PROBABILISTIC METHODS APPLIED TO FRACTURE CONTROL OF SPACEFLIGHT STRUCTURES

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Abstract

In the deterministic fracture control verification of critical spacecraft structures, as applied by ESA, the inherent scatter involved in structural parameters is currently intended to be covered by a scatter factor on life, lower bound fracture toughness values, average crack growth rates and a conservatively defined fatigue load spectrum enveloping the specified structural life. In this paper, the probabilistic fracture mechanics software ESACRACK-Prob Version 1.0 is used to explore the effect of the scatter of the parameters on the structural reliability and to gain insight into the methodology behind the derivation of the probability of failure. The investigation is carried out on the basis of representative examples of spacecraft structures designed to meet the fracture control verification requirements. In the scope of this work, the application of probabilistic fracture control methodology focuses on the investigation of the reliability associated with current deterministic design and verification rules, the identification of potential sources of unconservatism and the exploration of proposed modifications and exceptions. The effect of the deterministic life scatter factor on the structural probability of failure and the determination of parameters having the largest influence on the final reliability are investigated.

Keywords: Probabilistic fracture mechanics; fatigue crack growth; space structures, ESACRACK, NASGRO.

1. INTRODUCTION

Fatigue crack growth is of major importance for the dimensioning and maintenance of critical aerospace structures. The present work investigates structural design mainly driven by fatigue crack growth, which is known to be a stochastic phenomenon.
Inherent scatter in crack sizes, material properties and loading has been widely investigated and discussed [1, 2]. Comparison between the traditional deterministic safety factor approach and a more sophisticated probabilistic methodology has been investigated [3]. Development of numerical procedures for the probabilistic assessment of the probability of failure of structural parts containing crack-like defects has been carried out and is described in several publications [4, 5].

A deterministic approach based on parameters represented by single values and safety factors expected to cover the uncertainties in the design parameters is currently in use. However, for this approach the degree of conservatism introduced in the design remains unknown, which represents a major drawback of such an approach. Consequently, it is not possible to enforce a desired degree of reliability on the final design. In addition, safety factors do not provide insight into the sensitivity of the responses to the variations in the model parameters, and could potentially lead to unconservative designs in cases for which the response exhibits a high sensitivity to one of the variables.

In this context, probabilistic models provide tools to enforce a desired reliability and offer an insight into the conservatism and robustness inherent to the classical deterministic damage tolerance verification rules.
The application of probabilistic design methodology requires precise characterization of the variability of the parameters, implying the collection and statistical evaluation of significant amount of data.

ESA has been involved in the development of structural analysis software's and methodologies to deal with the probabilistic aspects of fracture mechanics and the scatter associated with existence and detection of cracks, service loads, material properties. The stochastic version of ESACRACK-Prob, based on ISPUD [1] probabilistic software and NASGRO© v.3 [2] crack growth software, allows reliability assessments for systems where the performance is mainly driven by fatigue crack growth.

[1] Importance Sampling Procedure using Design Points developed at the Institute of Engineering Mechanics, IfM, University of Innsbruck, Austria.
[2] Fatigue Crack Growth Computer Program, originally developed at National Aeronautics and Space Administration, NASA, Johnson Space Center (now further developed in cooperation with Southwest Research Institute).

2. DETERMINISTIC AND PROBABILISTIC APPROACHES

2.1. Conventional deterministic approach

In the conventional deterministic approach for crack growth prediction of spaceflight structures in linear elastic fracture mechanics (LEFM) domain, ESA uses the NASGRO module of ESACRACK for safe life analysis. In this software, a functional relationship is implemented, describing the crack growth rate *da/dN* and the stress intensity factor range *ΔK,* which combines the influences of loading applied to the cracked part, crack sizes and material properties. This so called NASGRO equation is:

(1) $$\frac{da}{dN} = C.\left[\left(\frac{1-f}{1-R}\right).\Delta K\right]^n . \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_c}\right)^q}$$

where N is the number of applied fatigue cycles, C is the Paris region constant, R is the stress ratio, f is the Newman crack opening function, ΔK_{th} is the threshold of the stress intensity factor range, K_c is the critical stress intensity factor and n, p and q are model parameters.

2.1.1. Deterministic design rules

Crack growth calculations are based on the assumed existence of an initial defect, the use of typical or conservative material properties values and a conservatively defined fatigue load spectrum. Additionally, a lower bound value for K_c (with a factor of 0.7 on average values) and the use of scatter factor of 4 on the number of cycles are applied.

The minimum detectable crack size, a_{NDE}, which is a characteristic of the specific non destructive inspection technique, is typically defined as the defect size value for which 90 percent of all cracks in the component are detected with a confidence level of 95 percent.
Conservative values for the material parameters (A-values, B-values) are commonly used to envelop the inherent variability.
The scatter factor to be applied on the nominal life aims to cover the variations in initial crack sizes evaluation, material properties (C, n, K_c, σ_{flow}, ΔK_{th}) and the uncertainties related with the loading definition.

