BENDING-WRINKLING CHARACTERISTICS OF THE INFLATED BOOM

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OVERVIEW

Gossamer space structures have received the widest attentions because they can meet the structural requirements for space application at a low cost. Gossamer structures are often partially wrinkled and the formation of wrinkles drastically degrades the structural performance and the surface precision. Inflated booms are the main load-carrying structures in the gossamer space system[1]. The load-carrying ability is the main design calibration. The compressive and bending loads are two main loading conditions. They are also the main reason to initiate the wrinkles. A better understanding of the effects of the wrinkles on the structural performance of these space structures is essential and desirable.

Wrinkling is the basic problem in the membrane structures. Wagner[2] firstly studied the membrane wrinkling using the tension field theory. In his work, only the in-plane wrinkling information can be obtained. After the inflatable antenna experiment[3] on orbit, the studies on the deployment control, the surface precision and the wrinkles have become the hot research topics. Where, studies on the wrinkles have upgraded to the level of the understandings of the out-of-plane wrinkling information, such as wrinkling numbers, amplitude, and wavelength. Some previous methods, such as tension field theory[2], variable Poisson's ratio method[4], have lost the effective prediction ability on the detail wrinkling characteristics. At present, a method, based on the nonlinear shell calculation, has rise up to obtain the expected wrinkling behaviours[5-7]. In this method, the membrane is modelled as a thin shell or thin plate with near-zero bending stiffness. The focus of the membrane wrinkling has been put on the studies on the singularity of the stiffness and the convergence.

Inflated booms are the main load-carrying parts in the space gossamer structures or the inflatable space structures. Membranes are also the main construction materials. Different from the common plane membrane structures, such as sun shield and solar sail, in the inflated boom, the inflation pressure is the main part to make the membrane form a structure. Thus, the inflation pressure must be right and carefully considered in the analysis. For the inflated boom, its function is mainly support the major part of the inflated structures, for example the antenna reflector. So, the axial compressive and the bending loads are the key load cases during its working conditions on orbit. For an inflated boom, the buckling and the local wrinkling are the possible failure modes.

Since 1960, some studies on the bending and buckling characteristics of the inflated booms had been carried on. In these studies, the inflated booms are assumed as a cantilever beam with the internal pressure. Comer et al.[8] used this assumption to study the buckling behaviours of the inflated boom under bending loads. They found that the wrinkles occurred when the bending loads reached the critical value, and the wrinkles are the local deformation behaviours. William[9] used the small deflection theory to study the bending stiffness of the cantilever inflated boom. In this method, the large deflection characteristics of the local wrinkling behaviours can not be obtained due to the adopted small deflection theory. Main et al.[10,11] used the similar method to study the bending and the wrinkling characteristics of the inflated boom with the textile material. They found that the bending behaviours have the same characteristics with the general solid elastic beam before the inflated boom wrinkled.

In this paper, we firstly calculate the stress field and the deformation of the inflated boom under bending based on the membrane theory. These results are then used to obtain the region and the distribution of the wrinkles. We have a further study on the wrinkling and collapse moment with respect to the internal pressure of such inflated boom under compressive and bending conditions. We extend our studies to regard the material as a very thin shell. The bending stiffness is introduced into our analysis to obtain the critical wrinkling load and the deflection. The results reveal that the wrinkles decrease the structural stiffness, and they rapidly extend in the structure. The internal pressure increases the transverse shearing stiffness and the resistance to the bending deformation. The influences of the wrinkling angles and the inflated pressure on the moment are also performed.

1. MEMBRANE THEORY WITH WRINKLES

In this section, we assumed the membrane has no bending stiffness. And we define that the wrinkles occur when the minor principal stress in the membrane reaches zero. According to the membrane theory and the elastic mechanics, we can obtain the principal stress, $\sigma_{1,2}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{1}$$

Thus, we expressed the wrinkling criterion as

$$\sigma_2 = 0 \; or \; \sigma_x \sigma_y = \tau_{xy}^2 \tag{2}$$

The stress equilibrium are given by

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0; \quad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \tag{3}$$

Combined with wrinkling condition, we can obtain the stress distribution in the wrinkled membrane based on Eq.3.

