COMPARISON OF DIFFERENT PARAMETERIZATION AND OPTIMIZATION APPROACHES IN THE FIELD OF AERODYNAMIC COMPRESSOR BLADE DESIGN

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ABSTRACT

Aero engine companies are working to meet challenging requirements for emission and fuel consumption reduction as e.g. declared by ACARE [1]. This effort includes the further refinement of the component design processes including aerodynamic compressor design which is characterized by a time-consuming blade geometry finding process. Many design iterations between different design tools for geometry generation and numerical flow analysis are required in order to achieve the best blade geometries which fulfill the global design intent and meet the global performance requirements.

The paper shows how the time-consuming aerodynamic blade design process can be supported and accelerated by means of process integration, automation, and the application of numerical optimization. Special emphasis lies on the blade geometry definition, where two different parameterization methods for describing the complex compressor blade geometry will be discussed which provide required flexibility and smoothness by a minimum set of describing parameters. The contradicting design goals within the aerodynamic blade design process are formulated as mathematical problem definitions and stochastic as well as deterministic optimization techniques are applied to solve the design problem based on two-dimensional flow analyses. Both developed parameterization methods and optimization techniques are used for a redesign of a stator blade of an industrial high pressure compressor. Two optimized blade geometries are selected from the set of optimal designs and full three-dimensional CFD calculations including turbulence modeling are performed for multiple stages and boundary conditions. The results are summarized and compared with the datum design and conclusions with respect to the parameterization methods and optimization techniques will be made.

NOMENCLATURE

$\mathbf{b}$	control point coordinate
c	curve segment
$\mathbf{p}$	design vector
A	cross section area
C	chord length
$\bar{H}_E$	boundary layer shape factor at exit
$PMXC$	position of max. thickness
PS	pressure side
SS	suction side
$S1$	blade-to-blade surface
$S2$	hub-to-tip surface
T	max. profile thickness
Tu	turbulence intensity
WR	working range
α_I, α_E	inlet and exit flow angle
ω	total pressure loss coefficient, $\omega = \frac{P_{0,E}^{isen} - \overline{P}_{0,E}}{P_{0,I} - P_I}$
γ	tangent angle

1 INTRODUCTION

Aerodynamic design of an axial compressor is a challenging design task with different contradicting requirements like high efficiency, high surge margin, low number of stages and wide operating range, [2]. The aerodynamic design process is typically subdivided into different design tasks with a various number of design freedom. The individual processes are run subsequently starting with a rather simple one-dimensional preliminary design predicting global parameters along the mid-height line of the compressor annulus and a more complex throughflow calculation considering parameter variation in radial direction. The aerodynamic design process ends-up with the blade design in which the engineer aims to find best blade geometries for each blade rows.

There is no straightforward approach to design three-dimensional blade geometries directly, since the flow in a turbo machinery is too complicated and 3D-CFD calculation is even on today's computers still too time-consuming to be used in an iterative design environment, [3]. In order to break down the complex blade design problem to more manageable portions, the three-dimensional blade geometry is approximated by a set of two-dimensional sections which are defined on stream surfaces ($S1$), [4]. According to given flow conditions from the previous throughflow calculation, two-dimensional blade sections have to be designed which are then stacked along a radial stacking line in order to achieve the three-dimensional blade geometry and the desired flow turning, Figure 1.

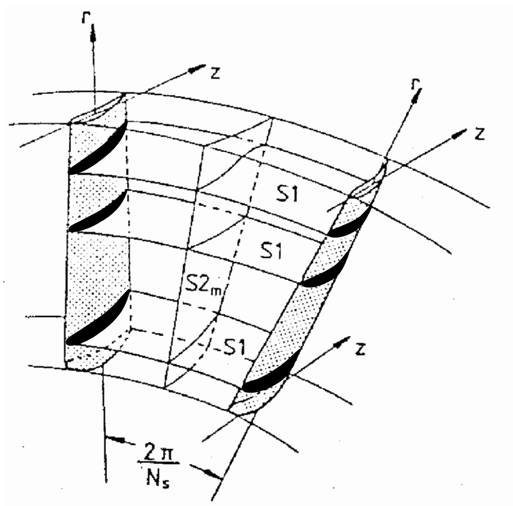

Figure 1. Definition of hub-to-tip ($S2$) and blade-to-blade surfaces ($S1$) according to Wu, [4]

This aerodynamic blade design process is rather time-consuming due to many geometry modifications on multiple blade sections and required design evaluation using two-dimensional flow analyses. Hence, the industrial aim is to integrate existing design tools into a common design environment and to automate the blade design process in order to reduce design cycle time and corresponding costs. This also includes new three-dimensional parameterization methods for describing the entire blade geometries and to avoid expensive two-dimensional geometry modifications. Due to the automated process flow numerical optimization algorithms can be used for finding better solutions and to release the human design engineer from time-consuming solution finding process.

