STATISTICAL PROCESSING OF SHOCK TEST DATA

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OVERVIEW

Statistical processing of shock test data are rarely employed in the aerospace industry because relevant database are not often available. Nevertheless this processing is one method to define a maximum expected environment (MEE) or to build a zoning of a spacecraft that constitute crucial phases of a space project with regards to the shock environment. They result indeed in shock specifications for equipment that are committing the project and the associated subcontractors.

Some statistical tools are reviewed in this paper. Among them, the classical Normal Tolerance Limit (NTL) method is examined. It aims at computing with a given confidence level a maximum environment corresponding to a specified probability of having a defined percentile of data under this level. The main drawback of this method resides in the strong assumption on the underlying distribution of the data that have to be normally distributed. As empirical data are rarely perfectly normally or log-normally distributed, alternative methods are described.

1. INTRODUCTION

The availability of a meaningful valid set of shock test data is often difficult to obtain. However, with spacecrafts families, satellite manufacturers can more and more dispose of a shock test database allowing thus a statistical processing of the SRS in order to define or refine statistically a maximum expected shock environment.

The specification process begins by defining some zones or regions where it is expected that the shock environment within that zone will be reasonably similar. Similar means that some limited scattering is expected within the particular zone, which is necessary to characterize. This phase is of paramount importance for the statistical processing because it is of no use to perform some statistical computation on very dissimilar or scattered data since the results will not be meaningful.

Besides it is well known that for a given launch or ground shock test, there will be some flight-to-flight or spacecraft-to-spacecraft variability. Some of this variability may be due to unavoidable differences between the flights, payload configuration ... But some can also be linked to the randomness of the launch or ground test events themselves. These uncertainties have also to be characterized if one wants to define the so-called maximum expected environment as a level that would typically not be exceeded or be exceeded exceptionally.

Hence, the MEE should account for both the expected spatial variation within a particular zone as well as the flight-to-flight changeability. This MEE is usually described in terms of the spectrum of a motion parameter, commonly acceleration.

Traditionally, NASA and U.S. Air Force Space Systems Division (AFSSD) have defined the one-sided NTL method to compute statistically the MEE.

Alternative methods are described in this paper. The first proposed method is trying to bring back to the gaussian case by modelling the real data set with a simple function (continuously differentiable with positive derivative) of a single gaussian data set. The transformed Gaussian model can substantially improve the result of the NTL method at a very low cost. Nevertheless, if the original data set is too far from the normal hypothesis, then the transformation may not perform as expected. Hence, a second method has been investigated and is based on the Bootstrap technique. The Bootstrap is a statistical subsampling method which uses sample data to generate replicates that are utilized for parameter and confidence interval estimation. An important feature of this method is that it does not make any assumption on the underlying distribution of data.

These statistical processing methods have been implemented and constitute now practical tools for defining standard environmental shock specification. Real shock data set extracted from clampband shock test on SPACEBUS family are used to illustrate these methods.

2. NORMAL TOLERANCE LIMIT METHOD

The NTL method is described in numerous American NASA standards [3][4][5] and military standards [6] and is used to derive the MEE from random vibration, vibroacoustic, and shock environments. It consists in computing a normal tolerance limit for the predicted spectra in each frequency resolution bandwidth. NTL method should be applied only to normally distributed random variables. If this is true, then the one-sided normal tolerance limit for the set of y variables, y_i , i = 1, 2,…, n is given by :

(1) $$NTL_y(n,p,\gamma) = \overline{Y} + K_{n,p,\gamma}.S_y$$

where

(2) $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is the sample mean,

(3) $S_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{Y})^2$ is the sample standard deviation,

(4) $K_{n,p,\gamma}$ the one-sided tolerance factor with p the percentile of data (e.g. 95%) and g the confidence level (e.g. 50%).

