# THE INTEGRATION OF MODAL ANALYSIS IN VIBRATION QUALIFICATION TESTING

**Laurent Britte(1), Bart Peeters (1), Herman Van der Auweraer (1), Jan Debille (1), Mark O'Grady (2), Raj Singhal(2)**

*(1) LMS International, Interleuvenlaan 68, Researchpark Z1, 3001 Leuven, Belgium*
*laurent.britte@lms.be, bart.peeters@lms.be,herman.vanderauweraer@lms.be, jan.debille@lms.be*
*(2) Canadian Space Agency, 3701 Carling Avenue, Ottawa, Ontario, Canada K2H 8S2*
*mark.ogrady@space.gc.ca, raj.singhal@space.gc.ca*

## ABSTRACT

In the process of the structural integrity and launch-environment survivability assessment of satellite structures, dynamic mathematical models are used for load prediction. These analytical models need to be test-verified. Therefore it is of vital importance to correlate the Finite Element model with experimental vibration data and to further fine-tuned and update the model. In the classical modal parameter estimation approach, the baseline data are Frequency Response Functions measured in laboratory conditions with low-level excitation. However, real operating conditions may differ significantly. The integration of the modal survey with the qualification shaker test combines the advantage of more realistic excitation level with productivity. This paper investigates the validity of modal analysis based on vibration control tests. This issue is complicated by the fact that during the qualification test often the load conditions are unknown. Modal analysis can only be based on transmissibilities in this case. Also the structure can only be excited in one direction at a time. A test structure was subjected to different typical vibration qualification test. The results of modal analysis based on Vibration Control data are compared with operational modal analysis and traditional input-output modal analysis.

## 1. INTRODUCTION

Space hardware development programs typically foresee 2 phases requiring laboratory tests. First a so-called modal survey test is carried out to obtain an experimental model of the structure or to update a Finite Element (FE) model. These models are essential in planning the second phase, namely the qualification test. Here, the structure is subjected to environments that are representative of flight or launch conditions. If the structure survives the qualification test, there is a high probability that it will also survive the launch. The structure is attached to a shaker table and excited by sine sweep or broadband forces. The acceleration at the interface between shaker and structure is controlled so that a prescribed reference profile is reproduced by adapting the excitation amplitudes. For large structures, the sine sweep and broadband vibration tests are sometimes complemented by a high-level acoustic test.

For carrying out the modal test that precedes the qualification test, several configurations are possible:

- The structure is freely suspended and modal shakers or impact hammers are used to excite the structure.
- Again, modal shakers or impact hammers are used, but the structure is attached to the shaker table as in the qualification test.
- The same shaker used in the qualification test excites the structure, but at a lower level (so-called base-driven test).

It is evident that a significant amount of expensive space program time could be saved if the hardware validation cycle could be done with only one test set-up: the one from the qualification test (i.e. the third possibility mentioned above). It is this integrated approach to modal and environmental testing that will be discussed in this paper.

Füllekrug and Sinapius of the German Aerospace Center (DLR) carried out thorough investigations on the problem of modal parameter identification from base-driven vibration data. If only the outputs are measured, i.e. accelerations at the interface between the shaker and the structure and accelerations of the structure, the eigenfrequencies, damping ratios and mode shapes of the fixed interface structure can be obtained; see also section 2. In [1] it is shown that if a Force Measurement Device (FMD) is also used to measure the 6 DOF force inputs between shaker and structure, both the free and fixed interface modes can be obtained from (multi-axial) base excitation test. In [2], [3] and also in Schedlinski and Link [4], it is shown how measurements of the interface forces can be used to obtain the modal participation factors (or equivalently the modal masses). The situation is summarized in Table 1.

*Table 1: Relation between measured quantities and modal parameters that can be identified*

| Measured Quantities | Fixed Interface Structure | Free Structure |
|---|---|---|
| Interface and structure accelerations (output-only) | Eigenfrequencies Damping ratios Mode shapes | - |
| Interface forces (+inputs) | + Modal participation factors | All modal parameters |

Van Langenhove et al. [5] illustrate the process of correlation and updating on the Olympus satellite. Instead of building another expensive and time-consuming classic modal survey set-up, the data from a low-level pre-qualification test is used, processed and fed into a conventional modal analysis package to estimate eigenfrequencies, damping ratios and mode shapes.

