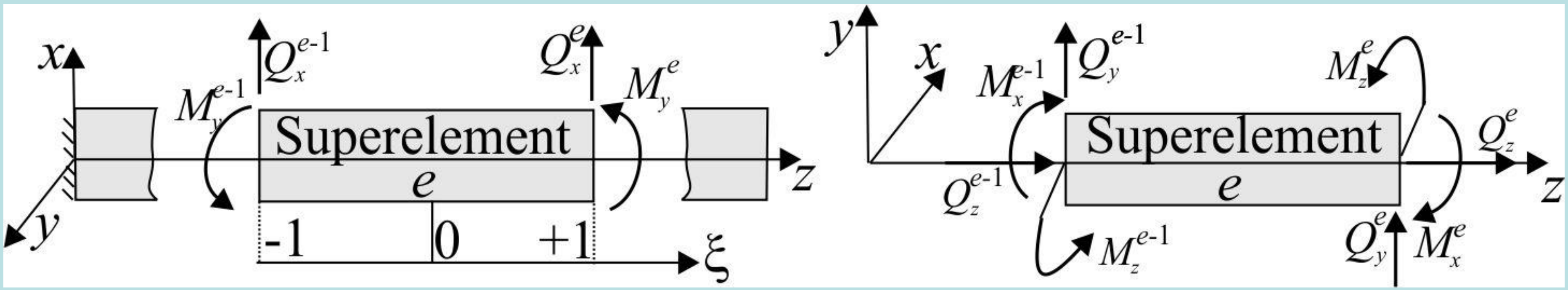
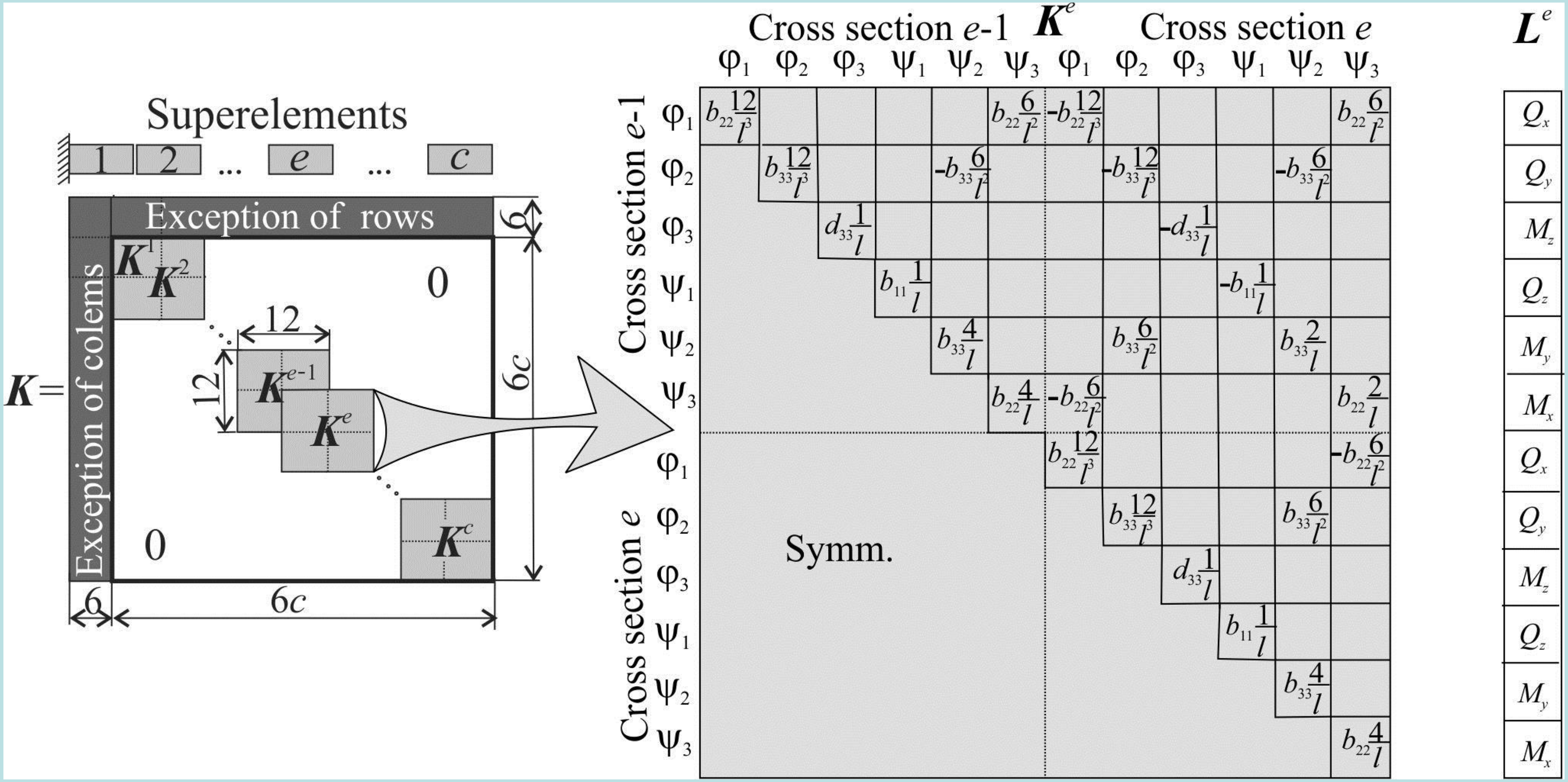


