STOCHASTIC SIMULATION FOR THE ROBUST DESIGN OF SPACE OPTICAL INSTRUMENTS

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OVERVIEW

In the recent years the requirements on the maximum allowed deformation of optical instruments have continuously been tightened, from mm to a μm or even nm scale. Thus the well known FEM Tools for structural analysis have to be improved. A more detailed numerical model (meaning an increase in terms of degrees of freedom) - will increase the level of realism. However an increase in the degrees of freedom only reduces the discretisation error. An improvement of the numerical model can only be reached by the integration of uncertainties in structural analysis, i.e., including the lack of knowledge or natural scattering in, e.g., material properties, load cases, manufacturing & integration processes.

This article presents the efforts made by Astrium GmbH to introduce a stochastic simulation process for the structural design of optical instruments. The paper presents the stochastic simulation and two examples of the successful application in the recent years.

1. INTRODUCTION

Space structures are designed to be compatible with large variations in the design parameters, e.g., deviations in the environment as well as scattering in material properties, manufacturing or integration tolerances. Since the consideration of such uncertainties during the whole design process is unfeasible, up to know, a common engineering approach is to reduce the problem to a deterministic level by defining “worst case” scenarios that cover all (or most) variations of the design parameters.

While well known and commonly used, this method has three major drawbacks. First, the worst case has to be identified, which especially in complex systems is not straight forward. Second, once the optimum design for a particular worst case has been identified, such design represents “the best” design merely for the fixed conditions of the made assumptions. In the most fortunate cases, “the best” design is at least over-engineered; if “the worst” case was not properly chosen, the design could even show insufficient performance. Which leads to the third drawback; the effect of the uncertainty introduced into the system as well as the degree of conservatism is not quantifiable.

With the request for further increased resolutions of space telescopes also the requirements for pointing accuracy have increased in the recent years. While Hubble had a pointing accuracy of 0.007 arc-sec, the requirements for the next generation telescope, the James Webb Space Telescope (JWST) will even be further increased. Increasing requirements also mean increasing accuracy in the analyses during the design phase.

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The developments in the computer technologies, which are leading to increasing computational power, provide the ground for a better modelling and analysis capability. On contrary to the well known deterministic approach, non-deterministic based methods can handle uncertainties and variability in a rational way. Since 1990’s several different methods and approaches had been proposed and evaluated by the scientific community [1, 2]. The most popular approach known as Monte Carlo Simulation, also referred to as stochastic simulation [3], was chosen by Astrium because of its easy implementation with the existing infrastructure. Uncertainties are expressed as random variables and following their distribution many realizations of the problem are generated. Each realization is leading to a deterministic problem which can be solved by well developed deterministic FE code. The solutions can be post processed to obtain statistical results like the mean value, standard deviation, correlation, etc. Thus, instead of taking a single value as sufficient for the representation of the system behaviour, a greater number of values, each associated with a certain probability of occurrence, are calculated. The result is a more realistic representation of the physical system with additional information for the decision maker. Furthermore, this approach enables the selection of a robust design instead of the optimum design.

This article presents the experience gained by Astrium in the field of optical instruments. In the first part the stochastic simulation is shortly explained. In the second part two examples are presented.

2. STOCHASTIC SIMULATION

2.1. Simulation

The Monte Carlo simulation is a method to solve a probabilistic problem with an indirect approach. First the uncertain variables itself have to be selected and a random distribution has to be defined. This first step is part of the pre-processing. Then from the specified input

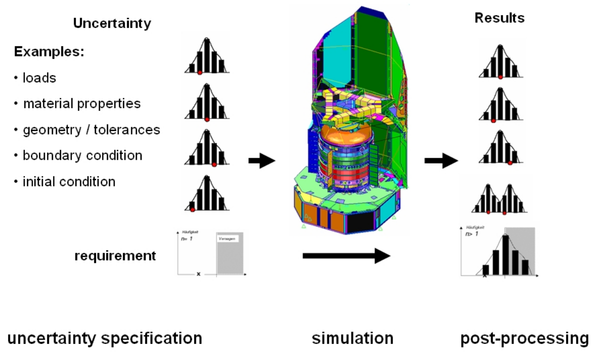

Figure 1. Stochastic simulation process.

distribution, samples are generated. This step is the first part of the Monte Carlo simulation (MCS) and is called sampling. Each input sample is a deterministic realisation of the problem, which is solved independently. This has the big advantage of enabling parallel processing. Each deterministic analysis results in one system response. All of these outputs are collected to form the so called meta model. This meta model is the result of the stochastic simulation and can be used for a statistical analysis.