In Hermans et al. [6] the possibility of applying operational modal analysis methods to vibration qualification test data is investigated. Operational means that the structure is measured while it is in normal operating conditions. In this case it is generally not possible to measure the forces that are exciting the structure and the parameter estimation methods must extract the modal parameters by using only the outputs.

Output-only or operational modal analysis has received a lot of attention during the last few years. Many successful applications are reported both in mechanical and civil engineering; see for instance [7], [8] and [9].

In this paper we restrict ourselves to the most common environmental test case, i.e. no forces have been measured and the structure is excited along a single axis. For this case, data processing and modal parameter estimation guidelines will be provided.

## 2. THEORY

Before acquiring and processing data, it is useful to review the equations that govern the structural behavior during the modal and environmental tests.

### 2.1 Dynamic equilibrium

The dynamic equilibrium can be expressed as:

$$M\ \ddot{q}(t) + C\ \dot{q}(t) + K\ q(t) = f(t) \qquad (1)$$

where $M, C, K \in \Upsilon^{n\times n}$ are the mass, damping and stiffness matrices; $q(t), \dot{q}(t), \ddot{q}(t) \in \Upsilon^{n}$ are the structural displacements, velocities and accelerations at continuous time $t$. The vector $f(t) \in \Upsilon^{n}$ contains the external forces. For systems with distributed parameters Eq.1 is obtained as the FE approximation of the system with only $n$ Degrees Of Freedom (DOFs) left. In the Laplace domain, Eq.1 can be written as:

$$Z(s)\,a(s) = f(s) \qquad (2)$$

where $s$ is the Laplace variable; $a(s)$ is the Laplace transform of the acceleration vector $\ddot{q}(t)$; and $Z(s) \in \leq^{n\times n}$ is the apparent mass, defined as:

$$Z(s) = M + \frac{1}{s}C + \frac{1}{s^2}K \qquad (3)$$

In the present case of base excitation, the only external force is applied at the interface. If it assumed that the control accelerometer is at the same location, Eq.2 can be partitioned as:

$$\begin{array}{r c} \begin{array}{r} 1 \updownarrow \\ n-1 \updownarrow \end{array} & \begin{bmatrix} z_{cc} & z_{cm} \\ z_{mc} & Z_{mm} \end{bmatrix} \\ & \begin{array}{cc} \underset{1}{\leftrightarrow} & \underset{n-1}{\leftrightarrow} \end{array} \end{array} \begin{bmatrix} a_c \\ a_m \end{bmatrix} = \begin{bmatrix} f_c \\ 0 \end{bmatrix} \qquad (4)$$

where $a_c \in \leq$ is the control acceleration; and $a_m \in \leq^{n-1}$ are the remaining $n-1$ accelerations that have been measured.

### 2.2 Driving point FRF

The FRF matrix $H(s) \in \leq^{n\times n}$ from forces to accelerations is the inverse of the apparent mass matrix:

$$H = Z^{-1} = \frac{1}{|Z|}\,\mathbf{adj}(Z) \qquad (5)$$

where $|\bullet|$ denotes the determinant and **adj(•)** the adjoint of a matrix. Elaborating Eq.5 for the upper left element, which is the driving point FRF $h_{cc} \in \leq$, leads to:

$$h_{cc} = \frac{|Z_{mm}|}{|Z|} \qquad (6)$$

The zeros of the numerator $|Z_{mm}|$ are the so-called anti-resonances of the driving point FRF. The zeros of the denominator $|Z|$ are the resonances of the structure (the denominator is common to all elements of the FRF matrix).

### 2.3 Transmissibilities

In case of an unknown input $f_c$ Eq.4 remains valid of course, but the useful part is the lower part, which can be rewritten as:

$$a_m = -Z_{mm}^{-1}\, z_{mc}\, a_c \tag{7}$$

Transfer functions between accelerations are called transmissibilities. Therefore, Eq.7 provides an expression for the transmissibility vector $T_{mc} \in \leq^{n-1}$ between the control accelerometer and the measurement accelerometers:

$$a_m = T_{mc}\, a_c\,, \quad T_{mc} = -\frac{1}{|Z_{mm}|}\mathbf{adj}(Z_{mm})\, z_{mc} \tag{8}$$

When comparing the position of $|Z_{mm}|$ in Eq.6 and Eq.8, it is observed that the anti-resonances of the driving point FRF correspond to the resonances of the transmissibilities.

