A HIERARCHICAL APPROACH FOR THE BUCKLING ANALYSIS OF THE VEGA 1/2 INTERSTAGE

E. Jansen[1], J. Wijker[2], J. Arbocz[1]

[1]Delft University of Technology, Faculty of Aerospace Engineering
Kluyverweg 1, 2629 HS Delft
The Netherlands

[2]Dutch Space BV
Newtonweg 1, 2333 CP Leiden
The Netherlands

ABSTRACT

Many practical shell structures can be idealized as cylindrical shells, conical shells, and more in general, shells of revolution. In order to demonstrate the accuracy and reliability of the nonlinear buckling Finite Element analysis of the real, not-idealized structure, one can use complementary analyses (analytical and semi-analytical methods) that are available for the corresponding idealized structure. A step-by-step approach is presented for the buckling analysis of an important class of aerospace structures using methods with different levels of analysis complexity. These methods give information and provide reference solutions that are prerequisites for the preparation and interpretation of the Finite Element calculations of the real structure. This step-by-step, Hierarchical Approach is illustrated for the buckling analysis of the 1/2 Interstage of the Vega Launcher, idealized as a conical shell.

1. INTRODUCTION

The light-weight shell structures used in aerospace industry are often buckling critical. The buckling load calculations are usually carried out by one of the many available Finite Element codes. In principle, these numerical codes make it possible to simulate the behaviour of the structure accurately through detailed discretizations and advanced procedures. When one uses this advanced analysis capability, referred to here as High-Fidelity Analysis, to carry out buckling load calculations, it must be demonstrated beyond reasonable doubt that the results obtained are indeed accurate and reliable. In order to arrive at a reliable prediction of the critical buckling load and to make an estimate of its imperfection sensitivity which can be used with confidence, one must proceed step by step from relatively simple analytical solutions (denoted as Level-1 Analysis), via more refined semi-analytical models (Level-2 Analysis), to High-Fidelity Finite Element solution procedures (Level-3 Analysis).[1,2] Central in this so-called Hierarchical Approach is the possibility of identification of an *idealized structure* that corresponds to the real structure under consideration. The real, not-idealized structure typically has stiffeners, cut-outs, and reinforcements. An idealized structure is a *reference model* that is a simplification of the real structure under consideration but represents important characteristics of the geometry and behaviour of the real structure and for which analytical and/or semi-analytical methods have been developed.

For many practical structural components this reference model is available in a natural way. Typical idealized structures are main structural parts such as beams, plates, panels, and shells of revolution. Shells of revolution are at the upper level of idealized structures and they form the idealization of important structural components in various branches of engineering. Also for space applications (spacecraft and launch vehicles), different types of shells of revolution constitute the idealized structures. The analysis of shells of revolution is therefore playing a key role in the Hierarchical Approach, which consists of methods at three levels of complexity.

The strategy for a step-by-step, Hierarchical Approach is illustrated in Figure 1. Methods with a reduced level of complexity and the Finite Element model of the idealized structure give information and provide reference solutions that are essential for the preparation and interpretation of the High Fidelity Finite Element calculations for the real structure. Only when the correlation of “Level 3 – Idealized structure” results with the Level-2 results is satisfactory, one can proceed with the execution of the Finite Element analysis of the real structure (“Level-3 – Real structure”).

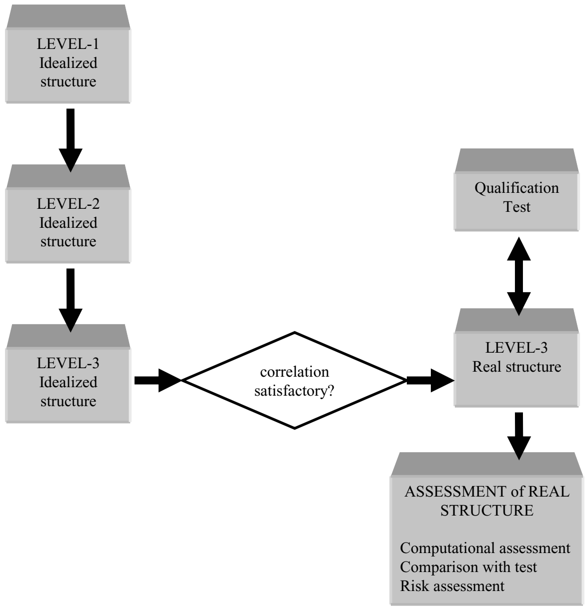

FIG 1. Hierarchical Approach for High-Fidelity Buckling Analysis

2. HIERARCHICAL APPROACH FOR BUCKLING ANALYSIS

A classification of Reduced Complexity levels based on the type of discretization makes use of three analysis levels. The different components in the Hierarchical Approach will be outlined in this section.