2.1.2. Failure conditions in deterministic verification

Crack instability is assumed to occur when the maximum stress intensity factor, K_{max}, exceeds the critical stress intensity factor $K_{c,\text{average}}$ i.e.:

(2) $$K_{\max} \geq 0.7 * K_{c,average}$$

Failure is also assumed to occur if the net section stress exceeds the flow stress, σ_{flow}, of the specified material, defined as the average of the yield and ultimate strengths:

(3) $$\sigma_n \geq \sigma_{flow}$$

The appropriate use of material fracture toughness properties is essential for consistent and accurate crack growth assessments. This includes plane strain fracture toughness (K_{Ic}) values, part-through fracture toughness (K_{Ie}) values and fracture toughness (K_c) values given as a function of thickness.

2.2. Probabilistic approach

The ESACRACK-Prob software has been developed to facilitate the use of probabilistic methods for fracture mechanics analysis of spaceflight structures.
Variance reducing simulation procedures like Importance Sampling and Adaptive Sampling, which are based on modified Monte Carlo simulation [6], allow a considerable reduction of the necessary number of simulation points for the evaluation of the probability of failure. These numerical procedures employ weight density functions, enforcing the simulation to focus in the vicinity of the so-called design point, which contributes the most to the probability of failure (PoF) [7]. Thus, it is possible to obtain high accuracy estimations for the PoF with relatively low effort in computations.

The probabilistic safe life analysis approach makes use of conventional deterministic calculations, randomizing the input parameters and statistically analyzing the output. The probabilistic methodology describes the selected parameters uncertainty by means of an appropriate distribution despite the use of single values.
Material properties scatter is assessed based on experimental data available in FRAMES2 [3] software [8].
Sensitivity analysis can be carried out performing individual simulations for the considered random variables. The results allow identifying the design variables having the highest effect on the probability of

[3] FRAMES2 Materials Database software is developed and distributed by ESA to store experimental data from tests performed in metallic alloys.

failure, therefore requiring detailed surveillance and accurate probabilistic characterization.

Probabilistic analyses are performed to explore the structural reliability associated with current deterministic verification rules and to quantify the degree of conservatism introduced in the design by the use of common life scatter factor.
The effect of combining different random parameters is investigated.
Furthermore, detailed investigation is carried out to comprehend the impact of potential variation of the lower bound fracture toughness (considered as 70% according to present design practice) and of the materials' flow strength (for which typical values are used).

This approach can be used to assess the reliability of structural parts and to identify unconservatism in the design verification introduced by the parameters' uncertainties.

For the current probabilistic approach the interaction between the failure modes is considered by means of an empirical equation, which is an interpolation function like the R6-Curve. It results in a somewhat smoother limit state function. This so-called *Two-Criteria Approach* leads to the definition of the following limit state function:

(4) $$g(\sigma_r, K_r) = \sigma_r . \left\{ \frac{8}{\pi^2} . \ln \sec\left[\sigma_r . \frac{\pi}{2} \right] \right\}^{-\frac{1}{2}} - K_r$$

where:

(6) $$\sigma_r = \frac{\sigma_n}{\sigma_{flow}} \text{ and } K_r = \frac{K_{\max}}{K_c}$$

Hence, failure corresponds to the condition:

(5) $$g\left(\sigma_r, K_r\right) \leq 0$$

For probabilistic calculations, the lifetime of the structure is one single life, i.e. the nominal service life.

3. EXAMPLE FOR TYPICAL ISS PAYLOAD STRUCTURES

The methodologies previously described are applied for damage tolerance verification of specific structural parts of typical equipment of the International Space Station (ISS). Particularly, it has been applied for the structural verification of the attachment area of a small electronic box (mass below 4 kg) typically used in ECLSS equipment (Environmental Control and Life Support System). The fatigue spectrum of the equipment is dominated by random vibration loads.

3.1. Model description

For investigation purposes, both Aluminium Al 7075 T7351 (Plate and Sheet, T-L, LA & DA) and Titanium Ti6 Al 4V (MA forging) alloys are considered for the attachment areas [9].
The statistical evaluation of the materials' crack growth data stored in FRAMES2 database is carried out, resulting in the definition of mean and standard deviation values for the Paris region constants, i.e. the parameters *C* and *n*.

According to damage tolerance verification principles, the presence of a single initial crack in the most critical area of the structure and with the worst possible orientation is assumed.
The verification of the attachment part is based on a plate model subjected to tension-compression cyclic loading, with a surface crack located at the center, i.e. SC01 model of NASGRO software. Initial crack dimensions are assumed according to the values indicated in ECSS-E30-01A standard for Fracture Control [10] and the NASGRO v.3 manual [11] for specific Non Destructive Evaluation techniques. The component is assumed dye-penetrant inspected.

Material	**Thickness of the part, t [mm]**	**Flaw shape, a_i/c_i**	**Crack depth, a_i [mm]**
Al 7075 T7351	4	1	1.91
Ti6 AL 4V	4	0.2	0.81(*)

* Based on assumption of Special NDE

TABLE 1. Initial crack dimensions

The fatigue load spectrum is dominated by random vibration loading, following Rayleigh distribution, and envelops the random acceptance test event (3 axes x 120 seconds per axis, 90 000 cycles) and the random vibration during lift-off phase (25 missions corresponding to 200 000 cycles). Stress levels are derived from exceedance curves as presented in Figure1. Spectrum loading is assumed to be fully reversed, i.e. stress ratio R equals -1.
Crack growth predictions consider no interaction phenomenon.

