STUDY AND EMPLOYMENT OF THE MONTE CARLO SIMULATION FOR THE ROBUST DESIGN OF SPACE STRUCTURES

A. Weber
Dresden, Germany
E-mail: andreas.weber@tu-dresden.de

OVERVIEW

In the design of space structures "worst case" scenarios and factors of safety are commonly used. An alternative approach is the Monte Carlo simulation which allows considering uncertainty in the design and thus provides the way for robustness assessments. The Monte Carlo Simulation and other methods have been studied extensively in the past. Nevertheless, neither method is yet established in the daily design process in the aerospace industry.

This article summarizes the theoretical background from an application point of view and introduces a new implementation of a stochastic simulation environment within existing solver and modelling techniques including assessment of robustness and dominating variables. Finally the application of the stochastic simulation is demonstrated using a model of a telescope structure provided by Astrium GmbH.

1. INTRODUCTION

Virtual prototyping has become a substantial component of the modern design process, especially as finite element methods (FEM) have become one of the standards for design validation and optimization. Uncertainties are generally addressed with various factors of safety. The simplicity and the wealth of experience in the usage of safety factors have made them a practical engineering tool. But a major drawback is that the effect of the uncertainty and variability introduced into the system as well as the degree of conservatism is not quantifiable.

Additionally, it is generally expected that a more detailed numerical model (meaning an increase in terms of degrees of freedom) - will increase the level of realism. However an increase in the degrees of freedom only reduces the discretisation error. A numerical model remains an idealization, representing the physical behaviour with limited accuracy.

In the recent years various new methods have been developed or studied to overcome these obstacles. The most popular method is the stochastic simulation. In the 1990's, the first simulations were used in automotive crash analysis and in the analysis of satellite micro vibration. Due to the high computational time even for a deterministic analysis, it was necessary to develop a simulation environment. This software tool was called PROMENVIR [1]. It allowed performing simulations on a European computer cluster. In 1999 Marchante [2] presented the application of the Monte Carlo simulation to the payload adapter of the Ariane 5 launcher. The aim was to establish the driving parameters for the dynamic characteristics of a typical payload on top of the launcher. Marczyk has published two books [3, 4] concerning the engineering process and the optimization using the simulation technique. With the increase in available computational power there are some recent applications. In 2005 Roy and Garcia de Paredes [5] published a stochastic approach to dynamic model validation of a Eurostar E3000 telecom satellite, Mary [6] published the application of a stochastic approach for the design improvement of a satellite and Pellissetti [7] demonstrated a static and dynamic reliability analysis of the Integral satellite. Thus the stochastic simulation itself is proven feasible.

In a stochastic simulation uncertainties are expressed as random variables and following their distribution many realizations of the problem are generated. Each realization is leading to a deterministic problem which can be solved by well developed deterministic FE code. The Monte Carlo simulation is a very simple and clean methodology. It has the following advantages and disadvantages [8]:

- full generality in terms of application,
- intrinsic parallelism because of the independence of the sampling units,
- does not require changing solver, algorithms or modelling techniques,
- avoids long and manual-type parametric studies,
- provides simultaneously:
    - statistical problem description,
    - correlation,
    - detection of dominating design variables,
    - robustness assessment,
- high computational effort for reasonable accuracy.

The solutions can be post processed to obtain statistical results like the mean value, standard deviation, correlation, etc. Furthermore the results can be used for an advanced design concept. The so called robust design concept introduces the robustness into the design process. Robustness is intended as a measure of the influence and effects of the uncertainties in the model parameters or the system itself. Instead of finding the optimal design this new approach is intended to find the most robust solution. This is a design which is able to cope with uncertainties during the whole lifetime in the best possible way. Additionally, the stochastic simulation can be used to obtain a better insight in the system behaviour.

This article presents an approach for a structural analysis including uncertainties and robustness evaluations. It is intended for the daily design process in the aerospace industry.

2. THEORY OF UNCERTAINTY AND MONTE CARLO SIMULATION

The theory of Monte Carlo Simulation and the definitions of uncertainty are well known. Basic definitions and facts are summarized in this section.

2.1. Definition of uncertainty

Uncertainty can be described as lack of certainty resulting from inaccuracy of input parameters, analysis process or both. Firstly, inaccuracy can be associated with variation inherent in the physical system or environment. This is called variability [9], or aleatoric uncertainty [10]. Secondly, inaccuracy can be associated with deficiency that originates in the lack of knowledge; this is called epistemic uncertainty [10]. In the scope of this work the term uncertainty is used to describe all kinds of inaccuracies. For a differentiation the terms variability and epistemic uncertainty are used.

In a structural context, following Kang [11] and Marczyk [1], uncertainties can be categorised in:

- model error,
- computational error,
- incomplete knowledge,
- manufacturing and assembly tolerances,
- material Imperfection and variability,
- environmental or boundary condition variation,
- loading fluctuation.

A different amount of knowledge can exist for these uncertainties. For example for the thickness of a plate, the manufacturing tolerance e.g. the boundary is usually known. Generally, there are three types of knowledge about inaccuracies:

- bounded quantities: information about the limits of quantities is known,
- fuzzy quantities: information about possible values is available,
- stochastic quantities: information about the probability of quantities is available.

These types are not strictly differentiated. Thus it is for example possible to describe a bounded quantity in a random framework with a uniform distribution.

