MULTI OBJECTIVE OPTIMIZATION OF A FAN BLADE USING AN ADVANCED PARAMETERIZATION METHOD

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Abstract. Optimal design techniques are not routinely used in the industry when dealing with complex physical phenomena, due to high computing costs. The parameterization method described in this paper is based on the differentiation and high-order Taylor-series expansion of the discretized Reynolds-Averaged Navier-Stokes equations. A flow database containing the derivatives of the physical variables with respect to the design variables is produced by the Turb'Opty[®] parameterization tool and thoroughly explored by a multi-objective Genetic Algorithm coupled to the extrapolation tool Turb'Post[®]. The optimization case of an automotive engine cooling fan blade is fully described. Five geometric parameters have been chosen to characterize the fan blade. Three objective functions have been taken into account: the minimization of the loss coefficient, the maximization of the static pressure rise and the minimization of the torque. Two geometric constraints have been imposed to the extrapolated profiles: the monotonicity of the thickness variation and the convexity of the pressure and suction sides. Quantitative results are finally discussed. A noticeable reduction in CPU time cost has been demonstrated.

NOMENCLATURE

ρ, E	Density and specific total energy
V	Velocity vector
k,ω	Specific turbulent kinetic energy and vorticity
$P_{\rm S}, P_{\rm T}$	Static and total pressures
q	Vector of conservative and turbulent variables
p	Vector of parameters
F	Discretized Reynolds-averaged Navier-Stokes flux vector
G	Jacobian matrix
R	First-order residual due to a variation Δp of p
$oldsymbol{q}^{(n)},oldsymbol{G}^{(n)},oldsymbol{R}^{(n)}$	$n^{\rm th}\text{-}{\rm order}$ total derivatives of ${\boldsymbol q},{\boldsymbol G}$ and ${\boldsymbol R}$ with respect to ${\boldsymbol p}$
\widetilde{q}	Reconstructed solution
$\widetilde{\omega}, \Delta P_{ m s}, M$	Cascade loss coefficient, static pressure rise and torque
$x_{\rm in}, x_{\rm out}$	Values of the variable x imposed at the inlet and outlet

1 INTRODUCTION

In the design of engine cooling systems, CFD plays a key role for the optimization of axial fans and, more recently, in the performance analysis of the whole fan system. Unfortunately, the computational times required by 3D simulations still limit the possibility of varying the necessary number of design parameters to cover a relevant map of fan performance characteristics. Indeed, for a given number P of geometrical and/or physical parameters (e.g. 5), the total number of numerical experiments would be $N^P = 10^5$, if each parameter was given N = 10 different values. Therefore, even the 2D preliminary design of a fan blade is still a challenge.

Simplified flow solvers, clever optimization schemes and interpolation tools, such as neural networks [9] or gradient methods [10], have been recently investigated to overcome the previous drawbacks. However, these approaches can only provide discrete solutions. On the contrary, the parameterization method described in [2, 7] provides all the results corresponding to a continuous variation of the design parameters in one single design step, which means one flow computation on one grid. This method is based on the differentiation and high-order Taylor-series expansion of the discretized Reynolds-Averaged Navier-Stokes (RANS) equations around an independently computed reference flow solution. It has been implemented by Fluorem in the CFD solver Turb'Flow[©] developed at the ECL/LMFA laboratory [1] to yield the parameterized code Turb'Opty[©].

The paper is organized as follows. The parameterization method and the Genetic Algorithm (GA) that has been used in the optimization loop are successively introduced in section 2 and section 3. Section 4 describes the coupling procedure between the different tools. The blade cascade test-case and the results are respectively presented and analyzed in section 5 and in section 6. Finally, some conclusions are drawn in section 7.

2 PARAMETERIZATION METHOD

The parameterization method is based on the differentiation of the RANS equations and the computation of a high-order Taylor-series expansion of their solution. The discretized steady Navier-Stokes equations are first written in the symbolic form:

. . .

$$\boldsymbol{F}(\boldsymbol{q}(\boldsymbol{p}), \boldsymbol{p}) = 0 \tag{1}$$

with \boldsymbol{F} being the flux vector expressing mass, momentum and energy conservation with respect to the conservative variables $(\rho, \rho \boldsymbol{V}, \rho E)$ and the transport of turbulent variables ρk and $\rho \omega$. \boldsymbol{F} includes both convective and viscous fluxes. From equation 1, the first-order derivation with respect to the design variables \boldsymbol{p} gives:

