IDENTIFICATION OF NUMERICAL MODELLING UNCERTAINTY BASED ON DYNAMIC FUZZY FINITE ELEMENT ANALYSIS

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ABSTRACT

In order to extend the applicability of numerical modelling tools in the design process, thorough validation of the used models is required. Uncertainty and variability are always to some extent present in numerical simulations. Especially in the model definition phase, lack of information or scatter in material properties or environmental conditions imply that a designer incorporates the possible effects of this non-determinism into the design procedure. This can be achieved by incorporation of non-determinism in the numerical model. This work shows how model uncertainty defined as intervals on model properties can be identified based on a limited number of measurements. The procedure is based on numerical fuzzy analysis. By defining fuzzy membership functions for the uncertain model properties, a large scale sensitivity analysis on the range of the outcome of the analysis with respect to the interval on the uncertain inputs can be performed. This output range is than compared to the actual outcome of the physical testing. From this comparison, interval bounds on the numerical model properties are derived. The methodology is capable of handling partial initial information on properties by assigning membership function that are compatible with the available information to the uncertain properties. The presented procedure focuses on dynamic frequency response function analysis. As the procedure requires an efficient implementation of the fuzzy analysis algorithm, the first part of the paper deals with a new response surface based interval FRF analysis technique. The procedure for uncertainty identification is demonstrated on a spacecraft component.

1. INTRODUCTION

In current mechanical design engineering, numerical analysis tools play an important and often decisive role in the design process. Especially in space industry, a profound numerical analysis of a new design can reduce the need for prototype testing substantially, resulting in a proportional reduction in associated costs. In order to extend the applicability of numerical modelling tools in the design process, thorough validation of the used models is required. Today's structural models are validated based on deterministic approaches which do not take into account the natural dispersion or scatter inherent to all physical mechanical assemblies. This results in limited confidence in the model which can only be partially compensated by the use of safety factors. It is therefore necessary to consider the scatter as an integral part of the model and to establish correlation and validation techniques which take this scatter into account. A reliable and robust numerical model should exhibit a high correlation with measurement data. This work contributes to the development of such high fidelity numerical models in a non-deterministic context.

An important prerequisite is the availability of non-deterministic numerical modelling tools, i.e. numerical modelling techniques that incorporate the model non-determinism, and are able to process the uncertain information to non-deterministic analysis results. Over the past decades, several methodologies are established, among which the probabilistic approach is by far the most popular. Recent developments however have clearly indicated that also the possibilistic concept can be very valuable for non-deterministic numerical analysis, as it requires less information and often is computationally more efficient than the probabilistic approaches. Two techniques are currently gaining momentum is the context of possibilistic finite element modelling:

- The Interval FE (IFE) analysis is based on the interval concept for the description of non-deterministic model properties. The aim of an interval analysis is to calculate the range of possible outcomes of a numerical analysis, given that some of the model properties are contained within uncertainty intervals (see e.g. [1, 2, 3]).
- The Fuzzy FE (FFE) analysis is basically an extension of the IFE analysis, and has been studied in a number of specific research domains, as e.g. static structural analysis (see [4, 5, 6]) and dynamic analysis (see [7, 8]).

See [9] for a more general overview of non-probabilistic uncertainty treatment in finite element analysis.

Recently, an interval finite element methodology to calculate envelope frequency response functions (FRF) of uncertain structures has been developed by the authors [10]. This procedure forms the basis for the implementation of the fuzzy finite element method. The goal of the interval analysis is to calculate the envelope of the FRF taking into account that the input uncertainties can vary within the bounded space defined by their combined intervals. For this purpose, a hybrid procedure involving both a global optimisation step and an interval arithmetic step has been developed. The resulting envelope response function gives a clear view on the possible variation of the response in the frequency domain. This paper increases the state-of-use of this approach by further development of the fuzzy

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finite element method envisaging its application in non-deterministic model validation procedures.

In section 2, the main principles of fuzzy finite element analysis are briefly discussed, focusing on both the philosophy as well as the methodology for fuzzy FRF analysis. Next, section 3 concentrates on an efficient implementation strategy for performing the dynamic finite element analysis in a fuzzy context. The paper then discusses the possible use of the fuzzy concept for interval uncertainty identification in section 4. Finally, section 5 illustrates the methodology is illustrated using a numerical example from space industry.

2. FUZZY FINITE ELEMENT METHOD FOR DYNAMIC RESPONSE ANALYSIS

2.1. The fuzzy finite element method

Fuzzy sets were introduced by Zadeh [11] in 1965. They're capable of describing linguistic and other incomplete information in a non-probabilistic way. Where classical sets clearly distinguish between members and non-members, fuzzy sets introduce a degree of membership, represented by a *membership function*. The membership function $\mu_{\tilde{x}}(x)$ describes the degree of membership of each element x in the domain X to the fuzzy set $\tilde{x}$:

$$\tilde{x} = \left\{(x, \mu_{\tilde{x}}(x)) \mid (x \in X)(\mu_{\tilde{x}} \in [0,1])\right\} \tag{1}$$

If $\mu_{\tilde{x}}(x) = 1$, x is definitely a member of $\tilde{x}$. If $\mu_{\tilde{x}}(x) = 0$, x is definitely not a member of $\tilde{x}$. In between, the membership is uncertain. The most used membership function shape is the triangular shape. Such a fuzzy number with support $[a, b]$ – the interval for which $\mu_{\tilde{x}}(x) > 0$ – and core c – the point for which $\mu_{\tilde{x}}(x) = 1$ – is denoted $(a/c/b)$.

