STOCHASTIC APPROACH FOR THE SIZING OF SPACE LAUNCHERS COMPONENTS

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OVERVIEW

Currently the sizing of structural components of space launchers is mainly based on the determinist approach, namely the use of safety factors for the assessment of safety margins. At the same time new approaches are developed to introduce in models the notion of uncertainty and failure events probability at required reliability and safety levels. The scope of this study is to clarify the methodology of one of these innovative alternative approaches for applications to the structural sizing, in other words for exhibiting the potential benefit to be gained on structural items with limited strength margins.

The coupling between the mechanical finite element model (which describes the physical problem) and the stochastic method (which introduces uncertainty by means of random variables) is performed with the software "Phimeca Soft".

FIG 1. Ariane 5 ECA

The industrial illustration is carried out on one major structural component of the ARIANE 5 launcher manufacturing with carbon laminate and aluminium rings at the junction with adjacent structures. Two rupture criteria, corresponding to assessed minimum margins of the structural analysis are selected. Material properties (ultimate strengths and modules) and loading are introduced as random variables replacing the safety factor introduced in the criteria formulation.

For the first phase addressing only the material uncertainties the results identified the significant parameter (the compression strength in fibre direction) having an influence on the reliability, the design point to be compared with allowable values, and some sensitivity measures like the mean elasticity.

The conclusions of this feasibility study show a relevant method for structures with weak margins. The limit-state function and the stochastic model have to be define carefully because of their influence on results. The evaluation of the loading uncertainties contribution will constitute the second phase of this exercise.

1. CONTEXT

The deterministic structural sizing rules for space launcher components introduce a safety factor (named j) for taking into account uncertainties on material properties and loading. Indeed the knowledge is today limited by the available amount of test data and by the inaccuracy of calculations.

If we consider the material property X, the allowable value X_{adm}, defined for the structural sizing, is assessed from the following formula :

$$X_{adm} = X_{mean} - k\,\sigma$$

X_{mean} and σ are the statistical properties of the X variable obtained from testing (the rules recommending to perform at least 10 samples from several batches). The k parameter is a function of the number of samples, the probability of the value to be exceeded and the confidence of this probability. The assumption of a standard normal distribution is always considered.

Two kinds of specific values are usually chosen :

- for the modules (Young and shear) and the Poisson coefficient a probability of 50% with a confidence of 90%
- for the ultimate stresses a probability of 99% with a confidence of 90%

The aim of the stochastic approach is to replace the safety factor (generally 1.25) and the allowable values by a probabilistic analysis with new input data : the random variables.

2. STOCHASTIC APPROACH WITH PHIMECA SOFT TOOL

The stochastic approach implemented in the Phimeca Soft tool (used for this study) is described hereafter.

2.1. Basic concept

The principle is built on the evaluation of :

- the significant parameters considered as random variables
- the determination for probability distribution functions
- the definition of the limit-state function

Each significant variable (material property, loading) is replaced by a probabilistic distribution and the safety factor is suppressed from the expression of the failure criterion. The limit-state G(X)=0 characterising the limit between both failure (G(X)<0) and safety (G(X)>0) domains is deduced from the expression of the criterion considered for the deterministic physical problem.

The goal of the stochastic analysis is to find the minimum distance β between the origin and the limit-state surface. This reliability index is directly linked with the failure probability P_f and the design point P^*, that is the most probable failure point.

The following synoptic scheme summarises the coupling methodology between the physical mechanical model and the reliability methods.

