HELICOPTER NOISE AND VIBRATION REDUCTION WITH ADAPTIVE FIBER COMPOSITES

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1. OVERVIEW

Acoustic emission to the environment as well as vibration load in the interior significantly restrains the operation of rotorcraft. Hence the future development of helicopters is deeply linked to systems with abilities to alleviate these effects. Such adaptive structural systems in conjunction with multifunctional materials facilitate technical solutions with a wide spectrum of applications and a high degree of integration. By virtue of combining the actuation and sensing capabilities of piezoelectric materials with the advantages of fiber composites, the anisotropic constitutive properties may be tailored according to requirements and the failure behavior can be improved. Such adaptive fiber composites are very well-suited for the task of noise and vibration reduction. In this respect the helicopter rotor system represents a very interesting and widely perceptible field of application. The occurring oscillations can be reduced with aid of aerodynamic couplings via fast manipulation of the angle of attack, being induced by twist actuation of the rotor blade. On the one hand the sensing properties may be used to determine the current state of deformation, while on the other hand the actuation properties may be used to attain the required state of deformation. The implementation of such concepts requires a broad range of knowledge. This includes the microelectromechanical modeling of piezoelectric composites as well as formulations for their employment in layered shells. Thereupon, the properties of thin-walled beams with actuation and sensing capabilities are derived for arbitrary cross-sections in consideration of shear flexibility and torsional warping. First, an analytic solution is obtained to gain insight into the static behavior and to be used for adjustment and optimization of beam configurations. Second, a general solution is derived via the development of spatial beam finite elements to capture the dynamic behavior of the rotating structure. After successful validation by comparison of solution components to those attainable in calculations with conventional shell finite elements, a comprehensive and versatile formulation is achieved.

2. INTRODUCTION

» The air was drowsy with the murmur of bees and helicopters. «

In The Brave New World of Huxley [1] the helicopter represents the dominant mean of personal transportation. The indeed shining but essentially dark vision of the future is still ahead of reality for the most part. This obviously applies also to the appealing aspect of replacing automobiles by helicopters, as no one today would seriously dare to compare their noise with the buzzing of bees. The spectrum of rotorcraft operation in the present time is limited by the impact of the occurring noise and vibrations. For civil applications the emitted noise constrains the operation in populous areas and for military use it leads to an easy detectability. The high noise and vibration levels inside the cabin stress the pilots and reduce passenger comfort. The vibration loads can cause component fatigue and require frequent and costly inspections. Moreover the performance concerning maneuver and high speed flight can be enhanced by the installation of adaptive structural systems. The problem of vibrations had to be addressed from the early beginning of rotorcraft development and the noise emission is coming more and more to the fore. Altogether these not separately treatable problems have great economic impact for this aircraft type. To give an impression of the most important interrelations, these effects are illustrated in FIG 1.

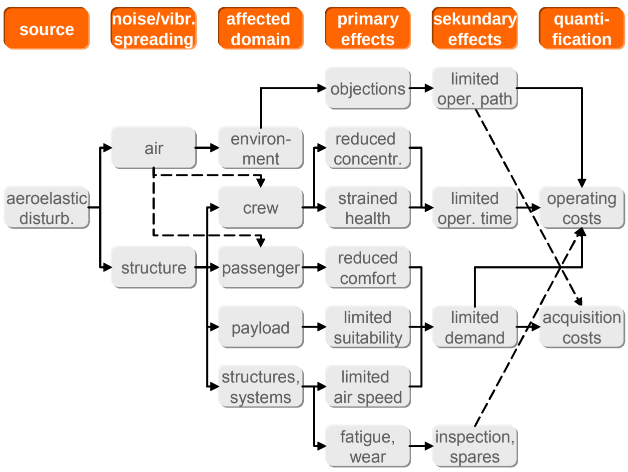

FIG 1. Complexity of noise and vibration related problems of the helicopter.

2.1. Noise and Vibration Problem

Naturally there exists a close relationship between the structural vibrations and the emitted noise of a rotorcraft. This concerns especially those parts with aeroelastic interaction, where aerodynamic loads and mechanical reactions excite the structure on the one hand and cause acoustic effects in the circumfluent air on the other hand. Furthermore the induced vibrations tend to spread over the entire system and might initiate noise emission or other perturbations at different locations. In this respect the main rotor can be identified as the most relevant origin for the noise and vibration problems of helicopters. From the structural point of view the slender, vibration susceptible blades with considerable aerodynamic damping only in flapwise direction have to be mentioned. Further on, there is the sophisticated rotor hub mechanism with the swash plate to induce angles of attack depending on the rotational position to account for the asymmetric flow conditions in non-hover flight. From the aerodynamic point of view it is dealt with complicated flow conditions leading to a number of undesirable effects. Several associated excitation mechanisms are denoted and located in FIG 2.

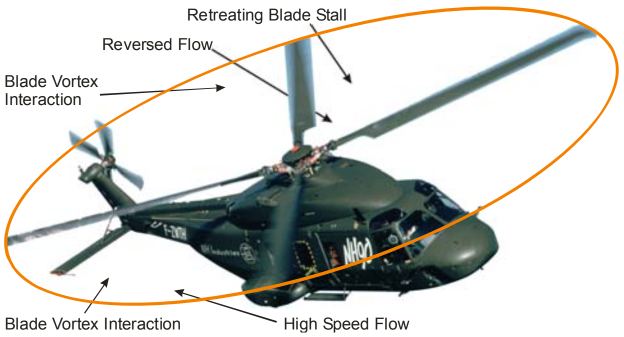

FIG 2. Aerodynamic sources of noise and vibrations at the helicopter main rotor.

2.2. Passive Measures

To produce relief a variety of partially very different approaches has been discussed and developed, ranging from external devices like pendulum absorbers and hydraulic dampers to modifications of the elastomechanic and aerodynamic behavior. The absorber devices improve the situation locally at the specific mount point and are adjusted to a specific frequency proportional to the rotor speed and thus only exhibit a certain degree of self tuning. The damper elements are employed to reduce the amplitudes of the oscillations below a critical margin. Most often they are positioned at the blade root in lead lag direction, as the oscillations in the rotor plane are only slightly damped by the aerodynamics. In general, these external devices are relatively simple in design and application, but imply additional weight, drag, and maintenance effort. Details may be found in [2] and [3]. When attempting to alter the elastomechanic and aerodynamic behavior of the rotor blade with its diverse couplings and thus the susceptibility to vibrations and noise, the complicated interrelations have to be kept in mind. As the blade responds to a composition of several excitation loads, these interrelations might force significant reduction or cancellation of vibrations, in principle just like an absorber. Regrettably this composition depends on the flight situation and therefore a beneficial coupling effect for a specific case might reverse to the contrary for another. Examples for such rotor modifications are given in [4].