The objective of the fuzzy finite element method is to introduce uncertainty as fuzzy numbers into the model definition, and to propagate this uncertainty to a fuzzy number describing the corresponding uncertainty on the analysis result. It is clear that the application of the fuzzy concept in a numerical modelling procedure requires a procedure for calculating the result of numerical operations on fuzzy numbers. A possible implementation of fuzzy functions is the α-level strategy. The intersection of the membership function of each input parameter with a discrete number of α-levels results in an interval $x^I_\alpha = [\underline{x}, \overline{x}]_\alpha$ for each input parameter at each α-level. Using these input intervals, an interval analysis is performed at each α-level. The fuzzy solution is finally assembled from the output intervals obtained at each α-level. Figure 1 shows this procedure for a function of two triangular parameters.

2.2. Fuzzy FRF analysis

Using the α-level procedure, it is clear that the fuzzy FE FRF analysis can be implemented as a sequence of interval FE FRF analyses. The goal of the interval FRF analysis is to calculate the bounds on the dynamic response of a structure in a specific frequency region given that a set of model parameters $\mathbf{x}$ is uncertain but bounded. The intervals on these parameters are specified in an interval vector $\mathbf{x^I}$. The methodology for the envelope dynamic response analysis as developed by the authors is based on a hybrid interval solution strategy, consisting of a preliminary optimisation step, followed by an interval arithmetic step. In the first part of this procedure, the optimisation is used to translate the interval properties defined on the finite element model to the exact interval modal stiffness and mass parameters of the structure. The calculation of the envelope FRFs in the second part is done by applying the interval arithmetic equivalent of the modal superposition procedure on these interval modal parameters. The final envelope FRFs have been proved to contain only a very limited amount of conservatism. A brief overview of the basic principles of the method is given in this section. The complete mathematical description can be found in [12].

2.2.1. The deterministic modal superposition principle

For undamped structures, the deterministic modal superposition principle states that, considering the first n_{modes} modes, the frequency response function between degrees of freedom j and k equals:

$$FRF_{jk} = \sum_{i=1}^{n_{modes}} \frac{\phi_{i_j}\phi_{i_k}}{\phi_\mathbf{i}{}^T\mathbf{K}\phi_\mathbf{i} - \omega^2\phi_\mathbf{i}{}^T\mathbf{M}\phi_\mathbf{i}} \tag{2}$$

with $\phi_\mathbf{i}$ the i^{th} eigenvector of the system and ϕ_{i_j} the j^{th} component of the i^{th} eigenvector. Simplification of equation (2) yields:

$$FRF_{jk} = \sum_{i=1}^{n_{modes}} \frac{1}{\hat{k}_i - \omega^2\hat{m}_i} \tag{3}$$

with $\hat{k}_i$ and $\hat{m}_i$ the modal parameters defined as:

$$\hat{k}_i = \frac{\phi_\mathbf{i}{}^T\mathbf{K}\phi_\mathbf{i}}{\phi_{i_j}\phi_{i_k}} = \frac{1}{\phi^K_{i_j}\phi^K_{i_k}} \tag{4}$$

$$\hat{m}_i = \frac{\phi_\mathbf{i}{}^T\mathbf{M}\phi_\mathbf{i}}{\phi_{i_j}\phi_{i_k}} = \frac{1}{\phi^M_{i_j}\phi^M_{i_k}} \tag{5}$$

with $\phi^\mathbf{K}_\mathbf{i}$ and $\phi^\mathbf{M}_\mathbf{i}$ the stiffness and mass normalised eigenvectors of the system.

2.2.2. Interval finite element FRF analysis

The modal superposition principle has been translated into an interval finite element method for FRF analysis. Figure 2 gives a graphical overview of the translation of the deterministic algorithm into an interval procedure. On the left-hand side is the deterministic algorithm as described in the previous section. On the right-hand side is the same procedure translated to an equivalent interval algorithm.

The interval method consists of the calculation of the result ranges of the sub functions appearing in the consecutive steps of the deterministic algorithm. Therefore, the deterministic algorithm is split into three sub functions. In the first step, step 1.1, the modal stiffness $\hat{k}_i$ and mass $\hat{m}_i$ are calculated for each considered mode. Step 1.2 then consists of the calculation of the modal FRF contributions FRF^i_{jk}. Step 1.1 and 1.2 have to be performed for each mode that is taken into consideration in the modal superposition. Therefore, it is referred to as the *modal part*. In step 2, the superposition is performed by a summation of the modal FRF contributions.

The interval procedure follows the same outline as the deterministic algorithm. Each step now concentrates on the derivation of the range of the sub functions in the deterministic algorithm:

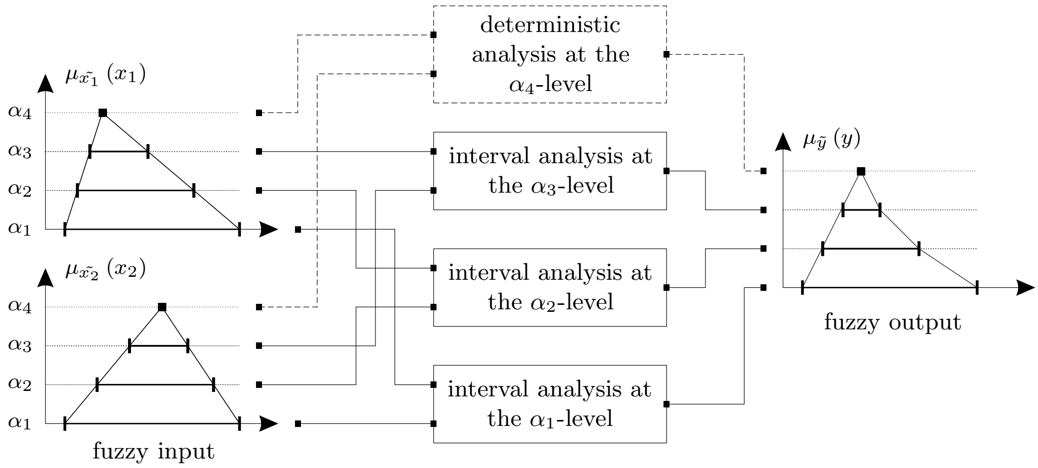

Figure 1. α-level strategy for a function of two triangular fuzzy parameters

deterministic algorithm	interval algorithm

modal part

step 1.1 (deterministic)