To express quantities in a stochastic framework their occurrence properties have to be defined. This can be done by using the probability density function (pdf) f(x) and the cumulative distribution function (cdf) F(x):

(1) $$F(x) = P(X \le x) \equiv \int_{-\infty}^{x} f(\tilde{x})d\tilde{x},$$

where X is the random variable and P(X ≤ x) is the probability that an event will occur. Additionally X can be characterized by its statistical moments. The first and second moment known as mean value μ(X) and variance σ^2(X) are given by:

(2) $$\mu(X) = \int_{-\infty}^{+\infty} x f(x)dx,$$
$$\sigma^2(x) = \int_{-\infty}^{+\infty} (x-\mu(X))^2 f(x)dx.$$

Often both moments are combined to the so called coefficient of variation:

(3) $$CV = \frac{\sigma(X)}{|\mu(X)|}$$

When the mean value is near zero, the coefficient of variation is sensitive to change in the standard deviation, limiting its usefulness.

2.2. Direct Monte Carlo Simulation

The MCS is a method to solve a probabilistic problem with an indirect approach. First the uncertain variables itself have to be selected and a random distribution has to be defined. This first step is part of the pre-processing. Then from the specified input distribution, samples are generated. This step is the first part of the MCS and is called sampling. Each input sample is a deterministic realisation of the problem, which is solved independently. Each deterministic analysis results in one system response. All of these outputs are collected to form the so called meta model. This meta model is the result of the stochastic simulation and can be used for a statistical analysis.

A well known method for generating the samples is the Latin Hypercube Technique (LHT) [12]. The sample is build by implementing criteria to optimise the filling of the input space. Therefore the space is divided into subsets of equal probability. In one population a subset of each random variable is combined with all the other variables only once. The main advantage of this method is the equal coverage of the input space even with small numbers of samples

2.3. Analysis of random systems

A stochastic simulation consists of n different samples, i.e. n observations of p random variables y. The physical quantities can be further divided in i variables specified with given probabilities (input variables) and o variables calculated as system response (output variables). Typical input variables are Young's modulus, density or geometric dimensions. Typical output variables are mass, displacements, forces or eigenfrequencies. The consideration is multivariate. Following Doltsinis [8] the n × p-matrix is called meta model and can be arranged as:

(4) $$\begin{array}{cc} \multicolumn{2}{c}{Variables \quad i+o=p} \\ \begin{array}{ccccc} 1 & \dots & i & i+1 & \dots \; p \end{array} & \\ M = \begin{bmatrix} x_{11} & \cdots x_{1i} & y_{1i+1} & \cdots y_{1p} \\ \vdots & & & \vdots \\ x_{n1} & \cdots x_{ni} & y_{ni+1} & \cdots y_{np} \end{bmatrix} & \begin{array}{c} 1 \\ \vdots \\ n \end{array} \end{array}$$

The comparison of different columns m_k in M regards the relationship between these variables, i.e. between these system quantities. The consideration of different rows m_l reveals variations between the samples.

The variance and covariance of all variables in M can be arranged in a p × p matrix:

(5) $$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ & s_{ss} & \cdots & s_{2p} \\ & & \ddots & \vdots \\ symm. & & & s_{pp} \end{bmatrix}$$

$$S = \frac{1}{n-1}\left[M^t M - n\overline{m}\overline{m}^t\right]$$

A characterisation of the resulting distribution can be done by using the measures of uncertainty presented in section 2. But instead of obtaining them analytically they are calculated in an empirical manner from the samples calculated during the simulation. The covariance can be a measure of the independence of two variables. Because the covariance depends on the unit of the quantities it is useful to define a normalisation. This can be achieved by using the product of standard deviations of the variables for the normalisation. The coefficient is called Pearson correlation or linear correlation coefficient.

(6) $$r_{kh} = \frac{s_{kh}}{s_k s_h}$$

The values of the correlation coefficients range between -1 and 1, inclusive. A value near 1 or -1 indicates a high correlation and thus a dependency between the two variables. A value near 0 indicates a small correlation but not necessarily the independence of the two variables.

3. IMPLEMENTATION OF A STOCHASTIC SIMULATION ENVIRONMENT

The transfer from theory to application in the aerospace industry is only possible while using existing software. Thus the objectives are:

- Usage of existing solver MSC.Nastran™ and FEM software MSC.Patran™,
- Easy to use interface for randomization
- Provide basic statistical results (histogram, scatter plots, standard deviation, …),
- Correlation analysis
- Extendable for further statistical evaluation like regression analysis.

On the basis of these objectives a simulation process is developed including robustness and system behaviour assessments.

3.1. Fields of application

The basic application of a stochastic approach is to determine the variability of the system performance due to the uncertainties and variability introduced into the system. These statistical results can be included directly into engineering decisions. Generally they can be grouped into five categories following [13]:

1) Model validation: The non-deterministic approach provides this variance also for the model and thus enables a comparison under consideration of uncertainties.
2) Design sensitivity: The results generated by a non-deterministic approach can be used to understand how and to which extend the system reacts to the inputs.
3) Robust design: The data provided by a simulation can be used to determine the robustness of a structure.
4) Reliability design: Reliability can be defined as a measure of the distance of the system response with respect to certain limits which are retained as critical.
5) Optimization: The robustness and reliability can also be included into an objective function. Additionally, using a stochastic simulation the minimum search is not based on the gradient of the objective function. This means a reduction in the convergence rate but an increase in the global search abilities.

