PARAMETRIC AIRCRAFT TRAJECTORY MODEL FOR TAKEOFF AND DEPARTURE

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ABSTRACT

The European Commission funded CREDOS (Cross wind reduced separations for departure operations) project aims to demonstrate the validity of reducing departure separations under conditions of sufficient crosswind. For this validation a safety assessment is required that computes encounter probabilities and encounter severity by means of Monte Carlo Simulation (MCS).
This paper describes the development of an aircraft trajectory model for departure of vortex generating and following aircraft. This model is a part of the MCS. The requirement to generate trajectories for different aircraft types can be fulfilled by a simple point mass model consisting of three main parts: (i) a database of all relevant aircraft parameters, nominal departure route information, meteorological conditions, (ii) pilot model, and (iii) the description of the aircraft dynamic by the equations of motion. The simulated flight path is subdivided into segments that are described by the following flight phases: lift-off, unsteady climb, steady climb, acceleration, and transitions between these phases.
Finally, results of a deterministic model validation based on high-fidelity flight simulation for a specific aircraft as well as the model validation based on aircraft trajectories measured at Frankfurt airport are presented.

1 INTRODUCTION

1.1 Background

The need to increase airport capacity is one of the major challenges for Air Traffic Management (ATM) research today. Major European airports are operating close to their maximum capacity, and often face large delays. Recent research suggests that such an increase could be achieved by reducing the current wake vortex separation minima while maintaining levels of safety.
For single-runway operations above a certain cross wind threshold, vortices are blown out of the flight corridor and pose no further threats to following aircraft. Therefore, such crosswind conditions offer a large potential for the tactical use of reduced aircraft separations, leading to increase airport capacity and reductions in mean airport delays.

1.2 Objective

The European Commission funded CREDOS (Cross wind reduced separations for departure operations) project aims to demonstrate the validity of reducing departure separations under conditions of sufficient crosswind. For this validation a safety assessment is required that computes encounter probabilities and encounter severity by means of Monte Carlo Simulation (MCS).

WakeScene, a software tool that is currently under development, was used to demonstrate aircraft arrivals under varying conditions, [8]. In CREDOS; WakeScene shall simulate departure trajectories of leading aircraft with generated wake vortices along the flight path and the trajectories of trailing aircraft to demonstrate and prove the CREDOS safety concept. Such simulations need validated parametric trajectory models.
Here, the objective is the development of such an aircraft trajectory model that will be integrated into WakeScene. The trajectory model simulates the aircraft dynamics, depending on the control inputs of the pilot or auto pilot, the aircraft type, weight, thrust mode and the atmospheric conditions. This paper describes the design of the trajectory model as well as verification and first validation results.

2 TRAJECTORY MODEL

2.1 Requirements

Aircraft trajectories shall be modelled for departure, beginning on the runway along a standard departure route until 3000 ft above ground level. Many environment and aircraft specific parameters influence an aircraft trajectory. The model is able to simulate sensitivities depending on parameters of main influence. The first step was to identify such parameters:

- Different standard departure routes and runways;
- Meteorological conditions, which include air temperature, density, pressure, wind direction and strength;
- Aircraft types;
- Aircraft takeoff weights;
- Takeoff thrust mode, takeoff go around (TOGA) thrust, or flex takeoff thrust (reduced thrust);
- Start position on the runway;
- Pilot behaviour.

These factors can be varied in between defined boundaries with given probability distributions in a Monte Carlo Simulation (MCS) to generate a set of trajectories for different aircraft types and departure conditions.
To describe the complete aircraft dynamics along all axes, the equations of motion for six degrees of freedom (DoF) are needed; three translational and three rotational. To describe an aircraft trajectory only, it is sufficient to model the translational degrees of freedom. Therefore, the objective to generate trajectories for different aircraft types can be fulfilled by a simple point mass model (3DoF).

2.2 Modules

The simulation of the aircraft trajectories is realised by the development of six software modules that receive their input from three databases, see FIG 1.

1) "A/C Database" contains all relevant aircraft performance and geometric parameters.
2) "Jeppesen Database" defines standard departure routes (SID) from German airports by waypoint lists, speed or altitude restrictions.
3) "Navigation" generates a nominal flight path for the chosen SID with the waypoint list and the altitude restriction given by the "Jeppesen Database", in advance of the trajectory simulation. The module commands the desired heading. For the nominal vertical profile, a phase table is created that contains the following segments: lift-off, unsteady climb, steady climb, acceleration, and transitions in between the phases. Depending on the actual phase table, the module provides a so called "Energy Sharing Index", which defines - multiplied with the potential aircraft climb angle - a commanded climb angle for the trajectory simulation.
4) "Guidance" determines the aircraft deviation from the nominal flight path, which is a required input for the "Pilot Model" to correct the aircraft flight path.
5) "Environment Database" contains meteorological and runway conditions.
6) "Performance" calculates the required aircraft speeds for each segment.
7) "Aircraft Dynamics" is the implementation of the Equations of Motion, which describe the aircraft's dynamic behaviour.
8) The "Pilot model" cares for aircraft guidance along the commanded nominal flight path during takeoff and departure.
9) "Flight Path" integrates aircraft velocities to positions.

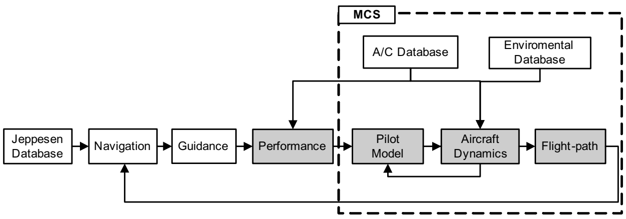

FIG 1: Modules of the Trajectory Model

2.3 Aircraft Database

All aircraft performance and geometric data i.e. lift, drag, thrust and fuel flow is based on the Eurocontrol's Base of Aircraft Data (BADA) [2].

2.4 Environmental Database

Aircraft trajectories depend on aircraft performance, wind, temperature, air pressure, and meteorological data. To be consistent, the same source of weather information has to be used for computation of trajectories and vortex evolutions.

DLR generated a database that contains nine meteorological parameters in 40 different altitudes predicted for every ten minutes over one year at 25 different positions along the centreline of the runway 07/25 at Frankfurt/Main airport. This database was generated with a software tool called NOWVIV [9]. It was generated under contract of Airbus and it is used in the CREDOS project by courtesy of Airbus.

As the departure lasts less then five minutes and the route leaves the centreline, the NOWVIV dataset at the airport itself is used during the whole simulation. Within the airspace used for the departure, this gives a vertical wind and temperature profile with a fine resolution of about 20 m.

The following parameters are used for trajectory calculation: air pressure at sea level (QNH), as well as wind speed, wind direction and temperature as a function of altitude. With these parameters all necessary inputs for the thrust and drag calculation like air density, speed of sound or dynamic pressure are calculated with the ICAO standard atmosphere model. The influence of vertical wind is assumed to be negligible for the trajectory prediction because up- or downwinds are usually spatially limited.