The whole simulation process includes additional steps to verify, e.g. the suitability (or stability) of the model for the stochastic simulation. For the application in the engineering process a stochastic simulation consists of four parts: the model health check (MHC), the randomization, the stochastic simulation itself and the evaluation of the results [4, 5]. This process is depicted in Fig. 1.

2.2. Computational effort

The number of samples should be chosen as a balance between the desired accuracy of the stochastic results and the required computational effort.

A reduction of the sample size can be obtained by implementing a criterion to optimise the filling of the input space with samples. One example is the Latin Hypercube Technique or its derivate the Updated Latin Hypercube Technique [5]. The input space is divided into subsets of equal probability. In one population a subset of each random variable is combined with all the other variables only once.

Will [7] suggests a sample size of:

(1) number of samples = 2 (input + output).

Mary [8] and Marchante [9] present stochastic simulations of large scale FE models using 100 and 200 samples. These assumptions are also verified with the in house experience. It seems reasonable to use 50 to 100 samples for a rough estimation of the correlation structure. Equation 1 should be fulfilled to obtain a good correlation structure. For large scale models, an upper bound of 400 samples seems reasonable but should be verified.

With modern computation cluster the computation time is no longer an obstacle of the stochastic simulation. Often the number of available solver licences is much more restrictive. Assuming one solver licence only, the computation time for one single analysis should not exceed 5 up to 10 minutes in order to reach a reasonable number of analysed samples over night.

2.3. Statistical Results

The evaluation of the statistical results leads to mean value and standard deviation of the performance parameter, quantifying the influence of the uncertainty on the system performance. Thus the degree of conservatism compared to a fixed requirement can be estimated. The arising dispersion of the performance parameter can be described mathematically by its distribution function. As the determination of the statistical parameters results from a finite number of samples, the results are not determinable definitely. This error can get assessed by the means of statistics by calculating confidence bounds for the mean value and the standard deviation. The assumed distribution function gets tested by the χ^2 adaptation test. This procedure corresponds to the standard methodology of a statistical analysis and is well investigated and documented for example in Hartung [11].

Additionally, the results can be used to estimate the robustness. Robustness is understood as a measure of uncertainty. Uncertainties introduce a variability Δx into the inputs variables of the system. The result is the scatter in the output values Δy. Thus instead of the system behaviour itself, the aim is to characterise the system behaviour due to uncertainties. Will [10] defines criteria for the evaluation of robustness. This list is supplemented with ideas mentioned by Marczyk [2]:

- exceedance of limit values,
- sudden changes of response quantities,
- occurrence of system instabilities,
- complexity,
- shift of the mean values,
- scatter of relevant parameters.

These criteria can be grouped in two parts: scatter and shape of the response [5].

First, the scatter itself and the bias of the mean values quantitatively characterize the effects of uncertainty on the system performance. A system with a low scatter of its responses has a high quality. Second, the shape or nature of the response can be used for characterization as well as to gain knowledge about the system itself. This is called vulnerability of the system. Vulnerability has to be understood as tendency to instability of the system performance of which complexity is one part. A high vulnerability combined with a low quality is a non-robust system. Vice-versa: a system with a high quality and low vulnerability is a robust system.

A closer insight in the system behaviour, thus a better understanding how much every input variable actually influences the system, is possible with the correlations and the regressions analysis.

The covariance can be a measure of the independence of two variables. The percentage of the calculated covariance on the maximal possible product results in the correlations coefficient:

(2) $$r_{xy} = \frac{\mathrm{cov}_{xy}}{\sigma_x \sigma_y}$$

If the relation between X and Y is totally linear the covariance has exactly the size of the product of the standard deviation ($|r_{xy}| = 1$). The covariance can also have a negative value which points at a negative linear relation than r_{xy} is less than zero. If the covariance is much less than the product of the standard deviations the relation between X and Y is either nonlinear or not existent ($r_{xy}=0$).

