FEASIBILITY STUDY OF ACCELERATION LIMIT SUBSTITUTION OF FORCE LIMIT VIBRATION TEST

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1. OVERVIEW

Force Limit Vibration Test (FLVT) is a method to limit the over testing of flight hardware, by controlling both input force and acceleration of a shaker to a test article. FLVT is developed to minimize the over testing, by limiting the maximum interface force which may be times of greater than the flight environment due to the mounting impedance differences in flight versus test. FLVT limits an interface force between a vibration table and a test article, therefore, force sensors and associated fixtures are needed to conduct FLVT in addition to conventional vibration test. Furthermore, FLVT configuration is more complex than conventional vibration test.

In this study, we propose a method to achieve FLVT replacement by acceleration limit for simplification of the configuration of FLVT test equipments. This method limits the inner acceleration of a test article instead of limiting interface force.

2. INTRODUCTION

FLVT is currently used in space engineering to reduce over test of space hardware by limit the maximum interface force to the test fixture, and force sensor are additional consideration than the conventional vibration test. Conventional vibration test with acceleration limit is easier to be achieved without additional consideration. In order to estimate the limit value of acceleration, the maximum inner acceleration is predicted by simplified model with Craig-Bampton method which is calculated from the acceleration transfer function of interface to inner acceleration of a test article in both flight and test configuration. Then, the response acceleration ratio between flight and test configuration at the coupled resonance frequency of flight configuration is calculated based on resonance frequency and Q factor of the test article. The coupled resonance frequency is calculated by a Two-degree-of-Freedom model in this study. The acceleration on the test article is limited during vibration test to the maximum response of flight obtained by the acceleration ratio.

A basic study of acceleration limit method, substitution of FLVT, is discussed in this paper, and several test results of structural model consists of main structure and substructure are used to verify the effectiveness and the trade-off of the method are also discussed.

3. A FEASIBILITY STUDY OF ACCELERATION LIMIT SUBSTITUTION OF FLVT

This chapter discusses basic theory of acceleration limit substitution of FLVT.

3.1 Theory Background

A basic structure model that boundary part is rigidly fixed is shown in FIG.1. The degrees of freedom (DOF) of boundary and inside have a relation shown in equation (1) by Craig-Bampton method (CB-method). Equation (1) shows that the transformation of inside DOF is a summation of a rigid and an elastic transformation. CB method is the popular method for Coupled Load Analysis (CLA) between rocket and satellite.

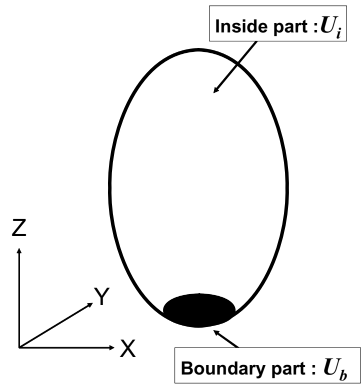

FIG.1 Basic structure model

C-B transformation matrix is;

$$\begin{Bmatrix} U_i \\ U_b \end{Bmatrix} = \begin{bmatrix} \phi_L & \phi_R \\ 0 & I_b \end{bmatrix} \begin{Bmatrix} q \\ U_b \end{Bmatrix}$$

$$U_i = \phi_L q + \phi_R U_b \qquad (1)$$

where U is the displacement, subscript i and b mean inside and boundary DOF of the structure model, respectively. ϕ_L is the elastic mode shape that the boundary is rigidly fixed. ϕ_R is the rigid mode shape, and I_b is the unit matrix.

The relationship kth (k=1, 2, 3... n) order of mode acceleration of inside q_k and acceleration of boundary $\ddot{U}_b$ is shown in equation (2). The transfer function of acceleration $\ddot{U}_b$ and force f_b of boundary is shown in equation (3). [1]

$$\ddot{q}_k = \frac{\omega^2 r_k^2}{(1-r_k^2)+j2\zeta_k r_k}\ddot{U}_b \quad , (k=1,2,3,...,n) \tag{2}$$

$$\frac{f_b(r_n)}{M\ddot{U}_b(r_n)} = 1+\sum_{k=1}^{n}\left\{\frac{m_{ek}}{M}\right\}\frac{r_k{}^2}{(1-r_k{}^2)+j2\zeta_k r_k} \tag{3}$$

where, $r_k=\omega/\omega_k$, ω_k is the kth mode angular frequency, and ζ_k is the damping ratio. m_{ek} and M are the effective mass and the rigid mass of the structure model, respectively.