$$\hat{k}_i = \frac{\phi_{\mathbf{i}}{}^T \mathbf{K} \phi_{\mathbf{i}}}{\phi_{i_j}\ \phi_{i_k}} = \frac{1}{\phi_{i_j}^K \phi_{i_k}^K}$$

$$\hat{m}_i = \frac{\phi_{\mathbf{i}}{}^T \mathbf{M} \phi_{\mathbf{i}}}{\phi_{i_j}\ \phi_{i_k}} = \frac{1}{\phi_{i_j}^M \phi_{i_k}^M}$$

step 1.1 (interval) — *optimalisation*

$$\hat{k}_i^S = \left[\min_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^K \phi_{i_k}^K \right), \max_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^K \phi_{i_k}^K \right) \right]^{-1}$$

$$\hat{m}_i^S = \left[\min_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^M \phi_{i_k}^M \right), \max_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^M \phi_{i_k}^M \right) \right]^{-1}$$

step 1.2 (deterministic)

$$FRF_{jk}^i = \frac{1}{\hat{k}_i - \omega^2 \hat{m}_i}$$

step 1.2 (interval) — *interval arithmetic*

$$\left(FRF_{jk}^i \right)^I = \frac{1}{\hat{k}_i^S - \omega^2 \hat{m}_i^S}$$

summation

step 2 (deterministic)

$$FRF_{jk} = \sum_{i=1}^{n} FRF_{jk}^i$$

step 2 (interval) — *interval arithmetic*

$$FRF_{jk}^I = \sum_{i=1}^{n} \left(FRF_{jk}^i \right)^I$$

Figure 2. Translation of the deterministic modal superposition algorithm to an equivalent interval procedure

step 1.1 For all n_{modes} taken into account, the ranges of possible values that the modal stiffness and mass can adopt have to be determined, taking into account that the uncertain parameters in $\mathbf{x}$ can vary within their respective intervals. These correct ranges of the modal parameters denoted by $\hat{k}_i^S$ and $\hat{m}_i^S$ are determined using a minimisation and maximisation over the uncertain interval space $\mathbf{x^I}$. For numerical convenience, the global optimisation is performed on the inverted modal parameters, after which the obtained intervals are inverted in order to obtain the actual modal parameter ranges:

$$\hat{k}_i^S = \left[\min_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^K \phi_{i_k}^K \right), \max_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^K \phi_{i_k}^K \right) \right]^{-1} \quad (6)$$

$$\hat{m}_i^S = \left[\min_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^M \phi_{i_k}^M \right), \max_{\mathbf{x} \in \mathbf{x^I}} \left(\phi_{i_j}^M \phi_{i_k}^M \right) \right]^{-1} \quad (7)$$

If the inverted modal parameter range contains zero, the inversion results in the union of two interval ranging from respectively plus and minus infinity to a finite value. These modes are referred to as *switch modes*. The modes for which the inverted modal parameter interval has a constant sign, are referred to as *strict modes*, and classified in either positive or negative modes, based on the sign of the interval.

step 1.2 The modal envelope FRF is calculated by substituting the ranges of the modal parameters in the deterministic expression of the modal FRF contribution:

$$\left(FRF_{jk}^i \right)^I = \frac{1}{\hat{k}_i^S - \omega^2 \hat{m}_i^S} \quad (8)$$

This is an analytical procedure performed using the interval arithmetic approach.

step 2 Finally, the total interval FRF is obtained by the summation of the contributions of all considered modes:

$$FRF^I_{jk} = \sum_{i=1}^{n} \left(FRF^i_{jk}\right)^I \tag{9}$$

Also this final step is performed using interval arithmetics.

2.2.3. Eigenvalue interval correction

The method as described above is enhanced based on a graphical interpretation of the modal part of the interval algorithm. For each mode, consider the domain of modal mass and stiffness pairs that are achieved by considering the complete range of models defined by the interval uncertainty space $\mathbf{x^I}$:

$$\langle \hat{k}_i, \hat{m}_i \rangle = \left\{ \left(\hat{k}_i, \hat{m}_i\right) \mid \left(\mathbf{x} \in \mathbf{x^I}\right)\right\} \tag{10}$$

This domain defines a bounded area in a $\hat{k}_i, \hat{m}_i$-workspace. The exact bounds of this domain however, are generally unknown. The modal part of the interval algorithm now is interpreted in this workspace. From the optimisation as described in step 1.1, it is clear that, for a strict mode, the calculated ranges on the modal parameters $\hat{k}_i^S$ and $\hat{m}_i^S$ represent a rectangular approximation of the actual $\langle \hat{k}_i, \hat{m}_i \rangle$-domain. Therefore, this method is referred to as the *Modal Rectangle* (MR) method. Figure 3 shows a general $\langle \hat{k}_i, \hat{m}_i \rangle$-domain and its approximation using the MR method.

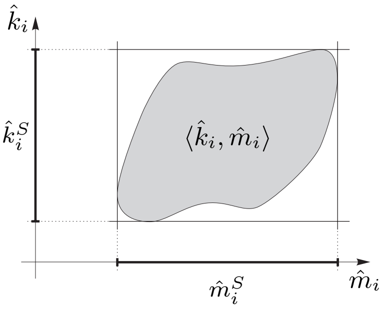

Figure 3. Graphical illustration of a mode's $\langle \hat{k}_i, \hat{m}_i \rangle$-domain and its approximation using the modal rectangle method

The interval arithmetic procedure for the calculation of the modal envelope FRF contributions in step 1.2 is interpreted in the same graphical domain. The goal in this step is to derive the bounds on the deterministic modal FRF, taking into account that $\hat{k}_i$ and $\hat{m}_i$ are located anywhere inside their intervals. By considering the modal FRF contribution as defined in equation (8) as an analytical function of $\hat{k}_i$ and $\hat{m}_i$, the bounds on this function over the modal rectangle have to be determined. It has been shown that this can be done analytically for the complete frequency domain, by considering only the function evaluations at the upper left and the lower right corner points of the rectangle. For switch modes, the modal domain spans out over the first and third quadrant of the modal parameter space. Still, a similar interpretation is possible (see [12]).