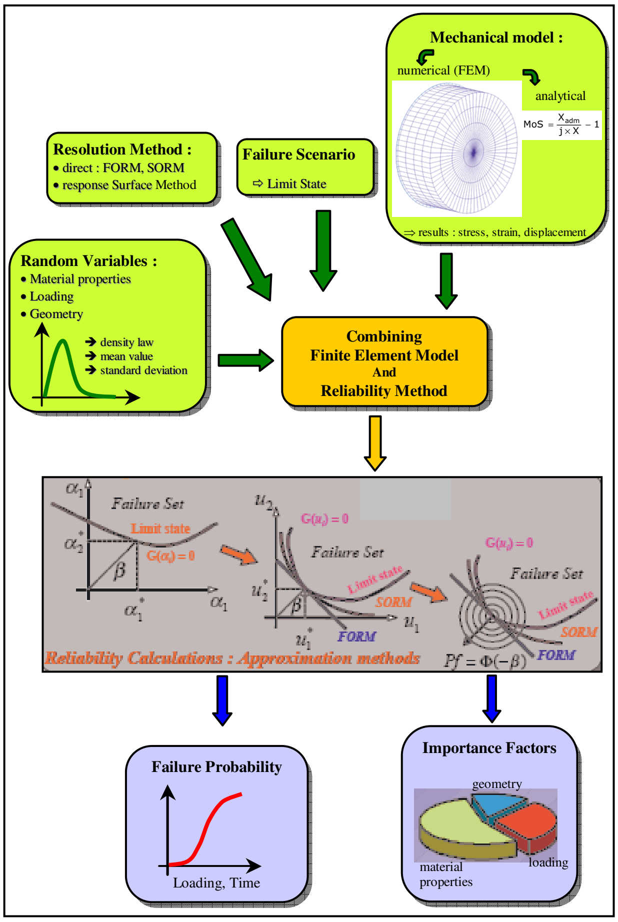

FIG 2. Combining Mechanical model and reliability method

2.2. Mathematical models

Several mathematical models are implemented in the tool for approximating the solution of the global problem :

- Monte-Carlo direct simulation algorithm involving a large and expensive number of calculations
- FORM (First Order Reliability Method) and SORM (Second Order Reliability Method) approximations
- Polynomial chaos

2.3. Provided results

Several kinds of results are proposed by this tool :

2.3.1. Reliability measures

Iterative steps are presented in a graph allowing to see possible difficulties during the calculation, especially the convergence and the number of steps.

Furthermore the failure probability P_f and the reliability index β are assessed for the specific study (uncertainties, failure criteria, density law …)

2.3.2. Design point

A set of data is provided for the design point. This result shows that instead of applying the formulation of the minimum guaranteed value (coming from the specification) on each significant parameter an interaction exists between each one and it is possible to combine the influence from all of them in determining a design point corresponding to the most probable failure of the structure.

2.3.3. Importance factors

The most critical parameters are identified among all input parameters characterised by a random variable. Important factors mean parameters driving the reliability.

2.3.4. Sensitivity measures

The tool provides also the sensitivity of the reliability index β due to the variations of the statistical properties (mean value, standard deviation). This result allows to know how improving the reliability. Indeed an influence of the mean value means that the material could be reconsidered whereas an influence of the standard deviation means rather that the manufacturing or testing process has to be adjusted.

3. APPLICATION ON ARIANE 5 COMPONENTS

The aim of this exercise is to estimate the feasibility of the stochastic approach in the frame of the space structural sizing.

The illustration is done by addressing the sizing of one Ariane 5 major product, the Vehicle Equipment Bay, manufacturing by EADS CASA Espacio with carbon fibre reinforced plastic (named CFRP) and linked to adjacent structures by means of metallic riveted junctions.

The following sketch shows a representation of the sandwich definition distinguishing both basic and joint areas.

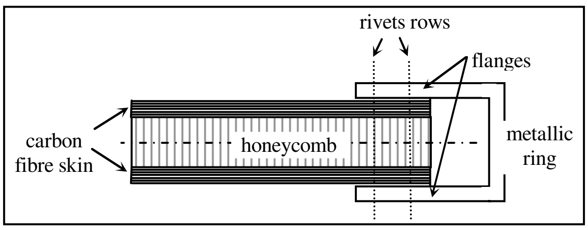

FIG 3. Lay-up sketch of a composite structure with a joint

The following subscript nomenclature is adopted concerning the composite definition : 1 for the direction parallel to fibre ; 2 for the direction normal to fibre.