If the two predictors are correlated with the response variable, but also correlated with themselves, the interference between them needs to be eliminated. If r_{yx1} and r_{yx2} are correlations coefficients of the predictors x_1 and y_1 with the response variable y and r_{x1x2} the correlation between them, the resulting correlation of x_1 and x_2 with y is:

(3) $$r_{Y(x1,x2)} = \sqrt{\frac{r_{yx1}^2 + r_{yx2}^2 - 2r_{yx1} \cdot r_{x1x2} r_{yx2}}{1 - r_{x1x2}^2}}$$

While the correlation analysis determines a quantitative degree of a relation, with the regressions analysis it is possible to find a functional context between the predictors (input variables) and the response variable.

If the relation between Y, X_1, X_2,…,X_n is investigated, it is possible to assume the following regression function:

(4) $$y_i(x_{1i},\ldots,x_{ki}) = \alpha + \beta_1 \cdot x_{1i} + \beta_2 \cdot x_{2i} + \ldots + \beta_k \cdot x_{ki} + \varepsilon_i$$

Further details about the multiple regression and multivariate statistics in general can be found in various textbooks, like Hartung [11].

2.4. Software Tools

The Monte Carlo Simulation uses deterministic simulations, thus the existing solvers can be used and the described simulation process can be easily integrated in the existing simulation infrastructure.

For the stochastic simulation several commercial tools and tool packages like iSight™, ST-ORM™ or RobustDesign™ are available. RobustDesign™ by MSC was chosen because it is special designed to interact with MSC.Patran™ / MSC.Nastran™. To increase the flexibility of the simulation environment and to include and add additional calculations a Matlab™ toolbox was created as depicted in Fig. 2. The so called Robust Design Toolbox (RDT) is used to link the different FEM Tools and for the result evaluation.

The finite element model created by MSC.Patran™ is handed to the RDT for pre-processing. This step is necessary because of a format incompatibility between MSC.Patran™ and MSC.RobustDesign™.

MSC.RobustDesign™ is used to specify the uncertainty of

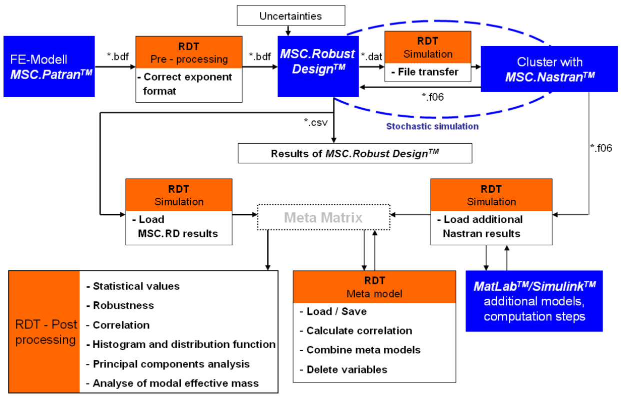

Figure 2. Flowchart of the Robust Design Toolbox (RDT) developed at Astrium.

the random variables and to generate the samples. MSC.RobustDesign™ can export the samples directly to MSC.Nastran™ input files which are passed back to the simulation part of the RDT.

In the next step all available results have to be collected. Therefore the results available in MSC.RobustDesign™ are exported to a *.csv file. The RD Toolbox is able to load this file and create a preliminary meta model. Additional output data can be added by directly accessing the MSC.Nastran™ results with the aid of the RDT.

In the final step the meta model can be analyzed using different tools available within the RDT:

- creation of excel worksheet with statistical values of all parameters,
- histogram, pdf, cdf, scatter plots,
- correlation map, correlation plots,
- principal component analysis,
- robustness evaluation,
- specific output for the modal effective mess.

Additionally, using the Matlab™ environment, the post processing can be arranged to the specific needs of the problem actually solved.

3. CARTOSAT-2 CAMERA STRUCTURE

The first step was the application of the proposed stochastic simulation to a well known problem, with the objective to demonstrate the advantages. To this end the camera structure of CARTOSAT-2 was chosen. During the qualification test of the structure the dominant influence of the upper and auxiliary titanium rings on the thermal deformation was exceeding the predictions and an additional compensation was added to the auxiliary ring. It is the aim of this simulation to show that this influence could have been discovered with a stochastic simulation.