2.3. Active and Adaptive Measures

The passive methods available for the reduction of noise and vibration are not able to achieve completely satisfying results. Thereupon different concepts involving control systems have been developed to facilitate more flexible intervention, ranging with increasing degree of structural integration from active, [2] and [3], to adaptive, [5]. As most of the characteristic perturbations occur with the blade passage frequency and its higher harmonics, the simplest approach is to modify the blade pitch correspondingly. The necessary motion is produced by stationary hydraulic actuators, inducing a vertical displacement of the swash plate and thus a collective actuation of the blades. Such an actuation mechanism may also modify the inclination of the swash plate, but it is still not possible to respond to events at an individual blade with this higher harmonic control (HHC). To improve this situation, the blade root actuation mechanism was advanced by inserting the hydraulic actuators between swash plate and blade roots in the control rods. With comprehension of adequate control algorithms it would be possible to implement a very flexible and powerful noise and vibration suppression system. Admittedly, the expenditure for this individual blade control (IBC) with a hydraulic system in the rotating part of the rotor hub is very high and therefore so far has been implemented only in prototype aircraft. Apart from further development of control algorithms, there is a need for efficient actuation mechanisms. The integration of flaps into rotor blades holds some not negligible challenges. In order to be aerodynamically effective, the intervention needs to be located in the outer blade region, where extreme centrifugals loads as well as spatial restrictions are present and the mass distribution should preferably not be altered. Under these conditions the application of hydraulic systems is hardly conceivable and multifunctional materials like piezoelectric ceramics come into operation. However, a discrete flap always disturbs the air flow and consequently reduces the aerodynamic performance, particularly in the extreme flow conditions exhibited by a helicopter rotor. Although the actuators themselves are designed to live without too many moving parts, quite a few hinges and connections are necessary, increasing complexity and maintenance effort. Multifunctional materials are currently not able to provide the performance needed for blade root actuation. In order to influence the aerodynamically interesting outer region of the blades, they can be applied to induce twist or manipulate the blade shape otherwise. A number of approaches for active blade twist have been developed, involving directionally attached monolithic piezoceramics or passive couplings of the anisotropic blade skin to convert the excitation of an actuator. The highest degree of integration is reached with the distributed application of adaptive fiber composites with tailorable active, sensory, and passive properties. In such a configuration adaptive layers are used as or merged into the blade skin and therefore provide actuation authority without moving parts or flow disturbance and only with a minor weight penalty, as they contribute to the passive structural behavior. Under this aspect for the case of piezoelectric fibers certain limitations in the material properties have to be considered. Due to their ceramic nature, they are relatively brittle and should carry loads in compression rather than in tension direction.

3. CONCEPTUAL CONSIDERATIONS

The essential noise and vibration phenomena occurring at the helicopter main rotor predominantly result in an excitation of bending oscillations of the blades. Thus, it is sought for strategies to compensate the associated displacements.

3.1. Actuation Schemes

The rotor blade may be modeled as a thin-walled beam. Equipping such a structure with adaptive fiber composites permits different actuation schemes to compensate for bending related displacements, see FIG 3. In a static environment it is self-suggesting to accomplish this by the induction of opposing displacements. Such bending actuation may be realized directly through expansion and contraction of opposing wall sectors and through shear deformation of transversely oriented wall sectors. Alternatively coupling effects due to constitutive anisotropy of the walls may be exploited, transforming for example a lengthwise expansion of piezoelectric layers consistent throughout the cross-section into the desired beam bending. In a rotating environment it is possible to amplify the in general rather small attainable displacements with aid of the aerodynamic forces. Since a small change in the angle of attack may lead to a significant change in lift and drag with the associated blade displacements, twist actuation becomes important. It can be achieved again either directly through the consistent induction of shear in the walls or via structural couplings related to the constitutive anisotropy of the walls as well as to the warping of the cross-section. For example, the prior couples extension with torsion and the latter warping with torsion. Naturally not all of the outlined actuation schemes are equally suitable to diminish the helicopter rotor problems. The research reported in the literature is clearly focused upon the direct torsion. Within the work at hand a general approach is developed, capable to describe all potential actuation schemes by means of a single theory of thin-walled beams with incorporated adaptive fiber composites.

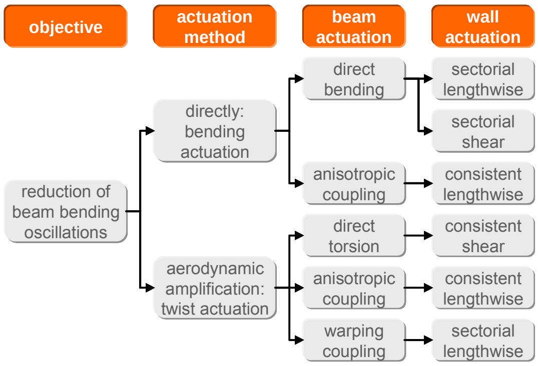

FIG 3. Actuation schemes for the reduction of beam bending oscillations in consideration of aerodynamic forces in a rotating environment.

3.2. Development Status

The initial examinations of directly induced torsion were conducted for monolithic piezoelectric ceramics being attached in +/- 45° direction to the top respectively bottom side of the blade. Modeling aspects thereof are reported for example in [6], while experiments including hover testing of a scaled rotor are covered in [7]. The application of piezoelectric fiber composites has been carried out in theory and experiment by means of a scaled rotor blade of a Boeing-Vertol CH-47D helicopter, [8]. Thereby predictions of the static twist performance of a box beam were gained with aid of a finite beam element model. Test results of this blade in the rotating environment are reported by [9]. Further development and testing of an active twist rotor with actuator patches of piezoelectric fiber composites attached to the spar has been conducted, [10]. In a series of wind tunnel experiments with open-loop control it has been shown, that the vibratory loads at the rotor hub can be reduced significantly, [11], and that there also is a potential for noise reduction, [12]. The blades have been modeled with two cells and thin walls, determining the cross-sectional properties in a linear analysis and the global dynamic behavior in a non-linear analysis for small strains and finite rotations with beam finite elements, [14], and with additional consideration of the aerodynamics, [15]. Thereby an asymptotic analysis approach leads to a beam description without explicit degrees of freedom for transverse shear and out-of-plane warping and thus it is desisted from the study of associated actuations concepts. A subsequent adjustment to a Timoshenko-like beam description was done by [16]. Further modeling approaches are reported in the context of an aeroelastic analysis, [17], and of a multi-body simulation, [18]. Thereby the prior is restricted to extension, torsion, and structurally uncoupled uniaxial bending, while the latter uses a finite element discretization of the cross-section to determine the beam stiffness and actuation properties, which subsequently enter a finite volume formulation. Preceding work to the publication at hand is presented for example in [19]-[22].