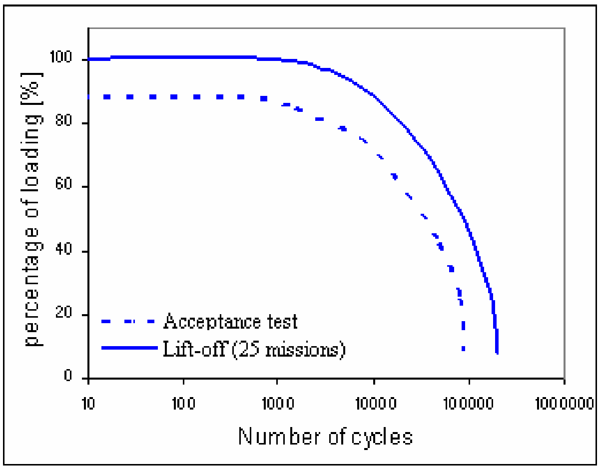

FIG 1. Exceedance curve for small ECLSS Equipments

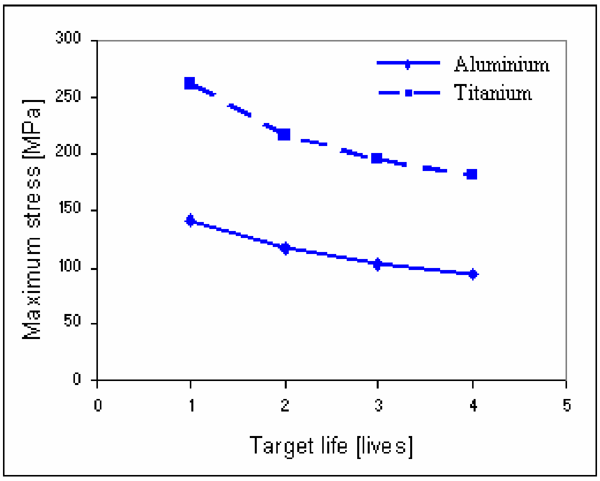

FIG 2. Loading Level for specified target life

Fatigue spectra are normalized by the maximal value from the exceedance curves. During the investigation several spectra are derived by scaling the stress components of the original spectrum with the Stress Scaling Factor (*SSF*) calculated for a previously defined target life.

3.2. Damage tolerance verification

Iterative deterministic calculations allow determining the stress levels for the spectrum that induces structural failure at the specified target life. A reduction on the intended target life results in the increase of the *SSF* and subsequently on the amplitude stress of the original spectrum. Therefore, designing for diverse target life allows investigating the structural reliability associated with different fatigue loading levels.
Values are given in Table 2 and 3 for Aluminium and Titanium alloys respectively and captured in Figure 2.

	Al 7075 T7351		
Target life [lives]	**Max. stress [MPa]**	**SSF_{design} [ ]**	**ΔSSF^* [%]**
4	95	0.95	0
3	103	1.03	8.4
2	116	1.16	22.1
1	142	1.42	49.5

* In comparison with target life equal to 4

TABLE 2. Loading level for specific target life, Aluminium alloy

	Ti 6Al 4V		
Target life [lives]	**Max. stress [MPa]**	**SSF_{design} [ ]**	**ΔSSF^* [%]**
4	181	1.81	0
3	195	1.95	7.7
2	217	2.17	19.9
1	262	2.62	44.8

* In comparison with target life equal to 4

TABLE 3. Loading level for specific target life, Titanium Alloy

3.3. Life assessment uncertainties

Regardless of the stochastic nature of the parameters involved in crack growth phenomenon, this investigation focus on the variables known to affect the most the crack growth behaviour. Scatter in loading and initial crack size are recognized to have major influence on the reliability. The effect of the variability in material properties is also assessed.

3.3.1. Initial crack size

The frequency of occurrence of an initial crack in the structure is assumed equal to one, i.e. the presence of a single initial crack is implied. This simplification adds conservatism to the analysis, because normally a detected crack is not allowed, i.e. the probability of occurrence of a defect is considerably lower than one.
Initial crack size distribution is assumed to follow Probability of Detection (POD) curves characteristic of common NDE techniques. By doing this, a potential defect larger than the NDE limit value (90% detection with 95% confidence level) undetected during the inspection or a smaller crack than the NDE limit value are situations covered by the statistical distribution. For all analyzed cases, the initial flaw shape is kept constant.

The initial crack size variation is modelled using log normal distribution. A series of POD curves for dye penetrant inspection are hypothesized based on a constant initial crack size based upon 90% probability of detection, with coefficients of variation (COV: ratio between the standard deviation and the mean value) derived from data available in literature [12]. A range of COV between 10 and 30% is investigated. It intends to cover the inherent variability due to inspection procedure, environment, geometry and surface condition of the component, location and geometry of the defect, and inspectors.