For a structural assessment and as first steps the second and third points are further addressed.

3.2. Simulation process

The whole simulation process includes more steps than a single simulation. For the application in the engineering process a stochastic simulation consists of four parts: the model health check (MHC), the randomization, the stochastic simulation itself and the evaluation of the results. The main steps are depicted in figure 1.

The MHC is suggested by Koch [14] and is intended to assure the quality of the model. A model with a poor quality can introduce analysis errors into the simulation. Additionally, equation 7 for the estimation of the number of observations is only valid in well-behaved systems. The behaviour can be checked by introducing a small uniform variation e.g. CV = 1% into all parameters, including parameters for modelling reasons. A well-behaved system should respond with small variations in the outputs. In case the coefficient of variation is higher than 10% it should be checked that the standard deviation is small. If the standard deviation is large and no physical explanation can be found for this behaviour of the system, the model should be checked for errors.

After the model itself was verified, the uncertainties can be assigned to the random variables. This is called randomisation. Generally, all parameters with a representation in the real system should be included. The computational effort does not depend on the number of random variables. Due to a priori selection a variable with an important but unexpected influence will remain undiscovered.

The next step is the simulation itself. This was further addressed in section 2. Once all deterministic calculations are finished the results and inputs of the simulation should

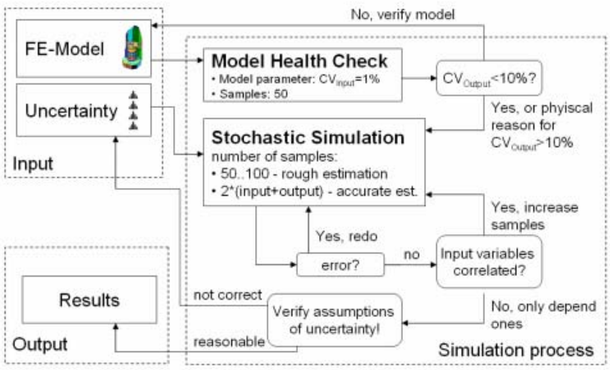

Figure 1. Main steps of a full stochastic simulation process.

be checked. Additionally it should be verified that no permutation of results during the simulation has occurred. This can happen if the result values are ordered due to their magnitude. The final step is to verify the assumptions for the most influencing variables. If the assumptions for uncertainties in these random variables were set too high, the results are possibly misleading.

Finally, the results can be evaluated, using the methods that are presented in section 2.

3.3. Quantification of uncertainty

The quantification of uncertainties is a difficult task. The variability is strictly dependant on the object considered, thus it depends on the design, manufacturing and installation. Additionally, only a few systematic studies are available. In general knowledge gained by experience should be translated into assumptions of a probability function. For major parameters these assumptions should be confirmed.

A study of the uncertainty of typical load cases in the field of space structures was conducted by Vidal [15, 16]. For the stress usually only the nominal or average levels are known. For geometric dimensions the boundaries are known, but assumptions about the distribution are vague. Assumptions about model and computational errors can only be derived from the model and should be based upon experience.

The variability in material properties like Young's Modulus or density is normally not known. Available data about some typical aerospace materials have been compared. The material properties have been taken from different sources [17–23]. The reasons of variability between these sources are the different methods of measurement and the variability of the material itself.

It can be seen in table 1 that the variability for metals is very low. A coefficient of variation around 1% seems appropriate. Meanwhile the variations for composite materials have a coefficient of variations of about 8% up to 12%. With three up to five different sources of one material property the calculated coefficient can only be regarded as a basis for approximations.

For epistemic uncertainty no general assumptions can be made. Engineering experience has to be used for an approximation.

As mentioned above, all parameters with a representation in the real system should be included, even if only a rough guess of its uncertainty is possible. In case the variable turns out to be important for the system performance the first assumption can be confirmed and maybe corrected, but the awareness is raised.

Material		Young's modulus		Shear modulus		Poisson's number	
		mean	CV	mean	CV	mean	CV
		$[kN/mm^2]$	$[-]$	$[kN/mm^2]$	$[-]$	$[-]$	$[-]$
Al 99.5	1200	68.2	1.12%	26.0	1.15%	0.34	0.75%
Al-Cu-Mg	2024-T4	72.3	1.36%	28.0	–	–	–
Al-ZN-Mg-Cu	7075-T6	71.2	1.19%	26.9	–	–	–
Ti6Al4V		118.3	1.96%	41.35	3.42%	0.33	2.55%
GY-70 Fibre / Epoxy		229.1	8.33%	3.69	11.84%	–	–
R-Glass/Epoxy		43.1	7.17%	6.35	6.16%	–	–
		Density		**CTE**			
		mean	cov	mean	cov		
		$[g/cm^3]$	$[-]$	$[1/K]$	$[-]$		
Al 99.5	1200	–	–	$23.8 \cdot 10^{-6}$	0.42%		
Al-Cu-Mg	2024-T4	2.79	0.18%	$23.0 \cdot 10^{-6}$	0.43%		
Al-ZN-Mg-Cu	7075-T6	–	–	–	–		
Ti6Al4V		4.43	0.06%	$8.65 \cdot 10^{-6}$	1.73%		

Table 1. Comparison of material properties for typical aerospace materials from different sources. CV includes variability due to manufacturing and measuring. A cut-off of 2σ is assumed for the calculation of the standard deviation. Material properties are taken from [17–23].