2.5 Navigation

The simulation of the trajectory starts at the beginning of the runway with engines at idle. Takeoff thrust is set and the aircraft accelerates. After lift-off the aircraft follows a predefined departure route called SID (Standard Instrument Departure). The SID comprises the complete routing from the runway to the surrounding en-route structure. It contains runway information, waypoints and other meta-information such as speed or altitude restrictions and. In this way, it represents the flight plan and under nominal conditions the aircraft will stay on this planned route.

For the CREDOS project, the RNAV SIDs from the Jeppesen Navigation Database for the Frankfurt/Main airport are used [10]. The nominal trajectory consisting of the horizontal SID route, the altitude and speed profile is derived prior to the actual calculation of the trajectory for a specific aircraft.

According to the waypoint specification from the SID; a list of straight and curved segments is generated as input for the trajectory simulation. The turn radii for the curved segments are calculated with a standard speed and bank angle for the specific aircraft.

For the vertical profile of the trajectory and for the speed schedule, the departure has been divided into five different phases, each of them characterized by a set of target or exit values for speed, altitude and position. Every phase has at least one exit condition so that the next phase starts when the aircraft has reached a certain speed, altitude or position. Together with these three constraints each phase contains parameter for the landing gear, the flaps, the power setting and the power distribution during phases where the aircraft accelerates and climbs in combination. As a control factor a so called energy sharing index (ESI) is used for a variable power distribution between acceleration and climb rate for each flight phase. The five departure phases are:

- Acceleration on ground to V_{ROT},
- Lift-off and climb to 35ft,
- Continue climb to 400ft, gear up,
- Accelerate to V_{FC}, climb configuration,
- Climb on track until 3000ft, max climb thrust setting.

While the altitude constraints for all aircraft are the same, the speed depends on aircraft type and takeoff weight.

2.6 Guidance

During initialization of the trajectory model the horizontal and vertical layout for the departure route is generated. During the actual calculation of the trajectory the position of the aircraft after each time step is compared with the nominal position on the predefined route. This is done in three steps:

1) Check whether the end of the current horizontal segment (straight or curve) has been reached – switch over to the next segment.
2) Check whether the exit conditions of the current vertical phase have been reached i.e. a certain altitude or speed has been reached and switch over to the next phase.
3) Calculation of the horizontal flight path deviation and the nominal track of the current route segment as input to the pilot model.

The flight path deviation during a straight segment is the perpendicular distance between the aircraft and the nominal track as shown in FIG 2.

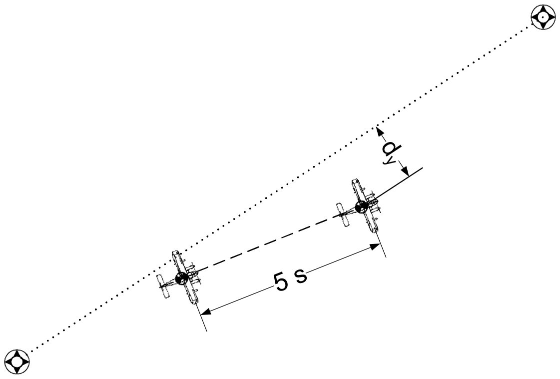

FIG 2: Flight path deviation to a straight segment

On a curved segment the flight path deviation is the distance from the aircraft to the nominal curve, which is mathematically the difference between the distance between the aircraft and the curve centre minus the curve radius.
The actual flight phase is also important for the calculation of the aircraft drag and the available thrust as it specifies the aircraft configuration (flaps and gear) and the thrust mode (e.g. takeoff thrust or maximum climb thrust). Together with the output values from the aircraft motion of the preceding time step these values are fed into the pilot model that generates command signals for the aircraft motion model.
All geographical calculations are based on a spherical coordinate system considering the earth as a perfect sphere. Only for spherical distances a local earth radius is calculated, taking the flattening of the earth as a function of the latitude into account.

2.7 Performance

The takeoff distance depends mainly on thrust, drag, and the maximum lift coefficient. Before takeoff the aircraft target speeds are delivered by the performance calculation. Target speeds, weight, thrust and flap settings define the departure profile. Furthermore, the vertical trajectory profile is influenced by desired climb gradients or acceleration demands.
The performance calculation is airline and aircraft specific. For major airlines, the dispatch does this calculation; otherwise pilots perform these calculations manually with the help of "Runway Weight Charts". This way of performance calculation requires detailed aircraft information. For a model that has to simulate trajectories of different aircraft types some simplifications have to be made. So, performance is computed by empirical equations and simplified flight mechanical equations. This allows covering a representative number of different aircraft.
Takeoff mass, air density, maximum lift coefficient, wing area, gravity are taken from the aircraft database and the environmental database. Rotation speed (V_R), safety speed V_2, and final climb speed $V_{FC.}$, are outputs of the simplified performance model. The performance calculation is based on [2] and [4]. The required speeds for takeoff performance can be derived from the aircraft stall speed.

(1) $$V_{Stall} = \sqrt{\frac{2 \cdot m_{initial} \cdot g}{S \cdot \rho \cdot C_{L\max}}}$$

The rotation speed is 1.2 times and the safety speed is 1.35 times the stall speed. The final climb speed is assumed to be 20 knots higher than the minimum safety climb speed.

(2) $$V_{ROT} = 1.2 \times V_{Stall}$$

(3) $$V_2 = 1.35 \times V_{Stall}$$

(4) $$V_{FC} = V_2 + 20\ \text{knots}$$

2.8 Aircraft Dynamics

To generate an aircraft trajectory it is sufficient to use an aircraft point mass model with three degrees of freedom. The model is build from the non-linear force equations in the three translational directions x, y, and z. The following simplifying assumptions have been made, see [7].

- All moments have to be in balance, which implies that all rotational degrees of freedom can be neglected.
- The lateral force Q in body axes and the angle of sideslip β are zero.
- The thrust vector is along the x-axis of the body-fixed system, thrust setting angle i_F=0.
- The angles α_K and β_K are small, to simplify the transformation matrices. The failure during linearization for this assumption is smaller than 1%.
- The aerodynamic drag of the landing gear was neglected.
- The rotation rate is set to a constant of 2.75°/s for all aircraft.

These simplifying assumptions allow the following formulation of the aircraft equations of motion.

2.8.1 Equations of Motion

The principal of d'Alembert leads to the equations of motion (EoM) in the flight path fixed coordinate system:

(5) $$-\underline{F}_k^K = \underline{\underline{T}}_{kg}\underline{G}_g + \underline{\underline{T}}_{kf}\underline{F}_f + \underline{\underline{T}}_{ka}\underline{R}_a^A + \underline{R}_k^{LG}$$

The aircraft trajectory is described by the flight path velocity V_K, flight path angle χ and climb angle γ.

(6) $$m\begin{bmatrix} \dot{V}_K \\ \dot{\chi}V_K\cos\gamma \\ -\dot{\gamma}V_K \end{bmatrix} = \underline{\underline{T}}_{kg}\begin{bmatrix} 0 \\ 0 \\ m \cdot g \end{bmatrix} + \underline{\underline{T}}_{kf}\begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} + \underline{\underline{T}}_{ka}\begin{bmatrix} -D \\ 0 \\ -L \end{bmatrix} + \begin{bmatrix} -\mu_R F_N \\ 0 \\ -F_N \end{bmatrix}$$

Thrust, angle of attack, and bank angle are the control inputs for the aircraft model. For solving the equations of motion thrust, drag, lift, and landing gear force need to be calculated.