2.1. Level-1 - Analytical solution of idealized structure

The Level-1 Analysis is characterized by a Fourier series discretization and results in analytical solutions. Methods based on series expansions (trial function methods) are used. The two most commonly used methods are the Galerkin method, which starts from the governing differential equations, and the Rayleigh-Ritz method, which starts from an energy expression. Sets of algebraic equations are obtained. Often simple support boundary conditions are assumed, in combination with a membrane prebuckling state.

2.2. Level-2 - Semi-analytical solution of idealized structure

The Level-2 Analysis is characterized by a Fourier series discretization in one direction (circumferential direction), and numerical discretization in the second direction (meridional direction), and results in semi-analytical solutions. Shell of revolution (axisymmetric structures) codes like BOSOR and SRA fall in this class.[3,4] A one-dimensional discretization is obtained after a Fourier decomposition in the circumferential direction of the shell has been carried out. Very accurate solutions can be obtained, including the effects of boundary conditions and a nonlinear prebuckling state, by solving the resulting sets of ordinary differential equations for the meridional direction numerically by means of the Shooting Method or the Finite Difference Method. These solutions form reference solutions for the Finite Element model of the *idealized structure*.

2.3. Level-3 - Finite Element solution of idealized structure

The Level-3 Analysis is characterized by a two-dimensional or three-dimensional numerical discretization. Refined nonlinear analysis of the idealized, axisymmetric structure using codes for shells with general shape like STAGS, MSC.Nastran, ABAQUS etc. are carried out.[5,6,7]

When the agreement between Finite Element results of the idealized structure are in satisfactory agreement with the results of the Level-2 analysis, the Finite Element model of the *real structure* can be developed on the basis of the Finite Element model of the idealized structure. This can be seen as a prerequisite for the development of the Finite Element model of the real structure.

2.4. Level-3 - Finite Element solution of real structure

The “Level-3 real structure” component of the Hierarchical Approach involves the High-Fidelity nonlinear analysis for the real structure under consideration using a two-dimensional or three-dimensional numerical discretization with Finite Elements (e.g. using STAGS, ABAQUS, MSC.Nastran). It is based on the discretization of the “Level-3 idealized structure” analysis, modified to include:

- Stiffeners
- Local discontinuities (holes, reinforcements)
- Measured or assumed initial imperfections
- Measured or assumed boundary initial imperfections

2.5. Test

Testing activities include

- Test plan (number and location of strain gauges, displacement measurement, load application, etc.)
- Test procedure (including test rig and load application)
- Initial imperfection measurements
- Establishing uncertainties (stochastic analysis)
- Material characterization for stochastic analysis
- Test execution and acquisition of test data

2.6. Assessment of real structure

The assessment of the real structure includes

- Computational assessment of real structure
- Test/analysis correlation
- Risk assessment

3. VEGA INTERSTAGE 1/2

In the present paper, the Hierarchical Approach is illustrated for the buckling analysis of the 1/2 Interstage of the Vega Launcher[8], idealized as a conical shell. In particular, the first three components in Figure 1, Level-1 Analysis, Level-2 Analysis, and Level-3 Analysis of the *idealized* structure, will be considered.

The functions of the VEGA launcher interstage 1/2 structure are as follows:

- To provide an intermediate structure to connect the cylindrical structures of stage 1 and stage 2 with sufficient global stiffness.
- To transfer the mechanical loads from stage 2 to stage 1 (normal load N, shear load D and bending moment M).
- To provide a separation plane (pyrotechnics).
- To provide mounting provisions for the 6 retro rockets to create a delta velocity between the first stage and the other part of the VEGA launch vehicle.
- To provide mounting provisions for other instrumentation and harness.
- To provide a smooth surface for the airflows along the launch vehicle.
- To provide mounting for the pressure plate.