In the wrinkled regions, the strain in the minor principal stress will include two parts: one is the material strain; the other is related to the out-of-plane wrinkling deflection. The strain related to the wrinkles are expressed as

$$\varepsilon' = -\lambda \frac{\sigma_1}{E} \tag{4}$$

Where, σ_1 is the principal stress in the major principal stress (also named as the wrinkling direction), E is the elastic modulus, and the λ is the contraction factor which is related the "over-contration" wrinkling deformation in the texture direction. The sign is the contraction.

In the wrinkled region, the stress-strain expression is given by

$$\varepsilon_x = \frac{\sigma_x}{E} - \lambda \frac{\sigma_y}{E}; \varepsilon_y = \frac{\sigma_y}{E} - \lambda \frac{\sigma_x}{E}; \quad \gamma_{xy} = 2\left(1+\lambda\right)\frac{\tau_{xy}}{E} \tag{5}$$

Based on the obtained the stress distribution in the wrinkled region, we can obtain the strain distribution in the wrinkled region bassed upon the Eq.5.

According to the strain compatible condition, we can obtain

$$\sigma_x \frac{\partial^2 \lambda}{\partial x^2} + 2\tau_{xy} \frac{\partial^2 \lambda}{\partial x \partial y} + \sigma_y \frac{\partial^2 \lambda}{\partial y^2} = \nabla^2 \left(\sigma_x + \sigma_y\right) \tag{6}$$

Combined with Eqs.1-6, we can obtain the wrinkling characteristics in the wrinkled region.

2. BENDING-WRINKLING BEHAVIORS OF THE INFLATED BOOM

Considered an inflated boom, the radius is r, the thickness is t, the internal pressure is p, and the bending load is M. The model script is shown in FIG.1. Where, x is the direction in length and y is the direction in radial. θ is the wrinkling direction angle.

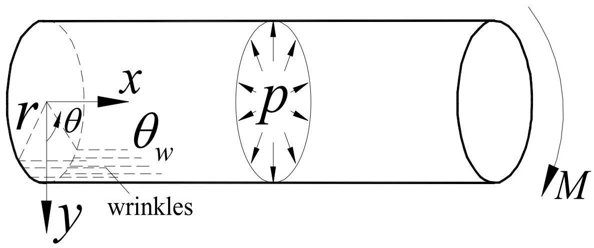

FIG 1. Bending-wrinkling analytical model

The equilibrium of the forces and the moment are given

$$\begin{cases} p\pi r^2 = rt\int_0^{2\pi} \sigma_x d\theta \\ M = -r^2 t \int_0^{2\pi} \sigma_x \cos\theta d\theta \end{cases} \tag{7}$$

We define θ_w as the wrinkling angle, and it ranges in $-\theta_w \le \theta \le \theta_w$. In our analysis, we ignore the pressure changes.

2.1. Membrane model

We treated the material as the true membrane with zero bending stiffness. The stress in the isotropic membrane is given by

$$\sigma_x = E\left(-kr\cos\theta + C_1\right) + \nu \frac{pr}{t} \tag{8}$$

At the critical wrinkling point, we can obtin the condtion $\sigma_x = 0$ and $\nu = \lambda$, which meets the wrinkling conditions Eq.2. According to above equation, we can obtain

$$\lambda = \frac{Et}{pr}\left(kr\cos\theta - C_1\right) \tag{9}$$

After the inflated boom wrinkled, we have $\theta = \theta_w$ and $\sigma_x = 0$. According to Eq.8, we can obtain

$$C_1 = kr\cos\theta_w - \nu\frac{pr}{Et} \tag{10}$$

Substituted Eq.10 into Eq.8, we have

$$\sigma_x = \begin{cases} Ekr\left(\cos\theta_w - \cos\theta\right); & \theta_w \le \theta \le 2\pi - \theta_w \\ 0 & ; -\theta_w \le \theta \le \theta_w \end{cases} \tag{11}$$