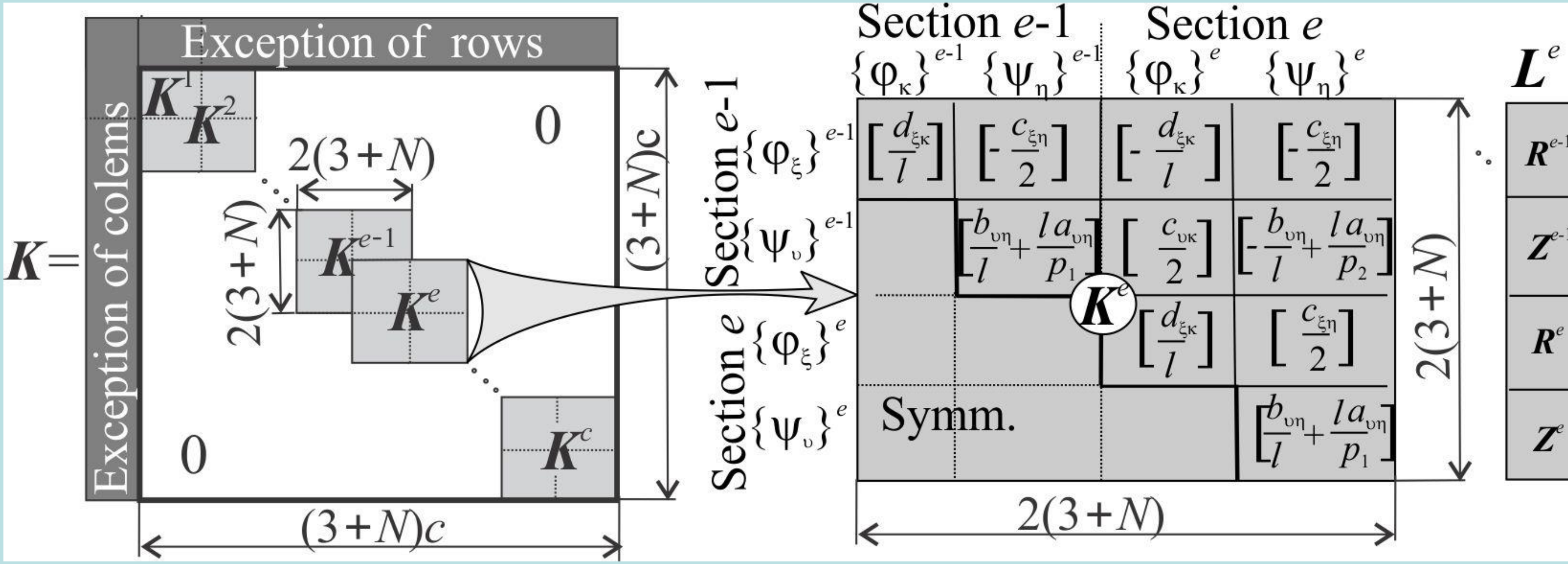
Fig.2. Beam's superelement and its loading: *a* – bending relative to the *y*-axis; *b* – bending relative to the *x*-axis, longitudinal and torsion loading

Classical solution for beam is from Euler-Bernoulli, which is obtained in the absence of shear in bending. First, we consider bending around the *y* axis (Fig.2, *a*). This solution under the given signs must satisfy the condition $d^4\varphi_1/dz^4=0$, and also $d\varphi_1/dz=\psi_2$.



a *b* *c*

Fig.3. Finite element formulation of beamed solutions: *a* is the global stiffness matrix; *b* is the stiffness matrix of superelement; *c* is the load's vector



a *b* *c*

Fig.4. FEM formulation of beam solutions: *a* – the global stiffness matrix; *b* – the stiffness matrix of superelement; *c* is the load’s vector

Verification is completed on the example of a beam, is rectangular cross-section with unit width, height *h*, which is loaded at the end by concentrated force *Qy* [1]. Theoretical values of the deflections at the free end, can be obtained by the beam model without taking into account the work on shear (theory of Euler-Bernoulli)