The telescope consists of two mirrors, a camera and the supporting structure. It is 1 273mm high and has a total mass of 46kg. The cylinder has a diameter of 729mm. The model consists of about 36 000 elements. The aim is to calculate thermal deformations caused by a 10K load. An important factor for the optical performance of the telescope is the relative displacements between the mirrors and the camera. The mirrors and the camera itself are not included in the model. Instead planes for the

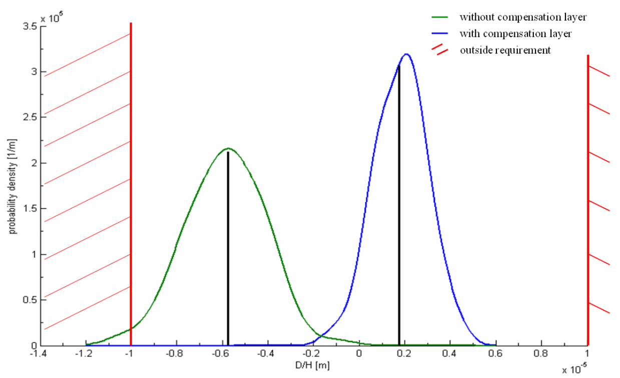

Figure 3. Probability density function of the relative displacement D/H of Cartosat 2 camera structure for both designs with and without compensation layer. The black line indicates the results of the deterministic analysis.

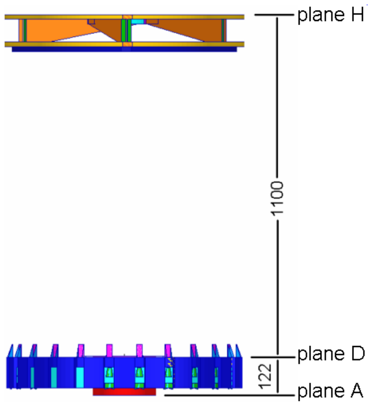

Figure 4. Definition of planes for the Cartosat 2 camera structure: Position of the primary mirror (D), secondary mirror (H) and camera (A).

primary mirror (PM), secondary mirror (SM) and camera (FP) are defined. The geometric tolerances and uncertainties in the material properties where randomized with conservative assumptions of uniform and normal distributions.

Two simulations with 100 observations each (samples) are performed: one without and one with the compensation. The computation on the Astrium Linux cluster takes less than 2 minutes per sample, i.e., about 3h for one stochastic simulation with one license.

The results of the deterministic analysis and the measurements are summarized in Fig. 3. D/H denotes the displacement between the primary and secondary mirror (see Fig. 4). For simplification only the random variables for the primary mirror base plate are selected and thus only the absolute displacement of plane H varies. The uncertainty inherent in the original structure without the compensation (green) causes the displacement D/H to vary of about 12µm. The standard deviation is 1.7µm. With a low probability of about 3% the requirement can be violated. The comparison with the measurements shows that the structure does not correspond to the nominal case. On the other side the measurements are still in the range predicted by the simulation. This demonstrates that the simulation is a better representation of the system behaviour than the analysis [5].

Adding the compensation layer shifts the relative displacement. This demonstrates the influence of the auxiliary ring. The standard deviation is reduced to 1.1µm which means a reduction of 35%. Thus the simulation demonstrates that the requirements are fulfilled even under the influence of the uncertainty affecting the system.

It has to be remarked that the difference between the analysis and test results found in this example may also lay in the exactitude of the FE model. The stochastic simulation can only represent the physical effects included in the model. If the model contains simplifications which lead to a bias, this bias will also be included in the stochastic simulation. Thus the difference can also be caused by remaining idealizations, e.g., imperfectly simulated parts and junctions.

4. JAMES WEBB SPACE TELESCOPE

In the frame of the James Webb Space Programme, Astrium GmbH develops the optical bench for the NIRSpec (Near Infrared Spectrograph) instrument. The stochastic simulation shall is applied in parallel to the conventional design activities. The intention was to prove the practicability and relevance of the method for the design process [12]. The investigated example is one of the larger mirrors (300x200mm) of the optical bench which is susceptible to critical deformations of the optical surface.