4. MATERIAL CONSIDERATIONS

Adaptive structural systems feature the capability to respond adequately to changing environmental conditions in terms of mechanical loads or displacements. Materials to be employed for the realization of such systems, like in case of the rotor blade concepts discussed above, shall be characterized in the following.

4.1. Multifunctional Materials

Especially suitable for the implementation of adaptive structural systems are materials which are able to take over several different tasks. Such multifunctional materials are able to provide actuation authority and might even have sensory capabilities in excess of their structural properties. Due to the reversibility of the piezoelectric effect, [13], materials exhibiting such an electromechanical coupling may be utilized to handle actuation as well as sensing tasks. The different piezoelectric materials are able to provide these functionalities in a frequency spectrum ranging beyond the level of acoustics. On the one hand, there are several monocrystals and polycrystalline ceramics, which are hard and brittle and therefore are suitable only for relatively small strains. On the other hand, there are semicrystalline polymers, which are soft and elastic but show less pronounced coupling properties. Another kind of electromechanical coupling is inherent to electrostrictive materials. This non-linear behavior is limited to actuation and typically applies also to some polycrystalline ceramics with similar consequences. Magnetostrictive materials may be employed for actuation and sensing by virtue of non-linear magnetomechanical coupling. Thereby alloys of iron and rare earth elements are capable to handle slightly higher strains than the above electromechanical coupling examples in a frequency range up to the level of acoustics. To establish respectively to detect the associated magnetic fields, comparatively massive devices need to be employed. Actuation with large strains may be realized by instigating phase changes of shape memory alloys. This highly non-linear thermomechanical coupling however is confined to very low frequencies. Carbon nanotubes possess excellent mechanical properties and their utilization for actuation as well as sensing is the promising subject of intense research activities in the field of material science.

4.2. Adaptive Fiber Composites

The concept of fiber composites is successfully applied to improve the structural properties of conventional materials. Likewise it may be adopted to increase the performance of multifunctional materials. For the anticipated rotor blade application frequency requirements and mass restrictions lead among the commercially available multifunctional materials to the selection of piezoelectric ceramics. Such materials, e.g. Lead Zirconate Titanate (PZT), feature in polarized state transversely isotropic mechanical as well as electric and piezoelectric properties with the plane of isotropy perpendicular to the polarization direction. In order to enable actuation and sensing of structures, involving laminar elements like plates and shells, their application is well established. Thereby in the plainest case monolithic ceramic materials are attached in form of thin patches on one or both sides. For any case of structural deformation the aspired effective direction of the piezoelectric effect lies in the plane of the piezoelectric laminae. Thereby initially areal electrodes on its top and bottom surface have been employed to provide respectively perceive the electrostatic field. To avoid unintentional and possibly creeping repolarization it should be refrained from diverging directions of polarization and electric field strength. Thus both of these directions are oriented normal to the plane of the considered patch and only the minor piezoelectric effect in the isotropic transverse plane can be exploited. To increase the electromechanical coupling and allow for anisotropic and thus directional actuation and sensing the concept of interdigitated electrodes has been introduced for monolithic piezoelectric laminae, [23]. Thereby the in-plane placement of the parallel directions of polarization and electric field strength is permitted. These directions jointly change sign from one interval between opposing polarity electrodes to the next and in this manner assure a uniform behavior. To compensate for the drawbacks caused by the brittle and inflexible nature of ceramic materials the embedding of piezoelectric materials in form of fibers into a polymer matrix has been implemented for areal electrodes, [24]. Thus besides increased strength and conformability also the advantageous possibility of optimizing the anisotropic properties is gained. Further improvements are achieved, when in addition interdigitated electrodes are applied, [25], as shown exemplarily in FIG 4.

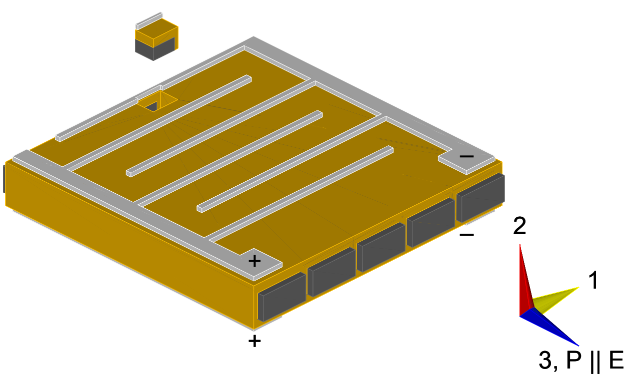

FIG 4. Piezoelectric fiber composite with interdigitated electrodes.

5. FORMULATION HIGHLIGHTS

Within the limits of the publication at hand the structure of the theoretical framework, developed to simulate the dynamic behavior of a rotating blade with integral actuation and sensing, shall be briefly summarized by means of outlining the associated major steps. For an in-depth presentation of the context and the complete derivation of formulations please refer to [26].

5.1. Piezoelectric Composites

To be able to tailor respectively optimize the properties of composites with piezoelectric fibers according to specific application conditions, insight into the constitutive interdependencies needs to be available. Therefore an enhanced micro-electromechanical model to determine the constitutive properties of piezoelectric composites based on the sequential stacking of constituents with uniform fields is proposed and compared with alternative approaches as well as validated with aid of experimental and finite element modeling results. An example thereof is shown in FIG 5.

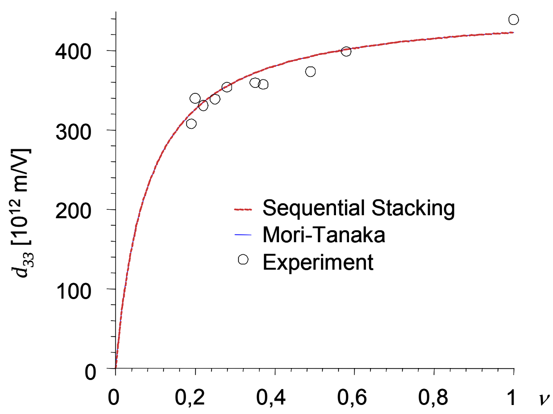

FIG 5. Variation of induced strain piezoelectric coupling coefficient d_{33} with the fiber volume fraction ν - experimental results from [27].