The idealized structure corresponding to the VEGA launch vehicle interstage structure that will be considered in Hierarchical Approach is a conical structure with the following characteristics:

- Top diameter 1874 mm
- Bottom diameter 2979 mm
- Height 2138 mm

An isotropic monocoque shell is used. The holes are not considered in the idealized model. The loading case of axial compression will be investigated. The following boundary conditions are applied:

- Bottom side simply supported: meridional displacement u, circumferential displacement v, and lateral displacement W are restrained, bending moment resultant $M_s = 0$.
- Top side simply supported, free to move uniformly in the vertical direction, radially restrained (stiff end ring): v is restrained, u and W are uniform in the circumferential direction, $W - u \tan\alpha = 0$, $M_s = 0$.

4. LEVEL-1 ANALYSIS

In the Level-1 Analysis, analytical solutions are used as a first characterization regarding the buckling behaviour. The critical axial compression load F_{crit} of an isotropic truncated cone with Young's modulus E, Poisson's ratio μ, wall thickness t and semi-vertex angle α can be obtained from NASA SP-8019 (Ref. 9) and Ref. 10,

(1) $$F_{crit} = \gamma \frac{2\pi E t^2 \cos^2\alpha}{\sqrt{3(1-\mu^2)}}$$

where γ is the knock down factor.

The expression for the critical buckling load is based on standard Donnell-type governing equations, and based on simplifying assumptions with respect to prebuckling state and boundary conditions. Depending on the knock-down factor used, the required wall thickness can be calculated. The order of the wall thickness that has been obtained using this formula is between 6.0 mm and 6.5 mm. The two values of the wall thickness that will be used in the present study are denoted with t_1 and a slightly higher value with t_2, respectively.

5. LEVEL-2 ANALYSIS

At the Level-2, a special purpose code for conical shells is used, BAAC (Bifurcation Analysis of Anisotropic Cones).[11] The code has been written as part of the shell buckling research and development of the suite of shell buckling codes DISDECO (Delft Interactive Shell Design Code) in the Aerospace Structures Group of the Delft University of Technology.[1] It is capable of accurately taking into account the effect of boundary conditions at the shell edges (corresponding to the Level-2 class). Using the analysis option 'membrane prebuckling' gives results that closely correspond to the Level-1 Analysis. Another code at the Level-2 used in this study is BOSOR4 (based on finite differences).[3] With these two semi-analytical methods, buckling and initial postbuckling calculations are performed. The reduction of the load carrying capability of the shell in the case of asymmetric imperfections is evaluated via initial-postbuckling and imperfection sensitivity analysis.

5.1. Numerical integration (Shooting Method)

In the present analysis, the boundary conditions at the shell edges can be taken into account accurately. The solutions of the sets of governing equations are represented by a Fourier decomposition in the circumferential direction of the shell. This reduces the problem to sets of boundary value problems with ordinary differential equations for the length direction. The specified boundary conditions at the shell edges are satisfied rigorously by solving the resulting two-point boundary value problems numerically via the parallel shooting method. The theory of the program for buckling analysis of laminated conical shells (BAAC) that will be used in this study is described in detail in Ref. 11.

The conical shell that will be used in this investigation is isotropic. However, the theory of buckling analysis of conical shells is presented in Ref. 11 for the general case of laminated shells, employing classical lamination theory. Nonlinear Donnell-type equations formulated in terms of the lateral displacement W and an Airy stress function F are used. Koiter's initial postbuckling theory, including the effect of a nonlinear prebuckling state, is applied in order to analyse the imperfection sensitivity.

Three different prebuckling state solutions can be distinguished,

- Membrane prebuckling: A membrane prebuckling state corresponds to the case that the prebuckling state is represented by the linear membrane equations. In this case bending is disregarded.
- Linear prebuckling: A linear prebuckling state corresponds to the case in which the nonlinear interaction between the membrane stress resultants and the out-of-plane deformations is neglected in the prebuckling state equations.
- Nonlinear prebuckling: A nonlinear prebuckling state corresponds to the case that the nonlinear interaction between the membrane stress resultants and the out-of-plane deformations is included in the prebuckling state equations. The full set of prebuckling state equations is taken into consideration.

An extensive list of different types of boundary conditions is included in Ref. 11. In the present semi-analytical approach, for the prebuckling state the top edge of the shell is assumed to be fixed, while the other edge is assumed to be movable. The lateral displacement W is assumed to be zero at the top edge of the cone. The boundary conditions in the prebuckling state is assumed to be “simply supported” (M_s = 0). For the buckling state, the boundary condition MSS4 (the meridional displacement is zero, u = 0) has been used in the calculations. The loading case considered is axial compression.