In-flight maximum interface force could be got to substitute in-flight interface acceleration specification for $\ddot{U}_b$ in equation (3). FLVT limits the interface force between a vibration table and a test article to in-flight maximum interface force.

On the other hand, the acceleration transfer function of inside and boundary DOF of structure model may be induced to limit the maximum acceleration of inside.

The acceleration transfer function of inside and boundary DOF could be obtained by substituting equation (2) to equation (1). The acceleration transfer function of noticed point p that the inside DOF to boundary DOF is:

$$\frac{\ddot{U}_i(p)}{\ddot{U}_b} = \phi_R(p)+\sum_{k=1}^{n}\phi_k(p)\ddot{q}_k = 1+\sum_{k=1}^{n}\{\phi_k(p)\}\frac{r_k^2}{(1-r_k^2)+j2\zeta_k r_k} \tag{4}$$

where, ϕ_R=1 due to the assumption that structure is excited in the translational direction only in this discussion. Comparing equation (3) and equation (4), the formula of these equations are the same except the term in the bracket value. This means that maximum value of these equations occurs at the same frequency. The difference is the maximum value of equation (3) is proportional to the effective mass, and the maximum value of equation (4) is proportional to the mode shape of noticed point p.

The same as FLVT, maximum in-flight acceleration level could be calculated to solve equation (4) if the couple resonance frequency and Q factor are known.

3.2. Acceleration reduction value calculation

This section discusses the procedure to calculate the acceleration limit value from equation (4). The acceleration reduction value is the ratio of maximum value of equation (4) in flight configuration and vibration test configuration.

In the vibration test configuration, the maximum value of equation (4) happens at $\omega=\omega_k$, i.e. r_k equals to one. The maximum value in vibration test configuration is:

$$\left|\frac{\ddot{U}_i(p)_{\_test}}{\ddot{U}_b}\right| = \left|\frac{\ddot{U}_i(p)}{\ddot{U}_b}\right|_{r_k=1}\Bigg| = 1+\sum_{k=1}^{n}(\phi_k(p))\frac{1}{2\xi_k} \approx \sum_{k=1}^{n}\phi_k(p)Q_k \tag{5}$$

where, Q_k is the amplification value of kth mode.

In flight configuration, the maximum value of equation (4) is at resonance frequency " ω_{cp} (or r_{cp})" of coupled system, which couples Load side (payloads; such as a satellite on a rocket or a component on satellite system) and Source side (careers; such as a rocket or satellite system).

There are some method to obtain the resonance frequency " ω_{cp} (or r_{cp})" of coupled system, following method from reference (2) is used in this discussion. A Two Degree of Freedom System (TDFS) to calculate the ω_{cp} (or r_{cp}) is shown in FIG.2. "m_{eS}" and "m_{eL}" in FIG.2 are an effective mass of Source and Load, respectively. "k" is the spring stiffness value which defined that the resonance frequency of Source and Load which is assumed to be equal before coupling. "r_{cp}" could calculate following equation, from reference (3).

$$r_{cp} = \left(\frac{\omega_{cp}}{\omega_0}\right)^2 = 1+\frac{1}{2}\frac{m_{eL}}{m_{eS}} \pm \sqrt{\frac{m_{eL}}{m_{eS}}+\frac{1}{4}\left(\frac{m_{eL}}{m_{eS}}\right)^2} = 1+\frac{1}{2}u \pm \sqrt{u+\frac{1}{4}u^2} \tag{6}$$

Equation (6) shows that "r_{cp}" is calculated by the ratio of effective mass of Source and Load, and there are two answers. Then, substitution these two "r_{cp}"s to the equation (3), and compare the value of equation at each "r_{cp}"s, and the larger value "r_{cp}" is adopted as "r_{cp_max}".

The maximum value of acceleration transfer function (equation (4)) in flight configuration could be obtained from equation (4) and (6).

$$\left|\frac{\ddot{U}_i(p)_{\_flight}}{\ddot{U}_b}\right| = \left|\frac{\ddot{U}_i(p)}{\ddot{U}_b}\right|_{r_k=r_{cp\_max}}\Bigg| \approx \sum_{k=1}^{n}\phi(p)\frac{r_{cp\_max}^2}{\sqrt{(1-r_{cp\_max}^2)+(2\xi_k r_{cp\_max})^2}} \tag{7}$$

Finally, acceleration reduction value which is the ratio of acceleration of in-flight to test at noticed kth mode is the ratio of equation (5) and (7).

$$A_{Acc\_reduction} = \left|\frac{\ddot{U}_i(p)_{\_flight}}{\ddot{U}_i(p)_{\_test}}\right| = \frac{r_{cp\_max}^2}{Q\sqrt{(1-r_{cp\_max}^2)+(2\xi_k r_{cp\_max})^2}} \tag{8}$$

The acceleration reduction value is a function of the Q factor and the ratio of effective mass.