5.2. Adaptive Laminated Composite Shells

In the next step a macro-electromechanical model is derived for shells consisting of several laminae of the above piezoelectric composites. Thereby the classical lamination theory is extended in consideration of laminae groups with piezoelectric properties and electrically paralleled electrodes. The resulting constitutive equations are accompanied by the appropriate kinematic relations for thin shells. Therewith a comprehensive description of composite shells with piezoelectric layers in arbitrary configuration for possibly combined actuation and sensing is supplied.

5.3. Adaptive Thin-Walled Beams

The deployment of the above adaptive laminated composite shells as thin walls of a beam structure is analyzed in the following step. Thereby the thin-walled beam kinematics and the set-up of warping functions for general anisotropic cross-sections with arbitrary open branches and respectively or closed cells is reconsidered. Consequently a novel theory for such thin-walled beams incorporating more than just membrane properties without additional degrees of freedom is developed for arbitrary cross-sections in consideration of shear flexibility and torsional warping as well as of the implications of a rotating environment.

5.4. Virtual Work Statements

For the subsequent derivation steps the principle of virtual work is employed respectively its components are examined. The constitutive relation and coefficients of the beam are deduced by comparing the virtual work of internal loads of beam and shell formulation. Equilibrium and boundary conditions are obtained with aid of the principle itself for the quasi-static case, where loads in principle may change over time but inertia effects are not considered. To complete the principle of virtual work for the general case, inertia load contributions, stemming from D'Alembert's principle in the Lagrangian version, are then established within the virtual work of external loads. Further on various sorts of pre-stress effects, including the stiffening due to centrifugal forces, are captured with an extended set of second order terms within the virtual work of internal loads.

5.5. Solution Variants

Besides the already complicated interactions due to arbitrary mechanical and electromechanical couplings the investigated system obviously requires the consideration of additional couplings due to the gyroscopic and second order theory effects. Consequently the general problem may only be solved by means of an approximation technique. Exact solutions of manageable complexity however may be found for simplified problems. Thus, solutions are obtained on the one hand for the statics of the non-rotating structure in analytic fashion and on the other hand for the dynamics of the rotating structure with aid of the finite element method.

Analytic Approach

In order to warrant the accessibility of the anticipated analytic solution, the mechanical couplings have to be limited to those, which are relevant for the considered application. Making use of equilibrium and boundary conditions of the quasi-static case in conjunction with the constitutive relation as obtained for the beam above, the coupled solution formulas for extension, torsion, and warping as well as for shear and bending are derived. The analytic solution may be utilized to gain insight into the behavior of structures with adaptive capabilities, to optimize the beam configuration, and to support the validation of the finite element solution

Beam Finite Elements

In order to obtain a solution without the substantial restrictions dictated by insisting on an analytical approach, the beam is divided into a finite number of elements. Discrete variables are introduced for each degree of freedom at the endpoints of these elements with an interpolation facilitated by shape functions in between. The beam displacements, cross-sectional rotations, and electric potential distributions are approximated with C_0 continuous linear Lagrange polynomials. In case of the beam twist the consideration of warping torsion is associated with the twist rate, so C_1 continuity is required and achieved by use of cubic Hermite polynomials. Thereafter this discretization is inserted into the principle of virtual work for the general case. To avoid the implications of the shear locking effect, which might appear in the context of the Timoshenko beam description, a reduced integration scheme is applied. For the simplified case of time invariant matrices the following equation of motion finally is derived:

$$\hat{\mathbf{M}}''\hat{\boldsymbol{v}}_{,tt}(t)+\hat{\mathbf{M}}'_{\Omega}(\Omega)\hat{\boldsymbol{v}}_{,t}(t)+\hat{\mathbf{P}}_{\Sigma}(\Omega)\hat{\boldsymbol{v}}(t)=\hat{\boldsymbol{p}}_{\Sigma}(t,\Omega) \tag{1}$$
$$\text{with}\quad \hat{\mathbf{P}}_{\Sigma}(\Omega)=\hat{\mathbf{P}}+\hat{\mathbf{G}}(\Omega)+\hat{\mathbf{M}}_{\Omega}(\Omega),\quad \hat{\boldsymbol{p}}_{\Sigma}(t,\Omega)=\hat{\boldsymbol{l}}(t)-\hat{\boldsymbol{p}}(t)-\hat{\boldsymbol{m}}_{\Omega}(\Omega).$$

This system of differential equations is of gyroscopic undamped type with time invariant matrices, see [28]. The mass matrix $\hat{\mathbf{M}}''$ is symmetric, the gyroscopic matrix $\hat{\mathbf{M}}'_{\Omega}(\Omega)$ is antimetric and the stiffness matrix $\hat{\mathbf{P}}_{\Sigma}(\Omega)$ is symmetric as well as positive definite. The latter summarizes constitutive properties, geometric stiffness influences, and deformation associated inertia effects. The applied loads, piezoelectric coupling implications of electric parameters, as well as initial state inertia effects are joined in the vector $\hat{\boldsymbol{p}}_{\Sigma}(t,\Omega)$. The problem at hand is solved with the separate treatment of steady state, homogeneous, and particular solution. So the derivation of the general solution is conducted via formulation of spatial beam finite elements, accounting for arbitrary combinations of actuator and sensor applications with voltage and current source respectively measurement, to capture the dynamic behavior of the rotating structure.

6. CALCULATIONS

The sequence of developed formulations, leading the way from piezoelectric composites via adaptive laminated composite shells to adaptive thin-walled beams, allows for examinations of almost arbitrary complexity. The attempt to provide examples for the full spectrum of capabilities shall be dropped in favor of an application oriented approach. Hence, the subsequent investigations shall be carried out in view of the integral actuation of structures with certain similarity to helicopter rotor blades.

6.1. Configuration

The developed theory is able to cope with thin-walled beams of arbitrary cross-section. This includes highly complex configurations with any combination of closed cells and open branches. In absence of correspondingly defined requirements and since such a complexity is not necessary for the purpose of elementary examination and validation, the focus shall be placed subsequently upon two rather unpretentious set-ups. These are a rectangular single-cell and a convex double-cell cross-section with constant wall properties all around the circumferential contour, see FIG 6.

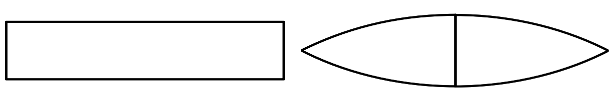

FIG 6. Rectangular single-cell and convex double-cell cross-section.