### 2.4 Cross spectra

From Eq.8, it can be derived that the cross spectra $S_{mc} = \mathbf{E}[a_m a_c^*] \in \leq^{n-1}$ ($\mathbf{E}[\bullet]$ is the expected value operator) between the measurement accelerations and the control acceleration can be written as:

$$S_{mc} = -\frac{1}{|Z_{mm}|}\mathbf{adj}(Z_{mm})\, z_{mc}\, S_{cc} \tag{9}$$

So the interpretation of the cross spectrum resonances is not unique as it depends on the shape of the control power spectrum $S_{cc} \in \leq$, which is related to the base excitation force spectrum $S_{ff} \in \leq$ via the driving point FRF:

$$S_{cc} = h_{cc}\, S_{ff}\, h_{cc}^* \tag{10}$$

If the control spectrum is flat, the resonances of the cross spectra coincide with the resonances of the transmissibilities (zeros of $|Z_{mm}|$). If on the other hand the base excitation forces have a flat spectrum (i.e. $S_{ff}$ is a constant), it can be seen that, after introducing Eq.6 and Eq.10, $|Z_{mm}|$ disappears from the denominator in Eq.9 and that the cross spectra resonances coincide with the resonances of the FRFs (Eq.5) (zeros of $|Z|$).

### 2.5 Free versus fixed interface

It is apparent that the resonance modes of the FRFs between excitation and accelerations (Eq.5) are the structural modes under free boundary conditions. The physical interpretation of the resonance modes of the transmissibilities (Eq.8) follows from the input-output relations of the structure that is fixed at the interface (control) point. The dynamic equilibrium of such a system is obtained by eliminating the row and column involving the control point from the equilibrium equations of the free structure (Eq.2). By consequence, the FRF of the fixed structure $H_{\mathbf{fixed}} \in \leq^{(n-1)\times(n-1)}$ equals:

$$H_{\mathbf{fixed}} = Z_{mm}^{-1} = \frac{1}{|Z_{mm}|}\mathbf{adj}(Z_{mm}) \tag{11}$$

By comparing Eq.11 with Eq.8, it is evident that the resonances of the transmissibilities determine the structural modes of the structure when it is fixed at the interface point. The interpretation of the theory of this section is summarized in Table 2. It can be concluded that only when using FRFs or transmissibilities, is it clear which modes will be identified: the modes of the free structure in case of FRFs and the modes of the fixed interface structure in case of transmissibilities. If cross spectra are used, some information about the input excitation is needed in order to interpret the modes correctly.

*Table 2 : Interpretation of resonances of FRFs, transmissibilities and cross spectra in terms of structural modes.*

| Type of base excitation | Measurement functions | | |
|---|---|---|---|
| | FRFs | Transmissibilities | Cross Spectra |
| White noise (flat force spectrum | Free modes | Fixed interface modes | Free modes |
| Shaped noise such that the control acceleration has a flat spectrum | Free modes | Fixed interface modes | Fixed interface modes |

## 3. DEMONSTRATION STRUCTURE

To illustrate the concepts of this paper, an environmental test was carried out on a demonstration structure (Fig.1 & Fig.2) at the Canadian Space Agency. In this experiment, eight force transducers were summed in two groups to measure the force at the interface structure / slip table in the two horizontal directions; X (the excitation direction) and Y. The vertical force Z and the moments, which were certainly as: present in this set-up, were not measured. Four control accelerometers were located as close as possible to the interface points. Although the force transducers and control accelerometers are not exactly at the same location, we will consider the Frequency Response Function (FRF) between them as the driving point FRF. Another 48 accelerometers measured the responses along the structure. Their purpose is to assess mode shape correspondence between different test and processing results. The demonstration structure was shaken horizontally in the X direction.