The physical displacements and deformations of the mirror surface have a direct interference with the optical performance of the mirror surface [13]. The deformed surface will reflect the light in a different way (see Fig. 5). It is assumed that the mirror bundles the light in the focus F_{ideal}. In the lower part the index of refraction is too strong. The wave front is bended too strong, and the beam of light moves to the point F_1. In the upper part the index of refraction is too weak and the light is bended less and the ray moves to point F_2. The optical retardation between the ideal wave front (gray solid line) and the actual wave front

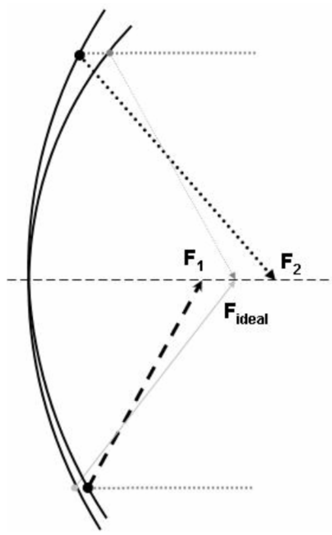

Figure 5. Definition of the wave front error, as optical retardation between the ideal wave front (gray solid line) and the actual wave front.

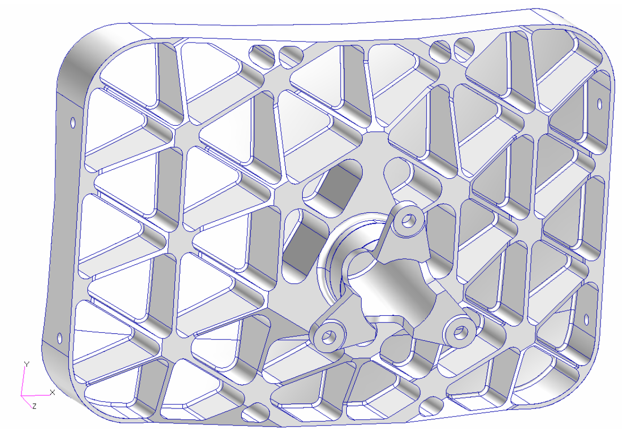

Figure 6. Backside view on the investigated mirror of the NIRSPEC optical bench for the James Webb Space Telescope.

is measured in nm and is called the wave front error (WFE). In the lower part of the mirror the wave precedes (dashed line), while the wave in the upper part is retarded (dotted line). For the calculation of the optical analysis the commercial software application Sigfit^{TM} was implemented into the routine for the stochastic simulation.

The relevant loads are mainly represented by a cool down to the operational temperature (40 K), gravity release as well as forces and moments caused by the manufacturing or integration tolerances. The stochastic simulation had to determine the probability that the budgets for the critical deformations will be not exceeded.

The FE model is derived from the CAD model in Fig. 6. To improve the performance of the calculation the number of nodes was limited to 10000. The time for the calculation for 4 load cases and a sample number of 200 was about 3 hours on the Astrium Linux cluster. The optical and statistical analysis caused additionally 30 minutes.

4.1. Deriving a distribution function for the performance criteria

Deformations caused by the cool down

The cool down causes a bending effect on the mirror surface due to different coefficients of thermal expansion of the material of the main structure and the materials which form the coating layer on the surface. The conventional design assumes largely equal CTE`s. The question was now how sensible the deformation will change if the CTE values vary. Another important uncertainty was the hardly predictable thickness of the coating after the chemical vaporization and the further mechanical processing.

The stochastic simulation, including the procedure for the statistical analysis, produces the distribution function of the performance criterion and the probability of remaining within the budget (see Fig. 7). For the investigated load case the probability of exceeding the budget is determined and the result is available for further steps. If the result is not satisfying e.g. if the probability of failure is too high, the statistical analysis may also produce the relations causing the dispersion to improve the design, see 4.4.

Deformation caused by the mounting distortion

Another interesting load case in relation with the stochastic approach was the deformation caused by the mounting of the mirror on the support structure. The pedestal of the mirror is mounted by the three flanges. The fabrication tolerance of the contact zones cause initial displacements and rotations in each flange (three degrees of freedom each), which in turn deform the surface of the mirror. Each contact zone consists of two opposite surfaces which have different angles and a different level to each other. The worst case would occur if the nine degrees of freedom have dedicated combinations of minimum and maximum values. It was a substantial effort to even estimate the worst case with conventional means. This problem can be avoided by a stochastic approach. The result is a distribution function for the investigated performance criterion, see Fig. 8. For the investigated load it was shown despite of very conservative assumptions that the arising deformation is far below the budget with a high probability [12].