In previous publications different approaches exist, where authors tried to solve the aerodynamic blade design problem appropriately. Trigg et al. [5] developed an automated and direct two-dimensional blade design system for steam turbines using a single-objective genetic optimizer and a parametric blade model. It has been shown that a significant reduction in profile loss can be achieved. However, blade design is a multi-objective task where beside the losses at design flow conditions the overall loss for off-design conditions plays a significant role. This second criterion can be formulated as the working range of the blade section and is contradicting to the first design criterion. Koeller et al. [6] developed a new family of subsonic compressor airfoils for heavy-duty gas turbines using the multi-objective approach. They have coupled a parametric blade modeler for geometry modifications with the MISES blade-to-blade flow solver [7] for design analyses and used a gradient based optimization algorithm for finding optimal geometries. It has been shown, that blade sections with lower losses and higher working ranges could be found compared to conventional controlled diffusion airfoils. Later, Sieverding et al. [8] used a different optimization algorithm to solve a comparable task. They also reported on an improvement in working range compared to an earlier design based on NACA65 profiles found by a genetic optimizer.

These investigations use the weighted-objectives approach to transform the multi-objective task to a single-objective optimization problem. Such an approach is very common for engineering applications, but typically results in a single solution point only. In terms of multi-objective optimization, however, this is not the best choice, [9]. Contradicting criteria offer more design information reflected in the Pareto-front, where optimal trade-off solutions can be found and discussed by the design engineer. A further point of these investigations is that designing on individual blade sections may lead to undesired three-dimensional blade shapes which are not smooth in radial direction.

Thus, in this investigation two automated blade optimization processes are developed in order to find optimized three-dimensional stator blade geometries. The two processes differ according to the blade geometry parameterization method and the optimization strategy. Solutions of both processes are used in order to perform a three-dimensional CFD calculation of multiple compressor rows and are compared with an existing industrial blade design which has been "optimized" manually by a human design engineer.

2 PROCESS FLOW

The blading process consists basically of two steps which are run subsequently. In the first step the geometry is modified within an appropriate design tool. In order to keep the number of design parameters small and the design flexibility high, parameterized geometry models are used. In the second step the new geometry has to be analyzed and design objectives has to extracted from the flow simulation. Figure 2 shows a generic process flow implemented in a common design environment using the commercial software package *iSight* [10]. It is used as process integration and automation as well as optimization tool, since several sophisticated deterministic and stochastic algorithms are available.

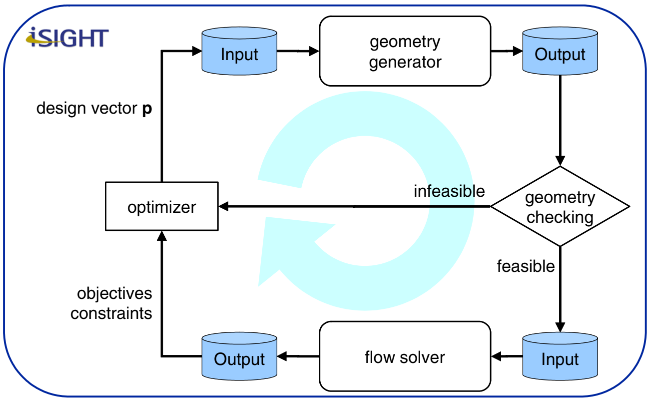

Figure 2. Generic process flow of aerodynamic blade geometry optimization integrated into the *iSight* environment

Once integrated in a common environment, the optimization process starts with a new design vector **p** which contains parameters for the parametric geometry description. The geometry generator creates a new blade geometry which is checked by a geometry checking procedure for admissible designs. Geometries which are not satisfying constraints on blade cross section area or position of maximum thickness are removed from the process in order to avoid expensive CFD calculations. Feasible geometries are evaluated by the two-dimensional CFD code MISES [11] and objective function and constraint values are extracted from the output file. The entire process runs automated until the maximum number of iterations is reached or a termination criterion is fulfilled.

3 DESIGN PARAMETERIZATION

In this work two different blade geometry parameterization techniques are investigated. The first one is focused on an accurate description of blade section geometries with many parameters providing high flexibility and high degree of design freedom. This method is very similar to the classical industrial approach, where three-dimensional blade geometries are obtained by individually designed and radially stacked blade sections. The second parameterization technique is concentrated on direct three-dimensional blade geometry description using radial design parameter distributions providing always radially smoothed geometries. This method requires less design parameters and hence it is less flexible. However, parameter reduction can also help optimization algorithms to converge quicker and to find optimal solutions. In the following sections both parameterization techniques will be described in more details.