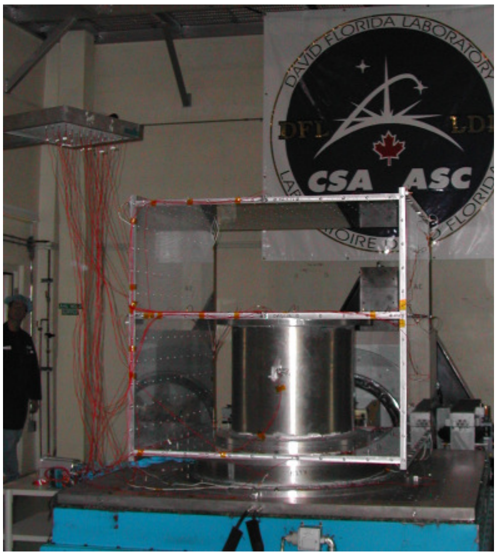
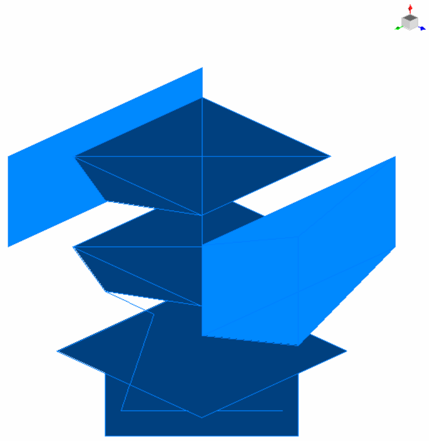



*Fig. 1 : SRS (Canadian Space Agency's demonstration*



*structure)*

*Fig. 2 : SRS (Geometry –
52 accelerometers and 8 force cells)*

## 4. PRACTICAL IMPLICATIONS

### 4.1 Environmental testing

In the environmental test the structure is subjected to vibration environments that correspond to the situation when the structure is in operation. An example of such a vibration environment specification is provided in Figure 3. This profile comes in the form of an acceleration Power Spectral Density (PSD) [$g^2$/Hz]. It is the aim to reproduce this spectrum at the interface between shaker and structure by using broadband excitation signals with well-defined amplitudes, but random phases. Such a test is commonly called a random control test. If on the other hand the vibration environment is reproduced by a sine sweep excitation, a similar reference profile is used, but expressed in [g] instead of [$g^2$/Hz]. In case of a sine sweep, the excitation signal is a sine tone with continuously changing frequency. Such a test is commonly called a sine control test. Using the LMS Test.Lab software [11] and LMS Scadas III hardware, both a random and a sine control test were carried out on the test structure introduced in section 2.1. The reference profile was in both cases a flat curve.

During the random control test the following measurements were taken:

- PSDs of control and measurement channels;
- Transmissibilities with respect to the control channel.

The force was included as a measurement channel. The transmissibilities are computed as least squares estimates assuming no noise on the control channel (the $H_1$ estimator); e.g. the transmissibility for channel $y$ is computed

$$\hat{T}_{yc} = \frac{\hat{S}_{yc}}{\hat{S}_{cc}} \qquad (12)$$

where $\hat{S}_{yc} \in \leq$ is the cross spectrum estimate. The classical FRFs between force and accelerations $\hat{H}_{yf} \in \leq$ are not directly available, but can be estimated from the measured transmissibilities:

$$\hat{H}_{yf} = \frac{\hat{T}_{yc}}{\hat{T}_{fc}} = \frac{\hat{S}_{yc}}{\hat{S}_{fc}} \qquad (13)$$

This estimate is the so-called instrumental-variable estimate, where the control channel serves as instrument. Finally, the cross spectra are obtained from measured transmissibilities and the control power spectrum:

$$\hat{S}_{yc} = \hat{T}_{yc}\,\hat{S}_{cc} \qquad (14)$$

In Fig.3, the measured driving point FRF (Eq.6) is compared with the sum of magnitudes of the transmissibilities (Eq.8). The resonances present in the measured transmissibilities will show up as peaks in the sum. As predicted by the theory (section 2), it is observed that the resonances of the transmissibilities correspond to the anti-resonances of the driving point FRF.