n	γ=0.50			γ=0.75			γ=0.90		
	β=0.90	β=0.95	β=0.99	β=0.90	β=0.95	β=0.99	β=0.90	β=0.95	β=0.99
3	1.50	1.94	2.76	2.50	3.15	4.40	4.26	5.31	7.34
4	1.42	1.83	2.60	2.13	2.68	3.73	3.19	3.96	5.44
5	1.38	1.78	2.53	1.96	2.46	3.42	2.74	3.40	4.67
6	1.36	1.75	2.48	1.86	2.34	3.24	2.49	3.09	4.24
7	1.35	1.73	2.46	1.79	2.25	3.13	2.33	2.89	3.97
8	1.34	1.72	2.44	1.74	2.19	3.04	2.22	2.76	3.78
9	1.33	1.71	2.42	1.70	2.14	2.98	2.13	2.65	3.64
10	1.32	1.70	2.41	1.67	2.10	2.93	2.06	2.57	3.53
12	1.32	1.69	2.40	1.62	2.05	2.85	1.97	2.45	3.37
14	1.31	1.68	2.39	1.59	2.01	2.80	1.90	2.36	3.26
16	1.31	1.68	2.38	1.57	1.98	2.76	1.84	2.30	3.17
17	1.31	1.68	2.37	1.55	1.96	2.74	1.82	2.27	3.14
18	1.30	1.67	2.37	1.54	1.95	2.72	1.80	2.25	3.11
20	1.30	1.67	2.37	1.53	1.93	2.70	1.76	2.21	3.05
25	1.30	1.67	2.36	1.50	1.90	2.65	1.70	2.13	2.95
30	1.29	1.66	2.35	1.48	1.87	2.61	1.66	2.08	2.88
35	1.29	1.66	2.35	1.46	1.85	2.59	1.62	2.04	2.83
40	1.29	1.66	2.35	1.44	1.83	2.57	1.60	2.01	2.79
50	1.29	1.65	2.34	1.43	1.81	2.54	1.56	1.96	2.74
∞	1.28	1.64	2.33	1.28	1.64	2.33	1.28	1.64	2.33

TAB 1. K factor for NTL method

A more complete K-factor table can be found in [2].

The K factor is both a function of the desired percentile and the chosen confidence. This uncertainty in the confidence results from using a sample mean and sample standard deviation instead of the true or entire population's mean and standard deviation values which are by nature unknown. For the special case where n is infinite, the K one-sided normal tolerance factor becomes the percentile point of the standardized gaussian distribution (last row of TAB 1). It means that there is no more uncertainty in the mean and standard deviation of the population. One notices that increasing the level of confidence, particularly when n is small is very costly and can lead to over-conservative results.

Both NASA and the U.S. AFSSD have standardized their MEE to be the p=0.95, γ=0.50 level. This is commonly referred to as the P95/C50 or P95/50 level. This level can be interpreted as follows : there is a 50-50 chance of one exceeding of the P95/50 level in 20 flights or ground test.

Some other statistical levels are known as type A value (p=0.99, γ=0.95) or type B value (p=0.90, γ=0.95) and are typically used for determining allowable values.

The qualification level is not yet completely standardized as the MEE. NASA and European Agencies use generally the following definition :

(5) QL=MEE + 3 dB.

AFSSD has defined the qualification level as the p=0.99, γ=0.90 level.

The return on experience using such method is that shock data set are generally not normally distributed but indeed lognormal [7]. In order to apply correctly the NTL method, one should use the following transformation on the data.

(6) $$z = \log_{10} y$$
$$NTL_y(n,p,\gamma) = 10^{NTL_z(n,p,\gamma)}$$

An illustration is given in FIG 1 (radial direction) and FIG 3 (longitudinal direction) representing the normal tolerance limits of clampband interface data for the SPACEBUS family, computed with p=95% and γ=50%.

The P95/50 level is logically exceeded on some frequency bandwidth by some measurements. It is interesting to verify the percentage of points that exceeds the defined level. This percentage should be compliant with the chosen percentile (i.e. 95%) with a confidence of 50 % if the main assumption of this method is valid, that is to say if the measurements follow a lognormal distribution (or equivalently if the logarithm of the data follow a normal distribution).

- For the radial direction : 7 % of the points are higher P95/50 shock level : the NTL method is slightly less conservative as expected.
- For the longitudinal direction : 5 % of the points are higher P95/50 shock level : the NTL method performs as expected.

To understand these numerical results, the best approach consists in plotting the empirical probability distribution function (PDF) and/or cumulative distribution function (CDF) of the data and compare them with the standardized normal one. FIG 2 and FIG 4 provide an example of such empirical distribution functions and the comparison with the theoretical standardized normal one (red curves). One notices that the radial direction data are not perfectly normal whereas the longitudinal ones are very close to the normal distribution.

As a general result, it is considered as very important to verify that the spectral test data (SRS for shock data) follow a lognormal distribution by computing the empirical distribution functions. The defined level can indeed be very conservative if the major hypothesis of normal distribution has a low degree of validity. If this method is used to specify a zone, then this recommended procedure should be carefully followed to avoid any over specification that can drive the structural hardware design (added mass …) or alternatively, lead to late and then costly mitigation treatments as isolation systems. If the distribution of the data is not lognormal, then an alternative solution has to be found or at least the NTL level has to be compared with another approach to determine the degree of conservatism it has generated.