In the case of a nonlinear prebuckling state, the lowest buckling load occurs for 10 waves in the circumferential direction. The postbuckling modes obtained from the initial-postbuckling analysis are shown for the case of nonlinear prebuckling in Figure 2. The results will be compared with the results from the Finite Difference program BOSOR in the next section.

Using BAAC, also an estimate of the imperfection sensitivity can be obtained. The so-called b-factor gives information about the initial curvature of the load versus deflection curve of the structure without ‘asymmetric’ imperfections. In the presence of asymmetric initial imperfections, buckling occurs at a limit-point in the load versus deflection curve. For imperfect shells the following expansion is used for the load parameter Λ,

$$(2)\qquad \begin{aligned}(\Lambda-\Lambda_c)\xi &= a\Lambda_c\xi^2 + b\Lambda_c\xi^3 + \cdots \\ &\quad -\alpha\Lambda\bar{\xi} - \beta(\Lambda-\Lambda_c)\bar{\xi} + O(\xi\bar{\xi})\end{aligned}$$

where $\bar{\xi}$ corresponds to the normalized amplitude of an asymmetric initial imperfection, and the imperfection form factors α and β depend on the assumed imperfection pattern. Expressions for α and β can be found in Ref. 11. Further, $\bar{\xi}$ has been normalized with respect to the shell thickness h. The reduction of the load carrying capacity with respect to the bifurcation buckling load (the case that the structure does not have asymmetric imperfections) can be estimated using the modified Koiter formula,

(3) $$(1-\rho_s)^2 = \frac{3}{2}\sqrt{-3\alpha^2 b}\left[1-\frac{\beta}{\alpha}(1-\rho_s)\right]|\bar{\xi}|$$

where

(4) $$\rho_s = \frac{\Lambda_s}{\Lambda_c}$$

is the ratio between the limit-point buckling load Λ_s and the bifurcation buckling load Λ_c.

Two types of imperfection shapes can be distinguished, an imperfection with the same shape as the buckling mode, referred to as affine imperfection, and an imperfection with a shape specified in advance, referred to as modal imperfection. In the case of an affine imperfection, for the Vega shell the reduction in buckling load is very significant (Table 1). Imperfection amplitudes of 0.1 to 0.5 of the wall thickness give considerable reductions as compared to the bifurcation buckling load. The results should be interpreted with caution. The initial postbuckling and imperfection sensitivity analysis provide approximations to the load deflection curves that are valid close to the bifurcation point.

Imperfection amplitude $\bar{\xi}$	$\rho_s = \frac{\Lambda_s}{\Lambda_c}$
0.1	0.81
0.5	0.29

TABLE 1. Reduction of load carrying capacity (t = t_2).

It is noted that when a modal imperfection is specified with one full wave in the meridional direction and 10 waves in the circumferential direction, the limit-point buckling load leads to more moderate reductions, in the order of a few percent.