Based on these observations, it becomes clear that the calculation based on the modal rectangle introduces conservatism in the procedure if the actual $\langle \hat{k}_i, \hat{m}_i \rangle$-domain differs strongly from the approximate rectangle. This is often the case, as the modal parameters are generally strongly coupled through the global system and therefore show a high degree of correlation. Therefore, an enhanced procedure has been introduced. The enhancement is based on an improved approximation of the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain. This is achieved by using information on the eigenvalue ranges, which are obtained using an additional eigenvalue optimisation step in the modal part of the algorithm. An eigenvalue interval λ_i^I introduces an extra restriction on the quotient of possible combinations of the modal parameters. This restriction is mathematically expressed as:

$$\underline{\lambda_i} \leq \frac{\hat{k}_i}{\hat{m}_i} \leq \overline{\lambda_i} \tag{11}$$

Graphically, the eigenvalue bounds represent lines through the origin of the $\hat{k}_i, \hat{m}_i$-space tangent to the actual $\langle \hat{k}_i, \hat{m}_i \rangle$-domain. These lines are extra delimiters for the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain approximation, and therefore give rise to an improved $\langle \hat{k}_i, \hat{m}_i \rangle$-domain approximation as illustrated in figure 4. This domain is referred to as the *Modal Rectangle with Eigenvalue interval correction* (MRE).

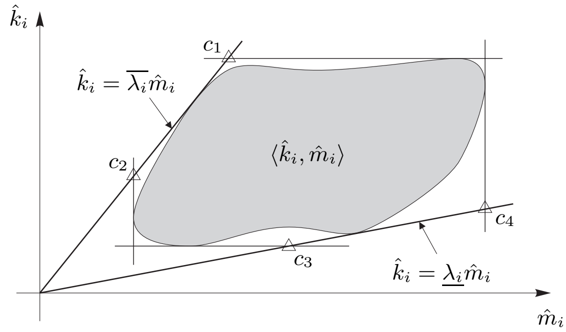

Figure 4. Effect of the introduction of the exact eigenvalue interval in the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain approximation of a positive mode

It has been shown that the conservatism in the modal envelope FRF contributions derived in step 1.2 is substantially reduced by considering the MRE domain instead of the MR domain as area of possible modal parameter pairs. The corresponding modal envelope FRF contributions are determined analytically by calculating the deterministic modal FRFs at the vertex points of the MRE-domain (indicated with c_i, $i = 1 \ldots 4$ in figure 4). This yields:

$$\left(FRF^i_{jk}\right)^I = \begin{cases} \left[\dfrac{\overline{\lambda_i}}{\overline{\hat{k}_i}\left(\overline{\lambda_i}-\omega^2\right)}, \dfrac{\underline{\lambda_i}}{\underline{\hat{k}_i}\left(\underline{\lambda_i}-\omega^2\right)}\right] \text{for } \omega^2 \leq \underline{\lambda_i} \\ \left[\dfrac{\overline{\lambda_i}}{\overline{\hat{k}_i}\left(\overline{\lambda_i}-\omega^2\right)}, \dfrac{1}{\overline{\hat{m}_i}\left(\underline{\lambda_i}-\omega^2\right)}\right] \text{for } \omega^2 \in \left[\underline{\lambda_i}, \overline{\lambda_i}\right] \\ \left[\dfrac{1}{\underline{\hat{m}_i}\left(\overline{\lambda_i}-\omega^2\right)}, \dfrac{1}{\overline{\hat{m}_i}\left(\underline{\lambda_i}-\omega^2\right)}\right] \text{for } \overline{\lambda_i} \leq \omega^2 \end{cases} \tag{12}$$

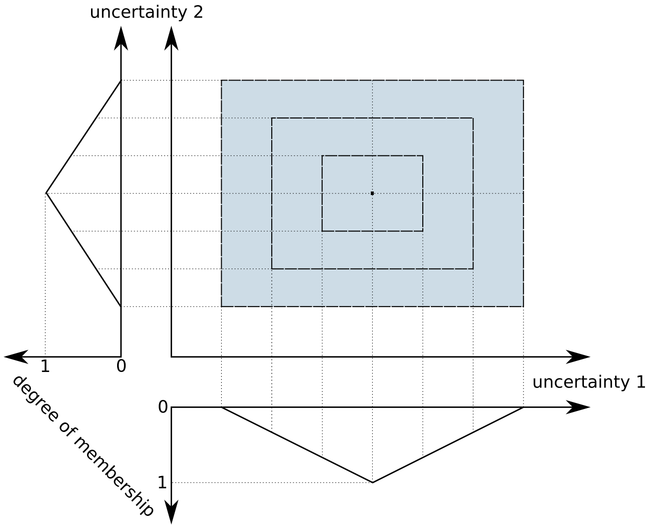

Figure 5. Optimisation bound constraints for an analysis with fuzzy uncertain parameters

for positive modes, and:

$$\left(FRF^{i}_{jk}\right)^{I} = \begin{cases} \left[\dfrac{\underline{\lambda_i}}{\overline{\hat{k}_i}\left(\underline{\lambda_i}-\omega^2\right)}, \dfrac{\overline{\lambda_i}}{\underline{\hat{k}_i}\left(\overline{\lambda_i}-\omega^2\right)}\right] \text{for } \omega^2 \leq \underline{\lambda_i} \\ \left[\dfrac{1}{\underline{\hat{m}_i}\left(\underline{\lambda_i}-\omega^2\right)}, \dfrac{\overline{\lambda_i}}{\underline{\hat{k}_i}\left(\overline{\lambda_i}-\omega^2\right)}\right] \text{for } \omega^2 \in \left[\underline{\lambda_i}, \overline{\lambda_i}\right] \\ \left[\dfrac{1}{\underline{\hat{m}_i}\left(\underline{\lambda_i}-\omega^2\right)}, \dfrac{1}{\overline{\hat{m}_i}\left(\overline{\lambda_i}-\omega^2\right)}\right] \text{for } \overline{\lambda_i} \leq \omega^2 \end{cases} \quad (13)$$

for negative modes. A similar procedure was derived for the bounds on the modal FRF contributions of switch modes. It has been shown that after applying the final summation step, this enhancement on the modal level of the algorithm leads to a close and guaranteed outer approximation of the actual modal envelope FRF contribution [12].