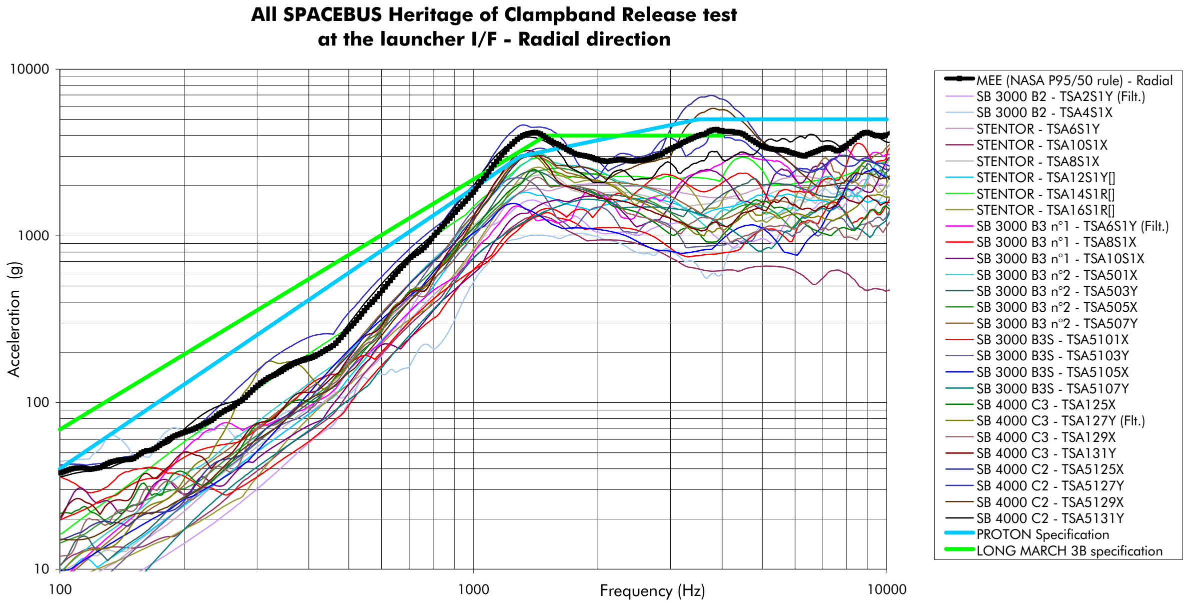

FIG 1. Statistical processing of Clamp band test data at S/C I/F - radial direction

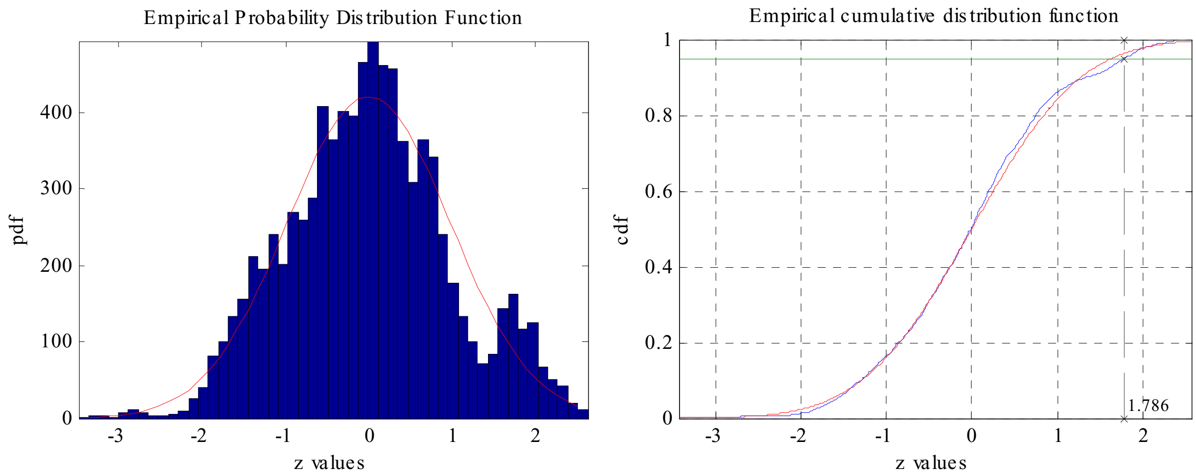

FIG 2. Empirical probability distribution function and cumulative distribution function – radial direction