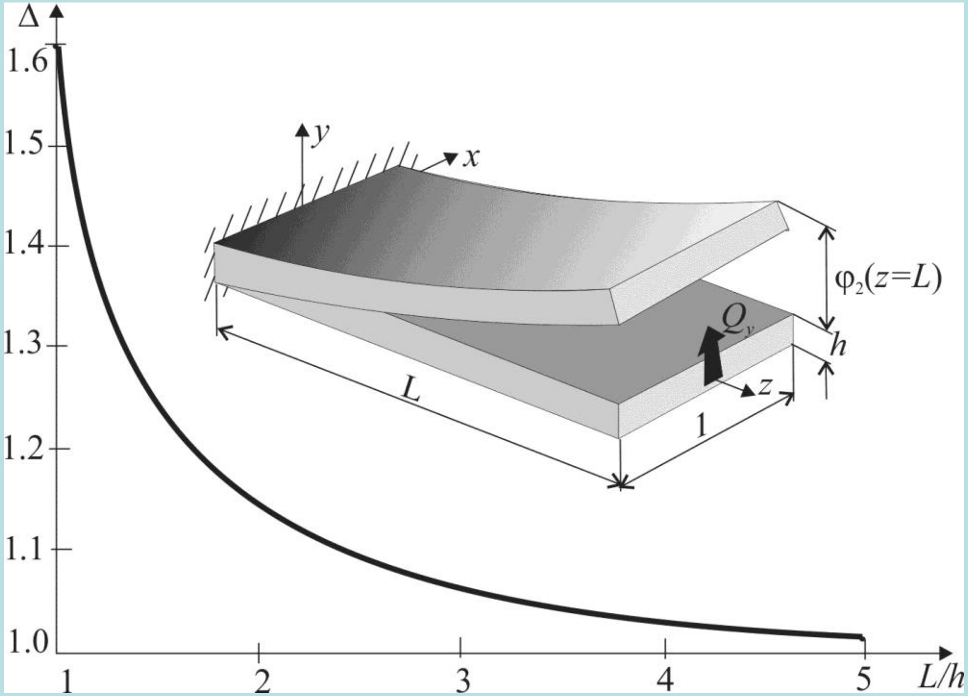


Fig.5. The effect of taking into account the shear of the deflection of the beam, depending on the relative length

Made comparing received results with classical one's of deflections $\Delta=\varphi_2(l)/\varphi_{2T}(l)$, where – $\varphi_2(l)$ are showed that oneo-point integration gives more better results than two-point integration