3. EFFICIENT IMPLEMENTATION OF THE FUZZY FRF ANALYSIS METHOD

The procedure as described above is in general computationally expensive. For each mode taken into account in the modal superposition scheme, the MRE analysis requires six global optimisations over the total uncertainty space spanned by the uncertainty intervals on the inputs. These optimisations are used to find the minimum and maximum modal stiffness ($\underline{\hat{k}_i}$ and $\overline{\hat{k}_i}$), modal mass ($\underline{\hat{m}_i}$ and $\overline{\hat{m}_i}$) and eigenvalue ($\underline{\lambda_i}$ and $\overline{\lambda_i}$). At this point an interval FRF at a certain α-level can be calculated. However, if we want to apply this interval procedure for a fuzzy analysis, the optimisations have to be performed using the input intervals on each of the α-levels of interest. Consequently, a fuzzy MRE analysis on 5 α-levels taking into account 10 modes requires 300 global optimisations. Each of the objective functions of these optimisation problems is the result of a classical deterministic modal FE analysis, which can be computationally expensive itself.

Generic non-linear optimisers can solve all optimisation problems independently. Theoretically, the optimisation problems can be non-convex, requiring global optimisation software, but analysis on different industrial sized applications [13] showed that in practical applications almost all the objective functions are convex or even monotonic, even with large uncertainty intervals. For these problems, local optimisation software gives accurate results. Because local optimisation problems are computationally far less expensive to solve, an efficient local optimiser is the best overall choice, but the results should be examined carefully to prevent false conclusions.

The efficient implementation of the fuzzy FRF procedure now takes advantage of, on the one hand, the strong coupling between the different goal functions, and, on the other hand, the relation between the interval problems at each α-level:

- The 6 objective function evaluations for all modes are all resulting from a deterministic modal analysis at a certain point in the uncertainty space. Consequently, when optimising a modal parameter of a specific mode, a goal function evaluation during the global search will not only return the corresponding goal function at this point, it also directly returns the value of all other goal functions at this point. These can be directly derived from the modal analysis results, as long as each modal analysis returns all the modes requested in the modal super-positioning.
- A fuzzy analysis requires the same objective functions to be minimised and maximised on different α-levels. In optimisation terms, this means that the same goal functions are optimised with different bound constraints. Figure 5 shows this for two fuzzy uncertain parameters. The shaded rectangle shows the bound constraints for the optimisation at $\alpha = 0.0$. It is clear that the search domains for all higher α-levels are contained within the lowest level. The rectangles inside this shaded rectangle show the bound constraints for the optimisations at higher α-levels.

Both these properties of the fuzzy FRF analysis enable a considerable increase in the computational efficiency of the total procedure, by including a response surface method (RSM) approach in the optimisation loop. First, the RSM builds approximations of the objective functions based on function evaluations in some well chosen points in the input parameter space at $\alpha = 0.0$. The response surfaces can be build simultaneously for all goal functions for all modes. This response surface now is valid in the complete search domain at $\alpha = 0.0$ and therefore, can be applied for the optimisations necessary on all higher α-levels. By using this approach, the computational cost of a fuzzy analysis is only slightly higher than the computational cost of an interval analysis when using a response surface based optimisation technique.

4. APPLICATION OF THE FUZZY FRF ANALYSIS METHOD FOR INTERVAL UNCERTAINTY IDENTIFICATION

The α-level strategy as described in section 2.1 and depicted in figure 1 forms the basis for the application of the fuzzy finite element methodology for interval uncertainty identification. Based on this strategy, a fuzzy FE analysis can be interpreted as a global sensitivity study the result of which reflects the influence of the input interval widths on the range of possible outcomes of the analysis. As such, fuzzy FE analysis becomes a tool to make numerical predictions on the changes in the range of possible physical design behaviour due to changes in tolerances on the design. A designer now is able to choose appropriate tolerance fields by choosing an α-level on the input side, which guarantees that the possible analysis results are located inside the allowable range for the physical behaviour (see e.g.[14]).

The application of the fuzzy FE method for interval uncertainty identification follows a similar path as the one described above for the tolerance analysis. In this case however, the range of allowable analysis results is no longer driven by design specifications, but results from collecting all physical testing results. The fuzzy FE analysis is now applied to derive an α-level at which the range of physical testing results is located within the range of the predicted analysis output. The corresponding intervals at the input side of the problem than constitute a set of interval values that will results in the observed physical behaviour.

It is clear that this procedure will result in a prediction of the actual uncertainty interval in case a single input interval is analysed with a fuzzy number that includes the actual uncertainty interval at a certain α-level. However, if this is not the case, in general a different input interval will be identified. This is due to the fact that a different choice of the fuzzy set at the input of the problem will generally result in a different fuzzy output, yielding a different allowable α-level and therefore a different identified interval uncertainty. These different interval sets can be assessed by specification of a performance measure based on the location of the measurements within the range of the numerical analysis results. For instance, the measurement cloud, while located within the analysis outcome range, could be strongly skewed to one of the bounds of the interval. This indicates that the input interval set possibly yields analysis results that were not found experimentally. Therefore, the corresponding input interval set is found to have a lower degree of possibility. The uncertainty interval identification therefore consists of assessing intervals for the uncertainty input property with respect to the obtained measurements. This can be achieved by repeating the identification for many different fuzzy sets which comply with the available information.