4.2. Investigation of superposed loads

A conventional analysis allows the superposition of e.g. deformations caused by different loads only for static assumptions. In reality the occurrence of a load (e.g. a

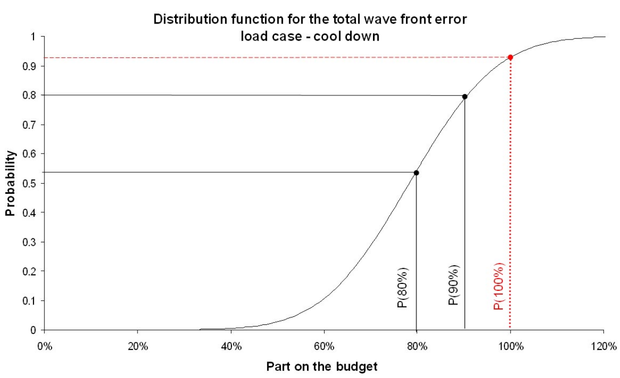

Figure 7. Probability distribution of the expected deformation in percentage of the budget due bending effect caused by the cool down.

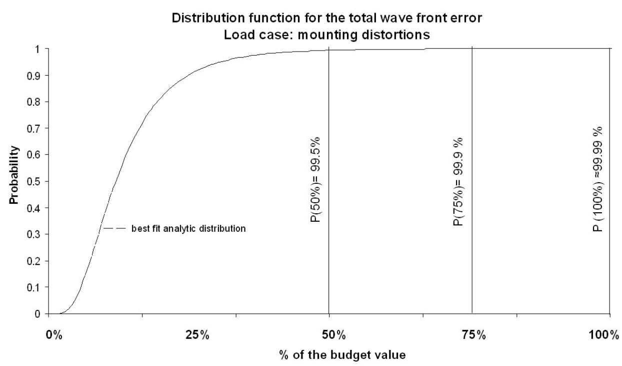

Figure 8. Probability distribution of the expected deformation in percentage of the budget due to mounting distortion caused by fabrication tolerances.

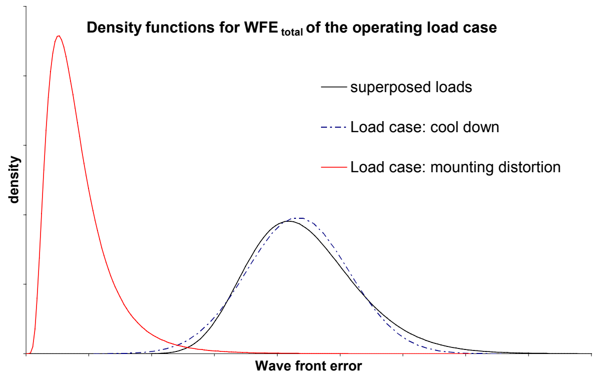

Figure 9. Density function for the single load cases and the superposed load, directly calculated with the FEM.

particular mounting distortion) and the effect (e.g. a particular CTE ratio) might depend on accidence as shown in the paragraphs above. The question is how the loads will statistically combine if all relevant load cases get superposed. In this example the superposition could be done by taking all possible combinations of possible deformations of the optical surface into account. The stochastic approach delivers an easier solution for this problem. So it becomes possible to calculate a performance criterion with the inclusion of the uncertainties of actually different load cases. The alternative would be the sum up of the results of every single load case with an approximation. Figure 8 shows the results for the superposition of the two load cases introduced above directly calculated with the stochastic simulation [13]. The functions demonstrated are the density functions, which arises of the differentiated distribution function.

$$(5) \qquad f(x) = \frac{dF(x)}{dx} \qquad F(x) = \int_{-\infty}^{\infty} f(x)dx$$

Fig.9 shows the dominance of the deformation caused by the thermal load, which is a fundamental piece of information for a further optimization.

In general the resulting density function for the superposed load case allows an assessment of the total performance of the design. This knowledge is an important basis of decision for the optimization of the design without the disadvantage of having to consider every load case apart.

4.3. Determining the influence of critical parameters

An arising problem in the application of a stochastic simulation is the often unknown dispersion of e.g. material parameters. It was proved that it is not necessary to know the distribution for every parameter because some of them have a low resulting influence. The application of the regressions analysis delivers a formula for the performance criterion which contains all gradients of the influencing parameters. With that formula it is possible to discuss the performance in dependence to a critical parameter.