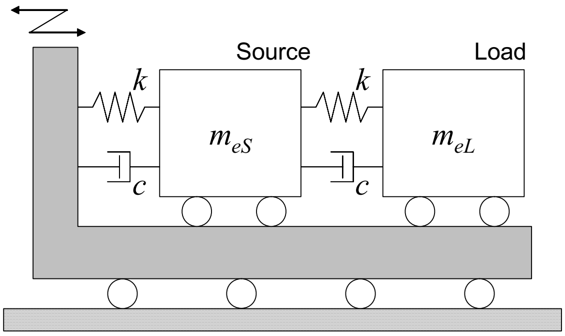

FIG.2 Two Degree of Freedom model

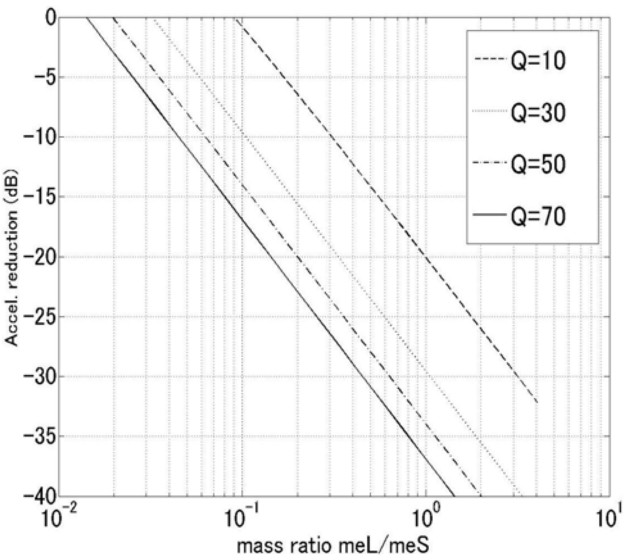

FIG.3 Acceleration reduction value vs. the ratio of effective mass (m_{eL}/m_{eS})

Examples of calculation results of equation (8) corresponding to Q values; Q equals to 10, 30, 50 and 70, are shown in FIG.3. The vertical axis of FIG.3 is the acceleration reduction value (dB), and the horizontal axis is the effective mass ratio (m_{eL}/m_{eS}). FIG.3 shows that the acceleration reduction value is in the direct proportion to the Q factor and the ratio of effective mass.

Most of the design of satellites, the first resonance frequency of Source is lower than the resonance frequency of Load. For example, the acceleration reduction value, the ratio of effective mass equals to 0.25, are about -9dB (Q=10), -18dB (Q=30), -22dB (Q=50) and -25dB (Q=70) from FIG.3.

4. EXPERIMENTS

This section discusses the acoustic test and vibration test performed for investigate the feasibility of acceleration limit method substitution of FLVT test proposed in this paper.

4.1. Test Article

The test article used in vibration test is shown in FIG.4, illustrates the exterior view. During the test, the force between the test article and vibration table, and also the acceleration at the specified locations on the test article are measured.

4.2. Vibration Test

Random Vibration tests by force limit control and acceleration limit control are executed and compared for the verification of acceleration limit method substitution of FLVT proposed in this paper. The situation of vibration test is shown in FIG.5. These vibration tests are executed vertical axis only.

At first, sine sweep vibration test is executed for the purpose of measurement and calculate the characteristics of the test article such as the resonance frequency, Q factor and effective mass (effective mass ratio). The acceleration reduction value is calculated by the procedures shown in section3 with these characteristics. The effective mass of Source is assumed to equal to the rigid mass of itself for the safety manner, in this vibration test. These results are shown in TAB.1.

And next, random vibration test is executed. The specification of random vibration test is defined by enveloping the interface acceleration of the acoustic test results which is executed with the test article attached on the main structure as Source. The acoustic test simulates the flight environment. The acceleration and force between Source and Load are also measured during the acoustic test, and the specification of random vibration test is defined to envelope the interface acceleration response during the acoustic test shown in FIG.6. The specification of random vibration test is shown in TAB.2.