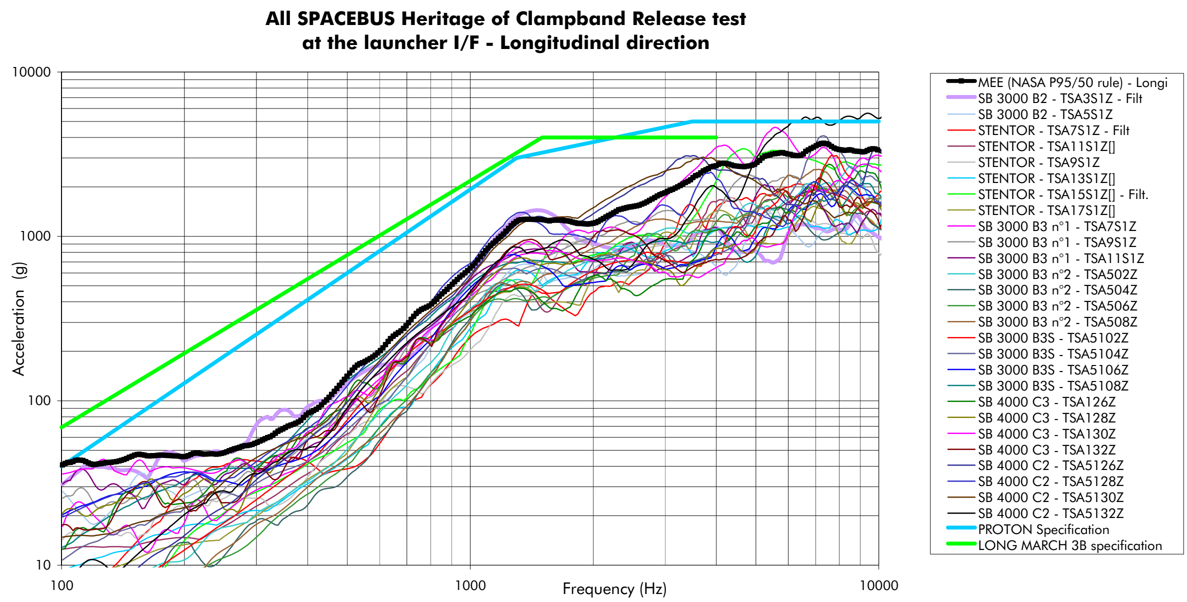

FIG 3. Statistical processing of Clamp band test data at S/C I/F - longitudinal direction

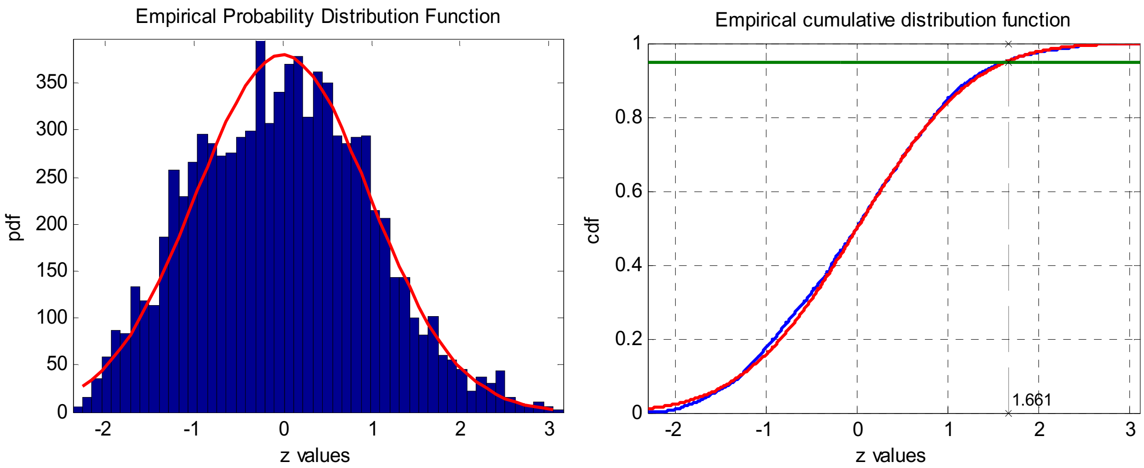

FIG 4. Empirical probability distribution function and cumulative distribution function– longitudinal direction

3. NORMAL TOLERANCE LIMIT WITH TRANSFORMED GAUSSIAN MODEL

3.1. Principle

Real measured data seldom perfectly support the gaussian assumption. However, since the normal case is well understood, it is often taken as a fact to perform some statistical computations (see NTL method).

If the usual gaussian assumption does not fit the observed data, a broader class of model should be used. The simplest model for a non-gaussian model is where *X* is a function of a gaussian model *Y* with variance one (i.e. $\mathrm{var}(Y) = 1$) [9].

$$X = G(Y)$$
$$\frac{dG}{dX} > 0 \qquad (7)$$
$$G(0) = 0$$

Different formulations (parametric and non-parametric) exist in literature to estimate the function G or its inverse $g = G^{-1}$. The followed approach in this study consists in estimating the function g from the empirical cumulative distribution of the observed data.