To obtain results within the same order of magnitude as expected from a real rotor blade, the data of an existing main rotor system shall be utilized. Then the cross-sectional dimensions and the wall set-ups may be adjusted to match the diagonal entries of the beam stiffness matrix of blades from the BO105 helicopter of Bölkow / MBB / Eurocopter. Since less parameters are specified than available, this is accomplished within the framework of an optimization. Therefore the analytic solution for the box beam is utilized in conjunction with a layer arrangement, which is able to depict all the twist actuation schemes of FIG 3. Not unexpectedly, the objective of maximum tip twist is achieved at the best with direct twist actuation. The associated results are shown exemplarily in FIG 7 with the optimal tip twist of 9.45° at a volume fraction of piezoelectric fibers amounting to 0.475 in two symmetric layer pairs with +/-17.6° orientation and a layer thickness of 1.82 mm. This configuration of the rectangular single-cell cross-section will be used for the subsequent calculation. For the convex double-cell cross-section a corresponding configuration is selected.

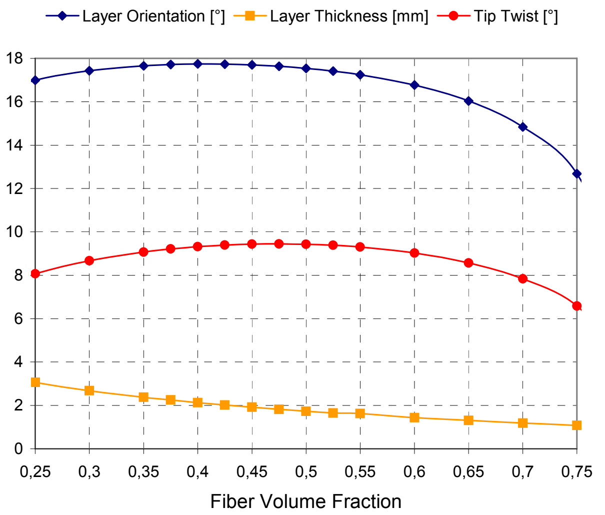

FIG 7. Influence of the fiber volume fraction on necessary layer orientation and thickness as well as on the resulting tip twist for the direct twist actuation.

6.2. Procedure

To ensure soundness of the various assumptions and simplifications made throughout the course of derivation as well as to exclude errors in the implementation the obtained results in absence of experiments have to be counterchecked with independent approaches. Thus, the outcome of calculations performed with the developed analytical approach and beam finite elements shall be compared with results from commercial shell finite elements. Another aspect to be checked is the utilization of a complete thin-shell description for the walls of the beam, as implemented within the derived formulations. Therefore, the results obtained for the shell description are to be compared to the outcome of the significantly simpler membrane description, which is deduced easily by neglecting the shell bending and twisting stiffness coefficients. The individual restrictions associated with the three different solution approaches shall be discussed in the following.

Analytic Approach

Within the bounds of the underlying theory the analytic solution can be regarded as exact. It is confined to the statics of the non-rotating structure as well as certain constitutive couplings and load configurations. Special cases like the elongation of the rotating beam due to centrifugal forces may be simulated with the appropriate choice of loads.

Beam Finite Elements

Due to the discretization and interpolation the finite element method is categorized as an approximation. The corresponding details are shown throughout the course of derivation of the beam finite element solution. With identical underlying theory but without all the restrictions necessary to obtain an analytical solution it is able provide answers to a wide range of problems.

Shell Finite Elements

The implementation of anisotropic thermal effects in commercial finite element codes may be utilized to simulate the implications of the piezoelectric effect. In order to capture the behavior of thin-walled beams with cross-sections as defined above, spatial shell elements may be employed. With this methodology however, it is not possible to examine problems with dynamic actuation. The beams with rectangular and convex cross-section have been discretized with 2200 respectively 2300 SHELL99 elements of ANSYS.

7. RESULTS

Following the structure of the beam finite element solution, first, the static behavior is checked by means of the steady state solution, second, the free vibrations governed by the homogeneous solution are analyzed, and third, the forced vibrations are simulated involving the particular solution.

7.1. Static Behavior

Since neither the analytic approach nor the application of shell finite elements is able to handle the general problem of dynamic actuation and response in the rotating environment, the developed beam finite elements need to be counter-checked by means of the individual solution components.

Beam Extension due to Centrifugal Forces

First, the steady state solution with the elongation of the blade resulting from the centrifugal forces shall be examined. For the analytic approach the required line force in lengthwise direction of the beam depicting the centrifugal effects are determined in consideration of individual cross-sectional geometry and employed materials. Although the stiffness properties are largely similar the total masses of the two beam variants differ significantly due to the diversity of construction and materials. While the single-cell rectangular cross-section possesses a mass of 33.8 kg, the double-cell convex cross-section beam gets by with only 20.0 kg. The beam elongation in response to the centrifugal force field is shown for the different calculation approaches in TAB 1. Since both beams are free of extension torsion coupling, no plate properties are involved in the solution to the lengthwise displacement. Therefore the comparison of the two analytic variants cannot show any divergence to be induced by the membrane response assumption. Both finite element solutions are very close to one another respectively show only minor deviations to the analytical solution. Therewith the vector of centrifugal forces $\hat{\boldsymbol{m}}_{\Omega}(\Omega)$ as well as the longitudinal components of the beam stiffness matrix $\hat{\mathbf{P}}_{\Sigma}(\Omega)$ of EQ (1) are checked successfully.

Method	Assumption	Rectangular Single-Cell Extension [cm]	Rectangular Single-Cell Error [%]	Convex Double-Cell Extension [cm]	Convex Double-Cell Error [%]
analytic	shell	1.4204		0.7760	
	membrane	1.4204	0	0.7760	0
beam FE	shell	1.4252	+0.34	0.7774	+0.18
shell FE	ANSYS SHELL99	1.4245	+0.30	0.7817	+0.74

TAB 1. Beam extension due to centrifugal forces.

Beam Torsion due to Piezoelectric Coupling

Next, a constant electric field shall be applied to the piezoelectric composites within the non-rotating structure in order to verify the constant factor of the piezoelectric actuation vector $\hat{p}(t)$ as well as the torsional components of the beam stiffness matrix $\hat{\mathbf{P}}_{\Sigma}(\Omega)$. This is done in consideration of the warping effect with cubic Hermite shape functions and without the warping effect using linear Lagrange polynomials. The first shows good agreement with the exact analytical solution and the shell finite element model, while in the latter case the torsional rigidity is notably smaller due to abandonment of the warping restraint at the clamped end. Naturally, the actuation vectors in both cases are identical. The values of the resulting blade tip rotation are given in TAB 2.