	Aluminium		Titanium	
COV (a_i) [%]	Mean a_i [mm]	St. dev. a_i [mm]	Mean a_i [mm]	St. dev. a_i [mm]
10	1.69	0.169	0.72	0.169
15	1.60	0.24	0.68	0.24
20	1.51	0.302	0.64	0.128
30	1.37	0.411	0.58	0.174

TABLE 4. Initial crack size distributions

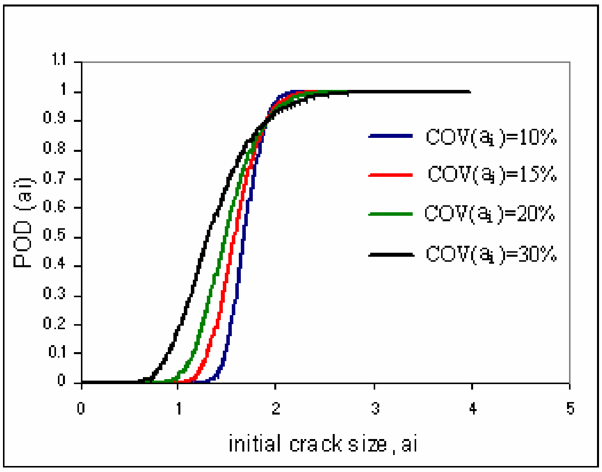

FIG 3. Initial crack depth distribution (for Aluminium)

3.3.2. Material properties

Description of material properties scatter is based on existing data for a number of available test batches of similar material, covering variations of homogeneity and material processing history.

The effect of material properties scatter in the Paris region is investigated for the Paris constant *C*, considered as random variable. Variability in the asymptotes of threshold (ΔK_{th}) and unstable crack growth (K_c) regions in addition to flow stress (σ_{flow}) is considered.

Close correlation between the Paris region constants, *C* and *n*, exists [13]. For the purpose of this study, correlation cannot be considered, which is known to induce an overestimation of the final probability of failure [14]. Therefore, a simplified approach is adopted, in which the growth exponent *n*, i.e. the slope of the Paris curve in log-log space, is kept constant while the *C* value is randomized. The variability of the *C* parameter is investigated through evaluation of crack growth rate curves, as illustrated in Figure 4.

For simplification purposes, the mean value of *C* is assumed to be located at the midpoint between the upper and lower bounds for the reported measurements. Standard deviation is set equal to 1/6 of the difference between upper and lower bound values.

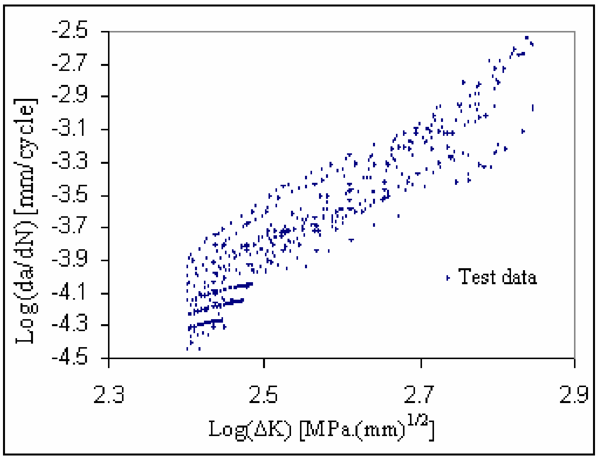

FIG 4. Crack growth rate data for similar Aluminium alloys (16 batches)

The scatter in fracture toughness and flow stress is assessed based on experimental data stored in FRAMES2 software. Results are in agreement with values commonly indicated in literature [1, 4, 15, 16, 17 and 18].
The randomization of K_c and ΔK_{th} is achieved through the variation of the plain strain fracture toughness, K_{Ic}, and the threshold stress intensity factor range at R=0, ΔK_0, respectively. All random variables investigated, i.e. C, ΔK_0, K_{Ic} and σ_{flow}, are modelled by log normal distributions. See Table 5.

	Al 7075 T7351		Ti 6 Al 4V	
Random variable	Mean value	COV [%]	Mean value	COV [%]
C [mm/(MPa.mm$^{1/2}$)^n]	$1.06.10^{-10}$	25	$4.89.10^{-14}$	24
K_{Ic} [MPa.mm$^{1/2}$]	869	10	1737	10
σ_{flow} [MPa]	452	5	966	5
ΔK_0 [MPa.mm$^{1/2}$]	104	10	122	10

TABLE 5. Material properties: scatter in random variables.

3.3.3. Loading

In current deterministic procedures the fatigue load spectrum is defined in a conservative way, generally with only 1% chance of exceeding the maximum peak stress.
For the probabilistic approach, loading uncertainties are treated in a simplified way. To get an indication of potential effect on PoF, only the effect of stress amplitude variability is investigated. No load history effects are considered: the SSF affects equally all cycles of the spectrum. Therefore, the deterministic *SSF* is now replaced by a probabilistic lognormal distribution, derived to reflect a probability of exceeding the maximum peak stress equal to 1%. Different COV

values are investigated. For normalized spectra, different *SSF* distributions are explored, as shown in Table 6.

COV (SSF) [%]	Mean SSF [mm]	St. dev. SSF [mm]
15	0.714	0.071
20	0.643	0.129
25	0.581	0.145

TABLE 6. *SSF* distributions for normalized spectra

3.4. Results

3.4.1. Sensitivity analysis

Sensitivity analyses for the material properties given in Table 5 together with initial crack size, a_i, and Stress Scaling Factor, *SSF*, are performed varying independently the COV of each parameter. Note that due to the assumptions for the initial crack size and Stress Scaling Factor's distributions, a variation in the COV implies a change in the mean values.