3.2. Estimation of the function g

An example is provided hereafter and is based on the clampband interface data, radial direction (FIG 1) for which the empirical distribution functions (FIG 2) are not perfectly normal.

FIG 5 shows the gaussian reference (red dashed line), the empirical estimated transform (blue line) and the smoothed estimated transform (red line). In this example, the function g is close to the normal distribution except in the tails of the distribution where some variations are clearly visible. This variations are also reflected in the smoothed estimation that is globally curved compared to the gaussian diagonal. The advantage of the smoothed estimation is thus to be able to treat data that are out of the empirical tails of the source data. In particular, if the source data do not exhibit any value in the tail of the distribution, the smoothed estimation has to be considered to complete the procedure.

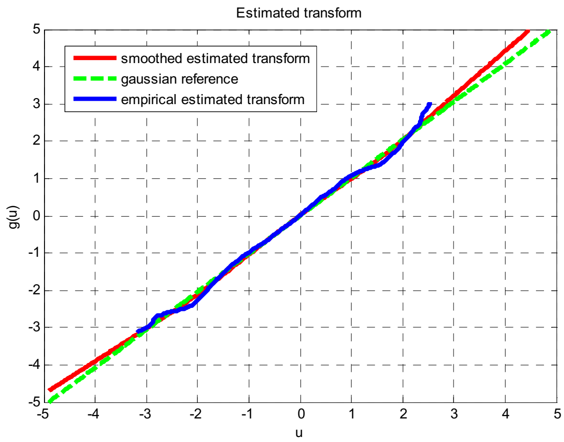

FIG 5. Identification of $g = G^{-1}$

FIG 6 shows the probability distribution function of the transformed data using the function g. The empirical distribution matches logically the standard normal one.

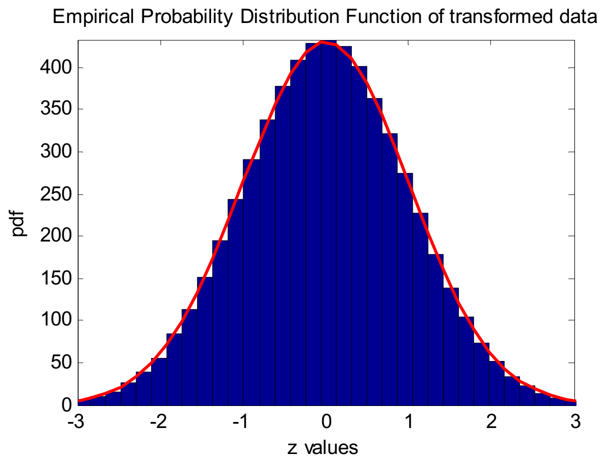

FIG 6. Probability distribution of transformed model

3.3. Use of the transformed model

Once defined the function g, it has to be used to compute the specified level. Mathematically, the problem of defining a level Pp/γ is, knowing $\bar{x}$ and s as the sample mean and standard deviation, to find k such that

$$\text{(8)} \qquad \Pr\{\Pr(X \le \bar{x} + ks) \ge p\} = \gamma$$

This problem is equivalent to the following

$$\text{(9)} \qquad \Pr\{\Pr(G(X) \le \bar{x} + ks) \ge p\} = \gamma$$

Equation (9) can be written

$$\text{(10)} \qquad \Pr\{\Pr(Y \le g(\bar{x} + ks)) \ge p\} = \gamma$$

where Y is a normal random variable of mean $\overline{x'}$ and standard deviation s'.

One defines then

$$\text{(11)} \qquad \overline{x'} + k's' = g(\bar{x} + ks)$$

One finally obtain the modified k coefficient by inverting (11).

$$\text{(12)} \qquad k = \frac{G(\overline{x'} + k's') - \bar{x}}{s}$$

with k' the one-sided tolerance factor $K_{n,p,\gamma}$.

3.4. Comparison with NTL method

The comparison between the NTL method and this method is shown hereunder and is based on the example of FIG 1.

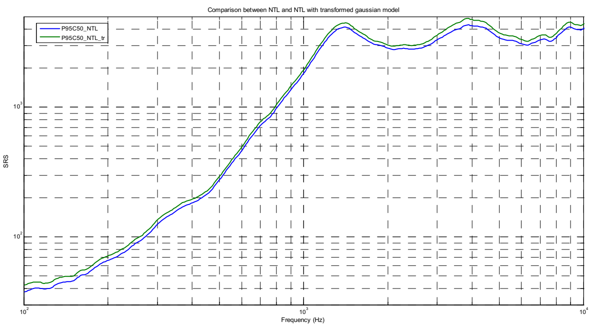

FIG 7. Comparison between P95/50 computed with NTL and NTL with gaussian transformed model

The NTL method, as explained in paragraph 2, performs not as expected since 7 % of the points exceeds the level instead of the theoretical 5%. With the gaussian transformed model, the K factor is adjusted to take into account the "non-normality" of the real data distribution. For that example, the new level is exceeded by only 4.4% of the points which is closer to 5%. Moreover, this level is slightly conservative compared to the theoretical 5% which can be adequate for a specification procedure.