This procedure requires the fuzzy analysis to be repeated for many different fuzzy input sets. The efficient implementation based on the RSM methodology as described in section 3 enables the above procedure to be performed at a very low cost. Indeed, if reliable response surfaces are built based on the support interval of a specific fuzzy number describing the input uncertainty, they enable the analysis of any fuzzy set with that support interval. Consequently, different fuzzy sets can be analysed easily without the need for a single additional goal function evaluation.

The same procedure now can be applied when there are several uncertain input parameters to be identified as intervals. By choosing fuzzy sets for their representation, different possible combinations of input sets can be derived. The next section illustrates this approach for a spacecraft structural part with 2 uncertain properties.

5. APPLICATION: VEGA INTERSTAGE 1/2

5.1. Numerical model

The developed algorithms are tested on a model of the VEGA interstage 1/2, kindly made available by Dutch Space. Figure 6 shows the finite element model (left) and a schematic view (right). The structure consists of five conical shell rings, connected and stiffened by stiffening rings. The finite element model consists of about 38000 nodes (228000 DOFs) and about 28000 elements (quadrilateral plate elements for the skin and five- and six-sided solid elements for the reinforcements). The model is subject to uncertainties in the thicknesses of the shell structures between the reinforcement rings.

This interstage serves as a connection between two relatively rigid structures, one rigidly bolted to the bottom ring and the other rigidly bolted to the top ring. These are modelled as rigid body elements. One is connected to all nodes on the bottom ring, the other one is connected to all nodes on the top ring. The acceleration transmittance FRF between the centres of these rigid body elements in the longitudinal direction is calculated. The large-mass method is applied to simulate base excitation.

5.2. Efficiency of the RSM based algorithm

In order to illustrate the performance of the new efficient implementation, the interstage model is subjected to five uncertain parameters. These uncertainties are described using fuzzy numbers: $(3/4/5)$ mm, $(3/4/6)$ mm, $(3/4/8)$ mm, $(3/4/8)$ mm and $(3/4/10)$ mm from the lower to the upper side of the structure. These uncertain parameters and uncertain parameter ranges are specified by the designer of this conical shell structure.

The model is analysed using the MRE method based on response surface based optimisation as described above. As the vertex analysis is often considered as a good approximation for the interval results, it is included here to assess the performance of the new algorithm.

The fuzzy analysis considers 6 α-levels $(0.0, 0.2, \ldots, 1.0)$. Based on the magnitude of the modal mass m_i and modal

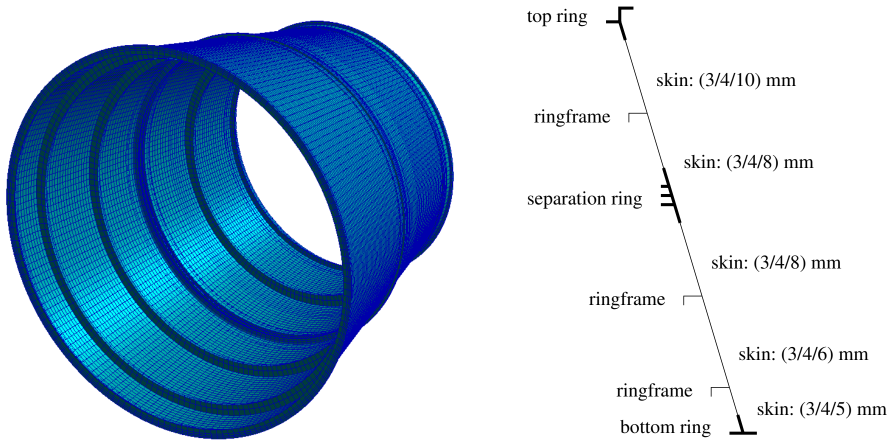

Figure 6. Finite element model (left) and schematic description (right) of the stiffened conical shell structure. (courtesy of Dutch Space, Leiden)

stiffness k_i, three modes (number 1, 2 and 13) with a non-negligible contribution to the observed FRF are selected. Because the order of the modes can change when the uncertain parameters change, the modes are tracked using a modal assurance criterion (MAC).

For this model, a comparison is made in the computational efficiency of the fuzzy FRF analysis based on three approaches:

- Using the original independent global optimisation strategy, the MRE analysis would require 6 optimisations for each mode at each α-level taken into account. Since three modes are considered at five α-levels (the analysis at level 1.0 is a deterministic analysis), 90 optimisations are required. The number of function evaluations per optimisation is unpredictable. However, due to the in general monotonic behaviour of the goal functions, it tends to be mainly dependent on the number of uncertain parameters.

- Using the automated response surface based optimisation procedure, 41 deterministic analyses are required to construct reliable response surfaces for all objective functions. The computational cost of the optimisation using the response surfaces is negligible compared to the computational cost to construct these response surfaces.

- The vertex analysis solves the deterministic model for all combinations of minima and maxima of the uncertain parameters. For this model with 5 uncertainties, 32 (2^5) deterministic analysis are required at each level, except for $\alpha = 1.0$, where only 1 deterministic analysis is required. In total, 161 deterministic analyses are required.

The top graph of figure 7 shows the upper bound on the FRF at $\alpha = 0.0$, calculated using the RSM-based MRE and vertex method. Additionally, all vertex samples are plotted. The bottom graph shows the relative difference between the MRE and vertex results. These results prove that the MRE method using the response surface based optimisation method is able to calculate the bounds on the FRF accurately.

