Design of Stable Fuzzy Control for a Flight Based on Popov-Lyapunov's Method

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Abstract

In this paper, a systematic procedure is presented to analyze and design a stable fuzzy controller for a class of nonlinear systems. Based on the Popov-Lyapunov's method, the flight model is described with the expert's linguistic information involved. The linguistic information for the flight model is represented as fuzzy sets. First, we transform a fuzzy control system into a flight control system with uncertainties or nonlinearities. Second, the Popov-Lyapunov's method is used to guarantee the stability of the flight control system, and a robustness measurement. The simulation results are included to show the effectiveness of the designed robust fuzzy controller.

1. Introduction

Fuzzy logic has been successfully applied to many engineering fields, especially, in the area of flight control engineering. The most advantageous property is that fuzzy controllers do not rely on the exact mathematical models of the processes. Based on fuzzy logic, an automatic control strategy has been converted the linguistic control strategy into fuzzy controllers. Experience shows that fuzzy controllers yield results superior to those obtained by conventional control methods when dealing with complex ill-defined processes that cannot be precisely described by mathematical formulations and have significant unmodeled effects and uncertainties. [1] The general analysis and synthesis techniques for them have not been well developed. Many methods have been investigated on the stability of fuzzy control systems. Batur and Leephakpreeda [3] have applied describing function methods to evaluate the stability of fuzzy control systems. Malki and Chen [10] showed that the fuzzy PD controller is similar to the conventional PD controller with nonconstant gains, and employed the small gain theorem for a global stability analysis. The Popov-Lyapunov's stable method is one of the most useful tools for handling the stability problems. First, we transform a fuzzy control system into a flight system with uncertainties or nonlinearities. Secondly, the Popov-Lyapunov's method is used to guarantee the global or local stability of the system, and then a robustness measurement that gives the bound on allowable uncertainties or nonlinearities is derived. The proposed method does not limit the types of inference method and control rules adopted in a fuzzy controller. The proposed method and theorem are valid and feasible in practical flight control systems.

2. Problem Formulation and Fuzzy Controller Design

Consider the nth-order dynamic systems expressed in the form of

$$\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= X_3 \\ &\vdots \\ \dot{X}_N &= f(X_1, X\ldots_1,, X_n, u) \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $u(t) \in R$ is the control input, $X = (X_1, X_2, \cdots, X_n)^T \in R^n$ is the state vector of the system and is assumed to be available for measurement, and $f(X,u)$ is a partially known continuous function representing system dynamics and unknown external disturbances. The control objective is to design fuzzy controller such that, starting from anywhere in the region, the state X tends to zero as $t \to \infty$. Assume that not all states $X_i (i = 1, 2, \cdots, n)$ are available for measurement, but y is measurable. Equation (1) can be rewritten in the following from:

$$\dot{X} = AX + B\left[f(x) + b(x)u + d(t)\right]$$
$$y = CX, \qquad \text{.................... (2)}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Generally, the fuzzy controller is designed in a heuristic way so that there is no definite stability analysis method on the overall control system [6]. In the following section, we will present a systematic design guideline for the fuzzy controller and then using the Lyapunov's stability method proves the stability of the control system.

The problem of the construction of Lyapunov functions for stability analysis is of great interest in general non-linear systems theory. Particularly, the search of quadratic Lyapunov functions for fuzzy systems is nowadays an active area of research [8]. Recently, a novel approach based on piecewise quadratic Lyapunov functions, has been proposed [9][10]. This approach can be applied to piecewise affine systems such as for example the system presented here.

In some cases, it is possible to prove the stability of piecewise linear systems using a globally quadratic Lyapunov function $V(X) = X^T PX$, if the following proposition is verified [9]:

For arbitrary b_0 and C^T, if the nominal open-loop transfer function of system (2) is relative degree 2 or more, it implies $C^T b_0 = 0$.

IF σ_1 is A_i AND σ_2 is B THEN ϕ_0 is C . (3)

The block diagram of the closed-loop fuzzy logic control system is shown in fig. (1) where K_1 , K_2 and K_u are scaling factors, and the universes of discourse of σ_1 and σ_2 are $[-\sigma_{1m}, \sigma_{1m}]$ and $[-\sigma_{2m}, \sigma_{2m}]$, in order to verify the reference input be zero. According tour assumption, the following relations can be obtained.

$$e(t) = r(t) - y(t);$$
$$c(t) = \dot{e}(t). \qquad \text{........................(4)}$$

$$\dot{X} = AX + B\left[K_1 \frac{e^{CE} - 1}{e^{CE} + 1} K_2 \dot{E}\right] \qquad \text{........................(5)}$$
$$y = C^T X$$