Three kinds of random vibration test are executed; without limit control, force limited control and acceleration limit control. The force limit control is based on Simple-TDFS method [2] defined by the characteristics of the test article in TAB.1. The acceleration reduction value is defined by FIG.3 (equation (8)) using characteristics of the test article shown in TAB.1. The acceleration reduction value is -26dB based on the parameters (Q and m_{eL}) measurement by interface force to acceleration ratio in sine sweep test (2Oct/min). However, if the effective mass is unknown, we treat it as m_{eL} equals to rigid mass to obtain the value -30dB shown in TAB.1. There is not large difference between actual measurement and this assumption.

FIG.4 Exterior view of the test article

TAB.1 Characteristics of the test article
(measured and calculated by Sine Sweep vibration test)

M_L [kg]	f_1 [Hz]	Q	m_{eL} [kg]
9.2	290	67	7.6

m_{eS} (= M_s) [kg]	m_{eL} / m_{eS}	Acc Reduction [dB] (*calculate by eq.(8)*)
26	0.3	-26 dB @290Hz
	0.35 *by the assumption of m_{eL} = M_L*	*-30dB @290Hz*

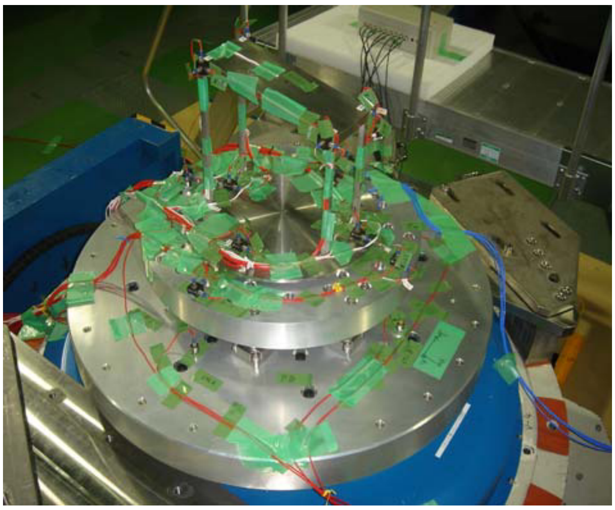

FIG.5 Situation of the vibration test

TAB.2 Specification of the random vibration test

Frequency [Hz]	PSD [$(m/s^2)^2$/Hz]
100	0.01
160	1.2
900	1.2
2000	0.15

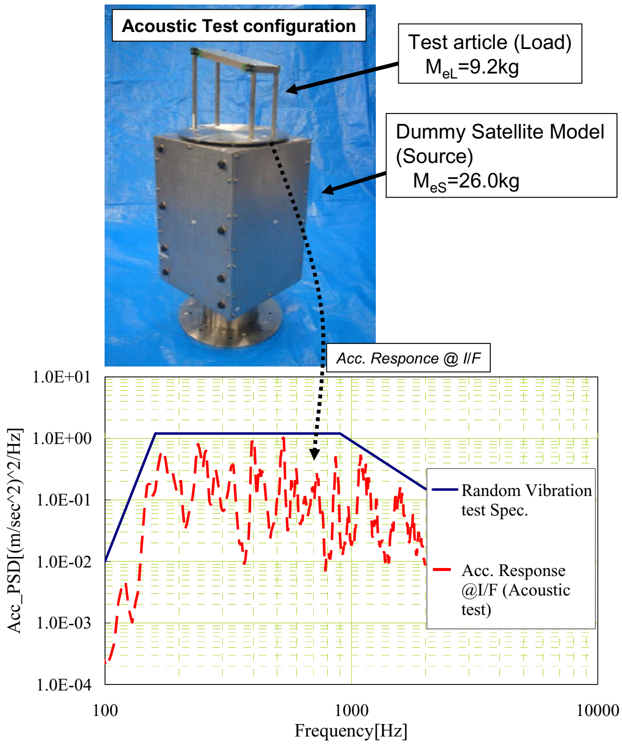

FIG.6 Configuration and result of the acoustic test

4.3. Results of the vibration test

The comparison results of random vibration tests and the acoustic test are shown in FIG.7 to FIG.9.

FIG.7 shows the Power Spectrum Density (PSD) results of the acceleration response on the top of the test article during the vibration test and the acoustic test. The limited controlled results are enveloped closer to the acoustic results and about 25dB reduced compare to the no-limit controlled result at resonance frequency (290Hz). The test results of force limit and acceleration limit controlled at the resonance frequency almost the same level. These results mean that acceleration limit is effective, and reduces about 25dB similar to FLVT.