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{q}}(\boldsymbol{q}, \boldsymbol{p}) \cdot \boldsymbol{q}^{(1)} \cdot \Delta \boldsymbol{p} = -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}}(\boldsymbol{q}, \boldsymbol{p}) \cdot \Delta \boldsymbol{p}$$
(2)

where $q^{(1)}$ is the desired first-order derivative of q with respect to the parameters vector p. Denoting $G = \partial F / \partial q$ the Jacobian matrix and $R(q, p, \Delta p)$ the right hand side of equation 2, the high-order derivatives $q^{(n)}$ of q with respect to p are then recursively built by subsequent multi-parameters total differentiations of equation 2:

$$\boldsymbol{G} \cdot \boldsymbol{q}^{(1)} \cdot \Delta \boldsymbol{p} = \boldsymbol{R}$$
(3)

$$\boldsymbol{G} \cdot \boldsymbol{q}^{(2)} \cdot \Delta \boldsymbol{p} = \boldsymbol{R}^{(1)} - \boldsymbol{G}^{(1)} \cdot \boldsymbol{q}^{(1)} \cdot \Delta \boldsymbol{p}$$
(4)

$$\boldsymbol{G} \cdot \boldsymbol{q}^{(n)} \cdot \Delta \boldsymbol{p} = \boldsymbol{R}^{(n-1)} - \sum_{i=1}^{i=n-1} C_{n-1}^{i} \boldsymbol{G}^{(i)} \cdot \boldsymbol{q}^{(n-i)} \cdot \Delta \boldsymbol{p}$$
(5)

Finally, the flow field corresponding to the modified parameters vector $\mathbf{p} + \Delta \mathbf{p}$ can be approximated by a multi-parameters high-order Taylor-series expansion [4]:

$$\widetilde{\boldsymbol{q}} = \boldsymbol{q}(\boldsymbol{p}) + \boldsymbol{q}^{(1)} \cdot \Delta \boldsymbol{p} + \dots + \frac{\boldsymbol{q}^{(n)}}{n!} \cdot \Delta \boldsymbol{p}^n = \boldsymbol{q}(\boldsymbol{p} + \Delta \boldsymbol{p}) + O(\Delta \boldsymbol{p}^{n+1})$$
(6)

In equation 6, the truncation error is of the order of magnitude of $\Delta \mathbf{p}^{(n+1)}$ and $\tilde{\mathbf{q}}$ satisfies the equilibrium condition to the n^{th} -order approximation:

$$\boldsymbol{F}(\boldsymbol{\tilde{q}}, \boldsymbol{p} + \Delta \boldsymbol{p}) = 0 + o(\Delta \boldsymbol{p}^n) \tag{7}$$

3 EFFICIENT GA FOR COMPLEX OPTIMIZATION PROBLEMS

The GA that has been used in this study is based on a previous version of a home-made computer code developed at the University of Liège/Turbomachinery Group. It has already been validated and successfully applied to design problems [5, 6]. This GA includes the classical genetic operators, and its main features are the following: a real-valued coding for the decision variables, a BLX-alpha and a one-point crossover respectively for the continuous and the discrete design variables, a mutation operator, and a Pareto based approach coupled with an efficient constraint-handling technique.

As we can see in figure 1, the constraints are firstly evaluated for each individual. The feasible solutions are ranked according to the MOGA algorithm proposed by Fonseca and Fleming [3]. Besides, the infeasible solutions are assigned a ranking factor according to MOGA on the basis of the violation of the constraints. At last, a selection, based on a "penalized tournament", is applied. This consists of randomly choosing and comparing the (generally two) individuals :

- if both are feasible OR infeasible, the best ranked element (according to MOGA) wins,
- if one is feasible and the others are infeasible, the feasible individual wins.

An archiving procedure has also been added to the GA. This new operator externally stores the non-dominated solutions found at each generation by copying all the individuals of the current Pareto front to the archive, removing any dominated solutions from the archive and applying a clustering strategy [8] if the number of non-dominated individuals in the archive is greater than a given maximum N_{arch} (figure 1). This archiving procedure has been inspired from the SPEA (Strength Pareto Evolutionary Algorithm) proposed by Zitzler [11]. However, in our implementation, the individuals stored in the archive do not participate to the selection phase, which results in a less disturbed evolution process.

The clustering step (e.g. reducing the size of the archive while maintaining its characteristics) is mandatory: the Pareto front (and the archive) could sometimes contain a huge number of non-dominated individuals. However, the designer is not interested in being offered too many solutions from which he has to choose.



Figure 1: Proposed GA flowchart

4 COUPLING PROCEDURE BETWEEN THE DIFFERENT TOOLS

The geometric parameterization is based on the preprocessing code *Turb'Mesh*[©] which provides the Taylor-series expansion of the computational grid with respect to the shape parameters, to a specified order, hence allowing smooth reconstruction of a new grid for any variation of the geometrical parameters.