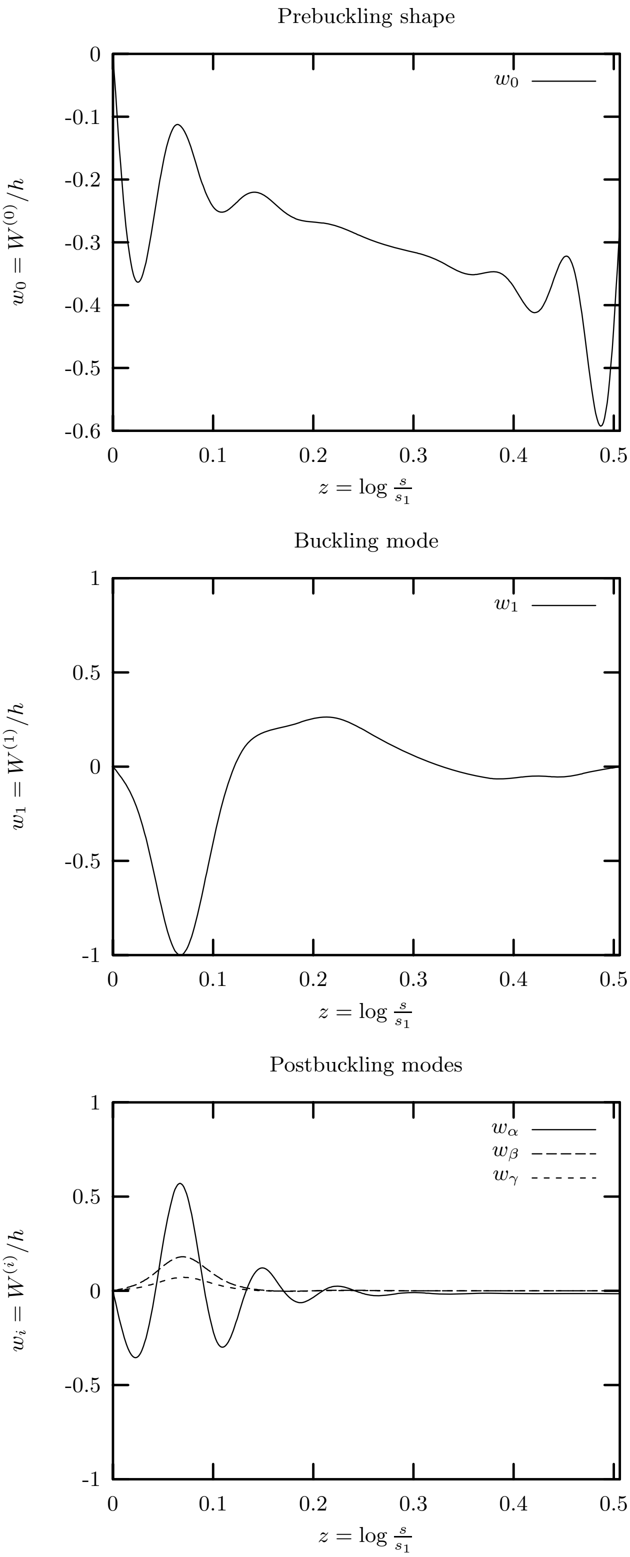

FIG 2. Prebuckling mode, buckling mode, and the different components of the postbuckling mode obtained with semi-analytical program BAAC using nonlinear prebuckling (t = t_2); s_1 is the meridional coordinate at the top edge.

5.2. Finite Difference Method: BOSOR

The BOSOR4 software package is a stability analysis program for axi-symmetric thin-walled structures based on the Finite Difference Method.[3] Buckling load and corresponding wave number N are determined, including nonlinear prebuckling effects.

Within the software package BOSOR4 (Buckling Of Shells Of Revolution) the applied loads are considered to be axi-symmetric. The loading case considered here is axial compression. The boundary conditions as specified in Section 3 are imposed.

Three BOSOR4 models have been made in order to study the convergence characteristics. Models with 100, 200, and 300 nodes have been used. The analysis results for these cases and for two different values of the wall thickness ($t = t_1$ and $t = t_2$) are given in Tables 2 and 3. In these tables, also the buckling loads obtained with the program BAAC are shown. The buckling mode obtained for the model with 300 nodes (for the thickness $t = t_1$) is shown in Figure 3.

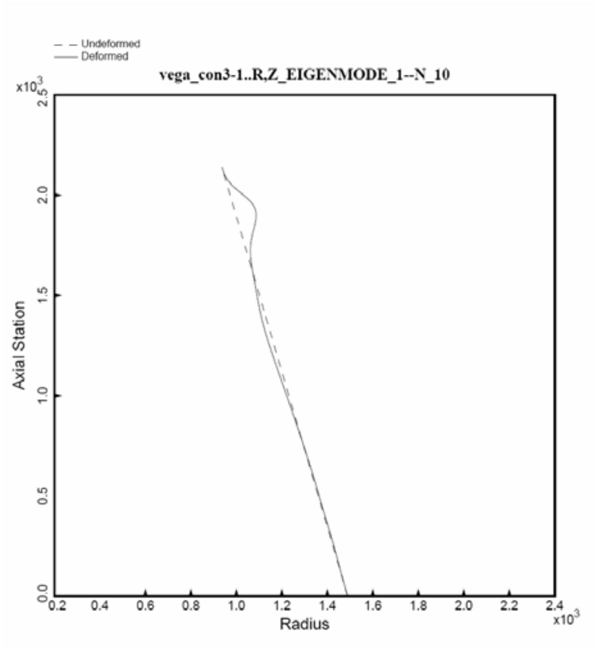

FIG 3. Buckling mode obtained with semi-analytical program BOSOR4 using nonlinear prebuckling ($t = t_1$).