For sensitivity investigation, individual randomization of each variable is carried out, keeping the remaining parameters constant and equal to the deterministic values, i.e. NDE value for the initial crack size, maximum Stress Scaling Factor (with 1% of exceedance probability) and material allowables. In the probabilistic calculations, if plane strain fracture toughness is not randomized, the average value is used, instead of the lower bound defined for the failure criterion. This results in a more conservative PoF estimation.

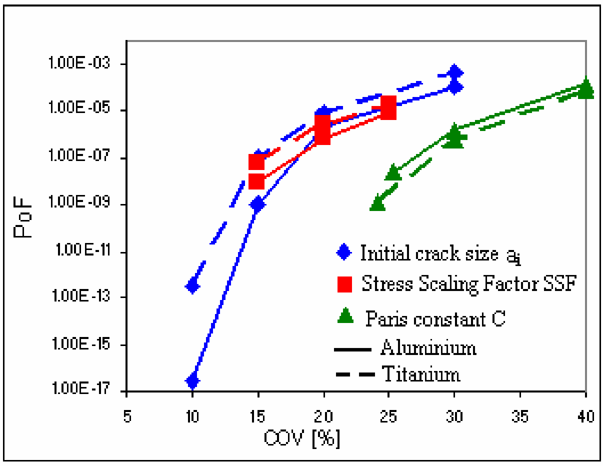

FIG 5. Sensitivity of PoF to crack depth, Stress Scaling Factor, Paris constant (Aluminium and Titanium alloys)

Numerical convergence problems (no simulation points in the failure domain) are identified when randomizing only K_{Ic} or ΔK_0 due to extremely low PoF.

Figure 5 illustrates the PoF sensitivity to variations in initial crack size, a_i, *SSF* and Paris region constant *C*.
In this specific example the crack growth behaviour is predominantly controlled by the initial crack size and the cyclic stress amplitude, here represented by the *SSF*.
The probability of failure rises with increasing COV values, for the investigated alloys. An increase of 5% in COV for the *SSF*, in particular from 20 to 25%, results in the increase of the PoF by one order of magnitude. This effect is stronger while changing the COV of *SSF* from 15 to 20%.
Initial crack size variations prove to have a major influence on the PoF, for this specific example. Lower values for the COV (around 10%) result in low PoF values. However, higher COV values (15% to 30%) result in a significant increase in the PoF, reaching a maximum of 10^{-4}, as illustrated in Figure 5.
Increasing scatter in Paris constant *C* leads to higher results of the PoF, being the maximum around 10^{-4} for a dispersion of 40%. Experience proves that manufacturing processes that are not tightly controlled may present dispersions that can reach 50% [14].
It is observed that the PoF for the Aluminium alloy case is more sensitive to variations in initial crack depth and loading, than for the Titanium structure. Conversely, variations in the Paris constant parameter *C* have higher influence on the PoF for the Titanium alloy case.
Sensitivity analysis carried out for ΔK_0 and K_{Ic} showed no noteworthy effect on the PoF, being therefore discarded as random parameters for calculations.

3.4.2. Random variables combination

Probabilistic simulations are performed taking into account combinations of initial crack size, a_i, and *SSF*'s scatter. For the initial crack size, a_i, COVs of 15 and 20% are investigated, whereas COV of *SSF* is assumed to vary between 15 and 25%. All the other parameters, and in particular the material properties are kept constant. They are defined according to the conditions of computation previously underlined.

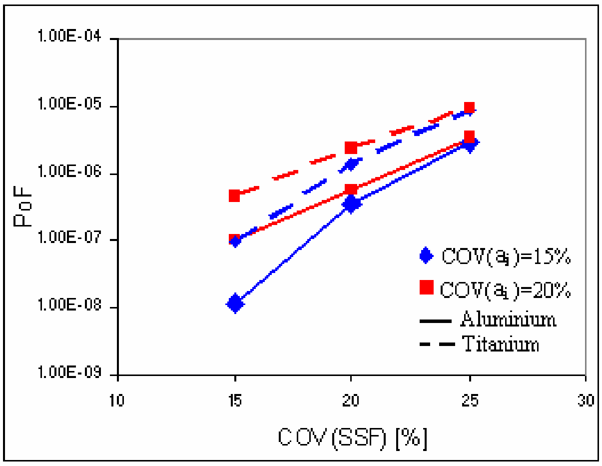

FIG 6. Probability of failure for combined scatter in a_i and *SSF*

Increasing the dispersion in both, initial crack size and *SSF*, results in the increase of PoF.
Probabilistic estimations considering the initial crack size as the only random variable, with 20% COV, resulted on PoF values of 2.1×10^{-6} and 9.0×10^{-6}, respectively for Aluminium and Titanium alloys. It is observed that for an initial crack size with COV of 20%, adding the *SSF* as a random variable, with COV of 15% results in one order of magnitude decrease for PoF, as shown in Figure 6. This effect is not verified while using initial crack size COV of 15%.

Figure 7 illustrates the results obtained while randomizing simultaneously the three parameters: the initial crack size, a_i, the Stress Scaling Factor, *SSF*, and the Paris constant *C*. The COV of *C* is set at 25% and 24% respectively for the Aluminium and Titanium alloys, whereas COV of 15% and 20% for the initial crack size and *SSF* are considered. The other parameters are kept constant.