Testing of the proposed model

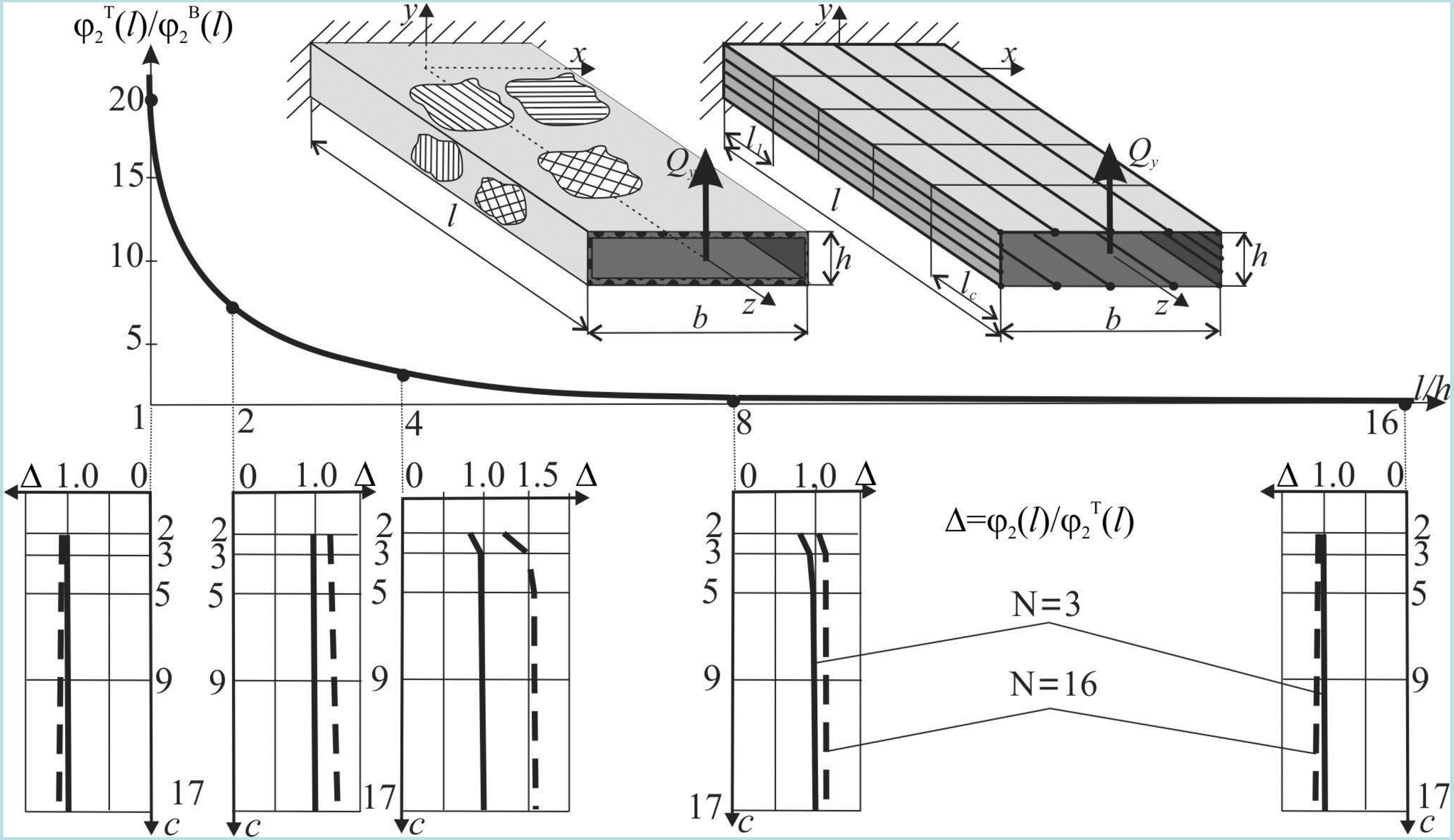


Fig.6. The study of deflection of the composite beam

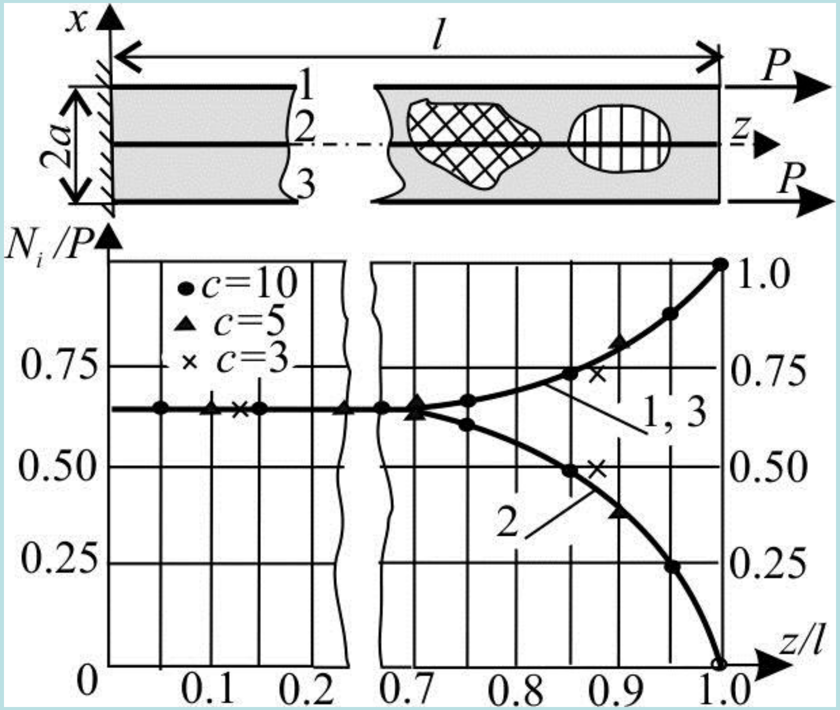


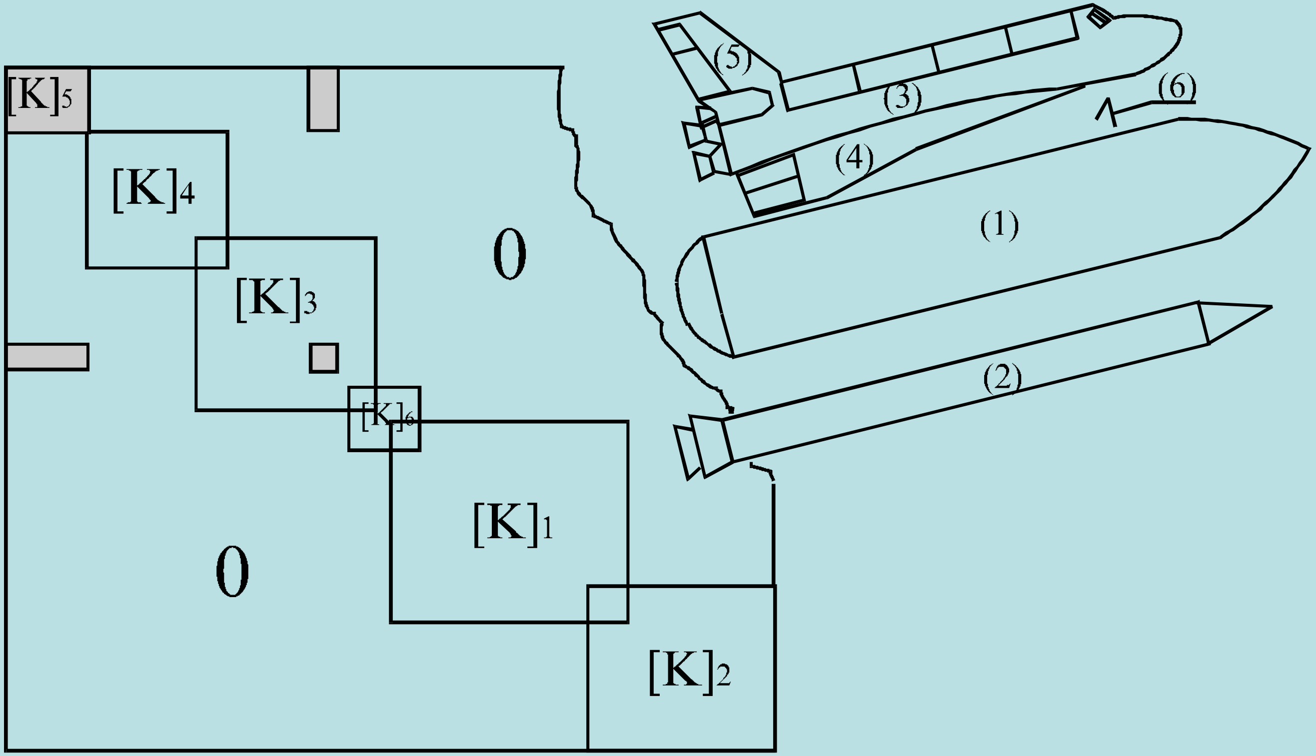
Fig.7. Calculation of composite panel on the tensile

As it is showed these calculations, four or five calculated cross-sections is sufficient in the case of common design and application by the single point integration acceptable accuracy can be obtained. The proposed model can be used in various design areas (including physical and geometrical nonlinearities, optimization, aeroelasticity and other), as well as in the educational process, because the application of the simplified calculation hypotheses contributes to the understanding of the physics of operation a forced structures.