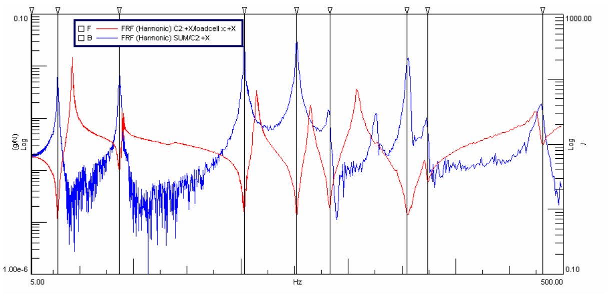



*Fig. 3: Driving point FRF (Red) and the sum of the magnitudes of the transmissibilities (blue). The anti-resonances of the driving point FRF correspond to the resonances of the transmissibilities.*

The PSD (Power Spectral Density) of a measurement channel and the PSD of the control channel are shown in Fig. 4: Random Control data magnitudes. (Top) Control and measurement PSD [g2/Hz]. (Bottom) Transmissibility [-]., together with the corresponding transmissibility, which is the ratio of both spectral densities.

During the sine control test the linear frequency spectra are measured. The obtained phases are relative phases between the different measurement channels. To obtain absolute phases, zero phase angles are assigned to the control spectrum. Although they are obtained in a different way, these sine sweep spectra can be considered as Fourier transforms of (virtual) time measurements. In case of a single excitation source, the transmissibilities $\hat{T}_{yc}$ and cross spectra $\hat{S}_{yc}$ can be computed from the linear spectra $\hat{S}_y$, $\hat{S}_c$ as:

$$\hat{T}_{yc} = \frac{\hat{S}_y}{\hat{S}_c}, \quad \hat{S}_{yc} = \hat{S}_y \hat{S}_c^* \qquad (15)$$

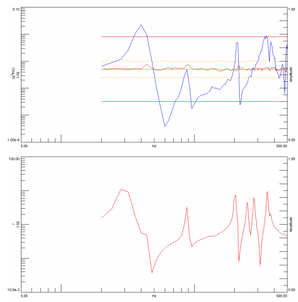


*Fig. 4: Random Control data magnitudes. (Top) Control and measurement PSD [$g^2$/Hz]. (Bottom) Transmissibility [-].*

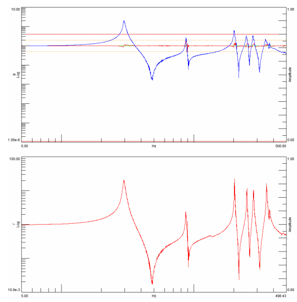


*Fig. 5: Sine Control data magnitudes. (Top) Control and measurement linear spectrum [g]. (Bottom) Transmissibility [-].*

The magnitudes of the linear spectra of the control and a measurement channel are shown in Fig. 5, together with the corresponding transmissibility. The shown random (Fig. 4: Random Control data magnitudes. (Top) Control and measurement PSD [g2/Hz]. (Bottom) Transmissibility [-].) and sine transmissibility (Fig.5) correspond well to each other.

## 4.2 Modal analysis

Classical modal analysis methods require FRF or impulse response data (obtained as the inverse Fourier transforms of FRFs). Operational modal analysis methods deal with measured cross spectra or cross correlations (i.e. the inverse Fourier transforms of cross spectra). In case of an environmental test, the use of transmissibilities as input data for parameter estimation methods also leads to sensible results. This is somewhat of a special case since transmissibilities are operational data (no input available), but classical parameter estimation methods can be used because they can be considered as the FRFs of the fixed interface system up to some scaling; see Eq.8, Eq.11.

As for practical reasons, the control spectrum is never perfectly flat, the interpretation of the cross spectra is generally not unambiguous (Table 2). Only if the control spectrum is flat (piecewise linear with not too sharp slopes should also work), are the fixed interface modes found. However, in environmental tests, the technique of response limiting is often used. In this case, the control spectrum can show sharp notches where the input signal is decreased at certain frequency regions because the response at a certain measurement channel exceeded pre-defined limits.