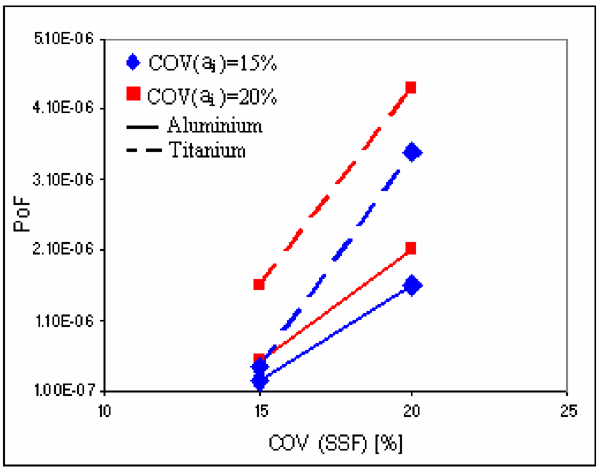

FIG 7. Probability of failure .vs. COV (*SSF*)

Considering the initial crack size and the Paris constant *C* as random variables, with COV of initial crack size equal to 20%, resulted on PoF values of 1.2×10^{-5} and 4.5×10^{-5}, respectively for Aluminium and Titanium alloys. For this situation, adding the *SSF* as a random variable, with COV of 15%, had a positive effect resulting in lower PoF values, respectively of 5.1×10^{-7} and 1.6×10^{-6} for Aluminium and Titanium alloys.

3.4.3. Reliability assessment for specific target life

The influence of the life scatter factor on the structural reliability is assessed. Both Aluminium and Titanium structures have been designed for different target life (1, 2, 3 and 4 lives). Figure 8 offers a summary of the obtained results.
Computations are carried out randomizing the initial crack size, a_i, the Stress Scaling Factor, *SSF*, and the Paris constant *C*. COVs of a_i and *SSF* are set at 20%. As before the COV of *C* is equal to 25 and 24 % respectively for Aluminium and Titanium alloys.

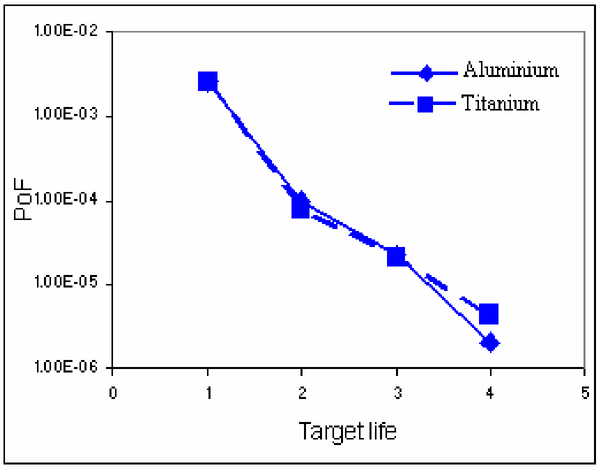

FIG 8. Probability of failure .vs. target life

Designing for a target life of 2, which represents an increase of the load level of 22% and 20% respectively for Aluminium and Titanium alloys, results in a decrease of the structural reliability of two orders of magnitude while compared with the traditional design for 4 lives.

4. EXAMPLE FOR PRESSURIZED ITEMS

In contrast to the previous case where fatigue crack growth played significant role in the structural integrity and where the fracture toughness value was not identified as relevant random parameter, a different situation is idealized for a structural part verified for a one-cycle-to-failure, representing an extreme condition. Even if this is unlikely to occur in most structural applications, it could be considered representative of certain pressurized items. Investigating the situation in which the initial crack size is close to the critical crack size allows studying the effect of lower bound values for fracture toughness and flow stress. The scatter in fracture toughness and flow stress is expected to have a significant effect on the structural reliability.

4.1. Model description

A pressurized item of 1.5 mm thickness made of Titanium alloy Ti6 Al 4V (MA forging) is subjected to one-cycle-to-failure. Statistical treatment of the material properties is carried out as before.
The presence of a through the thickness defect is assumed, as an extreme situation.

4.2. Damage tolerance verification

For damage tolerance verification a through-the-thickness crack located at the center of the plate is considered, corresponding to NASGRO configuration TC01. The initial crack size used for deterministic life assessment is equal to 2.31mm, as indicated in the NASGRO v.3 user's manual [11] for Titanium parts inspected with dye penetrant technique.
Since the structural component is verified for a single tensile cycle, the maximum stress level (see Table 7) is higher than the ones used before for fatigue loading case.
The classical deterministic design is carried out using NASGRO software, considering the failure criteria referred before in equations 2 and 3. In this specific example structural failure occurs due to criterion a), indicated in Table 7.
By means of probabilistic calculations, an investigation on the reliability associated with modified deterministic failure criteria is carried out, as summarized in Table 7.

Study Case	Investigated failure criterion	Max. Stress [MPa]
a)	$K_{\max} \geq 0.7 * K_{c,average}$	666
b)	$\sigma_n \geq \sigma_{flow}$	876
c)	$K_{\max} \geq 1 * K_{c,average}$	930
d)	$\sigma_n \geq 0.7 * \sigma_{flow}$	613

TABLE 7. Study cases investigated

4.3. Uncertainties

The uncertainties are treated in the same way as done in the former case.
Material properties uncertainties are described in Table 5. Table 8 summarizes the uncertainties for the initial crack size.