The distortion of the blade profile and the associated movement of the grid points are analytically defined by a devoted tool developed by Fluorem. Once the grid parameterization is achieved with $Turb'Mesh^{\odot}$, the flow parameterization can be computed with $Turb'Opty^{\odot}$, with the flow solution obtained by $Turb'Flow^{\odot}$ on the reference grid used as the reference flow field.

The GA explores the design space through an exploitation, with the *Turb'Post*[©] extrapolation tool, of the database provided by the parametric flow solver, which contains high-order derivatives of the physical variables with respect to the design variables.

5 DESCRIPTION OF THE INDUSTRIAL TEST-CASE

5.1 Blade cascade configuration

The configuration is a blade-to-blade bidimensional cut of an isolated rotor chosen by Valeo Motors and Actuators. A 13-domains grid in the (x, y) plane with a total of 30 400 points was created by Fluorem. The domains outline is drawn in figure 2. The grid x and y directions are respectively associated with the rotation and pitch periodicity, and with the upstream direction, the inlet being located at maximum y.



Figure 2: Grid configuration

5.2 Physical model and boundary conditions

The fluid is modeled as a viscous perfect gas. The heat capacity ratio is $\gamma = 1.4$ and the perfect gas constant is $r = 287 \,\mathrm{J \ kg^{-1} \ K^{-1}}$. The dynamic viscosity $\mu = 1.82 \,10^5 \,\mathrm{kg \ m^{-1} \ s^{-1}}$ and the thermal conductivity $\lambda = 2.54 \,\mathrm{kg \ m \ s^{-3} \ K^{-1}}$ are assumed independent of the temperature.

The density, the relative speed and the turbulent variables are imposed at the inlet:

$$\rho_{\rm in} = 1.164 \,\rm kg \, m^{-3} \tag{8}$$

$$U_{xin} = 31.41593 \text{ m s}^{-1} \tag{9}$$

$$V_{y_{\rm in}} = -7.72145 \,\mathrm{m \, s^{-1}}$$
 (10)

$$k_{\rm in} = 1.412885 \,\mathrm{m}^2 \,\mathrm{s}^{-2} \tag{11}$$

$$\omega_{\rm in} = 440.2401 \, {\rm s}^{-1} \tag{12}$$

The outlet static pressure is set to $P_{\text{sout}} = 10^5 \text{ Pa.}$

A no-slip condition is imposed on the blade profile which is assumed adiabatic. Periodic boundary conditions are applied in the pitch direction.

A low-Mach number turbulent flow is first computed around the reference fan blade cascade with $Turb'Flow^{\odot}$ using a low-mach preconditioning method in order to yield the reference flow field.

5.3 Geometric Parameterization

In this study, the geometric parameterization is led on the blade cascade case with five design parameters, which are:

- the stagger angle (δ) ,
- the tangent to the camber line at the leading edge angle (A),
- the tangent to the camber line at the trailing edge angle (B),
- the ratio of the maximum camber to the chordlength (d),
- the ratio of the maximum thickness to the chordlength (e).

The variation ranges of these parameters are summarized in table 1. They correspond to automotive engine cooling fan applications.

Parameter	Minimum	Reference	Maximum
δ (°)	-25.6212	-23.2920	-20.9628
A	0.1173	0.2346	0.3519
B	-0.2600	-0.1734	-0.0867
d	0.0353	0.0588	0.0883
e	0.0114	0.0229	0.0344

Table 1: Initial range for the blade cascade

The calculation of the reference flow field and the whole parameterization need about 30 hours CPU time. An optimization loop based on a GA and complete flow computations would require a tremendous amount of CPU time (several thousands of hours for about 3000 flow computations). The advantage provided by the parameterization method in this optimization approach is therefore clear.

5.4 Geometric constraints

Two geometric constraints have been defined for the extrapolated profiles. The first one enforces the monotonicity of the thickness variation along the profile. The second constraint ensures that the number of couples of inflexion points equal one.

6 RESULTS

6.1 Two-objective optimization: pressure rise and loss factor

In this first case, we consider a Pareto-based optimization of two objective functions. The first one contains the static pressure rise:

$$\Delta P_{\rm S} = P_{\rm Sout} - P_{\rm Sin} \tag{13}$$

which has to be maximized. The second objective corresponds to the loss coefficient:

$$\widetilde{\omega} = \frac{P_{\rm Tin} - P_{\rm Tout}}{P_{\rm Tin} - P_{\rm Sin}} \tag{14}$$

and has to be minimized.

The proposed GA (section 3) has been run with a population of $N_{pop} = 500$ individuals during $N_{gen} = 100$ generations (corresponding to 50 000 calls to the extrapolation tool Turb'Post[©] in order to exploit the database provided by the parametric flow solver).

