INVESTIGATIONS OF ATMOSPHERIC CONDITIONS IN FLUIDS ON SONIC BOOM

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Isotropic character of atmosphere turbulence was assumed to evaluate ground level parameters of sonic boom wave passing through the atmospheric turbulent boundary layer (this seems to be the most reasonable way).

In the frame of the Project elaboration of empirical method for analysis of relationships of overpressure and rise time of sonic boom and turbulence characteristics is based on (i) Yu.L. Zhilin's method of sonic boom calculation grounded on laws of geometric acoustics [1, 2], and (ii) Ph. Blanc-Benon et al. stochastic model of acoustic waves propagating through random scalar and vectorial fields [3, 4].

It was assumed that the atmosphere turbulence is "frozen", i.e. the time of acoustic wave passing through the turbulent media is considerably greater, than the time of evolution of atmosphere turbulence structures. The turbulence field is characterized as a series of independent realizations of random scalar and vector fields that are not related one with the other.

Height of HSST-2 cruise flight comes around 18–20 km. Hence, the wave front may be supposed to be plane for propagation of disturbances from airplane in the near-ground layer.

Velocity **V** of two-dimensional random isotropic vector field at any given point x has a fluctuating component that may be presented as a sum of n random Fourier-modes:

$$\mathbf{v}'(\mathbf{r}) = \sum_{i=1}^{n} \mathbf{u}_i(\mathbf{K}_i)\cos(\mathbf{K}_i\mathbf{r} + \varphi_i),$$
$$\mathbf{u}_i(\mathbf{K}_i)\cdot\mathbf{K}_i = 0,$$

where **r** is radius-vector of point with coordinates (x_1, x_2).

Direction of the wave vector $\mathbf{K}_i$ of each mode is random; in two-dimensional case, it is characterized by the random angle θ_i. Homogeneity of turbulence field is ensured by the randomness of the phase shift φ_i. θ_i and φ_i are independent random variables with uniform distributions. The amplitude of velocity fluctuations $|\mathbf{u}(\mathbf{K}_i)|$ is a deterministic variable whose value is set according energy spectrum $E(K)$, with $K=|\mathbf{K}_i|$:

$$|\mathbf{u}_i(\mathbf{K}_i)| = \sqrt{E(K)\cdot\Delta K}\ ,$$

where ΔK is a K increment.

In this work we consider the fields with a Gaussian correlation function $f(r) = \exp\left(-\frac{r^2}{L^2}\right)$. The length scale L is related to the longitudinal integral length scale L_f by $L_f = (\sqrt{\pi}/2)L$, and r is a distance between two arbitrary chosen points between which the correlation of velocity fluctuations is evaluated. The energy spectrum is determined by expression for two dimensional Gaussian random velocity fields:

$$E(K) = \frac{\overline{v'^2}}{8}\cdot K^3 L^4 \exp\left(-\frac{K^2L^2}{4}\right),$$

with $\overline{v'^2} = \overline{v_1'^2} = \overline{v_2'^2}$ is the mean square of the velocity fluctuations.

However, changing the kind of the energy spectrum presents no any difficulties.

It is assumed, that $K_{\min} = 0{,}1/L$, and $K_{\max} = 10/L$. We are restricted by 50 random Fourier-modes in our simulations. The averaging was carried out over ensembles of order of 100 realizations of stochastic field.

Two-dimensional random temperature fluctuations field is defined by the same way as random velocity fluctuations field:

$$T'(\mathbf{r}) = \sum_{j=1}^{n} \Theta_j(\mathbf{K}_j)\cos(\mathbf{K}_j\mathbf{r} + \psi_j)\ .$$

As before, direction of the wave vector $\mathbf{K}_i$ and phase shift ψ_j are independent random variables with uniform distributions, but $\Theta_j(\mathbf{K}_j)$ is defined by energy spectrum if temperature fluctuations

$$G(K) = \frac{\overline{T'^2}}{2}\cdot K\cdot L^2 \exp\left(-\frac{K^2L^2}{4}\right),$$

where $\overline{T'^2}$ is the mean square of the temperature fluctuations.

The linear geometric acoustics is the basis for determination of ray paths in each of turbulent layer realizations. It is known that rays are the line tangent to the group velocity $\mathbf{c}_g = c\cdot\mathbf{n} + \mathbf{V}$. Here c is local sound velocity, $\mathbf{n} = \mathbf{P}/P$ is the unit vector along the direction of wave front propagation, **V** is a medium velocity vector, **P** is nondimensional wave vector, $P = \frac{N}{(1+\mathbf{M}\cdot\mathbf{n})}$, $N=c_0/c$ is the refraction index and $\mathbf{M}=\mathbf{V}/c$ is the Mach number, c_0 is the sound velocities in undisturbed medium.