3.1. Deterministic model description

3.1.1. Allowable values

A testing campaign was performed at the beginning of the development program of this structure for determining the minimum guaranteed value of each material property : Young's modulus in tension for each direction (parallel and normal to fibre), shear modulus, Poisson coefficient, and all ultimate strengths related to used failure criteria, in both directions.

For all of them 12 unidirectional (UD) specimens were manufactured and tested according to international standards for material characterisation.

3.1.2. Finite Element Model

A finite element model exists for each Ariane 5 component : shell elements and nodes with 6 degrees of freedom are used by means of NASTRAN cards like CQUAD4, CTRIA3, RBE2 for elements and MAT1, MAT8, PSHELL, PCOMP for mechanical properties definition.

3.1.3. Failure criteria

Among all failure criteria considered for a structural analysis the two following have been retained.

3.1.3.1. "first ply failure" of the CFRP

For each structure in laminate material the main failure criterion in the basic area (skins) is the named "first ply failure". Several expressions of this criterion can be found in the literature : in the case of this study the "Tsaï-Wu" criterion is selected. The definition of the safety margin is :

$$\mathrm{MoS} = \frac{\lambda}{j} - 1$$

with j : safety factor
λ is the solution of the second degree equation

$$\lambda(F_x\sigma_x + F_y\sigma_y) + \lambda^2(F_{xx}\sigma_x^2 + F_{yy}\sigma_y^2 + 2F_{xy}\sigma_x\sigma_y + F_{SS}\tau_{xy}^2) - 1 = 0$$

where

$$F_x = \frac{1}{S_{1t}} - \frac{1}{S_{1c}} \qquad F_y = \frac{1}{S_{2t}} - \frac{1}{S_{2c}}$$

$$F_{xx} = \frac{1}{S_{1t}S_{1c}} \qquad F_{yy} = \frac{1}{S_{2t}S_{2c}}$$

$$F_{SS} = \frac{1}{S_{12}^2} \qquad F_{xy} = 0$$

$$F_{xy} = 0$$

and σ_x, σ_y and τ_{xy} are the stresses in the laminate

3.1.3.2. CFRP bearing

Concerning the junction areas another criterion is usually applied : the bearing of the CFRP around the rivet located in the flanges of the metallic ring.

The bearing means a deformation of 6% of the fastener diameter. In the case of 2 rivet rows, the definition of the safety margin is :

$$\mathrm{MoS} = \frac{\sigma_{bearing}}{\frac{j\times\phi_c}{2} \times \frac{W}{t\times d} \times FF} - 1$$

with j : safety factor
FF : deterministic fitting factor for covering a non uniform load distribution between both rivet rows
$\sigma_{bearing}$: bearing allowable
ϕ_c : maximum combined flux between both flanges, taken into account the axial load, the shear force and the circumferential moment
W, d : geometrical information about fasteners
t : skin thickness

3.1.4. Dimensioning load case

The second important information concerning the structural analysis is the determination of the load case for which the safety margins are the weakest.

In the frame of this study the load case associated to both above failure criteria is : "Wind & Gust".

3.2. Stochastic definitions

3.2.1. Random variables

In this paper only the material properties are considered for introducing the uncertainty. The list is constituted from the 10 parameters used in both selected failure criteria.

Each random variable of the following table is defined by 3 parameters : the mean value, the standard deviation and the distribution law. In a first time the assumption of a Gaussian law is considered. The subscript *t* (resp. *c*) corresponds to tension (resp. compression).