FIG.8 shows the PSD results of the interface force response of the test article during the tests. These results have the same tendency compare to the FIG.7.

FIG.9 shows the PSD results of the acceleration response on the interface of the test article during the tests. In order to the limit control, results of force limit and acceleration limit control have almost the same depth notch at resonance frequency (290Hz).

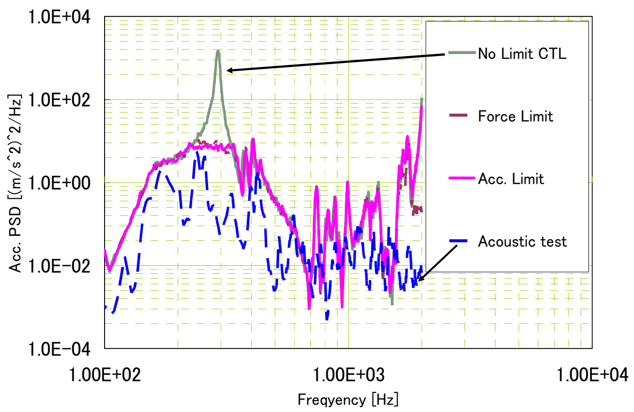

FIG.7 Acceleration PSD on the top of the test article

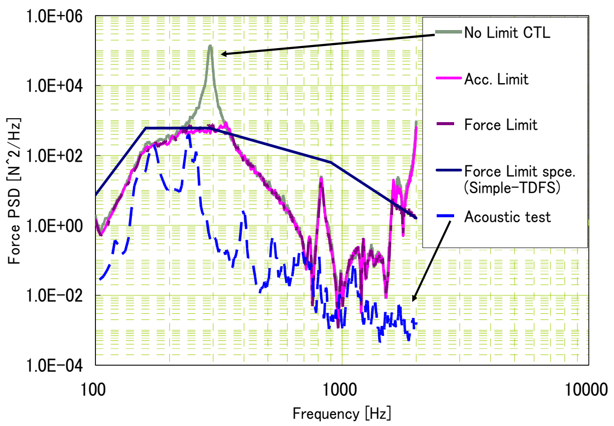

FIG.8 Interface-force PSD of the test article

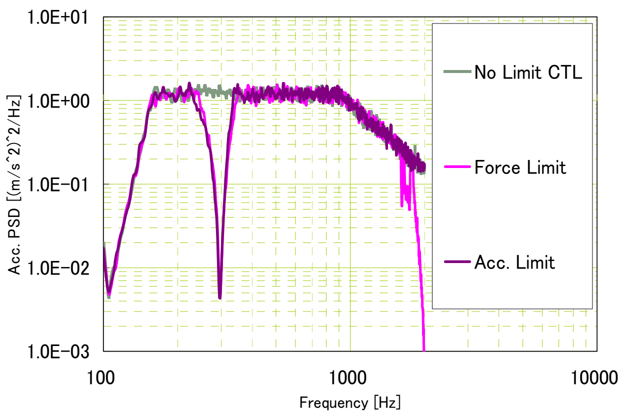

FIG.9 Acceleration PSD on the interface of the test article

5. THE VERIFICATION INVESTIGATION FOR ACTUAL COMPONENTS OF SATELLITE

This section discusses feasibility of the acceleration limit method substitution of FLVT proposed in this paper by using actual satellite components.

5.1. Components and satellite used in this study

Components used in this discussion are shown in TAB.3. These components are loaded on the large geostationary satellites.

TAB.3 components used in the discussion

Satellite-X	5,800 kg (= M_S)
Component-A	7.25 kg (= M_{La})
Component-B	54 kg (= M_{Lb})

5.2. Acceleration reduction value of components

Characteristics of components are calculated from the vibration test results of each component to calculate the acceleration reduction value.

There are some methods to obtain the effective mass m_{es}, the effective mass of Source (Satellite-X) assumes to equal to the residual effective mass over the 95Hz for the safety manner, which is estimated by the result of Finite Element Method (FEM) up to 100Hz, in this discussion. As a result, the effective mass of the Source (Satellite-X) is 290 kg (5% of the 5800 kg).

The effective mass of the Load (Components) assumes to equal to the rigid mass of itself because the mass of the components are very small compare to the satellite system.

Characteristics and calculation results of the acceleration reduction values are shown in TAB.4.