What opportunities has the proposed model?

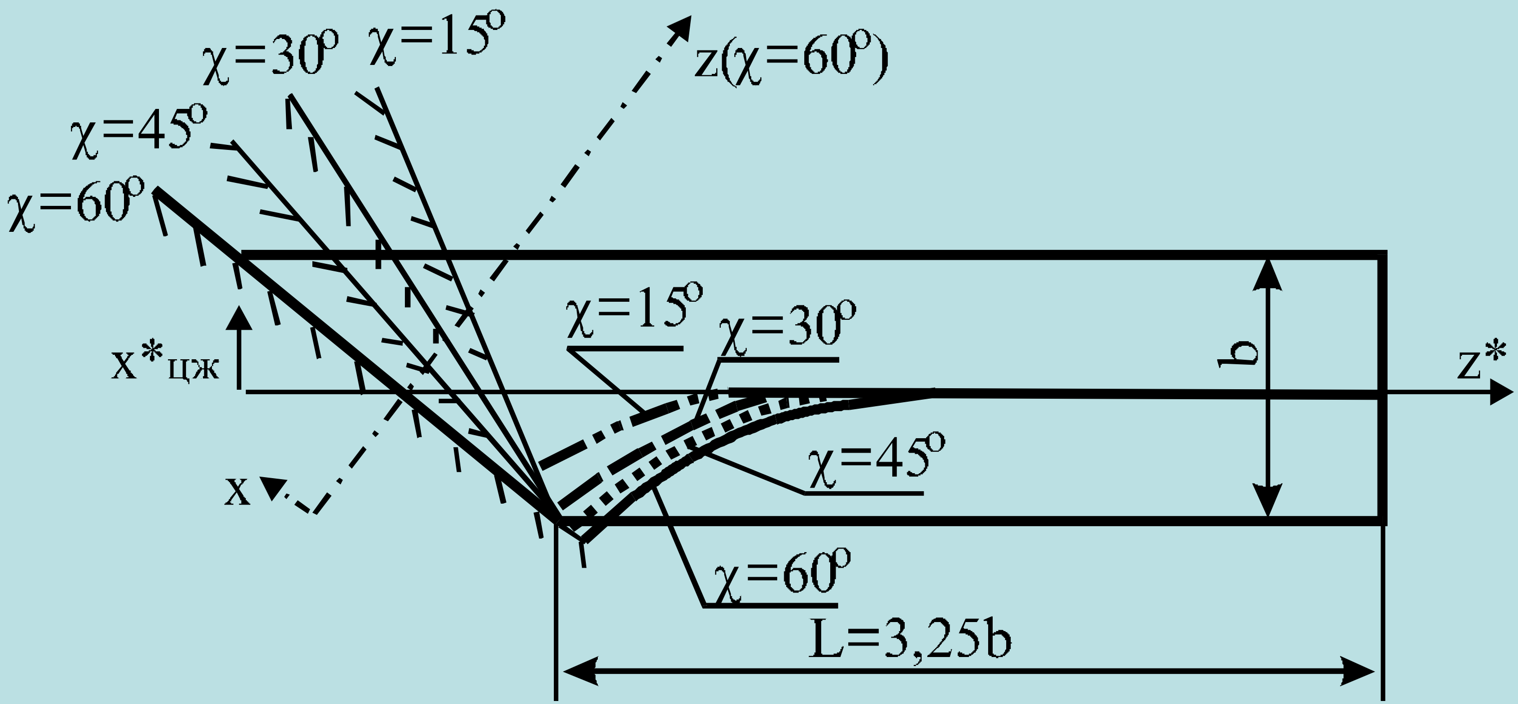
Conceptual Design and Educational Process