3.1 2D-BLADE SECTION APPROACH

In this parameterization technique each blade section is described by four segments, i.e. leading, trailing edges and suction, pressure sides, which are joint together at four blend points with first order continuity, [12]. Each side is represented by a cubic B-Spline curve with eight control points, $\mathbf{b}_i^{SS,PS}$ with $i \in \{1,2,\ldots,8\}$, which can be modified by changing design parameters controlling the control point coordinates, Figure 3.

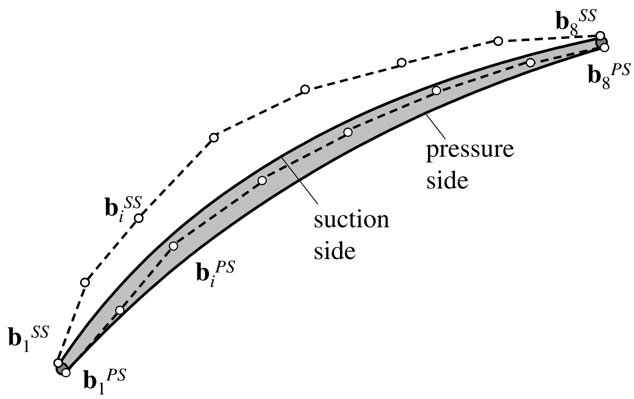

Figure 3. Sketch of 2D-blade parameterization

The parameterization is flexible and offers enough design freedom for a large variety of section geometries, [3]. Within this investigation the suction and pressure side as well as leading and trailing edges are modified in order to provide maximum design freedom. This results in a design vector **p** with 25 parameters for each considered blade section.

3.2 3D-BLADE GEOMETRY APPROACH

An additional approach is investigated for the generation and variation of the 3D-blade as a whole. The goals are to further reduce the set of defining parameters and to realize an extremely fair blade geometry. Furthermore, since the hub and tip regions sometimes vary rather rapidly, a smart flexibility of representation is introduced. Finally, the new parameter set still needs to be able to reproduce existing blade geometries with sufficient accuracy.

Based upon an analysis of the radial distributions for existing blades, the new parametric model utilizes a uniform description for all blade parameters. Basically, each radial distribution is defined via three combined curves. These curves provide the parameter values for each section as a function of the radius. Two fairness-optimized curves (see c_1 and c_3 in Figure 4) enable a high degree of freedom for the modeling process of the hub and tip region. A linear function c_2 that connects these two curves tangent-continuous is offered optionally. This is due to the fact that several quantities display almost linear characteristics within the inner region.

At most 12 parameters can be set for each radial quantity. Some of the parameters are optional, depending on the way of combining the three curves and the influence the design team wants to evoke. Within an optimization typically some of the parameters are fixed while others are varied, i.e. they become free variables of the process. For instance, if the hub region shall be modified while keeping other regions as is, the tangent angle γ of the corresponding fairness-optimized curve c_1 could be manipulated, see Figure 4.

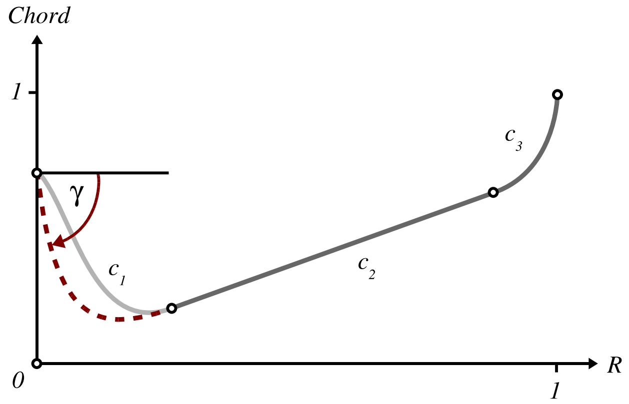

Figure 4. Setting the tangent value γ of the fairness-optimized parametric curve c_1 in order to change the chord lengths at the hub region

In Figure 5 two example distributions for chord length and blade outlet angle are shown, respectively. It can be seen that the parameterization is flexible enough to re-model the reference data for each distribution which leads to the aimed reduction in the number of design variables.

4 NUMERICAL SETTINGS

In order to keep the computational costs during the optimization process low, design evaluations are performed for each blade section with the two-dimensional flow solver MISES, [11]. The computational domain is discretized with a structured grid consisting of 60 grid lines in the inlet region, 70 grid lines on the blade section surface, and 45 lines in the outlet region, Figure 6. Within the blade passage 16 grid lines are used and a clustering function is applied to grid lines towards the blade surface and at regions of high local curvature.