TAB.4 The characteristics and calculation results of acceleration reduction value of components

	Component-A	Component-B
f_1	95 Hz	180 Hz
Q	15	5
m_{es}	290 kg	290 kg
m_{eL} (=M_L)	7.25 kg	54 kg
m_{eL} / m_{es}	0.025	0.186
Acc Reduction	-8.9 dB	-9.38 dB

5.3. Results

In this section, the acceleration reduction value calculated in section 6.2 is compared to the satellite system acoustic test results. These results are shown in FIG.10 and FIG.11. FIG.10 and FIG.11 are the PSD acceleration results of the component-A and component-B, respectively. The lines in the graph are the random vibration test specification and the acceleration response on the interface during acoustic test.

FIG.10 and FIG.11 shows that the acceleration reduction value is reasonable compare to the natural notch depth of the acoustic test result at the resonance frequency in each component. These results means that to limit by these acceleration reduction value in vibration test could produce appropriate vibration test specification for prevent over testing.

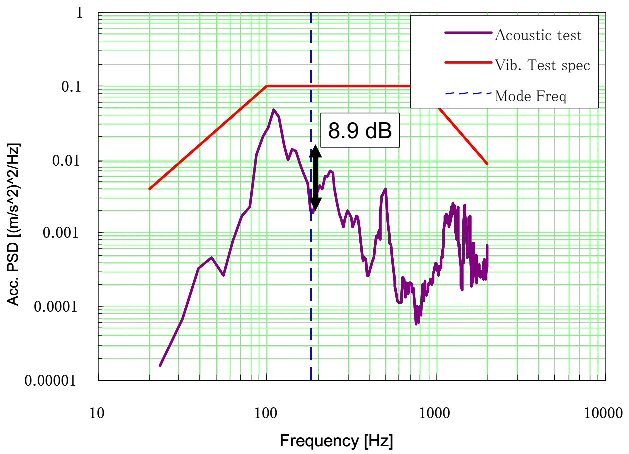

FIG.10 Results of the Component-A

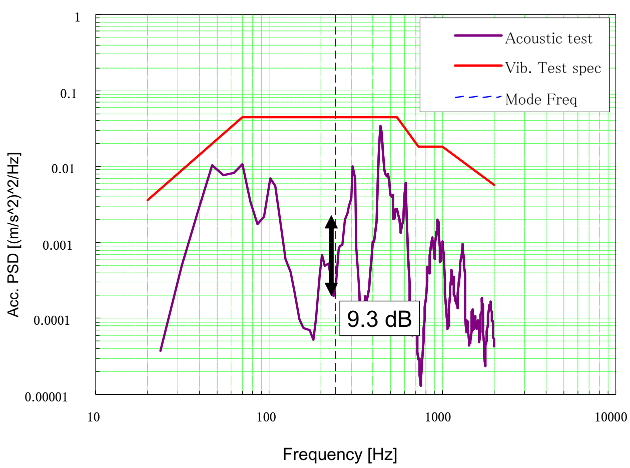

FIG.11 Results of Component-B

6. SUMMERY AND CONCLUSION REMARKS

The acceleration limit method substitution to FLVT is proposed in this paper, and discussed feasibility of this method. There are some methods to obtain the resonance frequency of coupled system "r_{cp}". In this paper, "r_{cp}" is calculated by simple TDFS model. The acceleration reduction value (dB) of inner test article is estimated by simple TDFS based on ratio of m_{eL}/m_{eS} and Q are known.

These facts discussed in this paper point to the conclusion that the acceleration limit method proposed in this paper has the possibility to execute the same limit control as FLVT.

This method is superior to the former FLVT in following points;

(1) This method needs acceleration only, force measurement is unnecessary.

(2) Test configuration is simple compare to the FLVT because force sensors and associated fixtures for the force measurement are unnecessary.

However, acceleration limit estimation may depend on the selection of acceleration location, as the equation (4) shows that the acceleration limit value depends on the mode shape of the structure. It is considered to have a risk of over testing or under testing for those points that are not select for limit due to the selecting of the limit point like anti-resonance point on the test article.

7. REFFERENCES

(1) Roy R. Craig Jr. and Mervyn C. C. Bampton, Coupling of Substructures for Dynamic Analysis, AIAA Journal, Vol. 6, No. 7, July 1968

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(3) Scharton, T.D., "Vibration-Test Force Limits Derived from Frequency –Shift Method", AIAA Journal of Spacecraft and Rockets, Vol.2, No.2, March 1995, pp.312-316