Thrust force

The BADA model [1] provides coefficients that allow the calculation of different thrust levels such as maximum climb, maximum cruise and descent. In the database there are up to 12 coefficients for the thrust calculation, some have different dimensions depending on the corresponding type of engine. The thrust for jet engines is calculated with a speed independent approach comprising only an altitude correction and a correction for temperature deviations from the standard atmosphere. As the model is designed to be used for trajectory calculations under nominal flight conditions, it always calculates the maximum available climb thrust which is less then the rated maximum output of the installed engines. Only for takeoff and emergency situations the engines can have a higher thrust, but only for a limited time. Different thrust levels like reduced climb thrust, cruise thrust or descent thrust are calculated by using a factor. For example the maximum cruise thrust is calculated with a cruise coefficient of C_{TCR}=0.95.

(7) $$T_{CRUISE,\max} = C_{Tcr} \cdot T_{CLIMB,\max}$$

As there are only coefficients for climb, cruise and descent in the database available, a factor of 1.44 is used to estimate realistic takeoff thrust levels.

Drag force

The drag of an aircraft is a function of the dimensionless drag coefficient C_D, the air density ρ, the true airspeed V_A and the wing area S

(8) $$D = C_D \cdot \frac{\rho}{2} \cdot V_A^2 \cdot S$$

The drag coefficient is calculated from a simple quadratic approach:

(9) $$C_D = C_{D0} + C_{D2} \cdot C_L^{\ 2}$$

The air density is calculated for the actual atmospheric conditions, and the airspeed comes from the actual aircraft state. The wing area is taken from the aircraft database as well as the parasitic drag coefficient C_{D0} and the induced drag coefficient C_{D2} that are specific to one of five available slat/flap configurations: cruise, initial climb, takeoff, approach and landing.

The lift coefficient is a function of the actual flight state and is calculated under the assumption of a stationary flight where the lift equals the aircraft weight.

Lift force

The lift force is defined by:

(10) $$L = \frac{\rho}{2} \cdot V_A^2 \cdot S \cdot C_L$$

Lift depends on dynamic pressure, wing area, aircraft speed, and the lift coefficient, which is determined by the zero lift coefficient C_{L0}, the lift coefficient derivative due to angle of attack $C_{L\alpha}$, and the incremental change of lift coefficient due to flap deflection ΔC_{LFlaps}.

(11) $$C_L = C_{L0} + C_{L\alpha} \cdot \alpha + \Delta C_{LFlaps}$$

According to Weissinger theory the lift slope for a swept wing in incompressible air is related to the aspect ratio Λ as follows:

(12) $$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha} = \frac{2 \cdot \pi \cdot \Lambda \cdot e}{2 + \Lambda \cdot e \sqrt{\frac{1}{\cos^2 \varphi} - Ma^2}}$$

Landing gear force

The normal force of the landing gear is given by:

(13) $$F_N = F_{N\_NoseLG} + F_{N\_MainLG} = m \cdot g \cdot \cos \gamma_{RWY} - L$$

2.8.2 Velocity Kinematics

The relation between true airspeed, flight path velocity, and wind velocity is described by the velocity equation. It is in the flight path fixed system (index k):

(14) $$\underline{\underline{T}}_{ka} \cdot \underline{V}_{Aa} = \underline{V}_{Kk} - \underline{\underline{T}}_{kg} \cdot \underline{V}_{Wg}$$

Expressed with the transformation matrices

(15) $$\begin{bmatrix} 1 \\ -\beta_W \\ \alpha_W \end{bmatrix} \cdot V_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix} \left\{ \begin{bmatrix} V_K \\ 0 \\ 0 \end{bmatrix} - \begin{pmatrix} \cos\gamma \cdot \cos\chi & \cos\gamma \cdot \sin\chi & -\sin\gamma \\ \sin\chi & \cos\chi & 0 \\ \sin\gamma \cdot \cos\chi & \sin\gamma \cdot \sin\chi & \cos\gamma \end{pmatrix} \cdot \begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix}_g \right\}$$

Flight path

With the equation of motion (6) and the velocity equation (15), the aircraft trajectory can be computed. The aircraft position is the integral of the flight path velocity. In the geodetic system it is:

(16) $$\frac{ds_g}{dt} = \begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{z}_g \end{bmatrix} = \begin{bmatrix} \cos\gamma \cdot \cos\chi \\ \cos\gamma \cdot \sin\chi \\ \sin\gamma \end{bmatrix} \cdot V_K$$

Here, the velocities in the three geodetic directions are expressed in terms of flight path velocity V_K, azimuth angle χ und climb angle γ.

Load factor

For passenger comfort, the load factor n_z should not exceed 1.15 and the bank angle should not exceed 25°. Therefore, the commanded bank angle Φ_c is limited in amplitude $\Phi \le 25^\circ$ and roll rate p is limited to p<10°/s. The load factor n_z in the body-fixed system can be calculated by:

(17) $$n_{zf} = \left(\frac{V_K \cdot \dot{\gamma}}{g} + \cos\gamma \right) \cdot \frac{1}{\cos\Phi}$$

and in the flight path system by:

(18) $$\underline{n}_k = \frac{\underline{F}_k^K - \underline{\underline{T}}_{kg} \cdot \underline{G}_g}{m \cdot g}$$

(19) $$\Leftrightarrow \underline{n}_k = \frac{1}{m \cdot g} \left\{ -m \begin{bmatrix} \dot{V}_K \\ \dot{\chi} V_K \cos\gamma \\ -\dot{\gamma} V_K \end{bmatrix} + \begin{bmatrix} -\sin\gamma \\ 0 \\ \cos\gamma \end{bmatrix} m \cdot g \right\} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}_k$$

2.9 Pilot Model

For standard departure route tracking, a pilot model is implemented that compensates deviations from the standard departure route. The requirements for this software module are to model how a pilot or an auto pilot tracks the standard departure route, with the following control tasks:

- Follow the desired flight path,
- Correct deviations from the lateral flight path,
- Match the stationary aircraft climb gradient to the actual thrust level,
- Maintain required aircraft speed by thrust control.

The pilot model is structured into three following sub-modules that are discussed below.

2.9.1 Thrust Control

The takeoff thrust setting, which is depending on airport altitude, aircraft velocity, and desired climb speed, as well as the maximum continuous thrust are delivered by the static thrust calculation during the initialization process, as described in section 2.8. The transfer function G(s) models the spooling-up of the engines.