The calculations are performed with constant inlet flow conditions, a turbulence intensity of $Tu = 4\%$ in order to simulate

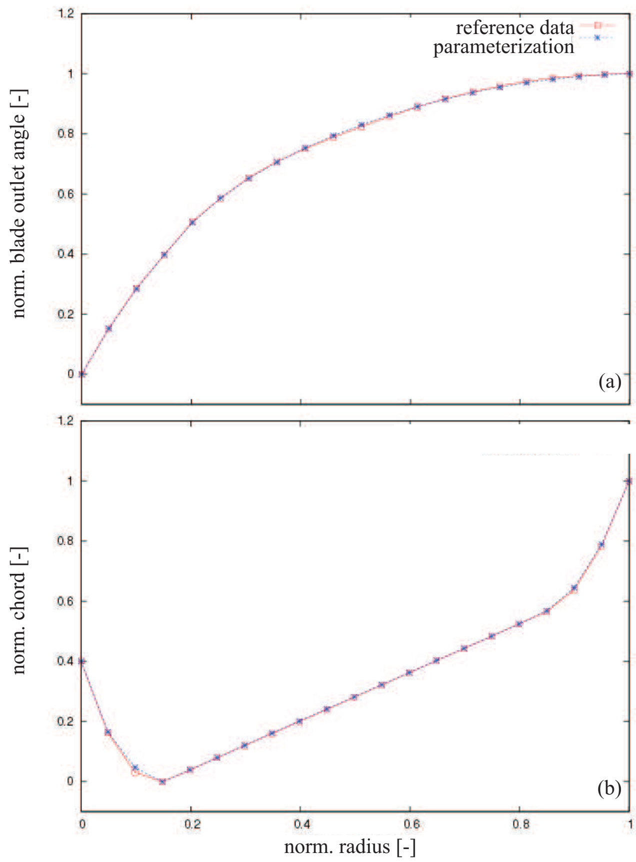

Figure 5. Comparison of original data and parametric re-modeling of blade outlet angle (a) and chord length (b)

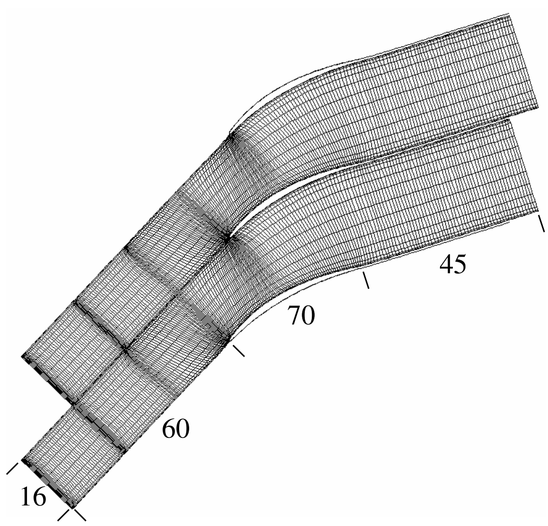

Figure 6. Computational grid used for 2D-blade flow analyses

turbomachinery environment, and the Abu-Ghannam/Shaw transition model for calculating the free transition position on suction and pressure side, [13].

At the end of the optimization processes with both approaches, best blade designs are selected and full multi-stage three-dimensional CFD calculations with variable flow conditions are performed. The computational domain consists of eight blade rows while the optimized stator 3 is embedded in order to consider up and down stream effects, Figure 7.

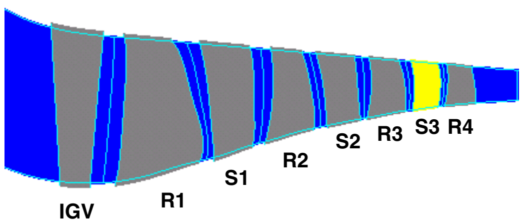

Figure 7. Computational domain for 3D multi-stage flow analyses

The tree-dimensional computations are carried out with the commercial software package NUMECA Fine^{TM}/Turbo [14] based on a structured grid with approximately 5 million grid points, Figure 8. Different flow conditions with increasing back pressure are conducted for solving the stationary, full turbulent flow with the Spalart-Allamaras turbulence model [15] in order to obtain a full characteristic of the axial compressor. The computational time for each of the different back pressure conditions is approximately 5 hours on a multi-processor Linux cluster so that a full characteristic with 8-9 points is obtained after 2 days for each design.