The range $-\theta_w \le \theta \le \theta_w$ is the wrinkled region, and the range $\theta_w \le \theta \le 2\pi - \theta_w$ is the taut region. Substituted Eq.11 into the equilibrium conditons Eq.7., we have

$$\begin{cases} p\pi r^2 = rt\int_{\theta_w}^{2\pi-\theta_w} Ekr\left(\cos\theta_w - \cos\theta\right)d\theta \\ M = -r^2 t\int_{\theta_w}^{2\pi-\theta_w} Ekr\left(\cos\theta_w - \cos\theta\right)\cos\theta d\theta \end{cases} \tag{12}$$

After derived, we obtain

$$\begin{cases} p\pi r^2 = 2kr^2 Et\left[\sin\theta_w + \left(\pi - \theta_w\right)\cos\theta_w\right] \\ M = kr^3 Et\left(\pi - \theta_w + \frac{1}{2}\sin 2\theta_w\right) \end{cases} \tag{13}$$

We further obtain

$$\frac{M}{p\pi r^2} = \frac{r\left(\pi - \theta_w + \frac{1}{2}\sin 2\theta_w\right)}{2\left[\sin\theta_w + \left(\pi - \theta_w\right)\cos\theta_w\right]} \tag{14}$$

The wrinkling occurs at $\theta_w = 0$, the critical wrinkling moment of the inflated boom is expressed as

$$M_w\big|_{\theta_w=0} = p\pi r^3/2 \tag{15}$$

We define the full wrinkling state as the failure state, the moment at this state is failure moment, which is given by

$$M_f\big|_{\theta_w=\pi} = \lim_{\theta_w\to\pi} M = p\pi r^3 \tag{16}$$

According to our analysis, the wrinkling moment is the half of the failure moment. The inflated boom can resist the loading action after the inflated boom wrinkled. The moment-wrinkling angle relationship is shown in Fig.2.

According to the results, we can find that the wrinkles accelerate the failure process of the inflated boom. The wrinkling and failure moment are only realted to the structural parameters, such as the inflated pressure and the radius, they are independent of the material properties.

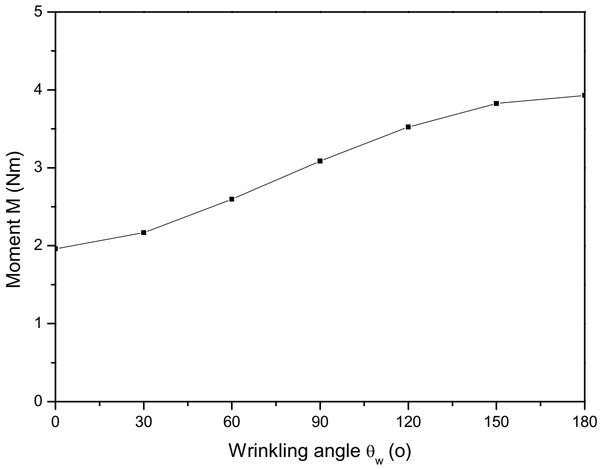

FIG 2. Moment-wrinkling angle relationship (Elastic modulus is 4910MPa; Poisson's ratio is 0.3; Length is 5m, radius is 0.05m, thickness is 0.05mm, inflated pressure is 10KPa)

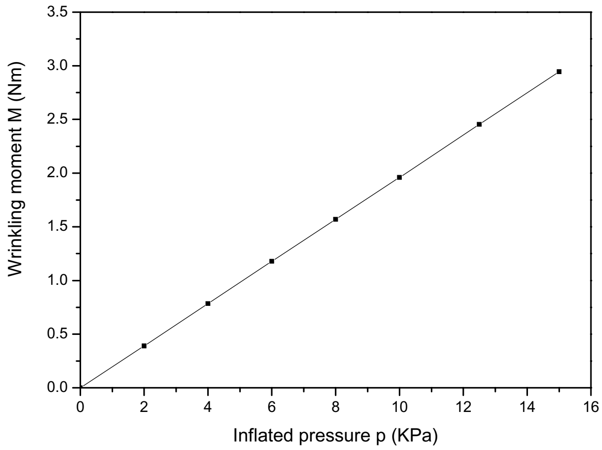

FIG 3. Moment-inflated pressure relationship (Elastic modulus is 4910MPa; Poisson's ratio is 0.3; Length is 5m, radius is 0.05m, thickness is 0.05mm, wrinkling angle is zero)

According to FIG3, the wrinkling moment is zero at the zero inflated pressure, which reveals that the inflated pressure is the major load-carrying factor in the membrane model. In fact, the function of the inflated pressure is to improve the in-plane tension and the out-of-plane shear stiffness of the inflated boom.