## 5. APPLICATION

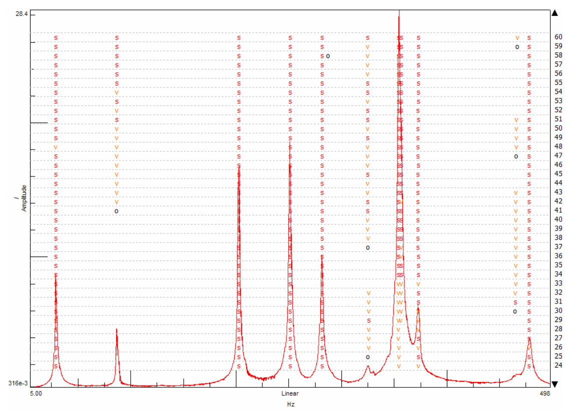


*Fig. 6 : Stabilization diagram obtained by applying the LMS PolyMax method to sine control transmissibilities .*

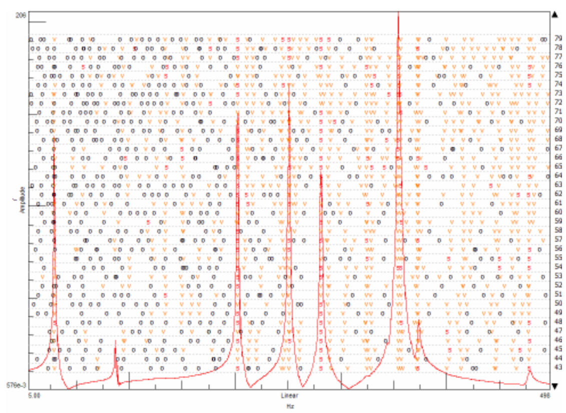


*Fig. 7 : Stabilization diagram obtained by applying stochastic subspace identification to sine control transmissibilities.*

This section presents the modal parameters that could be extracted from the different environmental test data sets. The LMS PolyMAX [13] method is applied to FRFs and transmissibilities. The LMS PolyMax (Fig.6) algorithm is a recently developed parameter estimation method known for its extremely clear stabilization diagram [11], [12]. For comparison, the Time MDOF (Fig.7) method was also applied on the same data. Both parameter estimation methods are implemented in the LMS Test.Lab software. The LMS PolyMAX gives a much cleaner stabilization diagram. If only the Time MDOF method were available, it would have been nearly impossible to extract the modal parameters.

*Table 3 : Free modes identified from Sine Control, Random Control and Burst Random FRFs (base-driven tests).*

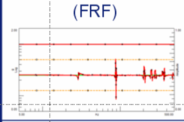
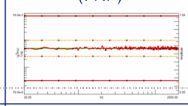
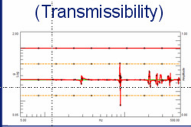
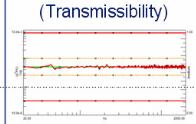
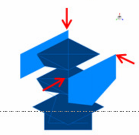
| Sine Control 0.25 g - Flat (FRF)  | Random Control 1 g RMS - Flat (FRF)  | Burst Random (FRF) |
|---|---|---|
| 43.25 Hz | 44.01 Hz | 43.60 Hz |
| 89.62 Hz | 89.72 Hz | 89.18 Hz |
| 214.67 Hz | 215.50 Hz | 215.31 Hz |
| 264.27 Hz | 263.86 Hz | 265.09 Hz |
| 307.69 Hz | 306.95 Hz | 308.85 Hz |
| 325.41 Hz | 322.86 Hz | 325.26 Hz |

*Table 4 : Fixed interface modes identified from Sine Control and Random Control transmissibilities, and from Modal Impact FRFs.*

| Sine Control 0.25 g - Flat (Transmissibility)  | Random Control 1 g RMS - Flat (Transmissibility)  | Modal Impact  |
|---|---|---|
| 29.24 Hz | 40.20 Hz | 40.77 Hz |
| 87.00 Hz | 86.567 Hz | 87.55 Hz |
| 202.56 Hz | 210.12 Hz | 204.75 Hz |
| 251.29 Hz | 233.12 Hz | 233.35 Hz |

The free-free modal parameters extracted from the FRFs are shown in Table 3. The fixed interface modal parameters are shown in Table 4. Evidently, the free-free modes are different from the fixed interface modes. The difference between the Sine Control test and the two other ones for the fixed interface modes can be explained by the presence of the force cells for the first case and not for the others. The mode shape correspondence between the different excitation modes was good as shown by the high diagonal MAC values in Fig.8. Some fixed interface mode shapes are shown in Fig.9.