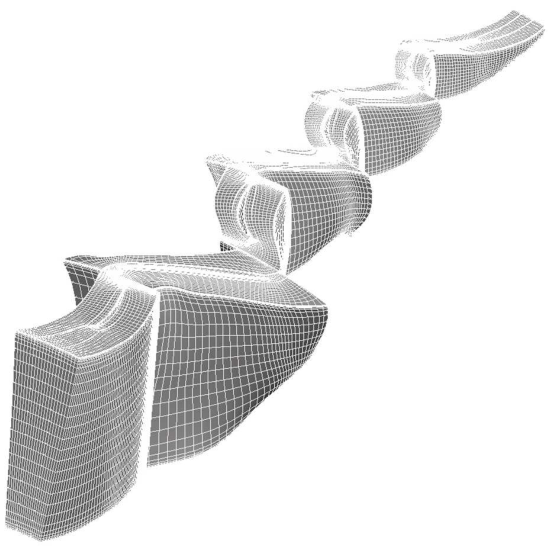

Figure 8. Grid topology for 3D multi-stage flow analyses (coarse grid)

5 BLADE DESIGN PROBLEM

The two-dimensional blade design problem is to find appropriate geometries which fulfill the required flow turning with a minimum loss for all flow conditions. Figure 9 shows the general loss behavior of a blade section geometry at variable inlet flow angle α_I. It can be seen that minimum loss is achieved at design flow angle α_I and that a variation caused by different operating conditions leads to higher losses. If an acceptable loss level ω_{WR} is defined, a maximum inlet flow angle α_I^R and a minimum inlet flow angle α_I^L can be found. The difference between these two angles describes the so-called working range $WR = \alpha_I^L - \alpha_I^R$ which should always be maximized in order to obtain insensitive geometries.

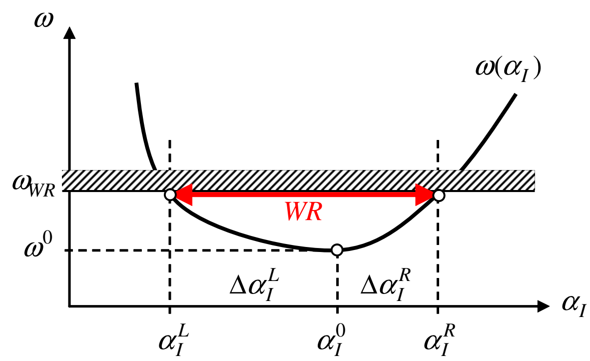

Figure 9. Loss curve for variable inlet flow

6 PROBLEM FORMULATION

For numerical optimization it is required to formulate the design problem in a mathematical formulation. The blade design problem from the previous section can be declared in different manners depending on the optimization objectives. Within this investigation two different optimization approaches are developed with different objective functions, but nearly same constraints. The first approach uses a multi-objective approach for solving the design problem of seven individual blade sections, while the second approach tries to find an optimized three-dimensional blade geometry by minimizing the losses at five different sections. The mathematical formulation of both approaches are described in the following.

6.1 MULTI-OBJECTIVE APPROACH

The vectorial objective function can be formulated as the minimization of the loss at the design inlet flow angle $\omega^0 = \omega(\alpha_I^0)$ and the maximization of the working range WR. The value for WR can be obtained by a variation of the inlet flow angle α_I^0 until the acceptable loss level ω_{WR} is reached for lower and higher values. This, however, requires multiple flow calculations determining the whole loss curve and would lead to an increasing optimization time. Hence, $\Delta\alpha_I^L$ and $\Delta\alpha_I^R$ are introduced as additional design variables which can be modified by the optimization algorithm in order to find the two intersecting points. A maximization of $\Delta\alpha_I^L + \Delta\alpha_I^R$ is equal to an increase in working range. As described before, not all geometries are acceptable, since geometric and aerodynamic constraints have to be fulfilled. It is required to keep the position of maximum thickness $PMXC$ within reasonable bounds, to guarantee values for cross section area A and thickness to chord ratio T/C being greater or equal the datum design. Beside these more stress driven constraints, it is necessary to avoid flow separation at the trailing edge by restricting the boundary layer shape factor $\bar{H}_E$ and the exit flow angle α_E. The latter one is important in order to maintain the correct inlet flow angle for the next down stream blade row. Summarizing, the multi-objective blade design problem for each considered blade section reads as

$$\min_{\mathbf{p},\Delta\alpha_I^L,\Delta\alpha_I^R} \begin{bmatrix} \omega^0 \\ -WR \end{bmatrix} \tag{1}$$

subject to

$$\begin{aligned} &\Delta\alpha_I^L \geq 0, \Delta\alpha_I^R \geq 0 \\ &0.2 \leq PMXC \leq 0.6 \\ &A \geq A_{datum} \\ &T/C \geq [T/C]_{datum} \\ &\omega\left(\alpha_I^0 - \Delta\alpha_I^L\right) \leq \omega_{WR} \\ &\omega\left(\alpha_I^0 + \Delta\alpha_I^R\right) \leq \omega_{WR} \\ &\bar{H}_E \leq 2.5 \\ &\alpha_E \leq \alpha_{E,datum} \end{aligned} \tag{2}$$

with the design vector $\mathbf{p}$ containing 25 parameters for each parametric blade section description.