Random variables
E_{1t}
E_{2t}
G_{12}
ν_{12}
σ_{1t}
σ_{2t}
σ_{1c}
σ_{2c}
τ_{12}
$\sigma_{bearing}$

TAB 1. Listing of the selected random variables

3.2.2. Limit state

Two limit states are defined according to each failure criterion. But in both cases the beginning of the failure is characterised by a margin of safety equal to 0. Therefore the limit state can be expressed by :

$$G = \min(\mathrm{MoS})$$

Let us recall here that the safety factor j has to be removed from the MoS formulation when the uncertainty is introduced by means of the stochastic approach. In our case the value is taken equal to 1 even if the only material uncertainties are introduced. Theoretically it would be righter to distinguish both contributions. But the lack of knowledge on the loading scatter oblige us to make this assumption in a first time (see chapter 3.3.5.1. for further explanations).

3.3. Results of the stochastic analysis

As the main goal of this study is to state the feasibility of this approach in the context of a structural sizing, the simplest approximation FORM is used for calculations. A verification is always performed by the way of Monte-Carlo simulations to show that this assumption is sufficient to give relevant results.

The results have to be linked with all assumptions considered in this study. Accordingly the figures for the failure probability and the reliability index don't represent the right configuration and have no relevant signification. That's why they are not displayed in this paper.

3.3.1. Importance factors

Concerning the 1st ply failure criterion, the results show that the most important factor is the strength in compression in the fibre direction σ_{1c}. This parameter is even the only one which has an non negligible influence on the reliability of the structure, as the following graph shows it.

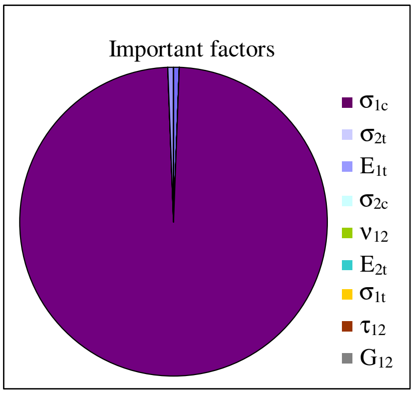

FIG 4. Important factors – 1st ply failure criterion

Concerning the CFRP bearing criterion, the bearing strength $\sigma_{bearing}$ is the only factor driving the failure of the structure.

3.3.2. Probability of occurrence of the failure event

In a deterministic approach the minimum margin combined with each criterion is determined from all concerned finite element. For the 1st ply failure criterion the critical area is the whole CFRP current zone. For the second criterion only elements defining the joint area (CFRP-ring) are examined. In the case of the stochastic approach the results can be plotted either on the critical element highlighted by the deterministic calculation or on all the area covered by the criterion without any restriction. In the frame of this study the second solution is chosen even if it implies an increase of the CPU time consuming and of the account of steps for converging.

Concerning the bearing criterion the results show that the element corresponding to the most likely location of the failure is the same for both deterministic and stochastic methods.

For the first criterion, the probability of occurrence of the failure event is identified by the stochastic calculation in another element in relation to the deterministic one. In fact this result is explained because the deterministic assessment highlights the worst case (i.e. the minimum margin) without a detailed listing of margins close to the minimum whereas a few values have a deviation lower than 1%. On the contrary the stochastic approach takes into account the fact that the failure can occur in any element among the selected area. Eventually this result shows that the 1st ply failure occurs in a critical area pointed out by the deterministic method but not worst.

3.3.3. Design point

The design point found for both criteria allows to conclude that the conservatism of the deterministic method comes from the use of safety factors but not only : the allowable values already contribute to this conservatism. The following table provides deviations between allowable values and the achieved design point regarding the 1st criterion.

	deviation (%)
E_{1t}	11.9
E_{2t}	-5.2
G_{12}	4.2
ν_{12}	6.5
σ_{1c}	-59.9
σ_{1t}	19.5
σ_{2c}	20.2
σ_{2t}	23.4
τ_{12}	4.4

TAB 2. Deviations allowable values/design point – 1st ply failure criterion

In other words the contribution of non important factors regarding the reliability through allowable values (like ultimate stresses σ_{1t}, σ_{2t} and σ_{2t}) penalizes the structural sizing. The conclusion is that the potential exists to recover margin.