2.2. Thin shell model

In this section, the membrane is modelled as the thin shell with near-zero bending stiffness. For this case, we need modify the wrinkling condition as

$$\sigma_x = \sigma_{cr} \tag{17}$$

Where, σ_{cr} is the critical compressive stress of the inflated boom. According to this modification, the wrinkles occur after the compressive stress in the inflated boom reaches the critical value. At this time, the modified contraction factor is

$$\lambda = \frac{Et}{pr}\left(kr\cos\theta - C_1 + \frac{\sigma_{cr}}{E}\right) \tag{18}$$

Where,

$$C_1 = kr\cos\theta_w - \nu\frac{pr}{Et} + \frac{\sigma_{cr}}{E} \tag{19}$$

For this case, the stress state of the inflated boom is

$$\sigma_x = \begin{cases} Ekr\left(\cos\theta_w - \cos\theta\right) + \sigma_{cr}; & \theta_w \le \theta \le 2\pi - \theta_w \\ \sigma_{cr} & ; -\theta_w \le \theta \le \theta_w \end{cases} \tag{20}$$

Substituted Eq.20 into Eq.7, we obtain

$$\begin{cases} p\pi r^2 = rt\int_{-\theta_w}^{\theta_w}\sigma_{cr}d\theta + rt\int_{\theta_w}^{2\pi-\theta_w}Ekr\left(\cos\theta_w - \cos\theta\right)d\theta \\ M = -r^2t\int_{-\theta_w}^{\theta_w}\sigma_{cr}\cos\theta d\theta \\ \quad -r^2t\int_{\theta_w}^{2\pi-\theta_w}Ekr\left(\cos\theta_w - \cos\theta\right)\cos\theta d\theta \end{cases} \tag{21}$$

After derived, we obtain

$$\begin{cases} p\pi r^2 = 2kr^2Et\left[\sin\theta_w + \left(\pi - \theta_w\right)\cos\theta_w\right] + 2\pi tr\sigma_{cr} \\ M = kr^3Et\left(\pi - \theta_w + \frac{1}{2}\sin 2\theta_w\right) \end{cases} \tag{22}$$

We found that the expression of the bending moment is same with the case of the membrane model. The load item has a small change due to the critical compressive stress. We further obtain

$$\frac{M}{p\pi r^2 - 2\pi tr\sigma_{cr}} = \frac{r\left(\pi - \theta_w + \frac{1}{2}\sin 2\theta_w\right)}{2\left[\sin\theta_w + \left(\pi - \theta_w\right)\cos\theta_w\right]} \tag{23}$$

For this case, the wrinkling moment is

$$M_w\big|_{\theta_w=0} = \frac{p\pi r^3}{2} - \pi tr^2\sigma_{cr} \tag{24}$$

The failure moment is

$$M_f\big|_{\theta_w=\pi} = \lim_{\theta_w\to\pi} M = p\pi r^3 - 2\pi tr^2\sigma_{cr} \tag{25}$$

According the results, we observed that the wrinkling moment is still the half of the failure moment, which is same to the membrane model. In this case, the critical compressive stress of the inflated boom may come from the design criterion of the buckling of the thin-wall inflated boom[12].

$$\sigma_{cr} = -\frac{Et}{r}\left[\frac{1}{\sqrt{3\left(1-\nu^2\right)}} + \frac{p}{2E}\left(\frac{r}{t}\right)^2\right] \tag{26}$$

Substitute Eq.26 into the Eqs.24 and 25, we obtain

$$M_w = p\pi r^3 + \frac{\pi rEt^2}{\sqrt{3\left(1-\nu^2\right)}} \tag{27}$$

$$M_f = 2p\pi r^3 + \frac{2\pi rEt^2}{\sqrt{3\left(1-\nu^2\right)}} \tag{28}$$

We found that the wrinkling and failure moment are related to the structural and material parameters. The comparisons of the influences of the wrinkling angle and the inflated pressure on the moment are shown in FIG.4 and 5, respectively.