6.2 SINGLE OBJECTIVE APPROACH

The second approach considers one objective function only. The aim is to find an optimal three-dimensional blade geometry by optimizing blade sections according to their loss at design flow conditions and to aggregate the losses ω_i^0 to an utility function $\bar{\omega}$. The single-objective optimization problem reads as

$$\min_{\mathbf{p}} \quad \bar{\omega} \tag{3}$$

with the aggregate function

$$\bar{\omega} = \sum_{i=1}^{N} \omega_i^0 \tag{4}$$

subject to the following constraints

$$\begin{aligned} &0.2 \leq PMXC \leq 0.6 \\ &A \geq A_{datum} \\ &T/C \geq [T/C]_{datum} \\ &\bar{H}_E \leq 2.5 \\ &\alpha_E \leq \alpha_{E,datum}. \end{aligned} \tag{5}$$

The aggregation of the individual objectives is a scalarization technique which is better known in the literature as the weighted-objective approach. This is basically not the best choice for solving a multi-objective optimization problem [16], however, it enables to use a classical optimization strategies rather than time-consuming multi-objective algorithms. Furthermore, it can be seen that in this case the number of constraints is much lower compared to the multi-objective approach due to the simplification of considering loss at design flow only. Finally, the number of design variables for the whole tree-dimensional blade geometry description in the design vector $\mathbf{p}$ is 20 and independent from the number of considered blade sections. This low number results from the applied three-dimensional blade geometry parameterization approach.

7 RESULTS

The multi-objective blade design problem Eq. (1)-(2) is solved for seven different blade sections. Due to the fact that the two-dimensional blade solver MISES does not consider any secondary flow phenomena, the calculated blade sections are selected off the end wall region in between 20 and 80% of the blade height. The optimization is performed using the Non-dominated Sorting Genetic Algorithm (NSGA-II) [17] which is characterized by a quick convergence towards the Pareto-curve mainly driven by a fast non-dominated sorting method, and a crowding distance method which is able to spread the solutions during the optimization process and to provide a better resolutions of the final Pareto-curve. The optimization is performed with a population size of 50 individuals proceeding over 200 generations which leads to an optimization time of approximately 5 days for each section. The reason for that is due to the three required design evaluations in the multi-objective problem definition in order to determine both objective function values. The final three-dimensional blade shape is obtained by stacking trade-off solutions providing the highest working range value from each Pareto-curve in radial direction.

The single-objective problem Eq. (3)-(5) is solved based on five different blade sections between 33 and 66% blade height. In order to gather first information about the single objective in the specified design space, 400 initial designs are generated using a Sobol sequence [18]. This quasi-random or low discrepancy algorithm is "less random" than a (pseudo-)random number sequence. It tends to sample the design space more uniformly and reduces clustering effects which shows to be more effective, e.g. in terms of global optimization. Then, as a starting point for an optimization, promising designs are selected from the exploration phase. The so-called Tangent Search Method (TSM) is utilized to find the local minimum in the vicinity of each design, [19]. This search method works well in practice for a large variety of problems since it does not rely on gradient and curvature information like gradient-based methods. The major features of the TSM are exploratory moves to detect a decent search direction, global moves to ensure fast improvement in the promising search direction and tangent moves to keep the search in the feasible domain, [20]. If a constraint bound is approached, a tangent move in hyperspace is performed tangentially to the constraint. The overall optimization time for this single-objective optimization approach is approximately 24 hours on a standard Linux PC. The reason for this short optimization time is the deterministic optimization approach and the reduced parameter set of the three-dimensional blade shape parameterization method.

The resulting geometry and objective function values of both optimization approaches are shown in Figure 11 and Figure 10, respectively. A comparison with the datum design shows that both optimized solutions produces less loss at each considered blade section while the differences between the optimized designs are rather small. Approach 2 (single-objective) is better in loss for the upper four sections compared to approach 1. The reason for that is that the optimized solutions for approach 1 are selected from the set of Pareto-optimal results which are best in working range and not in loss. Therefore, a comparison between both optimized blades on loss at design point conditions only is not fair. Furthermore, it can be seen that the multi-objective optimization approach is also able to improve the working range WR as the second objective in the optimization process. In general it can be stated that the improvements in loss are between 0.1 and 0.25%, and up to 1.5 degree in working range.