It is worth noting that this method is really efficient when the discrepancy with the normal distribution is reduced, i.e. when the function g is relatively smooth and close to the gaussian case. For cases where this is not true, the method may still be conservative even if it generally improves the NTL computation.

4. BOOTSTRAP METHOD

4.1. Bootstrap origin and principle

The Bootstrap method has been introduced and developed by Efron in the late seventies [8]. The initial problem Efron wanted to solve was : "given a random sample X =(X_1, X_2,….X_n) from an unknown probability distribution f, estimate the sampling distribution of some pre-specified random variable R(X,f), on the basis of the observed data x." This method is directly linked to the computational improvements of this period that allowed for numerous advances in applied statistics.

The main idea of the model-based sampling theory approach to statistical inference is that the data arise as a sample from some existing probability distribution, f (most of the time, the normal distribution is assumed). Uncertainties of our inferences can be measured if f can be estimated. The most fundamental idea of the Bootstrap method is that one computes measures of our inference uncertainty from that estimated sampling distribution of f.

The method consists of :

1) generating B Bootstrap replicate samples of the same size n as the original data sample. Each value in the original data sample is assigned an equal probability of 1/n. The elements of these Bootstrap samples are randomly chosen from the original data, with replacements. It means that a particular data can be chosen several times or not at all in a given replicate.

2) evaluating a parameter or statistic of interest for each of the B Bootstrap samples generated. Each computation produces a Bootstrap replicate of the statistic of interest (mean, standard deviation, P95, P99 …).

3) estimating an empirical cumulative distribution function (CDF) for this parameter by using the numerous Bootstrap replicates of this parameter.

4) establishing confidence intervals from this distribution.

The method principle is summarized in FIG 8.

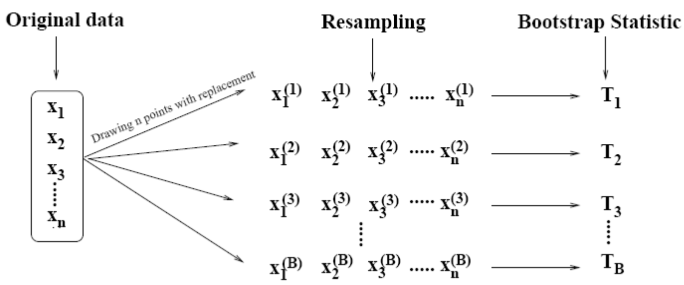

FIG 8. Bootstrap method principle

The set of B Bootstrap samples is a proxy for a set of B independent real samples from f (in reality we have only one actual sample of data). Properties expected from replicate real samples are inferred from the Bootstrap samples by analyzing each Bootstrap sample exactly as the real data sample were first analyzed. From the set of results of sample size B we measure our inference uncertainties from sample to population. In other words, the Bootstrap allows assessment of the accuracy and uncertainty of estimated parameters (even if no closed form exists) from small samples, without any prior assumptions about the underlying distribution (particularly the usual normal distribution). In that case, when no assumption or inference on the distribution f is made, the bootstrap is called non parametric. Other forms of parametric Bootstrap have been described in literature but have not been treated in the frame of this work.

4.2. Application to shock data

As explained in [1] for vibroacoustic data, the statistics of interest for the Bootstrap analysis are the P95 or P99 probability levels. This requires that the bootstrap mean and bootstrap standard deviation be used as a bootstrap pair to compute the desired probability level. In other words, the statistic of interest mentioned in paragraph 4.1.2) is not just the mean and standard deviation but instead the values generated by these that are associated with the P95 or P99 probability levels.

In practice, the Bootstrap mean and standard deviation are computed for each replicate. Then, in order to derive the level P95 or P99 levels which can be expressed as $P95 = \bar{x}_b + k\sigma_b$, one needs to determine the coefficient k. Several methods can be used. The k factor could be chosen as the $K_{n,p,\gamma}$ one-sided tolerance factor. However, that method would assume a normal distribution of the data which is not known, reducing consequently the generality of the problem. The k factor can be computed thanks to the original data sample itself. By using the CDF of the standard normalized data $z = \frac{x - \bar{x}}{\sigma}$ (about which the underlying distribution is not known). This solution has been retained for the present study.

For SRS, the size of each sample at a given frequency is usually very small and does not allow to correctly define a CDF and its tails per frequency. It is assumed thus that the CDF can be computed by combining all the frequencies of the SRS together. This assumption introduces an error that is generally found acceptable.