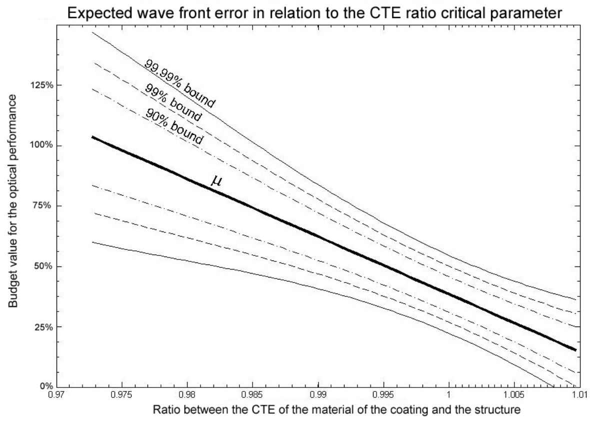

Figure 10. Probability of fulfilling the budget for the optical performance in dependence to the CTE ratio of two materials.

Figure 10 shows the expected optical performance of the mirror if all loads arise simultaneously in dependence to the ratio of the coefficients of thermal expansion of the material of the actual structure and the material of a coating layer. This ratio is the leading reason for the thermal deformation as derived in 4.2. In the vertical direction one can read the dispersion of all other uncertain parameters together for a particular value of the critical parameter. The intention of that demonstration was to determine how far a more precise determination of the material parameter would increase the expected performance [12].

4.4. The contribution to the optimization of the design

The structure of the stochastic simulation allows an optimization without the restrictions of every single analysis tool. For the statistic analysis the values for the input variables and the observables are available as a meta model. Therefore the optimization can be performed without taking the intermediate analysis steps into account. For example the geometrical input variables implemented in MSC.Patran^{TM} are directly comparable with the results of the optical analysis in Sigfit.

The stochastic simulation produces not only the dispersion of a performance criterion, but also the influence of each parameter. This influence is represented by its gradient and its initial dispersion. The gradients are calculated by the regressions analysis and could be regarded as a by-product of the statistical evaluation.

The influence of the thickness of each rib of the stiffening on the optical performance and the mass of the mirror is demonstrated in Fig.11. The diverging effectiveness of each rib becomes comprehensible with the means of statistics. Therefore a statistic analysis includes an analysis of the gradients without any significantly increasing effort.

With the information showed in Fig 11, a manipulation of the thickness of each rib could lead either to an improvement of the optical performance or a decrease of the mass.

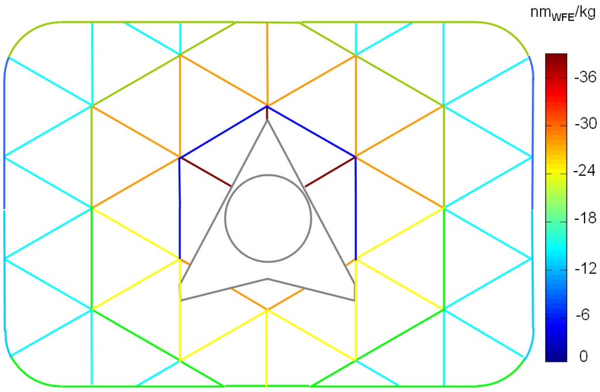

Figure 11. Demonstration of the optical performance improvement (Wave front error in nm) per mass increase for the ribs of the stiffening structure.