While the Sine Sweep excitation gives cleaner results which are easier to interpret and to correlate with finite element models, the Random Control is probably closer to the real operational excitation. Due to the accumulation of three directions for the excitation and the possibility to excite the structure at any point, the Modal Impact test also shows complex and torsion modes (Fig.10).

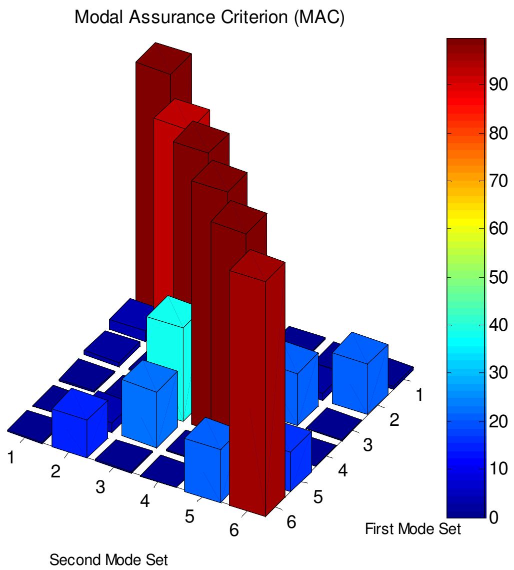



*Fig. 8 : MAC values assessing the mode shape correletion between sine and random transmissibilities*

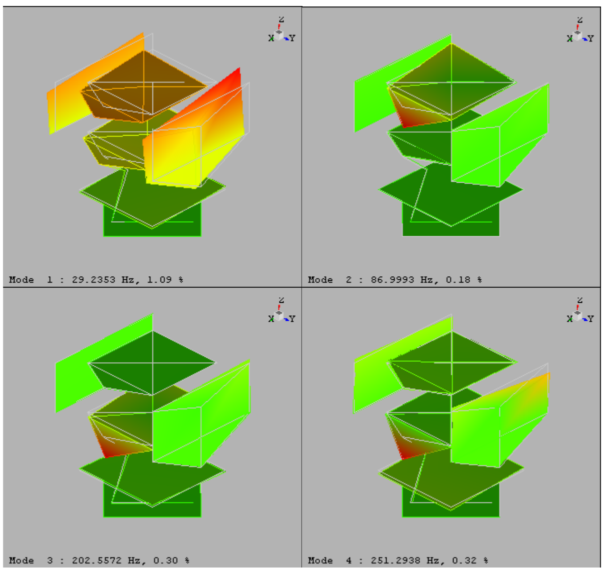



*Fig. 9 : First 4 fixed interface mode shapes.*

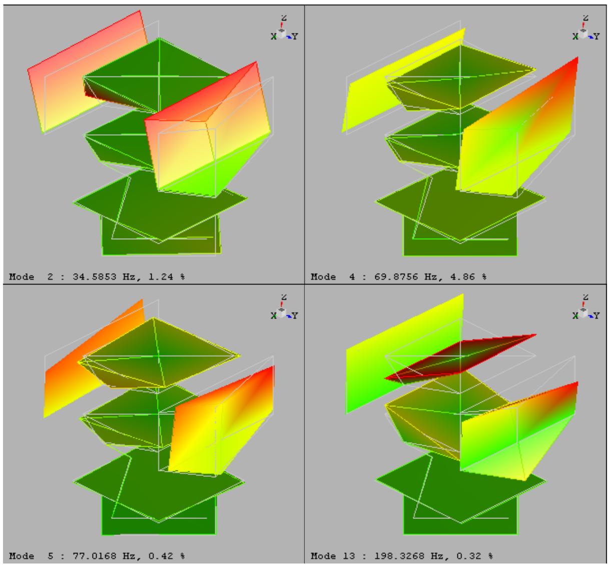



*Fig. 10 : Mode shapes not shown by single axis base excitation.*

## 6. CONCLUSIONS

This paper discussed the theory behind the application of modal analysis to vibration qualification test data. Experimental results from a demonstration structure illustrated the concepts. If no input measurements are available, it is recommended to use transmissibilities (either obtained from a random or sine control test) as operational measurement functions from which the modal parameters of the fixed interface structure can be extracted by classical parameter estimation methods. Using cross spectra combined with operational parameter estimation methods leads to stabilization diagrams that are more difficult to interpret and, without information about the shape of the excitation, it is uncertain whether the identified modes are the modes from the fixed interface structure.