(20) $$G(s) = \frac{K\omega_0^2}{s^2 + s2D\omega_0 + \omega_0^2} \quad \text{with} \quad K = 1; \omega_0 = 1$$

With increasing speed, engine thrust reduces. This speed dependency of the thrust is approximated by:

(21) $$F(V) = F_{Static}(0{,}17 \cdot V_A - 0{,}18)$$

The commanded thrust is controlled by the speed error (final climb speed minus airspeed), as shown in FIG 3. The thrust control module is activated after lift-off. The thrust control function reduces the thrust after takeoff, to avoid exceeding the target final climb speed provided by the performance calculation. The thrust will be reduced as soon as the true airspeed exceeds the default final climb speed with empirical factors K_F and K_V .

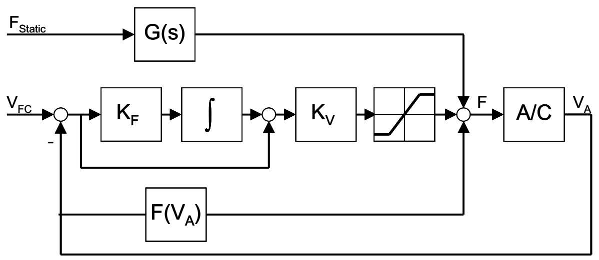

FIG 3: Thrust control

2.9.2 Azimuth and Position Control

For lateral SID tracking, the pilot model computes bank angle commands depending on lateral aircraft deviation d_y. Control becomes active after lift-off.

FIG 4 shows the assembly of the aircraft azimuth and position control. The desired heading Ψ_c is computed by the software module "Navigation" and the lateral aircraft deviation from the nominal flight path is determined in the "Guidance" module. The outer loop controls lateral deviations and commands azimuth angles to the inner loop. Integration and proportional feedback ensures the aircraft position on the nominal flight path. The gain K_y is empirically derived. The wind sideslip angle is required for disturbance suppression and the true airspeed is needed for computing a heading command that compensates the lateral deviation.

In the inner loop, roll dynamics are represented by a first order transfer function:

(22) $$F(s) = \frac{K}{1 + sT} \quad \text{with} \quad K = 1, \mathrm{T} = 1$$

Bank angle commands are limited in rate and amplitude.

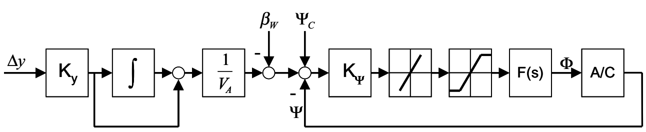

FIG 4: Azimuth and position control

2.9.3 Altitude Control

For vertical path tracking, altitude and climb rate are controlled by angle of attack commands from the pilot model taking SID constraints and airline procedures into account.

Altitude control is subdivided in two modes: aircraft rotation phase and climb phase after lift-off.

The rotation control mode is active from the time the aircraft reaches the rotation speed until reaching the lift-off condition (Lift=Weight). During this time period the aircraft starts to rotate with a fixed rotation rate (ROR). The aircraft lift increases in accordance with the rising angle of attack. When lift and weight are equal, the angle of attack will pass to the integrator of the second control mode to avoid overshoots.

The angle of attack at lift-off defines the initial condition for the second integration block. The commanded climb angle is smoothed by a first order low pass filter *F(s)*, see (22), and is compared to the current aircraft climb angle.

The pitch rate is subtracted from the delta climb angle with a delay of 0.4 seconds, to damp the phugoid mode and to avoid instabilities. The integration block is implemented to achieve the desired final climb angle. The bypass of the second gain allows an accelerated aircraft reaction by a linear transformation of the commanded climb angle. The aircraft bank compensation is considered after the second integration block.

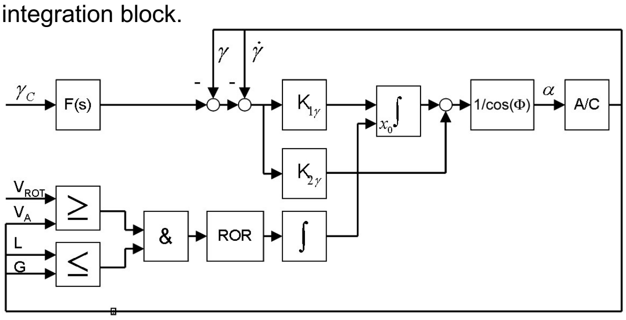

FIG 5: Altitude control

Climb angle command

The commanded climb angle for each flight segment is derived from the potential climb angle γ_{pot} (the max. climb angle for a given thrust setting that can be maintained in stationary flight) that is given by the drag equation for $\dot{V}_K = 0$:

(23) $$\gamma_{pot} = \arcsin\left(\frac{F - D}{m \cdot g}\right) V_A \geq V_{LOF}$$

While the aircraft is on ground γ_{pot} =0°. A so called Energy Sharing Index (ESI) is used to distribute thrust margin between climb angle and acceleration along the flight path. This leads to a commanded climb angle:

(24) $$\gamma_c = \gamma_{pot} \cdot ESI$$

3 MODEL VERIFICATION AND VALIDATION

The verification and validation of the trajectory model is accomplished as follows:

1) A deterministic verification by comparing results of the trajectory model with high-fidelity simulation data from departures that were simulated on the Level-D certified A330-300 full-flight simulator (A330-FFS) at TU Berlin.

2) A statistical validation by comparing MCS results of the trajectory model with 20,000 measured departures at Frankfurt airport provided by DFS.

In case of the deterministic verification, all parameters that the trajectory model computes can be compared with high-fidelity data from the A330-FFS.

The DFS data sets that are used for the statistical validation consist of measured trajectory data, which cover a large variety in aircraft types and other parameter variations.

3.1 Deterministic verification

The deterministic verification is based on recorded departures which were flown manually by airline pilots on the A330-FFS. As simulator data are only available for the A330-300, the deterministic verification is just performed for that aircraft. This verification demonstrates the correctness of the mathematical equations in the software that are also used for all other aircraft types.

The following figures compare recorded data of a manually flown standard instrumental departure with the A330-FFS to calculated data from the trajectory model. The simulation starts with the takeoff run and ends when the aircraft reaches 3000 ft. The initial conditions of the simulator flight and those used in the trajectory model are listed in TAB 1.

Furthermore, the rotation rates that influence the transition slope of the vertical profile are fixed in the trajectory model at 2.75°/s. The runway friction coefficient is assumed with 0.02. This parameter also has a significant influence on the needed takeoff distance and the transition angle.

Initial conditions	**A330 FFS**	**Trajectory model**
SID	TOBAK2F	TOBAK2F
Runway	25R	25R
Weather file	-	profil_00_20040 9230150
Aircraft type	A330-300	A330-300
Aircraft weight	2167029	2167029
Thrust mode	TOGA	TOGA
Start position	Begin runway	Begin runway
Pilot behaviour	Manually flown departure	Default conditions

TAB 1: Initial conditions of compared simulator output versus trajectory model

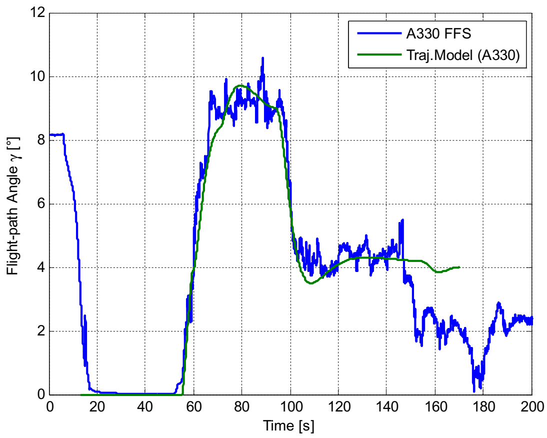

FIG 6: Flight path angle from A330-FFS versus trajectory model (A330)

The flight path angle, given in FIG 6, matches with the simulator data; leading to a good congruence of the climb profile with equalized aircraft weight.