3.4. Recommended number of samples

The number of samples should be chosen as a balance between the desired accuracy and computational effort. Will [24] suggests a sample size of:

(7) number of samples = 2 (input + output).

Mary [6] and Marchante [2] present stochastic simulations of large scale FE models using 100 and 200 samples. To obtain an impression of the possible accuracy with a specified number of samples several simulations were run on a medium sized model with about 120 random variables and 36000 elements. The accuracy is defined as the fraction between the sample mean value $\bar{y}$ and the population mean value μ_y or between the sample standard deviation s_y and the population standard deviation σ_y. The mean value converge within 50 observations, the standard deviation needs more than 150 for an accuracy of 95%. An accuracy of 99% is reached with 250 observations.

By contrast the shape of the pdf has a low accuracy (see Fig. 2). With only 200 to 300 observations evaluations based on the pdf or cdf are only rough estimations. Figure 3 shows the correlation defined in equation 6 of 12 randomly selected input variables with one output variable. It can be seen that at least 50 to 100 observations are needed for a rough estimation of the correlation. Simulations with 100 observations show that the correlation can vary about 0.2. About 200 to 400 samples seem to be enough for the calculation of the correlation structure. This corresponds to equation 7.

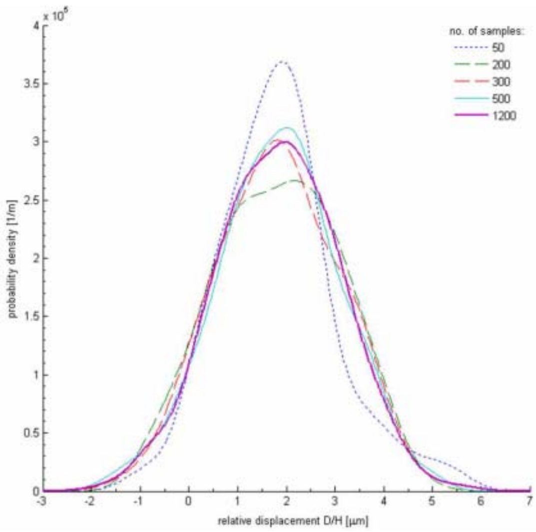

Figure 2. Variation in the shape of the pdf with increasing number of observations.

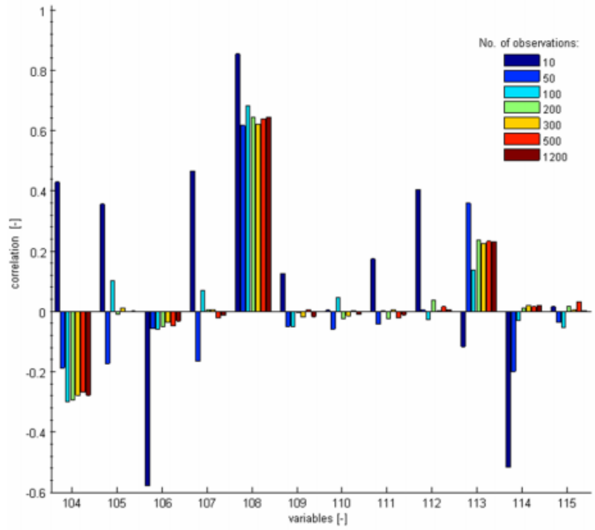

Figure 3. Variation in the shape of the pdf with increasing number of observations.

Taking the examples presented by Koch [14] in account, it seems reasonable to use 50 to 100 samples for a rough estimation of the correlation structure. Equation 1 should be fulfilled to obtain a good correlation structure. For large scale models, an upper bound of 400 samples seems reasonable but should be verified.

3.5. Computational effort

With modern computation cluster and the intrinsic parallelism of the method the computation time is no longer an obstacle of the stochastic simulation. Of course, as mentioned above, the computational time depends on the number of samples and thus is connected to the desired accuracy. As mentioned in the previous section even complex models can be simulated with a moderate number of samples using sampling techniques like LHT. Therefore, often the number of available solver licences is much more restrictive. Assuming one solver licence only, as a rule of thumb, the computation time for one single analysis should not exceed 5 up to 10 minutes in order to reach a reasonable number of analysed samples over night.

3.6. Simulation environment

In the scope of this work MSC.RobustDesign™ was used as stochastic simulation tool. While implementing a fast randomization process and a direct link to the solver MSC.Nastran™, MSC.RobustDesign™ can only deliver the basic statistic output and perform a correlation analysis.

Thus, in the scope of this work a Matlab™ toolbox called Robust Design Toolbox (RDT) was created. This toolbox allows a pre- and post processing of the simulation data. RDT is based on the Statistics Toolbox of Matlab™. In any case Matlab™ would be needed as an addition to MSC.RobustDesign™ for the communication with MSC.Nastran™. For FE-models with a high calculation time a computer cluster can be used. Because this cluster can not be accessed by MSC.AnalysisManager™, a solution was designed using Matlab™.