Figure 8 shows the upper part of the fuzzy FRF assembled from the interval FRFs at all 6 α-levels. This FRF shows that the uncertain parameters influence all modes more or less equally, so the uncertainty on the FRF is about equal over the full frequency range of interest. The RSM-based approach, while reducing the computational load by 75% compared to the vertex analysis, optaines a comparable accuracy. Furthermore, it has the advantage that intermediate α-levels can be included at a negligible cost. Also, a change in the membership function at the input of the problem can be done at no cost as long as the support inerval of the fuzzy numbers are retained. This last property is of specific importance in the application of the fuzzy analysis in uncertainty identification routines, as described in the next section.

5.3. Application for uncertainty identification

The procedure for uncertainty interval identification is now applied on the same model, considered however in different circumstances. In this case, the objective is to identify the uncertainty present in the model. The identification procedure focuses on two uncertainties. One represents the thickness of the three upper parts of the shell structure, the other the thickness of the two lower parts. this means that the five uncertain parameters as discussed in the previous section are lumped together to two uncertainties. As no physical testing data are available, simulated FRF data sampled using Monte Carlo simulation are used as measurement references throughout the procedure.

The uncertainty identification procedure starts from the following fuzzy sets for the identification: $(3/4/8)$ mm for the shell thickness of the upper parts, and $(3/6/6)$ for the shell thickness of the lower parts. The latter can be interpreted as an interval with a known upper bound but an unknown lower bound.

The fuzzy FRF procedure is first applied with these fuzzy input parameters. This results in the fuzzy FRF as shown in figure 9. The procedure now searches for the α-level that corresponds to the simulated test data.

The interval uncertainty identification is based on 10 sampled FRF's. These FRF's are compared to the interval FRF's resulting from intersecting the fuzzy FRF at levels $\alpha = 0.20, 0.40, 0.60$. This comparison is shown in fig-

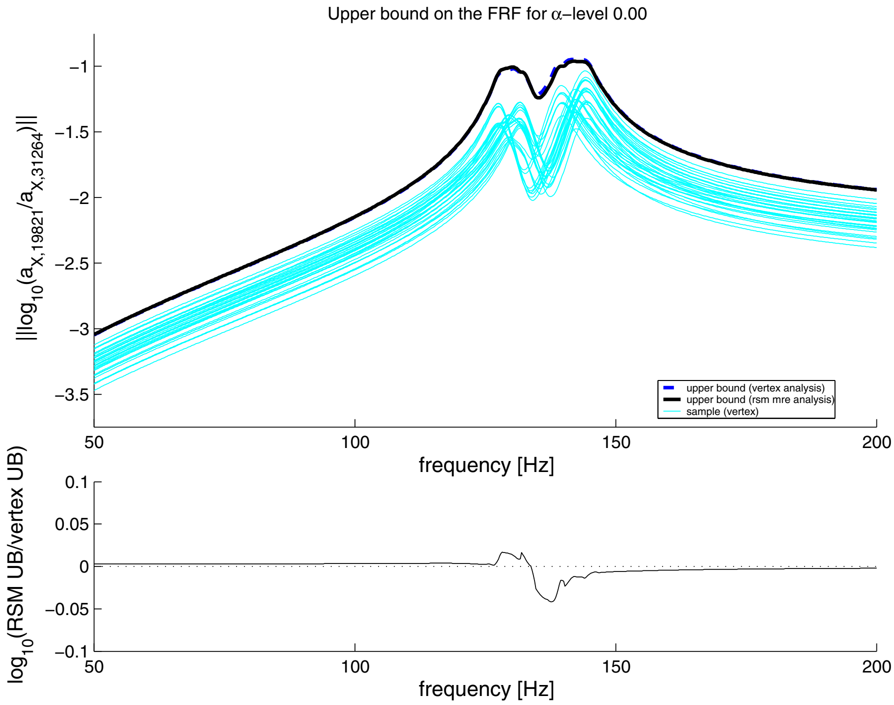

Figure 7. Upper bound on the FRF between the upper and lower ring of the structure

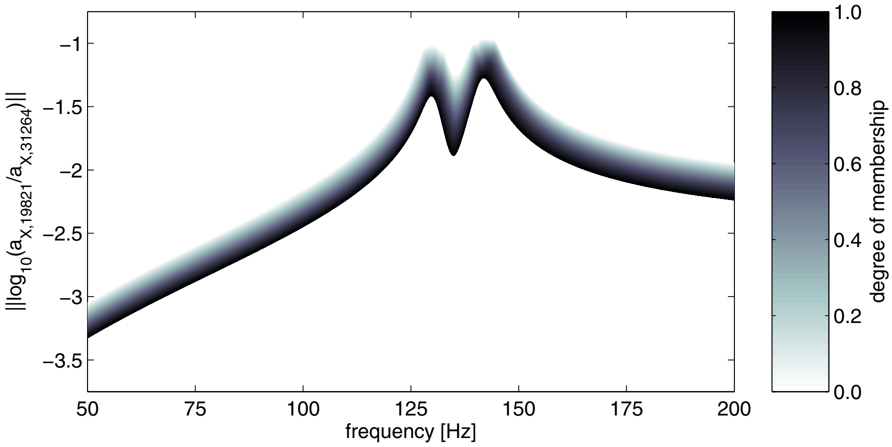

Figure 8. Fuzzy upper bound of the FRF between the upper and lower ring of the structure