If in space hardware development programs, the same physical set-up can be used for both the modal survey and the qualification test, a significant amount of time can be saved. This paper contributed to the idea of base excitation modal testing. A disadvantage of using base excitation is that not all modes may be excited, especially torsion and local modes. However, since the structure is attached to the shaker in the same way as it will be attached to the launch vehicle, one could argue that all important modes are excited. The modes that are missing during the test will probably not be excited during normal operation of the structure. This disadvantage can be partially compensated for by combining the transmissibilities of subsequent tests in different directions.

## 7. REFERENCES

[1] Sinapius J.M. *Identification of free and fixed interface normal modes by base excitation*. In Proceedings of IMAC 14, 23–31, Dearborn, USA, February 1996.

[2] Füllekrug U. *Determination of effective masses and modal masses from base-driven tests.* In Proceedings of IMAC 14, 671–681, Dearborn, USA, February 1996.

[3] Füllekrug U. and Sinapius J.M. *Force measurement in vibration testing – application to modal identification*. In Proceedings of the 7th International Congress on Sound and Vibration, 2567–2574, Garmisch-Partenkirchen, Germany, July 2000.

[4] Schedlinski C. and Link M. *Identification of frequency response function and modal data from base excitation tests under measured interface forces*. In Proceedings of the ASME Conference on Noise and Vibration, Boston, USA, 1995.

[5] Van Langenhove T., Fredö C. and Brunner O. *FE model correlation and mode shape updating using qualification test data – a case study on the Olympus satellite*. In Proceedings of the NAFEMS World Congress, Como, Italy, April 2001.

[6] Hermans L., Debille J. and Van der Auweraer H. *Extraction of modal parameters from closed-loop vibration control test data*. In Proceedings of the 45th Annual Technical Meeting of the Institute of Environmental Sciences and Technology, Ontario, CA, USA, 1999.

[7] Hermans L. and Van der Auweraer H. *Modal testing and analysis of structures under operational conditions: industrial applications*. Mechanical Systems and Signal Processing, 13(2), 193–216, 1999.

[8] Peeters B. *System Identification and Damage Detection in Civil Engineering*. PhD thesis, Department of Civil Engineering, K.U.Leuven, Belgium, [www.bwk. kuleuven.ac.be/bwm], December 2000.

[9] Peeters B. and De Roeck G. *Stochastic system identification for operational modal analysis: a review*. ASME Journal of Dynamic Systems, Measurement, and Control, 123(4), 2001.

[10] Kistler Aerospace Corporation. K-1 Vehicle Payload *User's Guide*, Kirkland (WA), USA, [www. kistleraerospace.com], 2001.

[11] LMS International. *LMS Test.Lab Environmental Testing, Release 7.A, User Documentation*, Leuven, Belgium, [www.lmsintl.com], 2006.

[12] LMS International. *LMS Test.Lab Modal Analysis, Release 7.A, User Documentation*, Leuven, Belgium, [www.lmsintl.com], 2006.

[13] B. Peeters, H. Van der Auweraer, P. Guillaume, J. Leuridan, *The PolyMAX frequency-domain method: a new standard for modal parameter estimation?* Shock and Vibration, Special Issue dedicated to Professor Bruno Piombo, 11, 395–409, 2004.

[14] H. Van der Auweraer, B. Peeters, *Discriminating physical poles from mathematical poles in high order systems: use and automation of the stabilization diagram.* In Proceedings of IMTC 2004, the IEEE Instrumentation and Measurement Technology Conference, Como, Italy, 18–20 May 2004.

[15] B. Peeters, A. Vecchio, H. Van der Auweraer, *PolyMAX modal parameter estimation from operational data.* In Proceedings of the ISMA 2004 International Conference on Noise and Vibration Engineering, Leuven, Belgium, September 2004.

[16] J. Lanslots, B. Rodiers, B. Peeters, *Automated pole-selection: proof-of-concept & validation*. In Proceedings of ISMA 2004, the International Conference on Noise and Vibration Engineering, Leuven, Belgium, 20–22 September 2004.