Figure 3 shows the 241 Pareto-optimal solutions that have been obtained at the end of the optimization process. The CPU time required by the optimization loop was about 1850 seconds. The $N_{arch} = 20$ solutions stored in the archive are also depicted in figure 3. All the non-dominated solutions clearly underline the conflicting nature of the two objectives.



Figure 3: Pareto front and Archive - Maximization of $\Delta P_{\rm S}$ with minimization of $\tilde{\omega}$

Two optimal configurations located near the extreme sides of the Pareto front have been retained. The first one maximizes the static pressure rise to the detriment of a higher loss factor, and the second one minimizes the loss coefficient, thus involving a lower static pressure gain. The values of the objective functions and the torque are given for both configurations in table 2.

For each solution, another profile has been finally tested with an increased thickness, because the optimal profiles did not meet the manufacturing requirements.

The profiles maximizing ΔP_s and minimizing $\tilde{\omega}$ are respectively displayed in blue and red in figure 4, where the optimized profiles with the increased thickness are drawn in bold. Compared to the reference profile, both new configurations have a decreased stagger angle. The maximum camber is also decreased, but in a stronger way in the case of the minimization of the loss factor. The absolute values of the tangent to the camber line at the leading and the trailing edges are increased.

	Reference	$\Delta P_{\rm Smax}$	$\Delta P_{\rm Smax}$	$\widetilde{\omega}_{\min}$	$\widetilde{\omega}_{\min}$
			(thick)		(thick)
$\Delta P_{ m s}$	269.5200	290.4900	287.1500	289.0000	285.6570
$\widetilde{\omega}$	0.1408	0.1265	0.1270	0.1248	0.1253
M	1.0750	1.1240	1.1120	1.1140	1.1020

M. N'Diaye, G. Grondin, V. Kelner, P. Ferrand, S. Moreau

Table 2:	Maximization	of $\Delta P_{\rm S}$	and	minimization	of $\widetilde{\omega}$
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Figure 5 respectively presents the iso-velocity contours of the reference flow and the extrapolated flow around the $\tilde{\omega}_{\min}$ optimized profile. One can observe that the flow separation area at the trailing edge is strongly reduced in thickness. This is the main reason why the fan blade performances are improved.



Figure 4: Profiles for $\widetilde{\omega}$ and $\Delta P_{\rm S}$ optimization



Figure 5: Velocity modulus for reference (left) and $\tilde{\omega}_{\min}$ -optimized (right) profiles

6.2 Three-objective optimization: pressure rise, loss factor and torque

In this last optimization case, the torque (M), which is the product of the force in the x direction by the fan radius at the bidimensional cut, has been added as a third objective function. It has to be minimized simultaneously with the optimization of the two previous objectives (pressure rise and loss factor).

The GA has been run with the same parameters $(N_{pop} = 500, N_{gen} = 100, N_{arch} = 20)$.

Figure 6 presents the archive and the 350 Pareto-optimal solutions found at the end of the optimization process. The CPU time required by the optimization loop was about 2600 seconds.



Figure 6: Pareto front and Archive - Maximization of $\Delta P_{\rm S}$ with minimization of $\tilde{\omega}$ and M

The Pareto surface describing the trade-off between the three objectives shows that $\Delta P_{\rm s}$ and $\tilde{\omega}$ are strongly correlated with each other in respect to M. One can observe that the pure minimization of the torque would yield a low static pressure rise and a high loss factor. Consequently, the optimal profiles are quite different from those obtained by the two-objective optimization. In that case, the torque was overvalued (table 2).

Three configurations have been hereby retained. The first one presents the same torque as the reference profile (M_{ref}) , the second one minimizes the torque (M_{\min}) , and the last one corresponds to an intermediate torque value (M_{mid}) . The values of the three objective functions are summarized for each retained configuration in table 3.

	$M_{\rm ref}$	M_{\min}	$M_{\rm mid}$
M	1.0770	0.9590	1.0160
$\Delta P_{ m s}$	279.3300	242.5300	261.5500
$\widetilde{\omega}$	0.1255	0.1348	0.1285

Table 3: Minimization of M

As usual in the framework of multi-objective evolutionary optimization, the final choice of a preferred configuration will be *a posteriori* determined by a smart compromise between the different objectives, based on engineering criteria.

7 CONCLUSIONS

The coupling procedure between a genetic optimizer, a flow parameterization method and an extrapolation tool has been proved feasible. All the results demonstrate the complementarity of these softwares. The performances of the optimized profiles have been noticeably improved. Moreover, a significant reduction of the computational time is offered by this cutting edge approach. The flexibility of the proposed method would allow a very quick implementation of other objectives.

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