Method	Assumption	Rectangular Single-Cell Twist [°]	Rectangular Single-Cell Error [%]	Convex Double-Cell Twist [°]	Convex Double-Cell Error [%]
analytic	shell	9.4479		9.1983	
	membrane	9.4484	+0.01	9.1987	+0.00
beam FE	shell, warping	9.4184	-0.31	9.2076	+0.10
	〃 , no warp.	9.5436	+1.01	9.2408	+0.46
shell FE	ANSYS SHELL99	9.4584	+0.11	9.2185	+0.22

TAB 2. Beam torsion due to piezoelectric coupling.

Different from above, plate properties are involved in the solution to the twisting angle. Due to the closed cross-sections and thin walls the implications of the membrane response assumption are very small for the cases at hand. However, this changes drastically with the consideration of open cross-section topologies just as well covered by the developed theory, where the torsional stiffness of the beam is solely governed by the twisting stiffness of the walls. Exemplarily, the box beam subjected to piezoelectrically induced torsion is shown for the shell finite element approach in FIG 8. Close to the clamped end the influence of the warping restraint on the beam twist becomes visible.

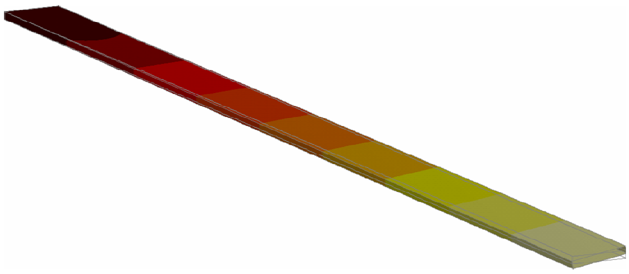

FIG 8. Torsion of the rectangular cross-section beam via piezoelectric coupling.

7.2. Free Vibrations

Since the essential parts of the right hand side of the differential equation system, given by EQ (1), have demonstrated their operability, the homogeneous solution shall be examined in detail thereafter to complete the inspection of the left hand side. As there is no analytic approach available to capture the dynamic behavior, the subsequent comparison comprises the formulations with the developed beam finite elements and with the commercial shell finite elements. The resulting natural frequencies for all modes up to the third torsional mode are given in TAB 3 for the non-rotating system as well as in 0 for the rotating system.

ω [1/s] Mode Shape		Rect. Single-Cell Beam FE	Rect. Single-Cell Shell FE	Conv. Double-Cell Beam FE	Conv. Double-Cell Shell FE
1st	lead-lag	25.92	25.88	22.56	22.61
2nd	lead-lag	159.39	158.69	139.79	139.81
3rd	lead-lag	433.67	430.11	384.80	383.51
1st	flap	5.17	5.16	6.70	6.72
2nd	flap	32.28	31.92	41.84	41.82
3rd	flap	89.80	87.37	116.42	115.08
4th	flap	174.33	164.96	226.13	217.91
5th	flap	284.84	258.11	369.70	340.62
6th	flap	419.67	357.34	545.08	470.48
7th	flap	577.01	452.55	750.04	569.63
1st	torsion	127.36	121.84	148.89	145.25
2nd	torsion	382.69	314.74	446.71	372.29
3rd	torsion	639.82	424.57	744.63	491.20

TAB 3. Natural angular frequencies of the non-rotating systems.

ω [1/s] Mode Shape		Rect. Single-Cell Beam FE	Rect. Single-Cell Shell FE	Conv. Double-Cell Beam FE	Conv. Double-Cell Shell FE
1st	lead-lag	34.73	34.99	32.27	32.53
2nd	lead-lag	193.13	192.98	177.46	177.86
3rd	lead-lag	473.34	470.79	428.97	428.58
1st	flap	48.23	48.41	48.92	48.93
2nd	flap	121.18	120.61	124.58	123.32
3rd	flap	210.86	208.71	225.04	217.71
4th	flap	323.10	315.72	355.98	330.90
5th	flap	456.24	435.27	515.80	453.23
1st	torsion	127.07	138.17	148.64	154.21
2nd	torsion	382.59	342.50	446.63	382.56
3rd	torsion	639.82	471.60	744.58	508.03

TAB 4. Natural angular frequencies of the rotating systems.

Influence of the Rotation

Comparing the natural frequencies of the non-rotating and rotating system consistently reveals a moderate increase for the lead-lag modes and a steep increase for the flapping modes. The deviating imprints of the rotational stiffening are founded on the significantly dissimilar structural stiffness properties with regard to the respective cross-sectional axis. This behavior is also being reflected in the associated mode shapes as visible exemplarily in FIG 9 for the fifth bending mode shape of the rectangular cross-section beam and the third lead-lag mode shape of the convex cross-section beam. With the increase of beam internal loads towards the center of rotation the oscillation amplitudes decrease while the wavelength is stretched. For the developed beam finite elements such effects are captured by virtue of the second order theory. Its derivation however indicates similar effects neither for rotation nor for warping. Consequently frequencies as well as shapes of the torsional modes are not sensitive to rotation in the beam finite element model. In contrast the shell finite element model shows at least a small dependence.

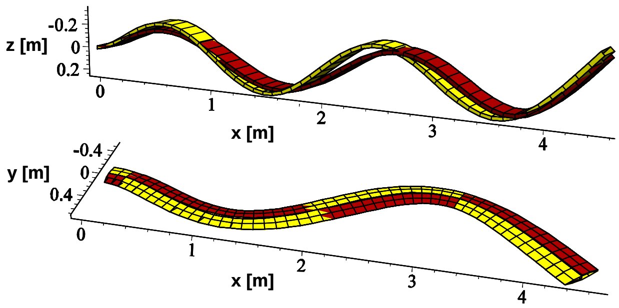

FIG 9. Fifth flapping mode of the rectangular cross-section beam and third lead-lag mode of the convex cross-section beam for the non-rotating (light) and rotating (dark) system (beam finite elements).

Influence of the Modeling Approach

The natural frequencies of the lower lead-lag and flapping modes agree very well across the models with beam and shell finite elements. The higher ones show an increasing divergence, which cannot be counteracted by a refined discretization. The reason thereof can be found in those effects exhibited by the shell description that are not included in the beam description. Most prominent among these are the implications of the classical assumption of beam theory. Stating the preservation of the cross-sectional shape in its plane is equivalent to infinite bending stiffness of the walls in this plane. Such properties obviously do not correspond to the observations made with the shell model visible in FIG 10. It becomes clear that the neglected warping deformation in the cross-sectional plane, capable to depict the local inertia effects of the walls, gain increasingly importance in comparison to the global inertia effects of the beam with higher frequencies and shorter wavelengths. The aspect ratio of the latter with the corresponding edge length of the cross-section may be considered in analogy to the buckling phenomena.