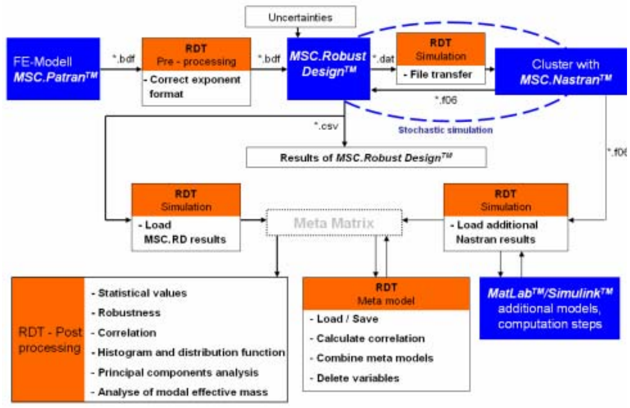

Figure 4. Flowchart of the Robust Design Toolbox (RDT) with its four parts: pre-processing, simulation, meta model and post processing.

The systematic is presented in figure 4. The RDT is wrapped around MSC.RobustDesign™, MSC.Nastran™ and the meta model. The toolbox is divided into four parts: the pre-processing, the simulation, the meta model and the post processing. Due to a restriction in MSC.RobustDesign™ only 255 random variables can be exported and used within the toolbox. Nevertheless additional variables can be directly loaded from Nastran Output files. Furthermore other calculations tools or algorithms can be easily included with Matlab™ interfaces.

3.7. Assessment of robustness

The evaluation of the statistical results leads to mean value and standard deviation of the performance parameter, quantifying the influence of the uncertainty on the system performance. Thus the degree of conservatism compared to a fixed requirement can be estimated.

Additionally, the results can be used to estimate the robustness. Robustness is understood as a measure of uncertainty. Uncertainties introduce a variability Δx into the system. The result is the scatter in the output values Δy. Thus instead of the system behaviour itself, the aim is to characterise the system behaviour due to uncertainties. Will [25] defines criteria for the evaluation of robustness. This list is supplemented with ideas mentioned by Marczyk [1]:

- exceedance of limit values,
- sudden changes of response quantities,
- occurrence of system instabilities,
- complexity,
- shift of the mean values,
- scatter of relevant parameters.

These criteria can be grouped in two parts: scatter and shape of the response. The concept is depicted in figure 5.

First, the scatter itself and the bias of the mean values quantitatively characterize the effects of uncertainty on the system performance. A system with a low scatter of its responses has a high quality. Second, the shape or nature of the response can be used for characterization as well as to gain knowledge about the system itself. This

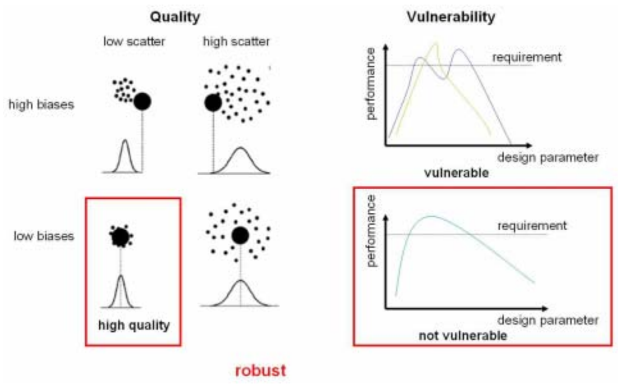

Figure 5. Definition of robustness.

is called vulnerability of the system. Vulnerability has to be understood as tendency to instability of the system performance of which complexity is one part. A high vulnerability combined with a low quality is a non-robust system. Vice-versa: a system with a high quality and low vulnerability is a robust system.

3.8. Identification of important parameter

A closer insight in the system behaviour, thus a better understanding how much every input variable actually influences the system, is possible with the correlation analysis. There are three different measures available for the identification of the importance e.g. sensitivity of an input parameter. First, the influence between two parameters can be identified with the correlation coefficient. If two variables are connected over a mean or strong correlation, than it is spoken of a link between these two variables [26]. Secondly, a further distinction can be made on the basis of the total number of links. For a better evaluation of a large finite element system a link factor is defined:

(8) $$LF = \frac{L}{L_{\max}}$$

with L denoting the number of links and L_{max} the total number of possible links.

The third measure for the importance of an input variable is the coefficient of variation. The correlation is a necessary criterion for the influence, but the magnitude is the variation. In multi-input multi-output systems both measures have to be combined. For each input variable j a vector can be defined, containing the correlations with other variables. The vector contains only correlations between input and output variables. Thus the sum of the scatter CV_k of all output variables that are linked with the input variable j can be calculated:

$$\sum_{k=i+1}^{p} |CV_k| \cdot l_{jk}^{\ 2}$$

$$with \quad l_{jk} = \begin{cases} 0 & for \quad (k \le i) \vee (j > i \wedge k > i) \\ r_{jk} & \end{cases}$$

The coefficient of variation of the parameter k is denoted CV_k. The correlation is squared to assure positive values and to decrease the influence of weak correlations. The defined measure can be normalized by the coefficient of variation of the input variable j, resulting in a factor of scatter:

(9) $$Sc_j = \frac{\sum_{k=i+1}^{p} CV_k \cdot l_{jk}^{\ 2}}{CV_j}$$

In case the parameter j is correlated with many output variables who has a high scatter the factor of scatter has a high value. Vice-versa for only a few correlations to output variables with a small coefficient of variation the factor of scatter is small.