There is a very good agreement between the results obtained by the two different programs for the 300 nodes BOSOR model. The BOSOR4 buckling analysis show a minimum buckling load for a mode with 10 waves in the circumferential direction and this is in accordance with the results of BAAC.

Analysis Tool	Buckling load (N/mm)	Wave number N
BOSOR4 (100 nodes)	1524.6	10
BOSOR4 (200 nodes)	1478.1	10
BOSOR4 (300 nodes)	1473.6	10
BAAC	1477.4	10

TABLE 2. Buckling loads including nonlinear prebuckling ($t = t_1$).

Analysis Tool	Buckling load (N/mm)	Wave number N
BOSOR4 (100 nodes)	1596.2	10
BOSOR4 (200 nodes)	1553.4	10
BOSOR4 (300 nodes)	1544.8	10
BAAC	1546.9	10

TABLE 3. Buckling loads including nonlinear prebuckling ($t = t_2$).

6. LEVEL-3 ANALYSIS (FINITE ELEMENTS)

At Level-3, Finite Element calculations for the conical shell under consideration have been carried out using the ABAQUS code. The ABAQUS general purpose finite element software package is capable of performing linear and nonlinear buckling analysis.[7] The results will be compared with the Level-2 analysis results.

The loading conditions are axisymmetric (axial compression). The running load q_0 (N/mm) is converted to discrete loads in 120 nodes. The boundary conditions that are used in the Finite Element model were specified in Section 3. The S4 shell element is applied in order to obtain reliable results.

Firstly, a linear prebuckling state was assumed. The linear bifurcation analysis results in a buckling load that is significantly higher than the buckling load obtained from the BOSOR4 analysis including a nonlinear prebuckling state, and the associated buckling mode is different. The ABAQUS bifurcation analysis compares very well with a linear bifurcation analysis using MSC.Nastran.[6] The buckling loads are given in Table 4. The buckling mode associated with a linear prebuckling state is illustrated in Figure 4.

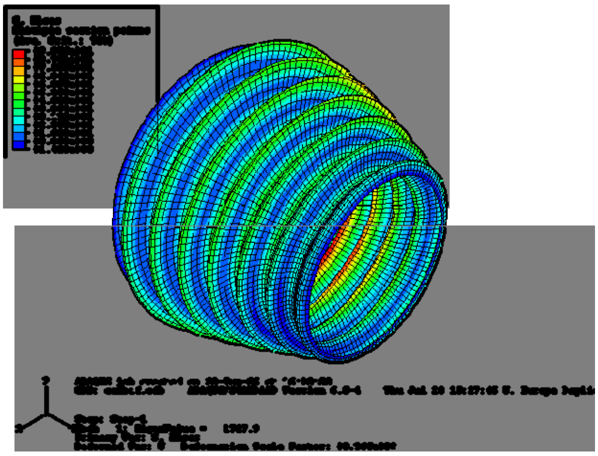

FIG 4. Buckling mode obtained with ABAQUS using a linear prebuckling state (t = t_1).

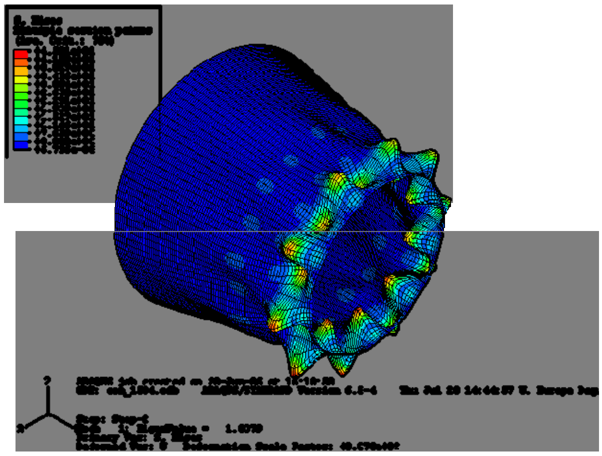

FIG 5. Buckling mode obtained with ABAQUS using a nonlinear prebuckling state (t = t_1).