FIG 7 shows the speed profile. The computed aircraft speed from the trajectory model has the same slope between start until rotation (12 s - 45 s) as the recorded one from the A330-FFS. In the first phase the aircraft accelerates on ground, at 54 seconds it starts to rotate. At 58 s the lift equals weight and the aircraft lifts off. The second phase between 60-90 s is climb with constant flight path velocity. Subsequently, after climbing through 1500 ft (~460m), the thrust is reduced to maximum continuous, the flight path angle is reduced and the aircraft starts to accelerate to its final climb speed. It is assumed that the small offset is due to small differences between the true aerodynamic drag coefficients and the drag coefficient delivered by BADA. Differences between the true thrust and the BADA thrust could also be the reason.

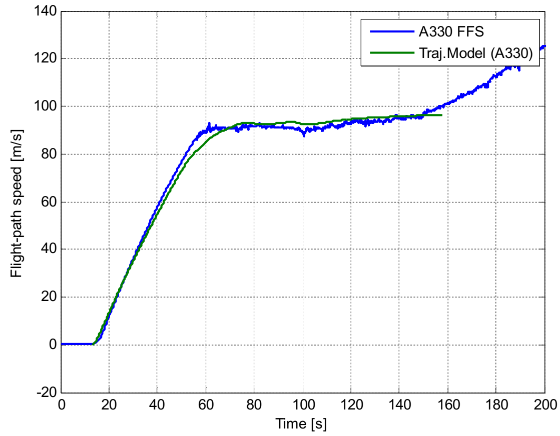

FIG 7: Flight path speed from A330 FFS versus trajectory model (A330)

FIG 8 shows the vertical aircraft profile, according to the modified Airport Traffic Area (ATA) departure procedure. After 54 s the aircraft starts to rotate, three seconds later it lifts off and starts to climb. After reaching 1500ft (~460m) thrust and climb angle are reduced and the aircraft starts to accelerate.

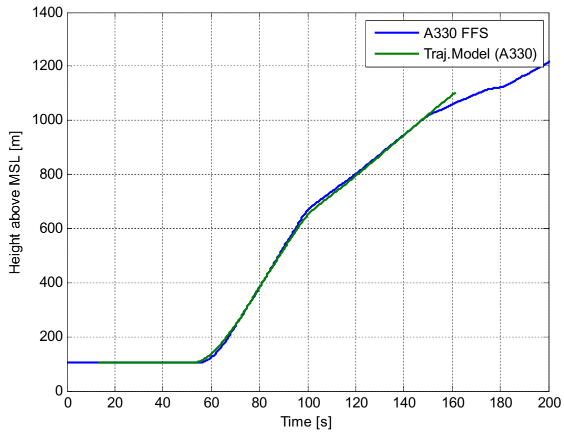

FIG 8: Height above MSL from A330 FFS versus trajectory model (A330)

FIG 9 shows the lateral flight path along the SID "TOBAK2F" from Frankfurt airport for the A330-300 until reaching 3000ft above ground level where the simulation is supposed to stop. For now the computed lateral profile can be only compared to the nominal SID route, delivered by the official Jeppesen waypoint list used by airlines [10], because of a problem with the recorded simulator data of lateral positions.

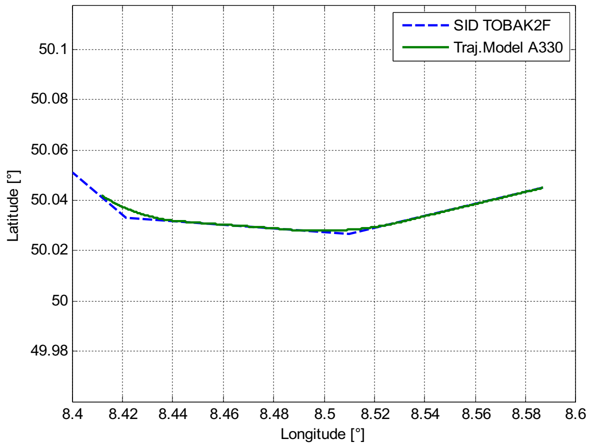

FIG 9: Lateral trajectory A330-300

The results from the deterministic validation are highly satisfying in view of a generic performance calculation and generic aerodynamic coefficients.

3.2 Statistical validation

The next sections describe how the measured Frankfurt data were prepared, which assumptions were made, and how the computed trajectories were compared to the recorded trajectories.
The objective is to validate that the trajectory model computes realistic and representative trajectories that can be used to assess wake vortex encounter risk.

3.2.1 Validation data

For the statistical validation, DFS provided 20,000 data sets of departures for different aircraft types, which represent typical traffic at Frankfurt airport. Each data set contains the following information, measured each 5-7 seconds:

- Aircraft type,
- Runway,
- x, y, z aircraft position, from start position up to 3000 ft above ground. The origin of the geodetic coordinate system is the runway threshold. A right hand coordinate system is used with its x axis in opposite direction to the runway heading. The aircraft position was tracked by Radar.
- Headwind and crosswind components which are measured 10m above ground level,
- Temperature profile as a function of altitude,
- Track angle.

Unfortunately no information is available about the aircraft weight and on the thrust setting, which have a significant influence on the vertical trajectory profile. In addition variations of wind velocity and direction are unknown. Hence, only computed mean trajectories (+/- standard deviation) for each aircraft type under varying conditions were compared to the mean measured trajectories (+/- standard deviation) of the DFS data. For this comparison the same aircraft type, runway, departure route and comparable average cross wind conditions are chosen. The initial condition and MCS parameters are listed in detail ahead of the validation result.

3.2.2 Preparation of validation data

It is assumed that horizontal and vertical aircraft trajectories are independent. This should result in a good approximation, as the gentle turns that are needed to follow the standard departure route do not have a significant influence on climb gradients and speed. Therefore, the validation process for the lateral and vertical aircraft route is performed independently.
In order to generate data sets for specific validation aspects, data sets were sorted and processed as follows:

1. The DFS data were sorted by aircraft type, the departure runway and the departure direction (north or south departure).
2. Departures were selected with crosswind conditions according to the “Initial Concept of Operation” That means departures with a crosswind above 5 knots (measured in 10 m ground level) were handled as “crosswind departures”.
3. Objectionable data sets were eliminated. Data sets for validation have to fulfil the following requirements for the first measured point:
   - The time stamp of the first measured position must be less than 23 s.
   - The recorded height above ground must be higher than zero meters.
   - The measured flight path speed must be lower than 250 but higher than 100 knots.
4. The measured trajectories were extrapolated to determine the x-position of the rotation point, because the first measured position is in most cases after aircraft rotation. This extrapolation was used to correct the measured lateral position by an assumed measurement failure. In case of an initial lateral data point deviation larger than 100 m from the extended centreline, the trajectory was straightened.
5. All parameters which were given as a function of time were transformed into a function of the distance along the route.
6. The measured data can be used up to 7000 m distance along the route, to guarantee a constant number of aircraft in the given period.