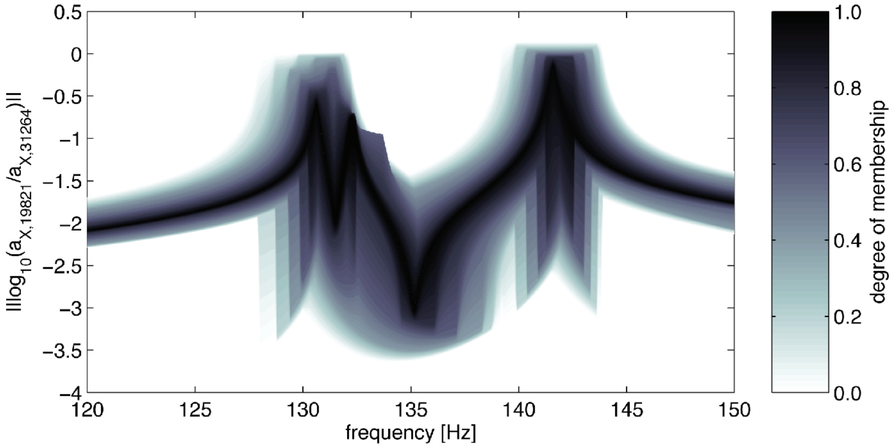

Figure 9. Fuzzy FRF between the upper and lower ring of the structure

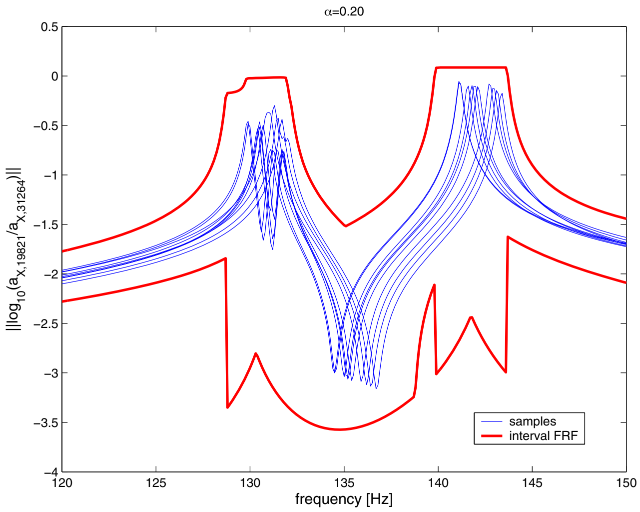

Figure 10. Comparison of the interval FRF at $\alpha = 0.20$ *with the sampled FRF's*

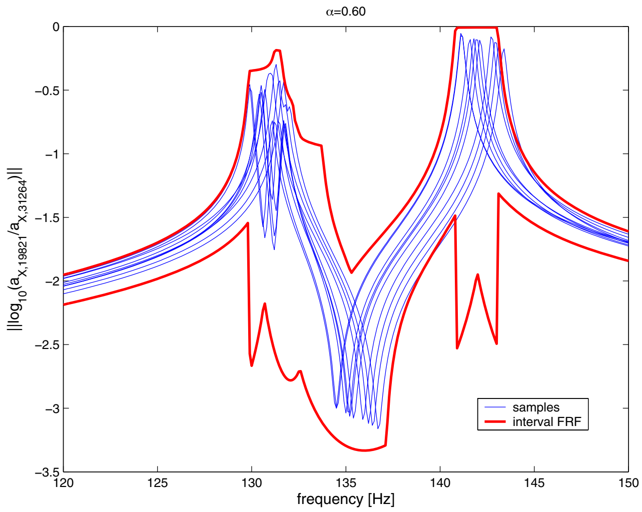

Figure 12. Comparison of the interval FRF at $\alpha = 0.60$ *with the sampled FRF's*

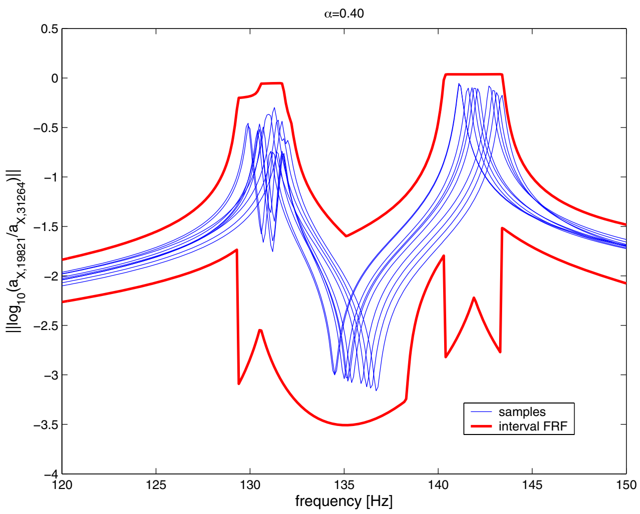

Figure 11. Comparison of the interval FRF at $\alpha = 0.40$ *with the sampled FRF's*

ures 10 to 12. From these figures, it is clear that the interval FRF at $\alpha = 0.40$ gives the best approximation of the actual range of the sample FRF's. Therefore, the uncertainty intervals at the input of the problem can be derived by intersecting the input fuzzy numbers at this level, resulting in the intervals $[3.4 \;\; 6.4]$ for the thickness of the upper part, and $[4.2 \;\; 6]$ for the thickness of the lower part of the shell structure.

6. CONCLUSIONS

This paper introduces a procedure for the identification of interval uncertainty on numerical models based on the fuzzy finite element method. The procedure is based on an initial assumption on membership functions for the fuzzy analysis. Starting from this initial assumption, numerical uncertainty intervals are gradually adapted based on the knowledge on dynamic response scatter resulting from a measurement campaign. The procedure compares the fuzzy outcome of the numerical analysis with the measured data. The level of membership at which the output quantity interval contains the reference data is selected. The intervals at this level at the input side of the problem are then the identified uncertainty ranges for the corresponding model properties.

This method requires a high amount of fuzzy analysis to be performed. For this purpose, this paper has introduced a highly efficient response surface based optimisation technique. This techniques proves to be extremely useful in the context of fuzzy analysis, as a fuzzy analysis requires the same objective functions to be minimised and maximised on different strongly related design spaces. Furthermore, the response surfaces can be reused when different assumptions on the initial fuzzy membership functions are made, as long as they have the same support. The validation case shows that the efficient implementation gives highly accurate results at a very low cost. It also illustrates the possible application of this technique for non-deterministic model updating.

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