If we compare the resulting geometries, huge differences are observable. The multi-objective approach produces a wiggly blade trailing edge caused by the individually optimized sections, while the result of the single-objective optimization approach is smooth due to the blade parameterization technique. This is an interesting result, since both geometries are very close to each other in their two-dimensional performance, but differ significantly in shape.

In order to judge the three-dimensional performance, both optimized geometries are selected and full multi-stage,

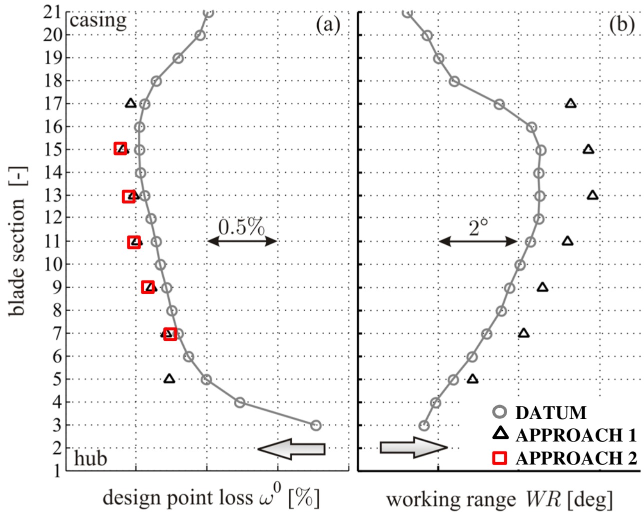

Figure 10. Radial distribution of loss at design flow angle ω^0 (a) and working range WR (b) for both optimized stator blades compared to the datum design

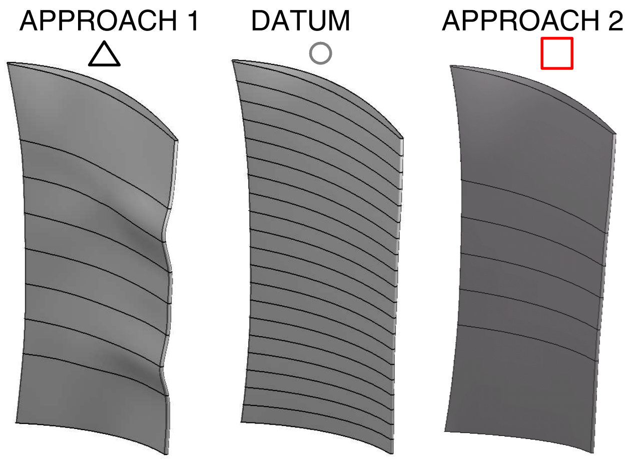

Figure 11. Comparison of optimized blade geometries with the datum design

three-dimensional flow analyses with outlet pressure variations are performed as described before. Figure 12 shows the pressure ratio and the polytropic efficiency distribution for both optimized cases in comparison to the datum design for the design shaft speed. It can be observed that in general the compressor behaves very similar in all cases, i.e. pressure ratio and efficiency are nearly similar for most of the calculated points. A closer look on the distributions shows that in the vicinity of the design point, which is indicated within a dashed circle, the optimized cases are superior in pressure ratio and efficiency.

For a better comparison of the results, pressure ratio

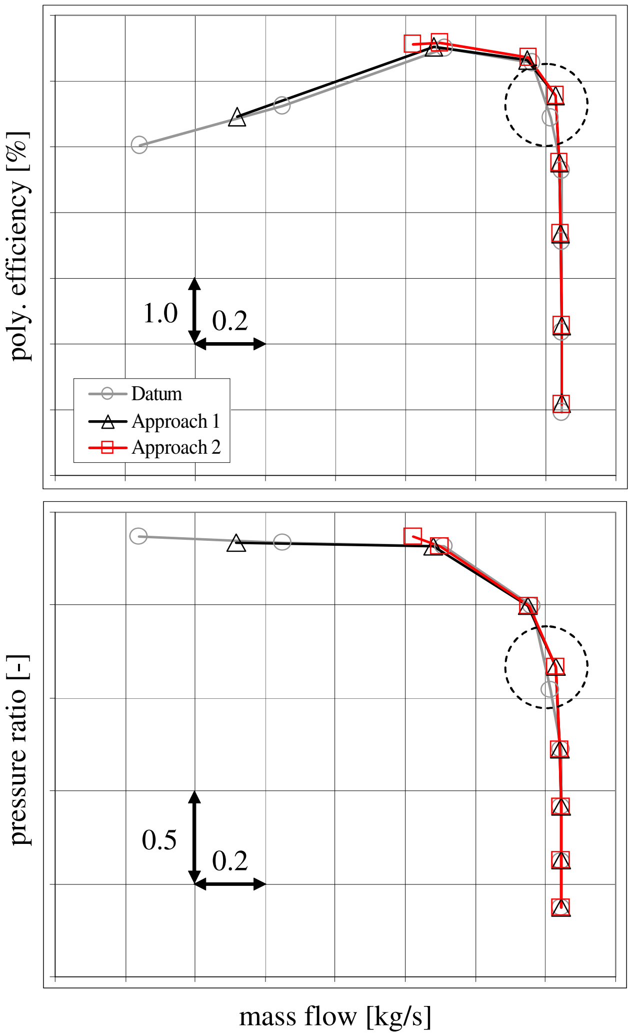

Figure 12. Pressure ratio and polytropic efficiency distributions versus inlet mass flow for optimized and datum blade design