After the described data processing, approximately a quarter of the data sets are left, mainly because the Frankfurt weather is not dominated by cross wind conditions.

3.2.3 Statistical evaluation

The statistical evaluation is based on mean trajectories and standard deviations. Mean trajectory deviations are determined for the lateral flight path and the aircraft height as a function of the distance along the route (DaR). The lateral and vertical deviations are determined at gates that are positioned every 100 meters along the DaR. The same computation is performed for the true airspeed.

$$\bar{z} = \frac{\sum \Delta z}{n} \qquad \bar{y} = \frac{\sum \Delta y}{n} \tag{25}$$

The standard deviations are calculated in gates that are normal to the mean trajectory.

(26)
$$s_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}}$$
$$s_z = \sqrt{\frac{\sum_{i=1}^{n}(z_i - \bar{z})^2}{n-1}}$$

The mean trajectory and its standard deviation are computed for aircraft that represent typical airport traffic under crosswind conditions.

3.2.4 Monte Carlo Simulation

The Monte Carlo Simulation allows computing a set of probabilistic trajectories with the "trajectory model" under varying conditions, to check if the distribution of computed trajectories fits the DFS data from Frankfurt airport. The following parameters have major influence on the results:

Aircraft type, weather condition, and SID route are character inputs. Behind these inputs are databases that dispose a set of parameters. For detailed information of the database content, see references [1], [11], and [12].

Aircraft mass can be varied between the minimum to the maximum takeoff weight (MTOW), with a factor from zero to one. Zero defines the minimum takeoff weight and one stands for MTOW. The minimum takeoff weight is defined as 50% loaded.

The thrust mode can be either one or zero. "One" stands for reduced takeoff (T/O) thrust or the so called "Flex T/O" thrust and "zero" stands for takeoff go around (TOGA) thrust mode.

The start position can be varied between the beginning of the runway until the runway intersection with the first taxiway, changeable with a factor between zero and one (0= beginning of runway, 1= start at the intersection with the first taxiway). The distance between begin of the runway until the first taxiway is assumed to be ~750 m.

The pilot factor represents a threshold in tolerable cross track deviation and pilot lag time. It can be varied between 0 and ±0.3°NM, which is the navigation accuracy limit, according to Required Navigation Performance (RNP) in controlled airspaces [12]. Whereas "0°NM" is the auto pilot that is supposed to fly the highest accuracy on the desired flight path.

The vertical profile is significantly influenced by the Energy Sharing Index (ESI) which is defined for each flight phase, according to the modified ATA departure procedure. These factors were hard coded inside the phase tables. As described in section 2.9.3, the "Energy Sharing Index" distributes thrust between potential climb angle and flight path acceleration. This leads to a commanded climb angle.

The following paragraphs describe the validation process for the A320 departures. All aircraft performance and geometric parameters used in the TM are based on BADA.

3.2.5 Assumptions

The following assumptions help to understand the interpretation of the validation sheets:

1) The aircraft weight has a negligible influence on the lateral trajectory profile. As the requirement for the first turn after takeoff is "5°NM DME or 800 ft whichever is later", all aircraft representing the normal Frankfurt traffic reach the 800°ft a long time before having 5°NM DME: Therefore, the lateral profile is not influenced by the aircraft weight.
2) The aircraft weight has no influence on the final climb speed; only the acceleration changes with weight.

3.2.6 Statistical validation results

Sensitivity analysis of aircraft trajectories due to weight, start point, thrust mode and weather variation were investigated for different kinds of aircraft. Only the validation results for the A320 are shown here and the sensitivity of generated trajectory due to changing aircraft weight, pilot behaviour and weather condition is discussed.

The first step was to estimate a realistic mean aircraft weight. This has been done with the help of further information about flight plans of departing aircraft at Frankfurt. In this way a typical average aircraft weight was derived for each aircraft type. Weight distribution was assumed to be normal. A normal distribution was assumed for a pilot reaction time due to lateral deviations between auto pilot and manually flown departure with the maximum allowed track deviation of ±0.3°NM.

For the validation a few NOWVIV weather files that fulfil the crosswind condition (>5knots) were selected. This allows trajectory simulations under comparable wind conditions. The mean crosswind and headwind can be found in TAB 2. One thousand departures were simulated with simultaneous variation of all three parameters. TAB 2 contains detailed information of the initial trajectory model conditions and its reference as well as the range and the kind of distribution that have been chosen for MCS parameters.

Input	**TM**	**DFS data**
Departures	1000	~1000
Airport	Frankfurt, EDDF	Frankfurt, EDDF
SID	TOBAK2F	TOBAK2F
Runway	25R	25R
Aircraft type	A320	A320
Takeoff thrust	Takeoff go around	Takeoff go around
Start point	263 m	?
Mean crosswind in 10 m	3.3715 m/s	3.4581 m/s
Mean headwind in 10 m	3.6923 m/s	3.4756 m/s
MCS parameter		
Input	**Range**	**Distribution**
Weather	-	Uniform
Weight	4555354 N- 755102 N	Normal (Mean=622984)
Pilot behaviour	Mean pilot lag= 0.16NM σ = 0.14 NM Mean cross track deviation= 0 NM σ = 0.03 NM	Normal

TAB 2: A320 trajectory model inputs versus initial conditions of DFS data

FIG 10 and FIG 12 compare the mean computed trajectory of the simulated A320 departures under varying weight, pilot behaviour, and weather conditions to the mean trajectory measured for all A320s (DFS data). There is no validation data available to compare the acceleration profile on ground, the measured trajectories start first at approximately 3000 m. The top diagram of FIG 10 shows the lateral flight path under varying pilot behaviours, aircraft weights and wind conditions. The radar track measurement error is around 50 m, which may explain the larger lateral spread in the measured data in FIG 12. The trajectory model simulates a lag in pilot reaction in terms of a delayed turn at defined waypoints and a lateral track deviation. This generates a deviation between the nominal waypoint track and the calculated lateral profile according to the trajectory model (TM), see FIG 12.

The middle diagram compares the trajectory model climb profile to the mean DFS A320 trajectory. The TM climb profile matches the DFS data well, and has a comparable standard deviation for a normal weight variation see FIG 12. The estimated average weight and range are given in TAB 2.

The mean speed profile is given in the bottom diagram. Shown DFS speeds are based on numerical differentiation calculated by the radar tracker (a system used by civil and military users for associating discrete radar plots with individual targets, and forming estimates of those targets' locations, headings and speeds). True airspeed is a vector sum of ground speed and WTR/RASS measured wind speed at the altitude of the aircraft. This way of getting the “true” airspeed leads to high deviation of individual speed measurement. The reference speeds contains aspects which were not considered in the trajectory model, therefore a comparison between speed deviations is not reasonable.