4. APPLICATION OF THE ROBUST DESIGN APPROACH

4.1. Problem description

The example is a model provided by the Astrium GmbH. It represents a real application of a Cassegrain telescope already flying in space. The telescope consists of two mirrors, a camera and the supporting structure. The FEM of the supporting structure is shown in figure 5. The model consists of about 36 000 elements. The aim is to calculate thermal deformations caused by a 10K load. An important factor for the optical performance of the telescope is the relative displacements between the mirrors and the camera. The mirrors and the camera itself are not included in the model. Instead planes for the primary mirror (PM), secondary mirror (SM) and camera (FP) are defined; see figure 6 for their definitions. During the qualification test of the structure the dominant influence of the upper and auxiliary titanium rings on the thermal deformation was exceeding the predictions and an

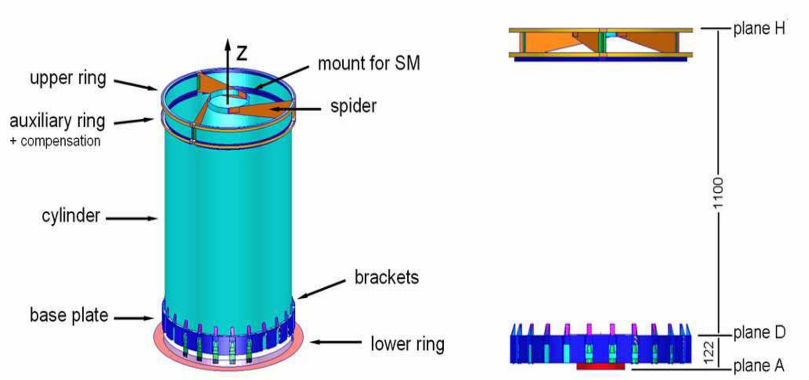

Figure 6. Structure of a Cassegrain telescope and definition of planes on the position of the primary mirror (D), secondary mirror (H) and camera (A).

Design	Measurement		FEM	
	D/H [μm]	A/H [μm]	D/H [μm]	A/H [μm]
without compensation	-9.4	-10.9	-5.9	-5
with compensation	0.8	-2.5	1.8	3.1

Table 2. Results of the deterministic analysis and the measurements during the qualification test.

additional compensation was added to the auxiliary ring. It is the aim of this simulation to show that this influence could have been better predicted with a stochastic simulation.

The results of the deterministic analysis and the measurements are summarized in table 2. Two simulations with 100 observations each are performed: one without and one with the compensation. The computation on the Linux cluster took about 3h for one simulation with one license.

4.2. Statistical results

The results of the deterministic analysis and the measurements are summarized in fig. 7. D/H denotes the displacement between the primary and secondary mirror. For simplification only the random variables for the Baseplate are selected and thus only the absolute displacement of plane H varies. The uncertainty inherent in the original structure without the compensation (green) causes the displacement D/H to vary of about 12µm. The standard deviation is 1.7µm. With a low probability of about 3% the requirement can be violated. The comparison with the measurements in table 2 shows that the structure does not correspond to the nominal case. On the other side the measurements are still in the range predicted by the simulation. This demonstrates that the simulation is a better representation of the system behaviour than the analysis.

Adding the compensation layer shifts the relative displacement. This demonstrates the influence of the auxiliary ring. The standard deviation is reduced to 1.1µm which means a reduction of 35%. Thus the simulation demonstrates that the requirements are fulfilled even under the influence of the uncertainty affecting the system. This statement is proved by the test results shown in table 2.

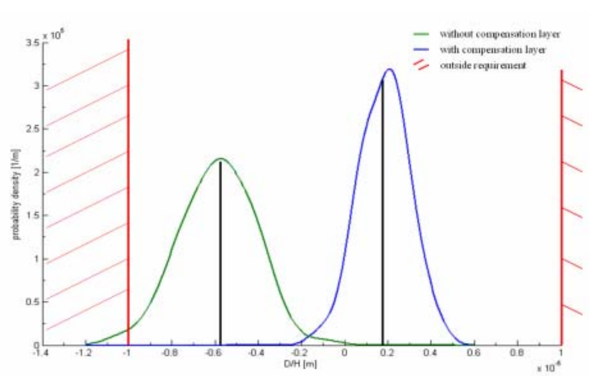

Figure 7. Probability density function of the relative displacement D/H for both designs with and without compensation layer, black line indicates nominal value.

It has to be remarked, that the difference between the analysis and test results found in this example may also lay in the exactitude of the FE model. The stochastic simulation can only represent the physical effects included in the model. If the model contains none parameterised simplifications which lead to a bias, this bias will also be included in the stochastic simulation. Thus a certain part of the difference between FEM and measurement can also be caused by remaining idealizations i.e. imperfectly simulated parts and junctions.

4.3. Important variables and robustness

Applying the definitions of the factor of scatter from equation 9 and the link factor from equation 8 a broad view of the influences can be obtained (see table 3). There are some parameters with a high link factor and a low factor of scatter or reversed. This visualizes the differences between the correlation, which is represented by the link factor and the combination of correlation and variation represented by the factor of scatter. The main parameters are those with a high link factor and a high factor of scatter.