Titanium TC01		
COV (a_i) [%]	Mean a_i [mm]	St. dev. a_i [mm]
10	2.04	0.204
15	1.93	0.29
20	1.83	0.37
30	1.65	0.50

TABLE 8. Initial crack size distributions for TC01 configuration.

The stochastic representation of *SSF* is derived as before, considering 1% chance of exceeding the maximum peak stress. Dispersions between 10 and 30% are investigated.

4.4. Results

4.4.1. Sensitivity analysis

Sensitivity analyses for the different variables are carried out. Results are illustrated in Figure 9.

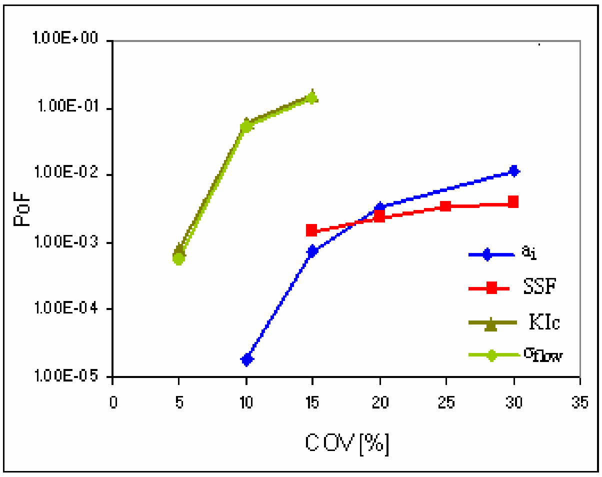

FIG 9. Sensitivity of PoF to crack depth, Stress Scaling Factor, fracture toughness and flow stress

The extreme conditions assumed in this example are reflected in the higher PoF values obtained.
While randomizing only Paris constant *C* or threshold stress intensity factor range at R=0, ΔK_0, low PoF values lead to numerical convergence problems (no simulation points in the failure domain).
Together with the initial crack size, uncertainties in fracture toughness and flow stress proved to be the most influent parameters. Considering the *SSF* as the only random variable, small sensitivity of the PoF is noticed, for this specific case. Opposite to what was seen in previous case, variability in material's resistance has a bigger influence on the structural reliability.

4.4.2. Random variables combination effect

Figure 10 illustrates the results obtained varying simultaneously the initial crack size and *SSF*.

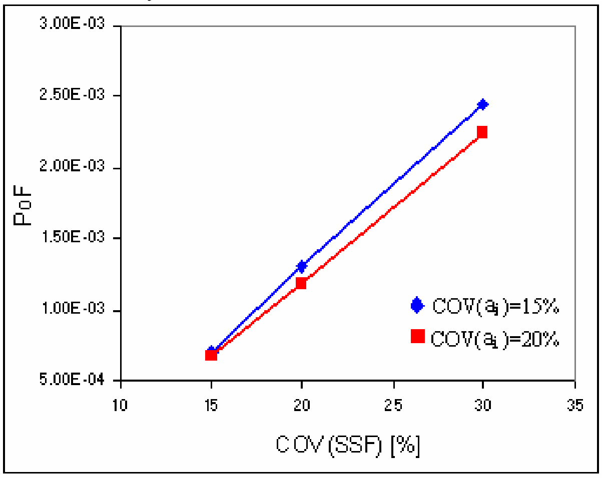

FIG 10. Probability of failure for combinations of scatter in a_i and *SSF* for case a)

Results show no significant effect identified for the studied variations.

The simultaneous randomization of the three parameters, i.e. initial crack size, *SSF* and plain strain fracture toughness or flow stress, depending on the investigated failure criteria (see Table 7) is carried out. Results are illustrated in Figure 11.
COV for the initial crack size and *SSF* is kept equal to 20%. The COV of the material's parameters varies from 5 to 20%.

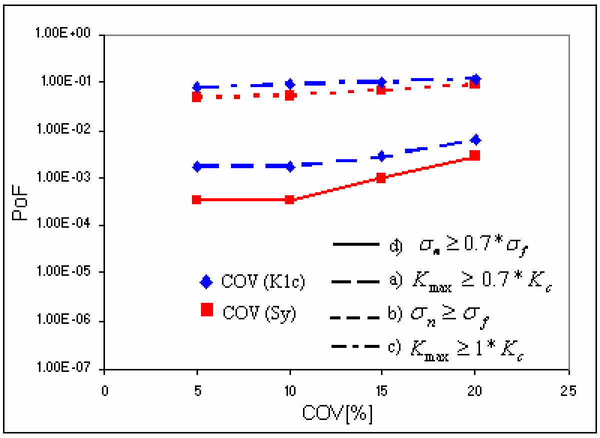

FIG 11. Probability of failure .vs. K_{Ic} or σ_{flow}

Sensitivity analysis showed that a change in COV of K_{Ic} and σ_{flow} of 5% has significant impact on the PoF. However, while randomizing simultaneously the three parameters (a_i, *SSF*, K_{Ic} or σ_{flow}), lower sensitivity is found for variations in K_{Ic} and σ_{flow}.