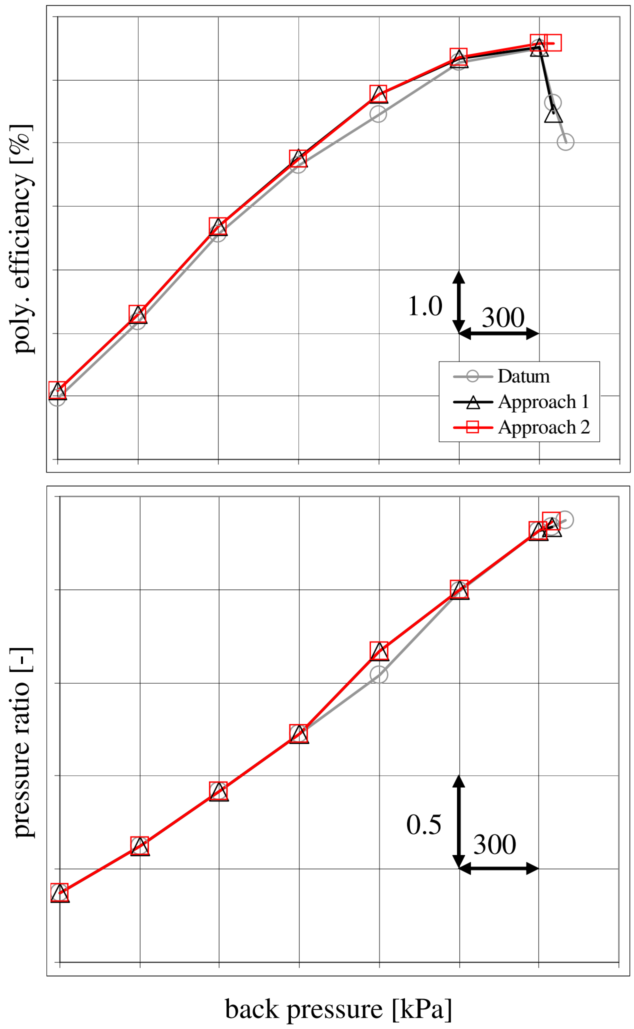

Figure 13. Pressure ratio and polytropic efficiency distributions versus back pressure for optimized and datum blade design

and efficiency are plotted against the back pressure variation for all three distributions, Figure 13. The optimized cases achieve a significant improvement in pressure ratio at design point conditions without being worst for the other calculated points. In terms of efficiency, an improvement for the same point is also observable which is an obvious result since in both optimized cases the loss at design flow condition is used as an objective to be minimized. More interesting is that the compressors with optimized stator 3 blades show benefits in efficiency for almost each of the calculated back pressure conditions. This was expected for the multi-objective approach since working range is considered as second objective. In the single-objective approach, however, no further objective or constraint are taken into account implicitly leading to this phenomena.

CONCLUSIONS

The aim of this paper is to investigate and discuss two different approaches for supporting the time-consuming industrial blade design process. The first approach is mainly characterized by a complex multi-objective optimization problem formulation and a highly flexible blade section geometry description. In contradictions to that, the second approach focuses on a more simple single-objective optimization problem definition with a reduced parameter set for the three-dimensional blade geometry description.

This investigation shows that the obtained results differ with respect to the final blade geometry, but the two-dimensional as well as the three-dimensional performances are very close to each other. Improvements in pressure ratio at design point conditions and in polytropic efficiency for almost each of the calculated back pressure points are achieved for both optimized cases compared to the datum design.

However, from the industrial point of view the second approach is more preferable. It produces smooth blade shapes which are better in terms of stress issues and the overall optimization time of approximately 24 hours for the entire blade geometry is extremely lower compared to the first approach.

As an outlook for further work, this investigation will be extended to rotor blades in order to find a general approach which is valid for rotor and stator blade shapes. Since the global aim of automated optimized blade geometries is to find best blade shapes which fulfill all requirements, it is planed to expand the current aerodynamic design process by introducing a stress analysis program and to consider multidisciplinary parameters within the optimization problem definition.

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