A look at figure 8 reveals that the correlation of the displacement D/H lead to the same result. Nevertheless, the advantage of the presented approach is that the influence on all output variables is taken into account. The reason that both approaches show the same results is that the variation of the mass and other relative displacements are very small. Thus they have no significant influence. Additionally both designs underlie the same uncertainty, thus the absolute results are comparable. The presented approach using L/Lmax and Sc do not has these restrictions.

No	Name	Sc_{CV}	L_{out}/L_{max}
1	Cylinder aluminium - CTE	4.95	80%
2	Upper/Auxiliary ring - CTE	4.61	60%
3	Brackets - thickness	2.86	0%
4	Cylinder CFK - CTE_x	1.84	60%
5	Cylinder CFK - CTE_y	1.58	40%
6	Cylinder CFK - E_y	1.23	100%

Table 3. Comparison of different global sensitivity measures for the telescope.

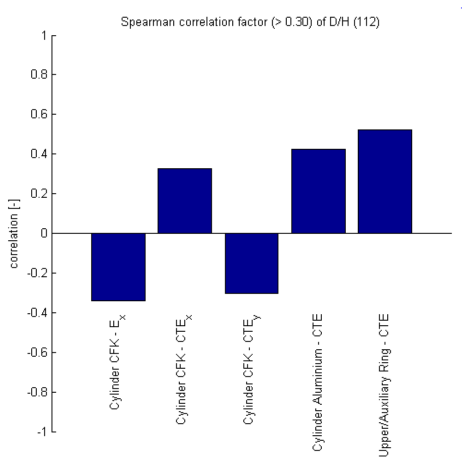

Figure 8. Correlations for the relative displacement D/H of the telescope without compensation layer.

As expected the stiffness and coefficient of thermal expansion (CTE) of the cylinder have an important influence. The cylinder consists of the composite material CFK and aluminium. Nevertheless also the CTE of the upper and auxiliary ring has a high factor of scatter and a high link factor thus it is a dominating parameter. With an increase of the CTE the relative displacement can be reduced. The spider act as a lever for the bracket of the secondary mirror. Therefore an increase in the CTE reduces the deformation. The same result can be obtained by increasing the stiffness of the rings as has been done by introducing the compensation layer.

The evaluation of robustness can be done by the analysis of a measure of quality. Due to the shift about the mean value zero the CV is sensitive to the mean value and can not be applied. Therefore other measures like the standard deviation have to be used. The standard deviation of the first design (without compensation layer) is 1.7µm. With the introduction of the compensation layer the standard deviation is reduced to 1.1µm and thus the robustness of the system is increased. This information cannot be obtained by a deterministic analysis. By contrast, using a stochastic simulation reveals that a slight change can improve the design.

4.4. Design scan

The Monte Carlo simulation can be used to evaluate the design space. This is demonstrated for the telescope. The thickness of the compensation layer is varied in a broad interval. The relative displacement D/H is used as a figure of merit. The other random variables are the same as within the previous simulations. The results are shown in figure 9. The first design e.g. the design without the compensation layer can be found for a layer thickness of zero. The final design has a layer thickness of 1.25mm and is marked in the figure. Minimizing the displacement D/H an even better solution can be found with a layer thickness of only 1.0mm. Additionally the possible scatter of the results can be estimated from the graph. For the optimal design the relative displacement can vary between −2µm and 2µm. Apart from this scatter bandwidth, in this example the problem is linear, thus the same results could have been obtained by interpolating between the two designs. This linearity can not be guaranteed for all designs. The design scan is a simple but powerful method without restrictions and therefore applicable on complex or non-linear systems.

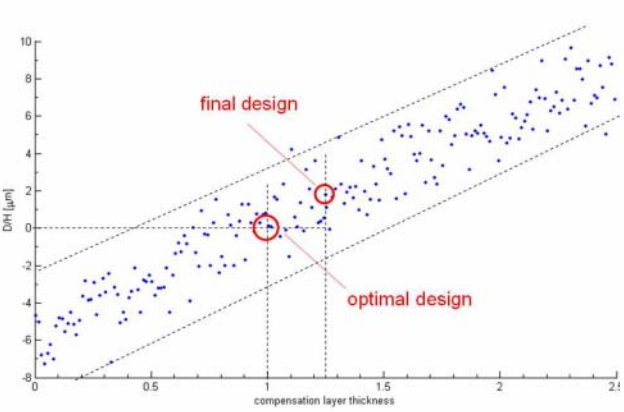

Figure 9. Design scan of the telescope for the thickness of the compensation layer.

5. CONCLUSION

The implementation of stochastic simulation to address mechanical problems was presented in the paper. The developed simulation environment includes a process chain and the assessment of question for the number of samples and the quantification of uncertainties are addressed. Furthermore an example illustrates the capability of the presented stochastic approach.

Three possible areas of application can be concluded. Firstly, the variation of performance parameters due to uncertainties in the design or lifetime can be calculated. Secondly, the design scan to evaluate the design space and find possible improvements. Thirdly, the sensitivity and robustness evaluation to assure that the system can handle possible uncertainties caused during the manufacturing, integration and operation.

In the scope of this work, material properties and geometric tolerances have been considered as uncertain. It might be useful to include uncertainties in connections and other boundary conditions as well as the uncertainties in load cases. A dedicated database with uncertainties for material properties or boundary conditions should be developed to increase the practicability of the method. Furthermore a user interface for the developed toolbox can improve the usability.