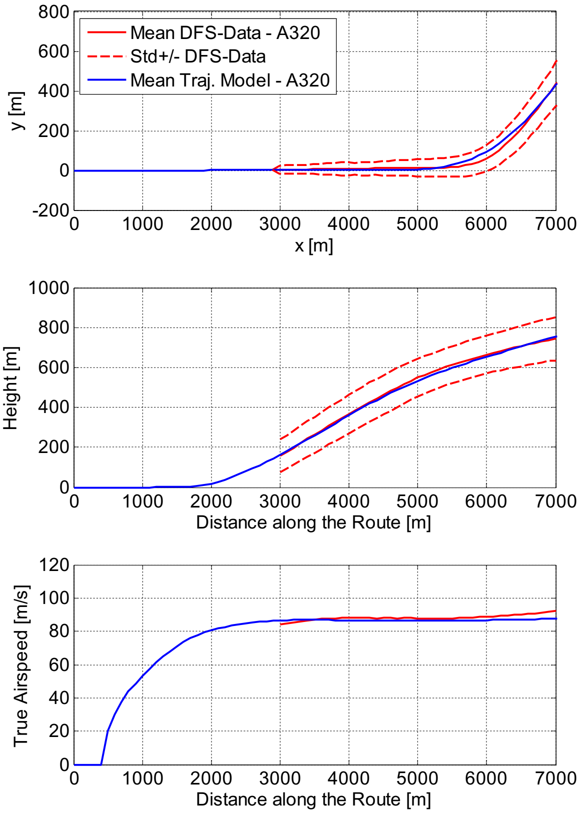

FIG 10: A320 lateral, vertical and speed profile

FIG 11 and FIG 13 show the measured distribution of the lateral and vertical A320 position at Frankfurt. Directly after lift-off (y@4000m) most aircraft stay exact on the extended runway centreline, which state the tight normal distribution in the second diagram. At 6000 m the mean value is displaced from runway heading (y=0m), because most A320 have already turned towards the next waypoint.

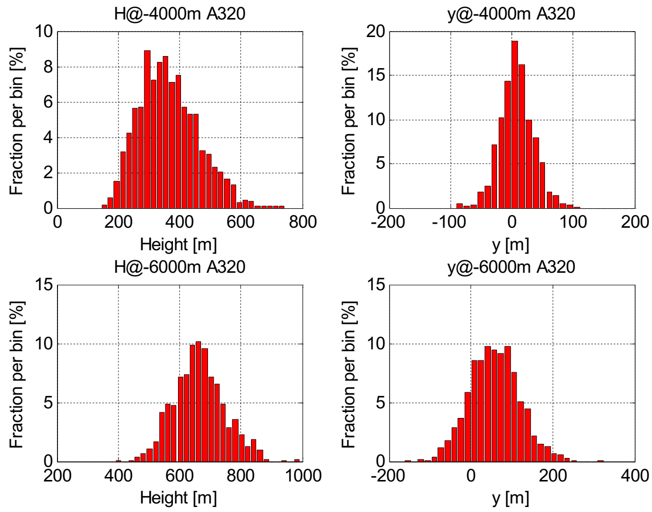

FIG 11: Distribution of height and lateral offset for 1000 A320 departures at 4000 m and 6000 m (DFS data)

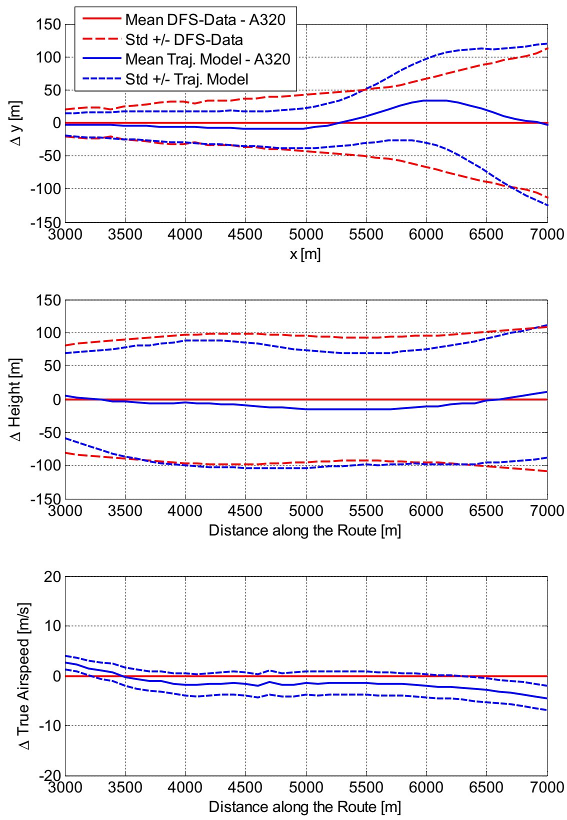

FIG 12: A320 trajectory model quantities referred to DFS data

The calculated A320 distribution is given in FIG 13. The inputs are normal weight and pilot behaviour distributions and a uniformly weather variation. Aircraft acceleration and therefore the point of rotation depend not linearly to the aircraft weight. Hence, the distribution of the vertical A320 position tends to lower heights a short-time after lift-off (H@4000m). This phenomenon is reduced for subsequent climb phases in the third diagram (H@6000m) shows.

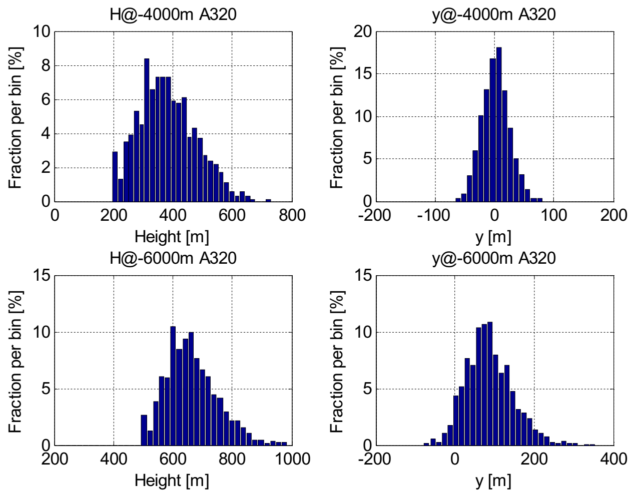

FIG 13: Distribution of height and lateral offset for 1000 A320 departures at 4000 m and 6000 m (TM data)

The calculated trajectory for lateral and vertical aircraft positions under described variations of defined input parameters show realistic distributions.

4 CONCLUSION

A trajectory model was developed, implemented, verified with results from a Level D certified Airbus A330-300 full flight simulator and validated on the basis of measured aircraft trajectories from Frankfurt airport.