Aircraft of complicated and compound scheme



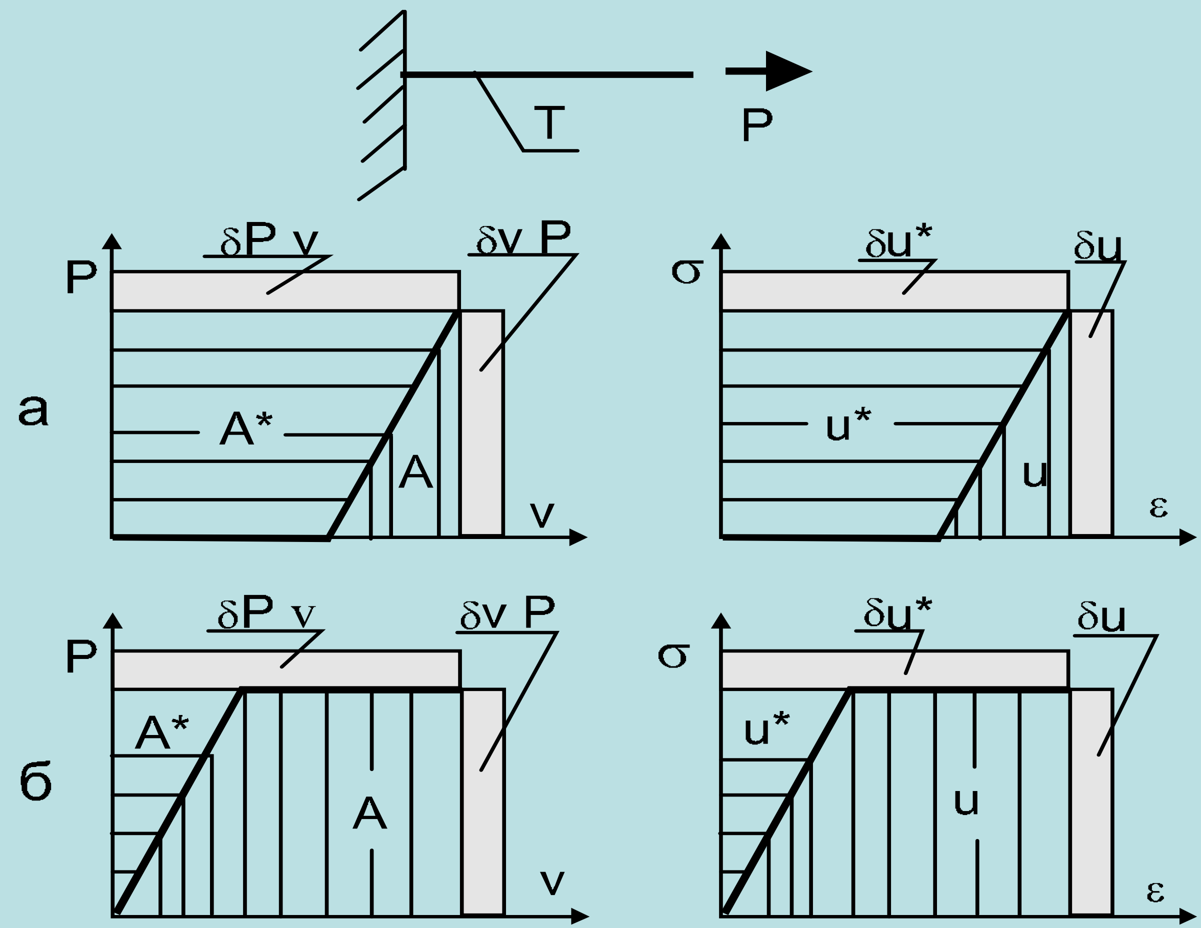
Conceptual Design and Educational Process

Calculation of the generalized characteristics of structures (for example Centrum of the Rigidity)



Conceptual Design and Educational Process

Optimization of taking into account the influence of heating



Conceptual Design and Educational Process

Optimization of taking into account the influence of heating.

Definition of the optimal parameters under the conditions of the total hardness

$$F = \left| U + U^* \right| - \min.$$

$$g = V - V_0 = 0,$$

$$h_i = \left| N_i \right| / \sqrt{\bar{e}_i} \, \{(V_0 - V^*) / \left| N_i \right| Si / \sqrt{\bar{e}_i} \},$$

$$h_i = h^*_i \text{, for elements with } e_i < 0 \; (i = 1 + m, \ldots, n)$$

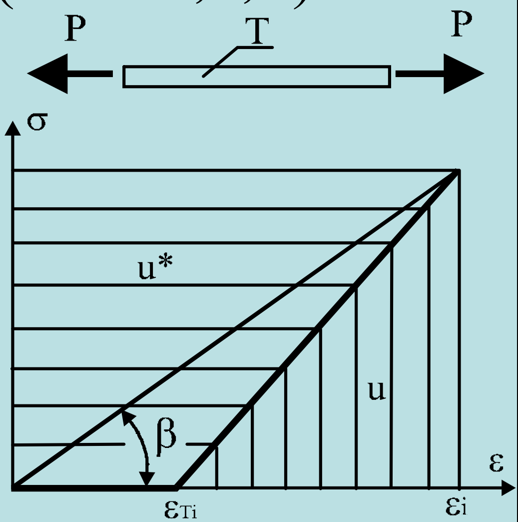
Where

$e_i = E_i / r_i$,

$r_i = (1 + \alpha_i \, T_i / \varepsilon_i)$ – for one-dimensional;