The previous procedure leads to B replicate values of the P95 or P99 level. The confidence levelis finally computed by sorting all the values of the statistic of interest and then selecting the percentage of the empirical sampling distribution of the P95 or P99 level. For instance, for a confidence level of 50% with 1000 Bootstrap replicates, one will choose the 500th=0.5x1000 sorted value.

1000 to 2000 replicates are usually found acceptable for defining correctly a Bootstrap statistic.

4.3. Comparison with previous methods

The bootstrap method has been used to compute the P95/50 and P99/90 levels of the clampband SRS data shown in FIG 1and FIG 3. For that cases, the k factor can be found using the CDF of the data. The k factors for the P95 level can be read on FIG 2 and FIG 4 and are respectively 1.786 and 1.661.

The computed levels are compared with the two other methods presented in paragraphs 2 and 3. FIG 9 and FIG 10 show the results. The three methods gives similar results especially in the longitudinal direction where the PDF of the data are quite close to the standard distribution (see FIG 4). This result is quite reassuring since the Bootstrap method is quite different from the first two.

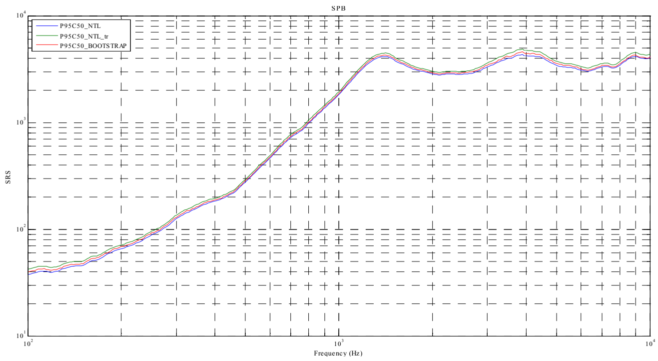

FIG 9. P95/50 level computed for Clamp band test data at S/C I/F - radial direction

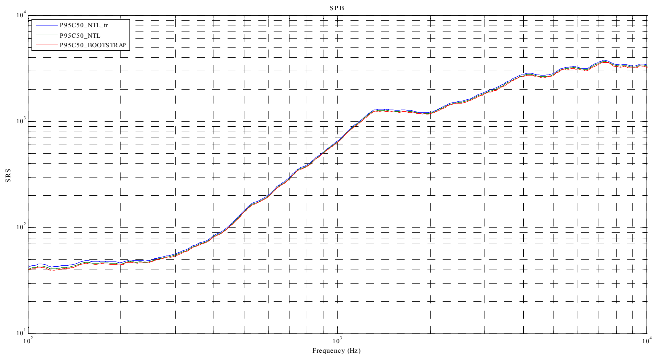

FIG 10. P95/50 level computed for Clamp band test data at S/C I/F - longitudinal direction

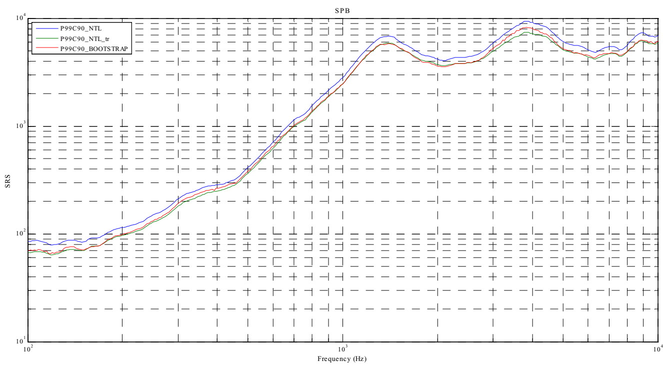

FIG 11. P99/90 level computed for Clamp band test data at S/C I/F - radial direction

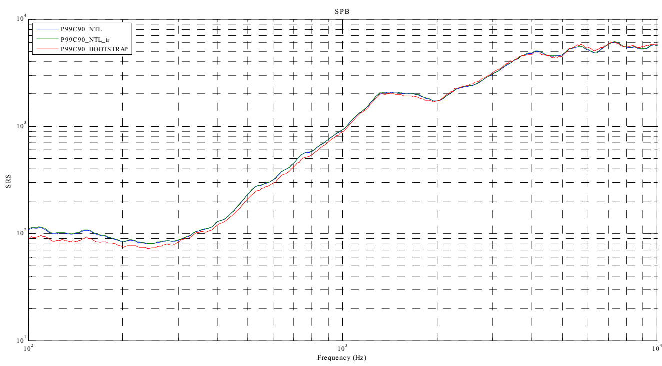

FIG 12. P99/90 level computed for Clamp band test data at S/C I/F - longitudinal direction

FIG 11 and FIG 12 are very interesting since they provide the influence of the non-normality of the original data sample on the probability levels. They clearly show that non-normality increases the scattering of probability levels between the three methods when using high percentile/confidence values. In the radial direction, the PDF are not so close to the standard distribution than in the longitudinal direction. As a consequence, the NTL method provides more conservative results for the P99/90 level in radial direction than the NTL method with a transformed gaussian model or than the Bootstrap method. For the longitudinal direction, the P99/90 levels given by the three methods are still in a good agreement because the original data sample is close to the normal distribution.