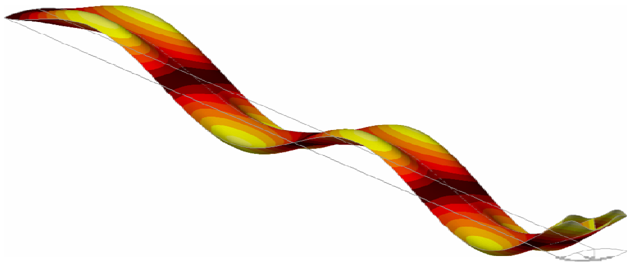

FIG 10. Fifth flapping mode of the convex cross-section beam subjected to rotation (shell finite elements).

In case of the torsional behavior the discrepancy of the natural frequencies with ascending mode shapes is even more articulate. The considered warping displacements are confined to proportionality with the twist rate and to the direction out of the cross-sectional plane. Just like in the case of the lead-lag and flapping modes there are no means to account for local inertia effects of the walls and resulting decrease of natural frequencies. In FIG 11 the cross-sectional deformations are already noticeable for the first torsional mode but become excessive for the third torsional mode. In addition to relatively thin walls both example configurations possess very slim cross-sections being fairly disadvantageous in this context.

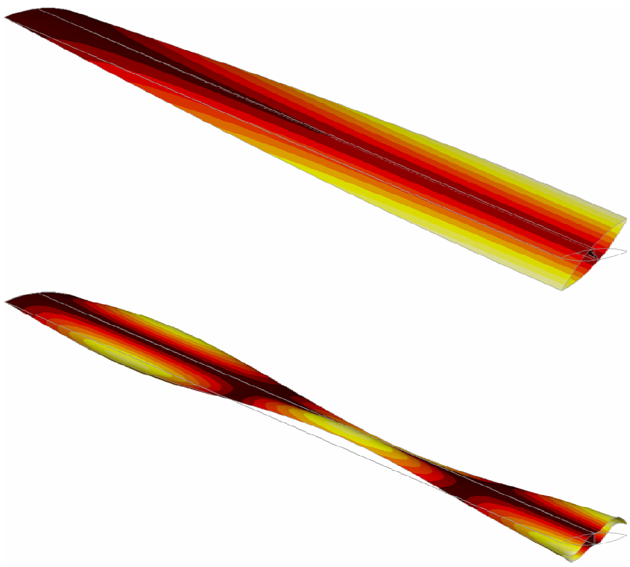

FIG 11. First and third torsional mode of the convex cross-section beam not subjected to rotation (shell finite elements).

Influence of the Cross-Section

In all cases the agreement of beam and shell finite element results for the convex double-cell cross-section is superior to the rectangular single-cell cross-section. On the one hand the outer walls of the prior are thinner and thus more flexible, but on the other hand they contribute only a part of the total mass. Further on the convex shape and particularly the stiff web stabilize the cross sectional shape. Consequently the beam with convex double-cell cross-section is less sensitive to the influence of local inertia effects of the walls in comparison to the global inertia effects of the beam. Moreover the not yet specially mentioned discrepancy of beam and shell finite element results for the first torsional mode as well as the rotation dependence of natural frequencies for torsional mode shapes in the shell model are diminished with the convex double-cell cross-section. Therefore it is permissible to conclude that the discovered deviations on the whole are due to the different handling of the cross-section with the use of beam respectively shell finite elements. So, which of these two modeling approaches is more appropriate? As always there are two faces to the truth. Regarding the results in correspondence to the input data the shell representation, presuming proper implementation, is clearly more precise. However, the input data already represents an idealization, since the actual structure, which in the given example is equipped with a foam core and thus is prevented from noteworthy deformations of the cross-sectional shape, has been replaced by its thin-walled likeness. To conclude, further pieces of the beam stiffness matrix $\widehat{\mathbf{P}}_{\Sigma}(\Omega)$ as well as mass matrix $\widehat{\mathbf{M}}''$ and gyroscopic matrix $\widehat{\mathbf{M}}'_{\Omega}(\Omega)$ may be regarded as validated.

7.3. Forced Vibrations

As far as available the counter-check of individual solution components has been successfully completed. The developed beam finite elements may now be employed for their proprietary task of simulating the universal dynamic behavior of adaptive thin-walled beams. In the most general case the developed theory is able to describe the mechanical as well as electric response to combined mechanical and electric excitation. The example configuration has been simplified to handle either actuation or sensing. For the helicopter rotor blade application especially the actuation aspect is critical and therefore has been pursued. Due to the reversibility of the piezoelectric effect this means no loss of generality towards validation. So finally it shall be sought for the response of the rotating beam to a piezoelectrically induced twist actuation. To visualize the result, the twist angle of the last node is selected from the overall solution and displayed over time in FIG 12. Since the harmonic excitation with the threefold of the rotor angular frequency is close to the frequency of the first torsional mode of the employed box beam the plot shows the characteristic beat pattern. With this last step the three-dimensional dynamic behavior of an adaptive thin-walled beam in a rotating environment is completely predictable.

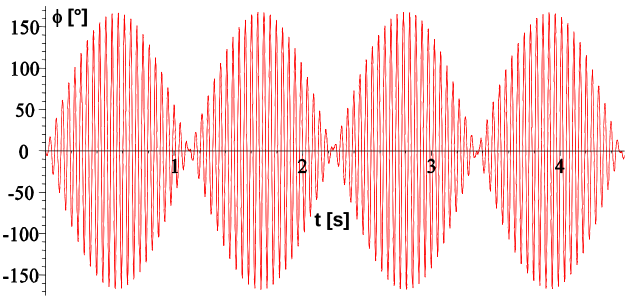

FIG 12. Tip twist of the box beam in response to harmonic excitation with the threefold of the rotor angular frequency.

8. CONCLUSION

With the presented comprehensive formulation of the theoretical framework and the associated elementary examinations the recognition and utilization of causal relationships in view of the manipulation of structural behavior with adaptive means is facilitated. With the resulting spatial beam finite elements a versatile modeling tool can be provided as a basis for further investigations. The focus of the work at hand is directed towards the structural aspects of adaptive systems with the associated actuation and sensing capabilities. The consequential next step therefore would consider the linkage with various control models, to make accessible the abilities of the complete system. With regard to the application case of helicopter rotor blades, the coupling with an aerodynamics model would allow to perform the aeroelastic analyses, necessary to prove the effectiveness of the concept. Furthermore noise emission and impact on the environment might be simulated by means of an acoustics model. To enhance the comparability of the developed adaptive thin-walled beam representation with the real rotor system, it could be refined with additional features, like pre-twist, non-rigid blade mounting, or cyclic pitch to cover forward flight conditions. Another direction for extension of the developed theory is the incorporation of alternative constitutive models to take other multifunctional materials into account. This could be interesting especially for the example of composites with electromechanical coupling through carbon nanotubes.