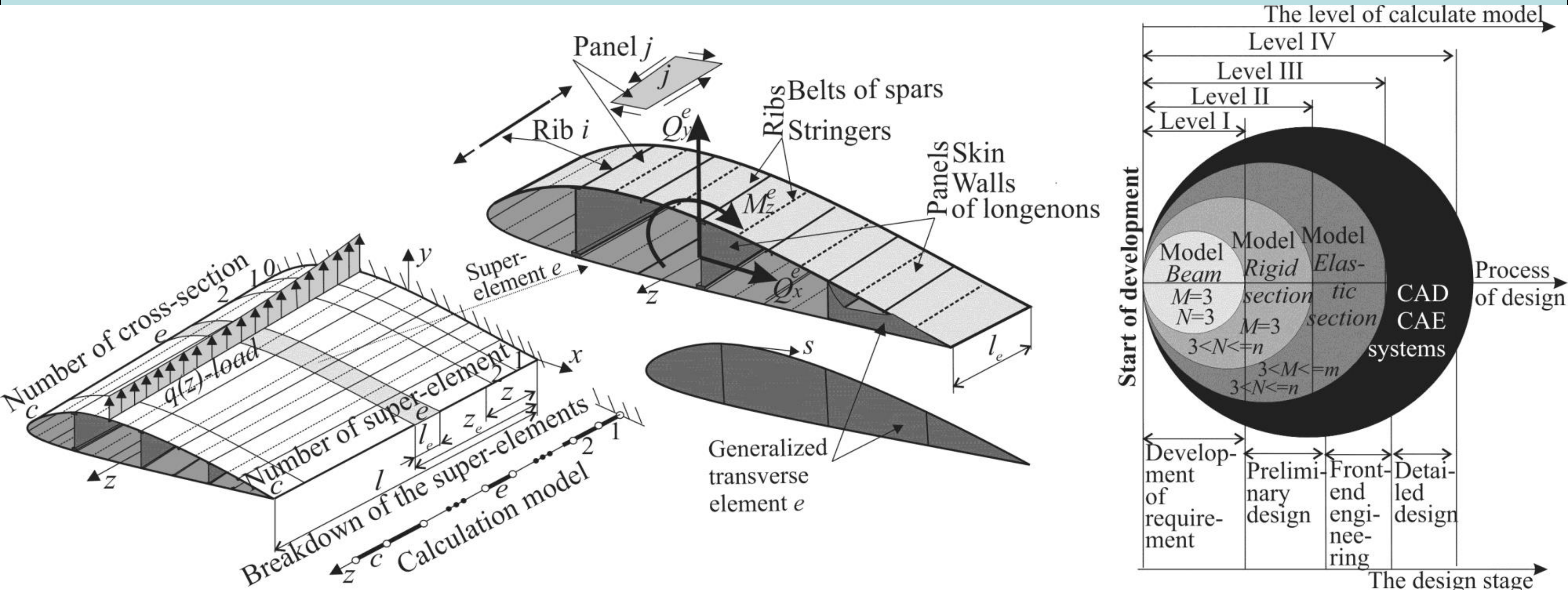
$r_i = (1 + N_{iT} \, (R_{xi} + R_{yi}) / N_i^2)$

– for two-dimensional elements.

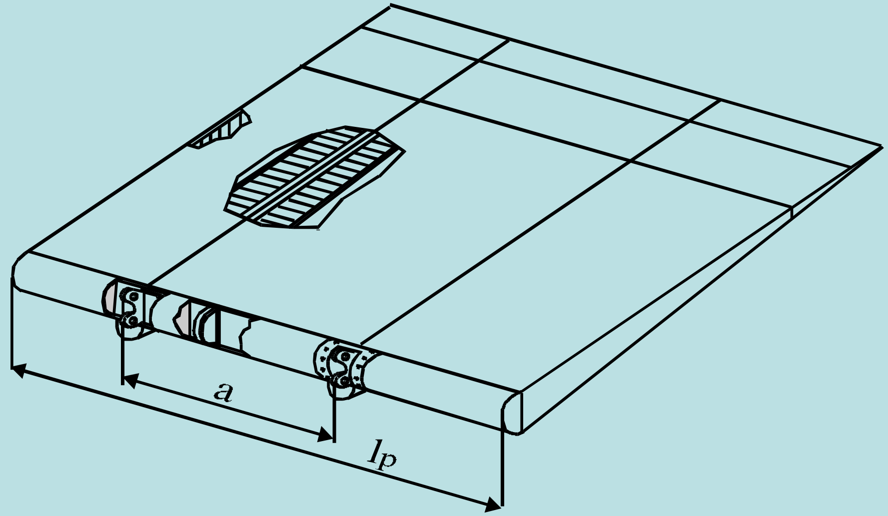


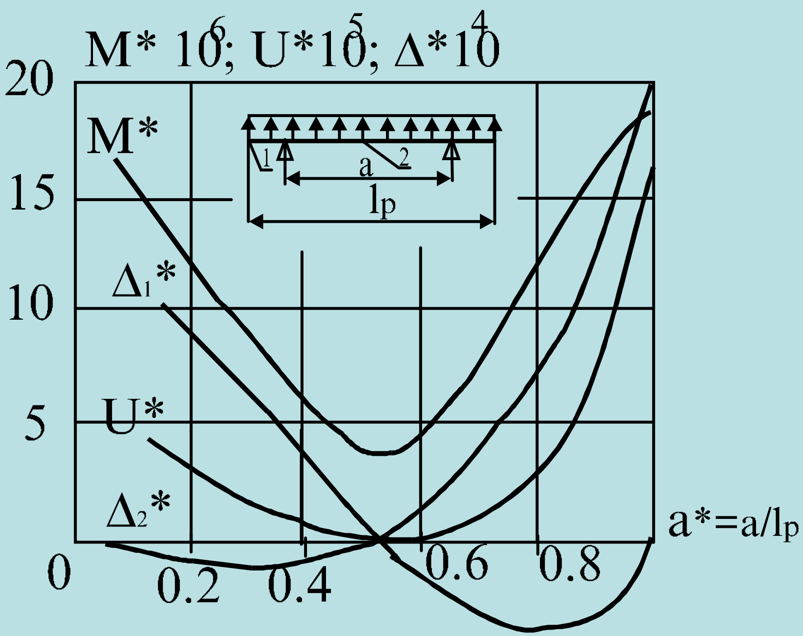
Definition of the optimal parameters under the conditions of the stiffness of a adjusted direction