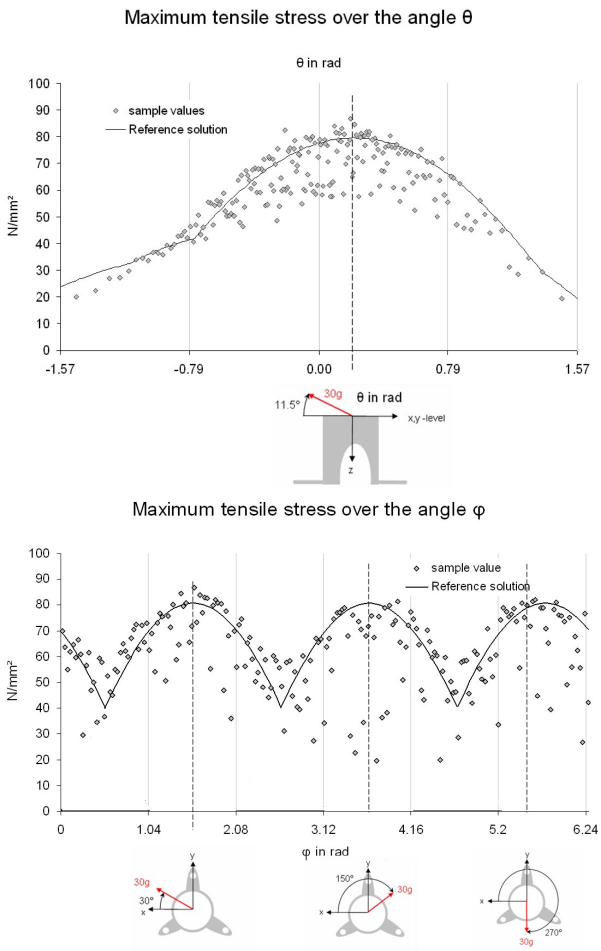

Figure 12. Distributions of the samples in dependence to the angle φ and angle θ, the continuous lines are the result of a two- dimensional reference solution.

It has to be remarked, that the initial dispersion of an input parameter must also be taken into account for a further optimization with respect to decrease the variance of the performance criterion, e.g. by limiting the influence of a very dispersing input parameter by a change of the design.

4.5. The stochastic simulation applied on an extremal problem

Another criterion for the design of this mirror is the required strength to withstand the occurring stresses during the launch which are mainly caused by a strong dynamic excitation. The evidence that the design withstands them is made if an acceleration vector with a constant absolute value in all possible directions will not cause over critical stresses. The arising question is, in which direction this acceleration will cause the greatest stress in the design. The stochastic simulation was applied to determine that. The calculation was repeated for a stochastic two dimensional acceleration vector with the intention to verify the direction which causes - in that case - the maximum tensile stress. The angle φ represents the rotation in the x,y- level, the angle θ the rotation in the y,z-level of a Cartesian coordinate system.

The resulting sample values form a structured picture which makes the run of a steady curve comprehensible, see Fig. 12. If this acceleration vector is used instead of the conventional approach, the propagated stress is about 10% lower [12].

A further consideration was made on the stresses implemented by the mounting, see above. The additional stress depends on the particular displacements and rotations of the contact zones and they are therefore stochastic. It was shown that the additional maximum stress is with 99.9% of probability only half as much as propagated with conventional means. Furthermore the material uncertainties, other geometrical tolerances as well as a varying stiffness of the mounting base were taken into account. In the conventional design they were taken into account by relative factors. It was shown that these factors do not match with the calculated dispersion. Especially the factor for the varying stiffness of the support structure was strongly overestimated.

The maximum tensile stress propagated with the stochastic simulation was after all 17% lower than with conventional means [12], besides the reliance of the propagation was increased by avoiding the inexact factorization of particular uncertainties.

5. CONCLUSION

The stochastic simulation environment developed at Astrium GmbH was presented and its applicability shown with the aid of two examples: The Cartosat 2 camera structure and the optical bench for the NIRSpec instrument.

The first example demonstrated the additional information available for engineering decision with a basic stochastic simulation even without extensive statistical evaluations. With the aid of a stochastic simulation the effects of uncertainty due to manufacture and tolerances can be predicted and the conservatism or the probability of remaining within a budget or requirement can be specified.

Also the second example demonstrated this simple and useful application for two different load cases. Additional addressed examples are the application to the combination of load cases or the determination of the influence of a known important parameter. Furthermore contributions to the structural optimisation were derived from the results of the stochastic simulation.

The stochastic simulation adds a sensible contribution to the analysis of a design. The application of proved statistic methods made the results comprehensible and reliable. Even very conservative assumptions in combination with a statistic calculation could lead to additional information. The different examples demonstrated the flexibility of the presented approach and proved the applicability of the method for a wide field of engineering problems.

The stochastic assumptions for the material parameters, the fabrication tolerances and for the probability of the arising loads can be developed further by creating dedicated databases. As proven, the computation time is no longer a difficulty of this method. Neither the theoretical methodology nor the difficulties caused by data transfer between the analysis tools represent a long-term obstacle for the regular application.

Astrium plan to integrate the stochastic simulation in the daily design process, thus reducing the gap between simulation and reality.

6. ACKNOWLEDGMENTS

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