3.3.4. Sensitivity analysis

For both criteria we state that an increase of the mean value tends to improve the reliability whereas the standard deviation has an weaker influence on the results. The following graph provides the results for the 1st ply failure criterion.

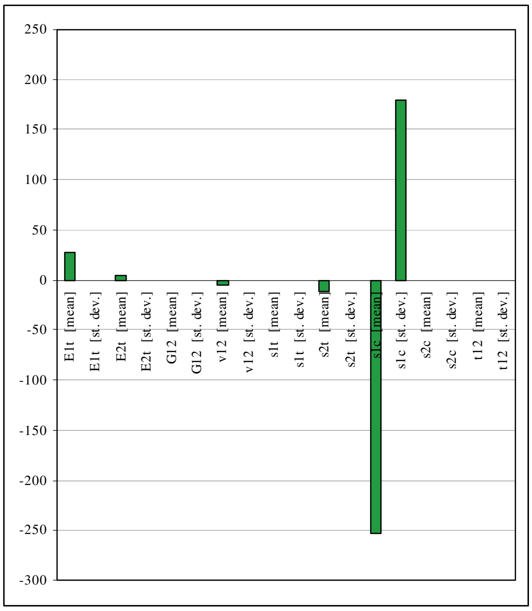

FIG 5. Elasticities – 1st ply failure criterion

3.3.5. Further results

3.3.5.1. Influence of the safety factor definition

For the previous results the safety factor was worth 1 because of no information was available at the beginning of the study related to uncertainties coming from the loading. As this second kind of uncertainty is not taken into account, the assumption “j=1” is not correct.

Therefore a second calculation with a safety factor equal to 1.2 is investigated assuming that the uncertainty on the material properties is less important compared to the loading.

The achieved results lead to the same conclusion (important factors, design point, sensitivity parameters) with a weaker reliability due to the direct link between the factor j and the margin MoS. Furthermore they confirm that the most likely location of the failure has not changed.

3.3.5.2. Failure occurrence

A stochastic calculation is investigated to go further into the results presented in chapter 3.3.2. The listing of elements where the failure can occur is reduced to the only critical element identified by the deterministic method. The results reveal firstly an overvalue of the reliability and secondly a new important factor namely σ_{2t}. In conclusion the choice of finite element for analysing the calculation is important for both methods too.

4. CONCLUSION

This exercise under CNES/DLA initiative has been performed in order to evaluate the potential interest of the stochastic approach in the frame of the cross checking activities for qualification and mission specific purposes.

In the frame of the structural sizing of a launcher component, the feasibility of such approach is demonstrated : the study point out the fact that all results are depending of the initial assumptions : the choice of the random variables, the probabilistic definition (mean value, standard deviation, density law, limit state function), the analysis area in the numerical model associated to the failure criteria. Accordingly the highlighted figures for the reliability and the β distance have to be understood with this restrictive application and always associated to the specific study. In other words even for a component (and *a posteriori* at system level) the complexity exists to clarify the reliability by taking into account all criteria corresponding to all safety margins assessed during a sizing.

Only one important parameter has been identified as driving the reliability of the structure considering uncertainties on material properties only : the compression strength in fibre direction (resp. the bearing strength) for the 1st ply failure criterion (resp. for the CFRP bearing criterion). For each of these random variables results have shown that an increase of the mean value leads to an increase of the reliability of the structure.

Further investigations are foreseen : firstly the introduction of uncertainties on loading for completing the study and taking into account all significant parameters in the definition of the safety factor ; secondly the evaluation of the influence of the density law definition (Gaussian, lognormal …) on the reliability.

5. REFERENCES

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