Analysis Tools	Linear Bifurcation Load (N/mm)
MSC.Nastran	1718.15
ABAQUS	1713.26

TABLE 4. Comparison of buckling loads using linear prebuckling (t = t_1).

Subsequently, a nonlinear prebuckling analysis was used in combination with a bifurcation analysis. In this case, the stability analysis in the ABAQUS finite element software package is done in two steps. The first step is a nonlinear geometric prebuckling analysis followed by a linearized eigenvalue analysis. The following eigenvalue problem is solved,

(5) $$([\mathrm{K}_0] + \lambda[\Delta\mathrm{K}])\{\Phi\} = \{0\},$$

where $[K_0]$ is the tangent stiffness matrix at the end of the nonlinear prebuckling analysis and $[\Delta K] = [K_{P_0+\Delta P} - K_0]$ is the geometric stiffness matrix, λ is the eigenvalue to be multiplied by the applied load (combination) ΔP, and $\{\Phi\}$ is the associated buckling mode.

For the wall thickness t = t_1, the buckling load including a nonlinear prebuckling state is found at $q_0 = 1505$ N/mm for a mode with 10 waves in the circumferential direction of the shell. The buckling mode is shown in Figure 5.

The buckling analysis including a nonlinear prebuckling state with ABAQUS gives results that agree reasonably well with the Level-2 Analysis results (BOSOR finite difference results and BAAC shooting method results). It is noted that in order to ensure the quality of the model one should carry out a convergence study. Using appropriate mesh refinement one can attain results that correspond closely to the Level-2 Analysis results.

The ABAQUS Finite Element results presented here, obtained using a mesh with 120 elements in the circumferential direction and 100 elements in the longitudinal direction, predict the buckling behaviour of the idealized Vega Interstage model including a nonlinear prebuckling state with reasonable accuracy. The mesh is able to capture the correct number of circumferential waves. The MSC.Nastran results were obtained with fewer elements in the circumferential direction (100 elements).

The buckling load level of the most accurate BOSOR model (300 nodes) and the result of the semi-analytical program BAAC are somewhat lower than the results obtained with the ABAQUS and MSC.Nastran Finite Element models. The buckling loads including a nonlinear prebuckling state are summarized in Table 5. The result of the BOSOR model with 100 nodes is included.

Analysis Tool	Bucklin g load (N/mm)	Wave numbe r N
BOSOR 4 (100 nodes)	1524.6	10
MSC.Nastra n	1517.3	9
ABAQUS	1505.0	10

TABLE 5. Comparison of buckling loads using nonlinear prebuckling (t = t_1).

A nonlinear buckling analysis (limit point analysis) was done assuming the first bifurcation buckling mode, with a nonlinear prebuckling analysis, as initial imperfection. The amplitude of the imperfection was scaled with respect to the wall thickness of the cone ($t = t_1$),

- 25% of the wall thickness ($\bar{\xi} = 0.25$)
- 50% of the wall thickness ($\bar{\xi} = 0.5$)

The results are tabulated in Table 6, where the limit-point buckling load is denoted as Λ_s and the bifurcation buckling load as Λ_c.

Imperfection amplitude $\bar{\xi}$	$\rho_s = \frac{\Lambda_s}{\Lambda_c}$
0.25	0.62
0.5	0.48

TABLE 6: Reduction of load carrying capacity ($t = t_1$) using ABAQUS.

The results of the imperfection sensitivity analysis (for a different wall thickness) using the semi-analytical program BAAC also showed a significant sensitivity to an affine initial imperfection (imperfection with the shape of the first buckling mode).

7. CONCLUDING REMARKS

The Hierarchical Approach for High-Fidelity Buckling Analysis has been illustrated for a typical case, a conical shell, representative of the 1/2 interstage of the Vega Launcher. The results provide reference solutions for the Finite Element modelling and calculations of the real structure with cut-outs and reinforcements. One should proceed with the Finite Element modelling of the real structure, only if the correlation between the Finite Element results of the idealized structure and the Level-2 results is satisfactory.

It should be noted that the approach requires good background knowledge about the buckling and post-buckling analysis of thin-walled shell structures, and about the dedicated and general purpose software codes that are available. In order to obtain reliable buckling predictions, it is strongly recommended that the Hierarchical Approach proposed in this paper is applied when performing High-Fidelity Analysis for practical aerospace applications.

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