6. ACKNOWLEDGMENT

The author would like to thank Prof. S. Fasoulas, TU Dresden for supervising this research work. Special thanks are directed to S. Lucarelli, Astrium GmbH who supervised this work with many useful comments and recommendations. The work was funded by Astrium GmbH; therefore the author would like to thank Dr. W. Konrad.

7. REFERENCES

[1] Marczyk, J. (Ed.): Computational Stochastic Mechanics in a Meta-Computing Perspective, CIMNE, Spain, 1997.
[2] Marchante, E.: Ethodology to Account for Uncertainties, Proceedings of the European Conference on Spacecraft Structures, Materials and Mechanical Testing, No. 428 in SP, 1999, pp. 187–192.
[3] Marczyk, J.: Principles of Simulation-Based Computer-Aided Engineering, FIM Publications, Spain, 1999.
[4] Marczyk, J.: Beyond Optimization in CAE, CIMNE, Spain, 2002.
[5] Roy, N.; de Paredes, S. G.: A stochastic approach to the validation of spacecraft structural dynamic models, Proceedings of the European Conference on Spacecraft Structures and Mechanical Testing, No. 581 in SP, ESA, 2005, CDROM.
[6] Mary, S.; Rivire, E.; Bricout, J.-N.: Stochastic Analysis, To Do What? Proceedings of the European Conference on Spacecraft Structures and Mechanical Testing, No. 581 in SP, ESA, 2005, CD-ROM.
[7] Pellissetti, M.; Schuëller, G.; Pradlwarter, H.; Calvi, A.; Fransen, S.; Klein, M.: Static and Dynamic Reliability Analysis of Integral Satellite in View of low Failure Probability, Proceedings of the European

Conference on Spacecraft Structures and Mechanical Testing, No. 581 in SP, ESA, 2005, CD-ROM.
[8] Doltsinis, I. (Ed.): Stochastic Analysis of Multivariate Systems in Computational Mechanics and Engineering. CIMNE, Spain, 1999
[9] Alvin, K.F.; Oberkampf, W.L.; Diegert, K.V.; Rutherford, B.M.: Uncertainty quantification in computational structural dynamics: A new paradigm for model validation. In: Proceedings of the International Modal Analysis Conference - IMAX, 1998, S. 1191–1197
[10] Eiermann, M.; Ernst, O.G.; Ullmann, E.: Computational aspects of the stochastic finite element method. In: Proceedings of Algoritmy, 2005, S. 1–10
[11] Kang, Z.: Robust Design Optimization of Structures under Uncertainties, Universität Stuttgart, Fakultät Luft- und Raumfahrttechnik und Geodäsie, Diss., June 2005
[12] Florian, A.: An efficient sampling scheme: Updated Latin Hypercube Sampling. In: Probabilistic Engineering Mechanics 7 (1992), S. 123–130
[13] Steilein, T.: Hillmann, J.: Stochastische Simulation und Optimierung am Beispiel des VM Phaeton. In: VDI-Tagung Würzburg, 2002
[14] Koch, A.: Study and Employment of Stochastic Methods for the Design of Space Structures, TU Ilmenau, Fakultät für Maschinenbau, Diplomarbeit, April 2006
[15] Klein, M.; Schuëller, G.I.; Deymarie, P.; et.al.: Probabilistic approach to structural factors of safety. In: Proceedings of the European Conference on Spacecraft Structures and Mechanical Testing CNES, Cépaduès-Editons, 1994, S. 679–693
[16] Vidal, C.; Mollerat, T. de; Klein, M.: Evaluation of Tests and Design Factors. In: Burke, W.R. (Ed.): Spacecraft Structures and Mechanical Testing ESA, 1989 (SP 289), S. 3–14
[17] Kammer, C.: Aluminiumtaschenbuch. Aluminium-Verlag, 1998
[18] Simmchen, E.: Skript Vorlesung Luft- und Raumfahrtwerkstoffe. : TU Dresden, Fakultät Maschinenwesen, 2005
[19] Boller, C.; Seeger, T.: Materials Data for Cyclic Loading. Elsevier, 1987
[20] Bäumel, A.; Seeger, T.: Materials Data for Cyclic Loading, Supplement 1. Elsevier, 1990
[21] Automation Creations Inc: MatWeb. http://www.matweb.com, last checked: 19.06.2006. –- online database of material properties.
[22] Beitz, W. ; Grote, K.-H.: Dubbel Taschenbuch für den Maschinenbau. Springer, 2001
[23] ESA (Ed.): Structural Materials Handbook, Vol. 1 Polymer Composites. : ESA, (PSS 03-203)
[24] Will, J.; Möller, J.-St.; Bauer, E.: Robustheitsbewertung des Fahrkomfortverhaltens an Gesamtfahrzeugmodellen mittels stochastischer Analyse. VDI, 2004 (VDI Berichte 1846)
[25] Will, J.; Roos, D.; Riedel, J.; Bucher, C.: Robustness Analysis in Stochastic Structural Mechanics. In: NAFEMS Seminar: Use of Stochastics in FEM Analyses, 2003
[26] Allen, G.: Complexity Management for Decision-Making. In: Proceedings International Conference on Intelligence Analysis, 2005