9. ACKNOWLEDGEMENT

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10. REFERENCES

[1] Huxley, A., Brave New World. Chatto and Windus, London, 1932.
[2] Bielawa, R.L., Rotary Wing Structural Dynamics and Aeroelasticity. AIAA, Washington DC, 1992.
[3] Bramwell, A.R.S., Gone, G., Balmford, D., Bramwell's Helicopter Dynamics. Butterworth Heinemann, Oxford, 2001.
[4] Edwards, B., Cox, C., Revolutionary Concepts for Helicopter Noise Reduction - SILENT Program. Report CR-2002-211650, NASA, Langley, 2002.
[5] Büter A. Untersuchung adaptiver Konzepte zur Reduktion von Hubschraubervibrationen, zur Minderung des Hubschrauberlärms und zur Steigerung der aerodynamischen Effizienz. PhD Thesis, RWTH Aachen. DLR, Braunschweig, 1998.
[6] Park, C., and I. Chopra, Modeling Piezoceramic Actuation of Beams in Torsion. AIAA Journal, 34, 2582–2589, 1996.
[7] Chen, P. C., and I. Chopra, Hover Testing of Smart Rotor with Induced-Strain Actuation of Blade Twist. AIAA Journal, 35, 6–16, 1997.
[8] Du Plessis, A. J., and N. Hagood, Modeling and Experimental Testing of Twist Actuated Single Cell Composite Beams for Helicopter Blade Control. Master’s Thesis, MIT, 1996.
[9] Rodgers, J., and N.W. Hagood, Preliminary Mach-Scale Hover Testing of an Integral Twist-Actuated Ro-

tor Blade. In Smart Structures and Materials 1998: Smart Structures and Integrated Systems, edited by M. E. Regelbrugge, vol. 3329 of Proceedings of SPIE, pp. 291–308, 1998.

[10] Cesnik, C. E. S., S. Shin, W. K. Wilkie, M. L. Wilbur, and P. H. Mirick, Modeling, Design, and Testing of the NASA/Army/MIT Active Twist Rotor Prototype Blade, In Proceedings of the American Helicopter Society 55th Annual Forum, Montreal, 1999.

[11] Wilbur, M. L., W. T. Yeager, and M. K. Sekula, Further Examination of the Vibratory Loads Reduction Results from the NASA/ARMY/MIT Active Twist Rotor Test. In Proceedings of the American Helicopter Society 58th Annual Forum, Montreal, 2002.

[12] Booth, E. R., and M. L. Wilbur, Acoustic Aspects of Active-Twist Rotor Control. In Proceedings of the American Helicopter Society 58th Annual Forum, Montreal, 2002.

[13] Cady, W. G.; Piezoelectricity - An Introduction to the Theory and Applications of Electromechanical Phenomena in Crystals; Dover Publications, New York; 1964.

[14] Cesnik, C. E. S., and S. J. Shin, On the Modeling of Integrally Actuated Helicopter Blades. International Journal of Solids and Structures, 38, 1765–1789, 2001.

[15] Cesnik, C. E. S., S. J. Shin, and M. L. Wilbur, Dynamic Response of Active Twist Rotor Blades. Smart Materials and Structures, 10, 62–76, 2001.

[16] Cesnik, C. E. S., and R. Palacios, Modeling Piezocomposite Actuators Embedded in Slender Structures. In 44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, AIAA 2003-1803, AIAA, Norfolk, 2003.

[17] Wilkie, W. K., and K. C. Park, An Aeroelastic Analysis of Helicopter Blades Incorporating Piezoelectric Fiber Composite Twist Actuation. TM 110252, NASA, 1996.

[18] Ghiringhelli, G. L., P. Masarati, and P. Mantegazza, Analysis of an Actively Twisted Rotor by Multi-Body Global Modeling. In ASME Mechanics and Materials Conference, Blacksburg, 1999.

[19] Brockmann, T. H., and R. Lammering, Finite Beam Elements for Rotating Piezoelectric Fiber Composite Structures. In Computational Fluid and Solid Mechanics 2003, edited by K. J. Bathe, Elsevier, Amsterdam, 2003.

[20] Brockmann, T. H., and R. Lammering, Helicopter Rotor Blade Dynamics with Adaptive Fiber Composites. In Jahrbuch 2004 der Deutschen Gesellschaft Für Luft- und Raumfahrt Lilienthal-Oberth E.V., DGLR, Bonn, 2004.

[21] Brockmann, T. H., and R. Lammering, Beam Finite Elements for Rotating Piezoelectric Fiber Composite Structures. Journal of Intelligent Material Systems and Structures, 17, 431–448, 2006.

[22] Brockmann, T. H., R. Lammering, and F. Yang, Modeling and Computational Analysis of Structures with Integrated Piezoelectric Material. Mechanics of Advanced Materials and Structures, 13, 371–378, 2006.

[23] Hagood, N., R. Kindel, K. Ghandi, and P. Gaudenzi, Improving Transverse Actuation of Piezoceramics Using Interdigitated Surface Electrodes. In Smart Structures and Materials 1993: Smart Structures and Intelligent Systems, edited by M. E. Regelbrugge, vol. 1917 of Proceedings of SPIE, pp. 341–352, 1993.

[24] Bent, A. A., N.W. Hagood, and J. P. Rodgers, Anisotropic Actuation with Piezoelectric Fiber Composites. Jounal of Intelligent Material Systems and Structures, 6, 338–349, 1995.

[25] Bent, A. A., and N. W. Hagood, Piezoelectric Fiber Composites with Interdigitated Electrodes. Journal of Intelligent Material Systems and Structures, 8, 903–919, 1997.

[26] Brockmann, T. H., Theory of Adaptive Fiber Composites - From Piezoelectric Material Behavior to Dynamics of Rotating Structures. PhD Thesis, to be published.

[27] Steinhausen, R., Charakterisierung und Modellierung piezoelektrischer 1-3 Komposite. PhD Thesis, Martin-Luther-Universität Halle-Wittenberg, 2002.

[28] Gasch, R., and K. Knothe, Strukturdynamik. Vol. 1 & 2. Springer-Verlag, Berlin, 1987 & 1989.