For this specific case and taking into account the investigated range of PoF, the application of the lower bound fracture toughness value (70% of typical value) as done in current determinist verification seems to be appropriate. This failure criterion brings additional conservatism to the design, resulting in lower PoF values.
In addition, for the investigated situation the adoption of a lower bound value for the material's flow stress causes a reduction of PoF by approximately one order of magnitude.
On the other hand, designing without taking into account lower bound values for fracture toughness and flow stress, as represented by situations b) and c) above, results on higher structural PoF, the difference being close to one order magnitude.

5. CONCLUSION

The present investigation attempts to assess the levels of conservatism associated with deterministic principles, as currently applied for damage tolerance verification of spacecraft structural parts. For this purpose, the probabilistic fracture mechanics software ESACRACK-Prob is used. It is a suitable tool for modelling uncertainties allowing efficient estimations of the probability of failure.
Random variables have been selected based on sensitivity analysis. Initial crack size and amplitude stress level proved to be the most influential parameters. The Paris region constant (*C*) has significant impact on reliability of structural applications withstanding cyclic fatigue loading. Fracture toughness and flow stress play a major role for structural components for which the initial crack size is close to the critical size. Furthermore, results indicate the adequacy and appropriateness of the practice to apply the lower bound fracture toughness, as commonly used in deterministic design verification.
The potential application of probabilistic analysis for adjusting deterministic verification parameters, i.e. life scatter factor and lower bounds values on fracture toughness and flow stress, is explored through representative examples.
The probabilistic methodology is known to have significant potential, but its implementation requires significant effort. It is currently not intended to substitute the current deterministic approach by a fully probabilistic approach in the design and verification process. The probabilistic methodology may however supplement the deterministic one for special cases, where such additional effort can be justified.
Further work should focus on gathering additional data for accurate description of the most influential random variables. Also, the effect of correlation between variables and inaccuracies in the mathematical models should be investigated further.

6. REFERENCES

[1] J.M Bloom, *Partial Safety Factor (PSF) and Their Impact on ASME Section XI, IWB-3610*. Conference on ASME Pressure vessel and Piping, Seattle, Washington, July 23-27 2000.
[2] G. Maymon, *Probabilistic crack growth behaviour of aluminium 2024-T351 alloy using the "unified" approach*, International Journal of Fatigue 27, 2005, 828-834.
[3] G. Maymon, *The Stochastic Factor of Safety – A different Approach To Structural design,* 42nd Israel Annual Conference on Aerospace Sciences, Tel-Aviv, Haifa, February 2002.
[4] G. Maymon, *On the probability of failure of structural elements with cracks*, 37th Israel Conference on Aeronautics and Astronautics, February 1997.
[5] W. Puatatsananon, V.E. Saouma, *Reliability analysis in fracture mechanics using the first-order reliability and Monte Carlo simulation,* 2006, J. Fatigue Fract Engng Mater. Struct 29, 959-975.
[6] G.I Schuëller, H.J. Pradlwarter, P.S. Koutsourelakis, *A critical appraisal of reliability estimation procedures for high dimensions*, Probabilistic Engineering Mechanics 19, 2004, 463-474.
[7] ESACRACK-Prob Version 1.0 User's Manual, 1994.
[8] F. Brunetti, *Frames 2 Software User Manual*, European Space Agency, 2005.
[9] R. Köper, *Technical note – Simplified Approach for ECLSS Damage Tolerance Verification*, 1996.
[10] ECSS secretariat, *ECSS-E-30-01A – Fracture Control*, Noordwijk, The Netherlands, 1999.
[11] National aeronautics and Space Administration (NASA), *Fatigue Crack Growth Computer Program "NASGRO" Version 3.0, Reference Manual*, JSC-22267B, August 2000.
[12] W.D. Rammel, G.A. Matzkanin, *NDE Capabilities book*, Nov. 1997.
[13] L. Tóth, *Reliability assessment of cracked structural elements under cyclic loading*, Handbook of Fatigue Crack Propagation in Metallic structures, Volume 2, edited by Andrea Carpinteri, 1994.
[14] D.F. Socie, *Probabilistic Aspect of Fatigue – Introduction*, University of Illinois at Urbana-Champaign, 2003 (www.fatiguecalculator.com).
[15] P.S. Veers, *Statistical Considerations in Fatigue*, in: ASM Handbook, vol.19, Fatigue and Fracture, ASM International, 1996.
[16] H.R. Millwater, Jr., P.H. Wirsching, *Analysis Methods for Probabilistic Life Assessment*, ASM Handbook Vol 11, Failure Analysis and Prevention, ASM International, 2002.
[17] J.N Yang, H.Y. Chen, C.M Chang and H.K. Shee, *Reliability analysis of pressure vessels based on proof test and multiple failure modes*, Structural Safety and reliability, G.I. Schuëller, M. Shinozuka, J.T.P Yao (editors), 1994.
[18] C.H. Tsai and W.F Wu, *Application of probabilistic fracture mechanics to the risk assessment of pressure vessels*, MG09/2, 12th International Conference on Structural Mechanics in Reactor Technology, 1993.
[19] M. Macke, T.K. Henriksen, G.I. Schuëller and M. Klein, *Extending ESAFATIGUE and ESACRACK for rational handling of uncertainties*, International Conference on Spacecraft Structures and Mechanical Testing, Paris, France, 1994.