Where

$r = 0$, $E = -y$, $\dot{X} = A'X + B'\dfrac{e^{-CX} - 1}{e^{-CX} + 1}$,

$A' = [I + K_2 BC]^{-1} A, B = K_1 [I + K_2 BC]^{-1}$

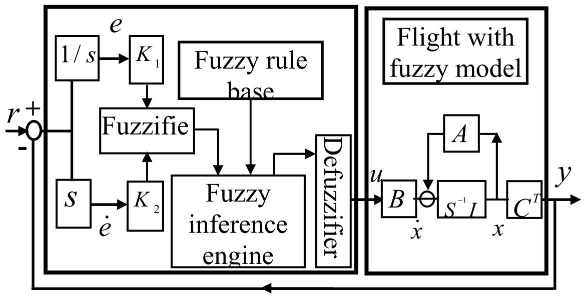


Fig. 1. Block diagram of the closed-loop fuzzy system control

IF the center $Y_1^{(j_1 \cdots j_n)}$ is given by $Y_1^{(j_1 \cdots j_n)} = k_1 X_1^{j_1} + k_2 X_2^{j_2} + \cdots + k_n X_n^{j_n}$, then the defuzzified output u_1 in the dynamic range of the fuzzy controller is:

$u_1 = k_1 x_1 + k_2 x_2 + \cdots + k_n x_n$, Where $K = (k_1, k_2 \cdots k_n)$.

IF the pair is controllable, we can choose a proper feedback gain so the eigenvalues of are all in the open left-half complex flight model. Let a lyapunov function be:

$$V(X) = X^T PX$$

Proposition: IF there exists a matrix $P = P^T \geq 0$ such that $A_i^T + PA_i < 0, i \in I$ then every trajectory of (2) with $a_i = 0, i \in I$ tends to zero exponentially [2][4]. However, the condition of Proposition is unnecessarily restrictive because the dynamics given by A_i is only valid inside a region of the state-space. Therefore, it is sufficient to prove that:

$$x^T \left(A_i^T P + PA_i\right) x < 0, \quad x \in X_i \qquad (6)$$

Condition (6) is in fact a set of matrix inequalities. To find a solution for these inequalities constitutes a so-called, feasibility problem for which different public software is available [6]. This software deals the problem as a convex optimization problem because as it is known, solving a matrix inequality like for example $P < 0$ is similar to solving the optimization problem minimize the scalar t subject to the constraint $P < tI$.

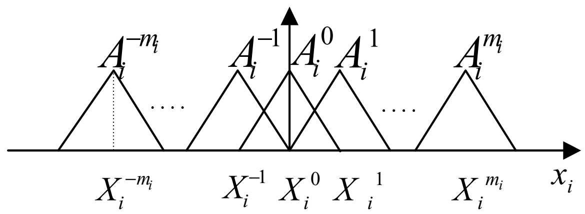


Fig. 2. Fuzzy membership functions of input fuzzy sets corresponding to x_i

3.Flight dynamics and problem statement

Nonlinear aircraft model

The aircraft model used is similar to performance fighter aircraft model as reported in. Rigid body dynamics of aircraft are described globally (over the full-flight envelope) by a set of 12 nonlinear differential equations, for six degrees of freedom [7]. We will summarize these equations as follow. Flight nonlinear dynamics formulation: The flight signals were acquired: altitude $-Z$, velocity V, linear accelerations and angular rates on all three channels (Ax, Ay, Az, p, q, r), roll and pitch attitude angles (ϕ,θ), angle of attack and sideslip angle (α,β), and speed in body axis (u,v,w).

$$X=[p \quad q \quad r \quad \phi \quad \theta \quad \varphi \quad u \quad v \quad w \quad X \quad Y \quad -Z]^T$$

$$\dot V = -D/m + T\cdot\cos\beta\cdot\cos\alpha/m + g\cdot(-\cos\alpha\cos\beta\sin\theta+\sin\beta\sin\phi\cos\theta+\sin\alpha\cos\beta\cos\phi\cos\theta) \quad \ldots\ldots\ldots\ldots(6)$$

$$\dot\alpha = -L/(mV\cos\beta) - T\cdot\sin\alpha/(mV\cos\beta) + g\cdot(-\sin\alpha\sin\theta+\cos\alpha\cos\phi\cos\theta)/(V\cos\beta) - (p\cos\alpha\sin\beta - q\cos\beta + r\sin\alpha\sin\beta)/\cos\beta \quad \ldots\ldots\ldots\ldots(7)$$

$$\dot\beta = (D\tan\beta + C\sec\beta)/(mV) - T\sin\beta\cos\alpha/(mV) + p\sin\alpha - r\cos\alpha \; g(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta)/V \quad \ldots\ldots\ldots\ldots(8)$$

$$\dot p = [I_z\cdot L^R + I_{zx}N^R + pq\cdot I_{zx}(I_z+I_x-I_y) + qr\cdot(I_yI_z - I_z^2 - I_{zx}^2)]/(I_xI_z - I_{zx}^2) \quad \ldots\ldots\ldots\ldots(9)$$

$$\dot q = M^R/I_y - pr(I_x - I_z)/I_y + (p^2 - r^2)I_{xz}/I_y \quad \ldots\ldots\ldots\ldots(10)$$

$$\dot r = [I_{zx}\cdot L^R + I_x\cdot N^R + pq\cdot(I_{zx}^2 + I_x^2 - I_yI_x) + qr\cdot I_{zx}(I_y - I_z - I_x)]/(I_xI_z - I_{zx}^2) \quad \ldots\ldots\ldots\ldots(11)$$