The trajectory model allows simulating aircraft departures beginning at the aircraft's start position on the runway. It takes into account wind conditions, standard departure routes, aircraft weights, start point positions, and pilot behaviour. These input dependencies can be specified by range and distribution and varied as MCS parameters for an investigation of their impact on aircraft departure trajectories.

The trajectory model demonstrates a correct physical behaviour in the deterministic verification process and the statistical validation showed a good agreement between simulated and measured distributions of departure trajectories.

5 ACKNOWLEDGEMENTS

The work reported on herein was performed within the CREDOS (Crosswind-Reduced Separations for Departure Operations) project, which was funded by the European Commission. Thanks to DLR that contributed with software modules for navigation and guidance and DFS Deutsche Flugsicherung GmbH that provided measured trajectories for validation.

6 REFERECES

[1] - ; *BADA 3.6 User Manual*; EUROCONTROL, 2003
[2] Wilhelm, K.; *Flugleistungen*, TU Berlin, Vorlesungs-skript, Institut für Luft- und Raumfahrttechnik, Flug-mechanik und Flugregelung, Technische Universität Berlin, Berlin, 1999/00
[3] Brüning, Hafer, Sachs; *Flugleistungen*; ISBN-10 3-540-30447-9, Springer-Verlag, Berlin Heidelberg New York, 2006
[4] Luckner, R.; *Flugmechanik I+II;* Vorlesungsskripte, Institut für Luft- und Raumfahrttechnik, Fach Flugmechanik, Flugregelung und Aeroelastizität, Technische Universität Berlin, Berlin, 2007
[5] Luckner, R.; *Methoden der Regelung in der Luft- und Raumfahrt*; Vorlesungsskript, Institut für Luft- und Raumfahrttechnik, Flugmechanik, Flugregelung und Aeroelastizität, Technische Universität Berlin, Berlin, 2007
[6] Press, S.; Teukolsky, W.; Vetterling, B.; Flannery: *Numerical Recipes*, University of Cambridge, Cambridge, 1992
[7] Brockhaus, R.; *Flugregelung*; ISBN 3-540-55416-5, Springer-Verlag Berlin Heidelberg New York, 1994
[8] Holzäpfel, F.; *Aircraft Wake Vortex Scenarios Simulation Package - Wakescene*, 25th Congress of the International Council of the Aeronautical Sciences, Paper 2006-863, Hamburg, 2006
[9] -; *NOWVIV*; Frankfurt weather database, Frankfurt, 2005
[10] - ; *Jeppesen*, Jeppesen Navigation Database, 2006
[11] - ; Aerodrome chart –ICAO, AIP Germany, 8 JUN 2006
[12] - ; Manual on Required Navigation Performance (RNP) (Doc 9613-AN/937), ICAO, Second Edition 1999
[13] -;http://www.fraport.de/cms/ infoservice_fluglaerm/dok/24/24484.14_was_unterscheidet_das_steilstartverfa.htm;7/6/2007

APPENDIX

SYMBOLS

(Wherever it is possible, the ISO 1151 is used)

C_D	Drag coefficient
C_L	Lift coefficient
$C_{L(\alpha=0)}$	Lift coefficient due to zero angle of attack
C_{Lmax}	Maximum lift coefficient
$C_{L\alpha}$	Lift coefficient due to angle of attack
D	Drag
F	Thrust
F_N	Normal force landing gear
G	Weight
g	Acceleration of gravity
L	Lift
m	Aircraft mass
R	Aerodynamic force
S	Wing area
T	time
$\underline{T}$	Transformation matrix
u	Aircraft velocity components in x direction
v	Aircraft velocity components in y direction
w	Aircraft velocity components in z direction
V_{Stall}	Stall speed
V_{LOF}	Lift-off speed
V_2	Takeoff safety speed
V_{FC}	Final takeoff climb speed
x	Position coordinate with reference to the longitudinal axis
y	Position coordinate with reference to the lateral axis
z	Position coordinate with reference to the vertical axis
ΔC_{LFlap}	Change in airplane lift coefficient due to flap deflection
Φ	Bank angle
Λ	Aspect ratio
Θ	Pitch altitude angle
Ψ	Azimuth angle
α	Angle of attack
β	Angle of sideslip
χ	Flight path azimuth angle (Track)
γ	Climb angle
μ	Air-path bank angle
μ_R	Friction coefficient RWY
α_W	Angle from x-axis of flight path coordinate system to aerodynamic system. Sign is defined according to [7], in [4] α_W is positive in opposite direction.

Indices

A	Aerodynamic
a	Aerodynamic axis system
ESI	Energy Sharing Index
f	Body axis system
g	Geodetic axis system
K	Kinematics
k	Flight path fixed axis system
TAS	True air speed
W	Wind

Atmosphere

H_{GND}	Height above ground
H_{MSL}	Height above mean sea level (Altitude)
Ma	Mach number
N	Polytropic exponent
T	Temperature
κ	Isentropic exponent
ρ	Air density

Aircraft control

K_Y	Pilot model gain
K_Ψ	Pilot model gain
K_ϕ	Pilot model gain
$K\gamma_\alpha$	Pilot model gain
Δy	Lateral deviation of nominal flight path
$\Delta\alpha_c$	Delta command of angle of attack
$\Delta\gamma$	Delta air-path climb angle
Ψ_c	Commanded heading
χ_c	Commanded track
γ_c	Command air-path climb angle

Abbreviations

A/C	Aircraft
ATA	Airport Traffic Area
ATM	Air Traffic Management
ESI	Energy Sharing Index
ICAO	International Civil Aviation Organization
MCS	Monte Carlo Simulation
MTOW	Maximum Takeoff Weight
NOWIV	Now Casting Wake Vortex Impact Variables
NTI	Anti-Ice
RWY	Runway
ROR	Rotation Rate
TAS	True Air Speed
WNTI	Wing Anti-Ice
WP	Work Package
WTR /RASS	Wind-Temperature Radar with Radio Acoustic Sounding System

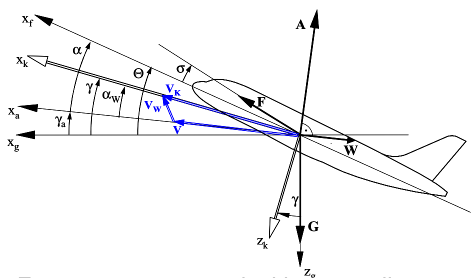

FIG 14: Forces, moments, velocities, coordinate system, and angles in the x-y-plane [4]

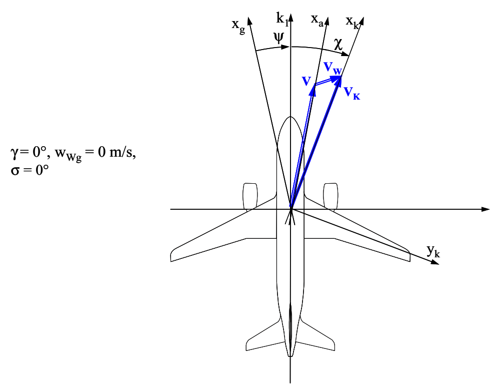

FIG 15: Forces, moments, velocities, coordinate system, and angles in x-z-plane [4]