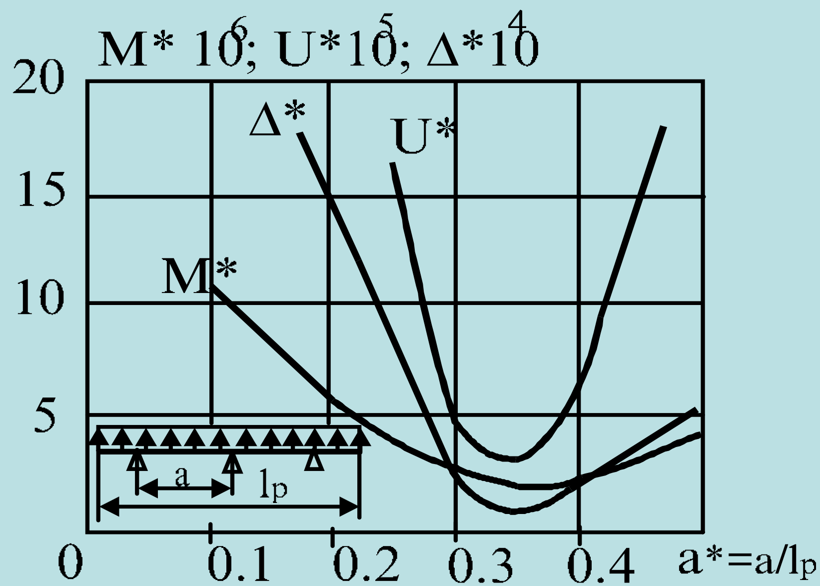
Optimization



Definition of the optimal position of the rudder surfaces

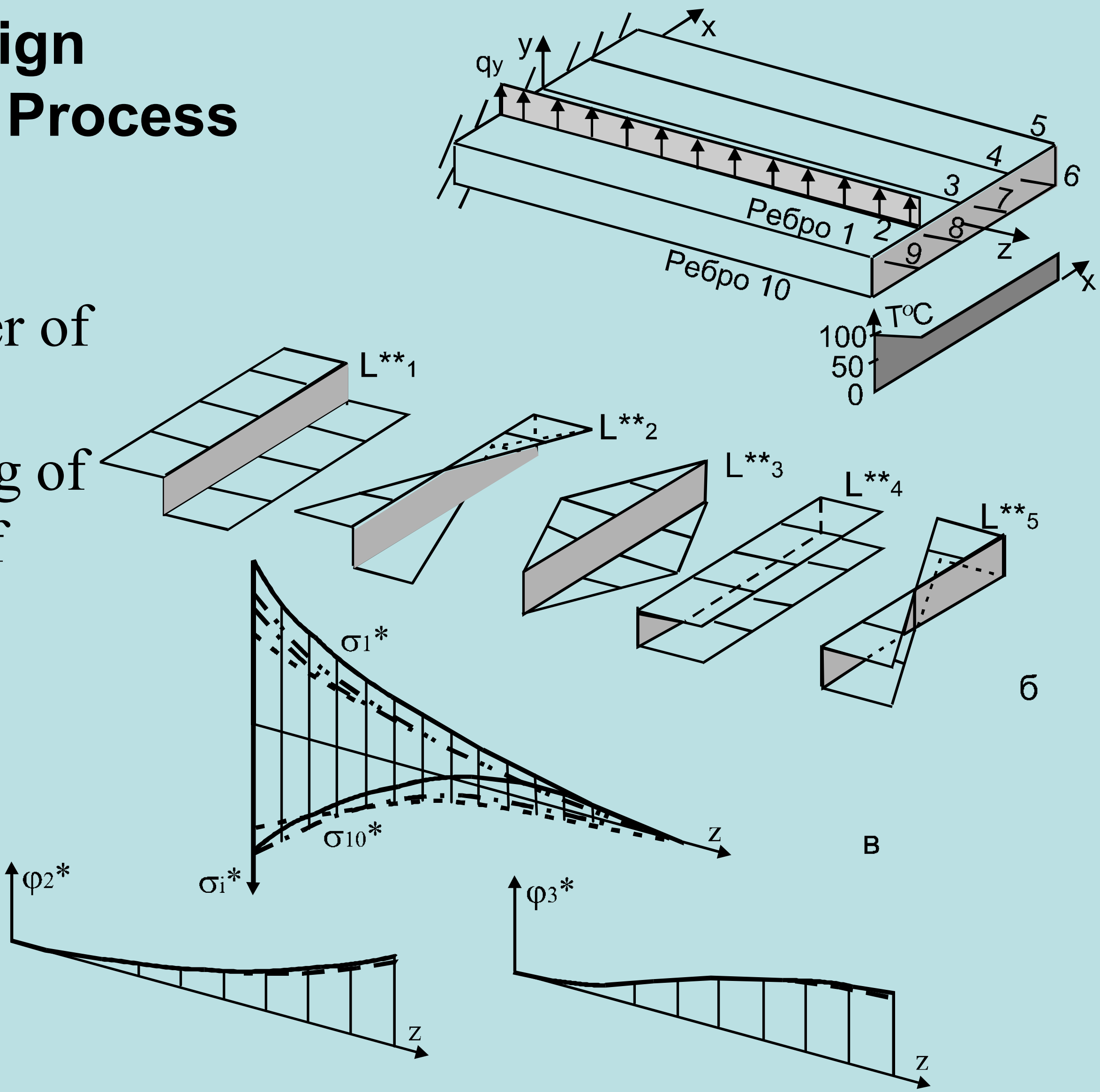






Conceptual Design and Educational Process

Increasing of number of degrees of freedom without of increasing of size of the system of equations



谢谢

Thank you for attention!

Спасибо за внимание