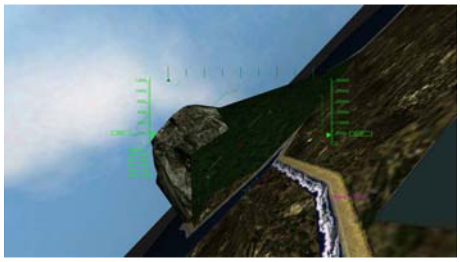
Airplane kinematics equation:

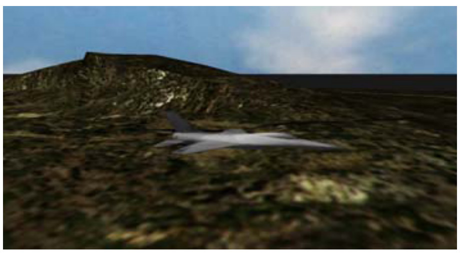
$$\dot\phi = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$
$$\dot\theta = q\cdot\cos\phi - r\sin\phi$$
$$\dot\psi = q\sin\phi\sec\theta + r\cos\phi\sec\theta$$

$$\dot x_e = V\cos\alpha\cos\beta\cos\theta\cos\psi + V\sin\beta(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + V\sin\alpha\cos\beta(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)$$
$$\dot y_e = V\cos\alpha\cos\beta\cos\theta\sin\phi + V\sin\beta(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + V\sin\alpha\cos\beta(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)$$
$$\dot z_e = -V\cos\alpha\cos\beta\sin\theta + V\sin\beta\sin\phi\cos\theta + V\sin\alpha\cos\beta\cos\phi\cos\theta$$

4. Simulation

For the flightier, state parameters are used in the simulation, which are mainly the velocity v of flight, angle α of attack, angle β of sideslip, rolling angle ϕ yaw angle φ, pitcher θ, rolling angular velocity p, yaw angular velocity r, pitcher angular velocity q, the air position coordinate of fighter $(X, Y, -Z)$.





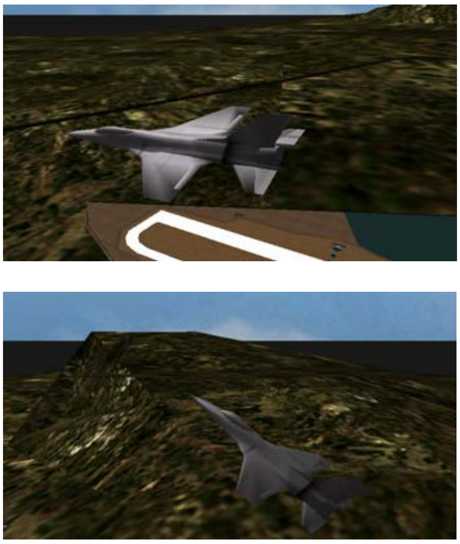


Fig.3 Simulation figure of cockpit vision

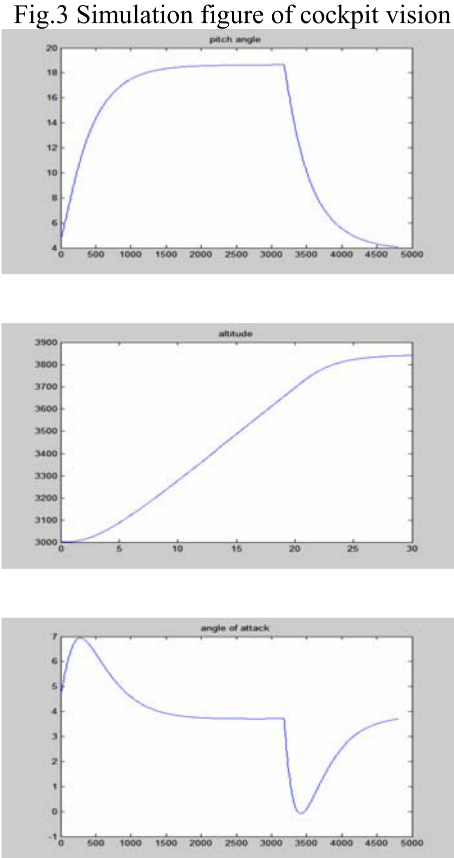


Fig.4.The fuzzy controller of flight α , θ , $-z$

5. Summary

In this paper, we have introduced a systematic analysis and design procedure for a class of flight nonlinear systems to guarantee the stability, and even the performance. The fuzzy controller is made to satisfy the Lyapunov's stability condition by assigning proper output centers of the associated fuzzy rules. Finally, the simulation results for a chaotic system are given and it shows that the proposed design procedure is an effective approach to design stable fuzzy controller for the nonlinear systems. That is applied to realize the operation for the purpose, including fly control matters. Thus, more research efforts are needed to produce new practical method for stability analysis of fuzzy systems.

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