According to the comparisons, we can conclude that the wrinkling and failure moment are higher than the case of the membrane model, due to the bending stiffness. And the bending stiffness is small (near zero), whereas, it has great influences on the wrinkling and failure moment. According to the FIG3, the wrinkling moment in the thin shell model is larger than the failure moment in the membrane theory. There is only small different (bending stiffness) in these two models. Thus, in some analysis, the membrane with larger thickness should be modelled as the thin plate or thin shell with near zero bending stiffness. While in other cases, the membrane with smaller thickness should be modelled as the true membrane with zero bending stiffness according to the membrane theory.

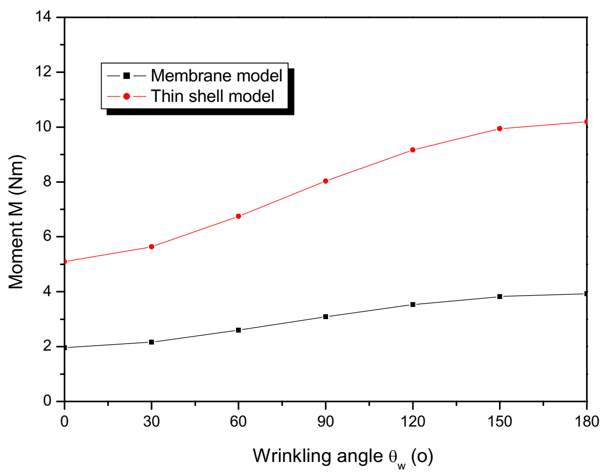

FIG 4. Comparison of the Moment-wrinkling angle relationship (Elastic modulus is 4910MPa; Poisson's ratio is 0.3; Length is 5m, radius is 0.05m, thickness is 0.05mm, inflated pressure is 10KPa)

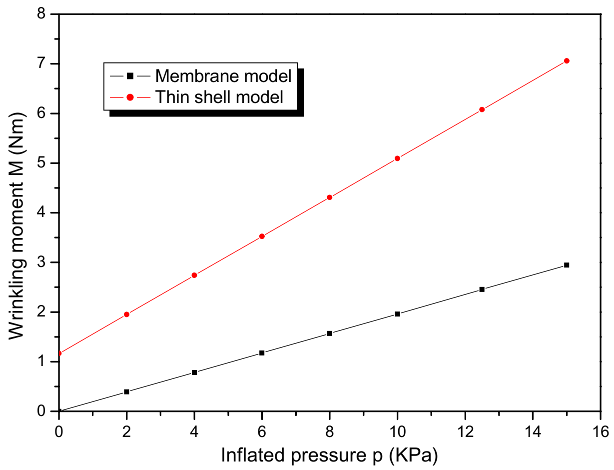

FIG 5. Comparison of the wrinkling moment-inflated pressure relationship (Elastic modulus is 4910MPa; Poisson's ratio is 0.3; Length is 5m, radius is 0.05m, thickness is 0.05mm, wrinkling angle is zero)

CONCLUSIONS

(1)- The wrinkled inflated boom trends to failure. Thus, the wrinkles are the main factor to accelerate the failure of the inflated boom. After wrinkled, the inflated boom may still carry the loads.

(2)- There is big difference between the membrane model and the thin shell model, the bending stiffness is the main reason. According to the results, the failure moment in the membrane model is smaller than the wrinkling moment in the thin shell model. The thickness/bending stiffness is the main reason to answer for this result. For the thin membrane with smaller thickness, we should model it as a true membrane with zero bending stiffness. The thick membrane should be modelled as a thin plate/shell with near-zero bending stiffness.

(3)- The inflated pressure is the main part to support the action of the bending loads, especially in the membrane model. The basic function of the inflated pressure is that it improves the in-plane tension and the out-of-plane shear stiffness.

(4)- In the membrane model, the moment is only related to the structural parameters. In the thin shell model, the moment is related to the structural and material parameters.

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