4.4. Comparison between P99/90 and P95/50+3dB levels

This comparison aims at determining the qualification level that is defined by NASA as MEE+3 dB, i.e. P95/50+3dB and by AFSSD as P99/90 level. As the data in general cannot be for sure associated with the normal distribution, it was decided to compare these probability levels thanks to the Bootstrap method.

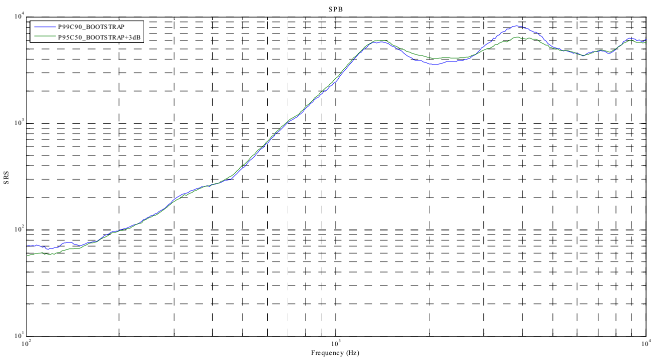

FIG 13. P95/50+3dB and P99/90 levels computed for Clamp band test data at S/C I/F - radial direction

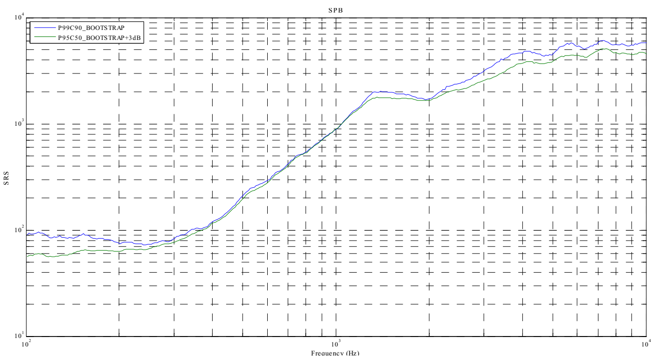

FIG 14. P95/50+3dB and P99/90 levels computed for Clamp band test data at S/C I/F - longitudinal direction

FIG 13 and FIG 14 show empirically that the P95/50+3dB and the P99/90 level are quite equivalent and can both be utilized to define a qualification level. The usual 3 dB margin for qualification [3] finds with the P99/90 level a statistical justification.

5. CONCLUSIONS AND FUTURE WORK

Statistical processing of shock test data constitutes one method to define the MEE with regards to the shock environment as other classical methods like extrapolation techniques or numerical analysis. The classical NTL method is widely used in the US aerospace industry with success since more than 40 years. The simplicity of the NTL method should not hide its fundamental assumption that the original data have to be normally distributed. If it is not the case, the NTL method can become over-conservative, especially for small samples. Two other statistical techniques have been presented to be used with shock test data aiming at correcting the NTL defaults.

The return of experience using these methods shows the following :

- The three methods give similar results for normally distributed data.
- Non-normality increases the scattering of evaluation between the three methods when using high percentile/confidence values (NTL seems too conservative).
- If the distribution of observed data is not perfectly normal, the NTL with transformed gaussian model method tends to partially correct the default of NTL method with almost no increase in computation time.
- If the data are not distributed normally, the Bootstrap method seems to give the most reliable results but with an increased computation time (linked to the number of replicates).
- The comparison between P95/50+3dB and P99/90 shock levels with the Bootstrap method shows a good convergence. It provides a good justification to the NASA choice to define the qualification environment as the MEE + 3 dB

Future work will concentrate on the Bootstrap method which seems very promising for applied statistics. Particularly, the methods to define accurate and not biased confidence intervals have been deeply studied, notably by Efron.

6. ACKNOWLEGDMENTS

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7. ACRONYMS

NTL	Normal Tolerance Limit
MEE	Maximum Expected Environment
SRS	Shock Response Spectrum
S/C	Spacecraft
I/F	Interface
PDF	Probability Distribution Function
CDF	Cumulative Distribution Function

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