For the plane wave propagating in random field the ray trace calculation consists in the solution of the system of eight ordinary differential equations:

1) four equations for determination of coordinates of radius-vector **r** and components of wave vector **P** with the initial conditions

$$\mathbf{r}_{t=0} = \begin{pmatrix} 0 \\ x_2^0 \end{pmatrix}; \qquad \mathbf{P}_{t=0} = \frac{N}{(1+\mathbf{M}\cdot\mathbf{n})} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

2) four equations for determination of components of geodesic elements $\mathbf{R}^{\alpha} = \left(\frac{\partial \mathbf{r}}{\partial x_2^0}\right)$ and $\mathbf{Q}^{\alpha} = \left(\frac{\partial \mathbf{P}}{\partial x_2^0}\right)$ with the initial conditions

$$\mathbf{R}^{\alpha}{}_{t=0} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \qquad \mathbf{Q}^{\alpha}{}_{t=0} = \frac{\partial \mathbf{P}_{t=0}}{\partial x_2^0} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}.\text{y}$$

Determination of geodesic elements $\mathbf{R}^{\alpha}$ and $\mathbf{Q}^{\alpha}$ describing the wave front evolution along each ray is necessary for calculation of elementary ray tube area.

Two limit cases are considered:

1) The turbulence is caused by temperature fluctuations only, i.e. Mach number **M**=0, and index of refraction $N = 1 - \frac{T'}{2T_0}$, where T_0 is the temperature of undisturbed medium. It is possible to represent the index of refraction with a good accuracy in the form $N = \exp\left(-\frac{T'}{2T_0}\right)$. Such representation allows to simplify the process of spatial derivatives calculation and, thereby, permits to accelerate the solution of the system of differential equations.

2) The turbulence is caused by velocity fluctuations only. In this case $N = 1$.

For the solution of equations the fourth-order Runge-Kutta scheme is used.

The nonlinear transport equation for the propagation of the wave along the eigenrays to obtain the solution for acoustic pressure p is used. This transport equation was derived by Robinson [5].

Several assumptions are made in the development of nonlinear equations and are listed below:

1) parameters of inhomogeneity of the medium vary slowly on the time scale of the characteristic signal duration;
2) the fluid motion is isentropic;
3) ray acoustics approximation is true;
4) first-order terms are sufficient in the ray equations while second-order terms are only important in the transport equation for acoustic pressure p;
5) weak shock approximation is used;
6) loss terms are neglected in the development of the ray path and transport equations.

This equation takes the form

$$(1) \quad \frac{\partial}{\partial s}\left[\frac{|A|}{\rho_0 c} \cdot |\mathbf{n}+\mathbf{M}| \cdot (1+\mathbf{M}\cdot\mathbf{n}) \cdot p^2\right] - \frac{2\gamma |A|}{\rho_0^2 c_0^4} p^2 \frac{\partial p}{\partial t'} = 0,$$

where s is the arc length along the ray trace, t' is the retarded time coordinate, $|A|$ is the elementary ray tube area, γ is the nonlinearity coefficient (γ depends on index of refraction, when $N = 1$ $\gamma = 1.2$), ρ_0 is the density of undisturbed medium.

As distinct from the approach of paper [4], the equation (1) is solved in time-domain (i.e. in s and t' coordinates). If the eigenray passes through a caustic, then $|A| \to 0$ in the neighborhood of this point and the equation (1) has a singularity.

In order to avoid troubles in the numerical solution of the equation (1) the regularizing known as the method of artificial viscosity is applied [6]: the term $\varepsilon \frac{\partial^2 p}{\partial t'^2}$ is added to the transport equation (the ε is small parameter). Then the modified equation (1) is solved with the use of algorithm described in [7]. To the some extent this artificial technique may be considered as absorption effects modeling [8].

The described method of evaluation of atmosphere turbulent boundary layer influence on the sonic boom wave parameters was applied to the solution of model task, namely the propagation of the plane N-wave through the random temperature or velocity fluctuation field. The parameters of initial N-wave are the same as in paper [4]. Peak overpressure is 500 Pa, duration is 15 μs, and the rise time τ_0 (time portion between 10% and 90% of peak pressure) is 1 μs. The linear scale of turbulence L is assumed to be equals to 0,1 m [3].

In fig. 1 the ray traces of acoustic wave propagating through the various realizations of random temperature field are shown. The distribution of index of refraction N in calculation domain is shown as a color map. The figure demonstrates the focusing and defocusing phenomena of the acoustic wave on the local inhomogeneities as well as the influence of temperature stochastic field parameters on this process.

The first caustics appear at $T'_{rms} = \sqrt{\overline{T'^2}} = 1{,}172 \cdot 10^{-2} \cdot T_0$ in the region 15< x_1/L <25. It is seen from the concentration and crossings of the ray traces. We note that the pattern of the wave propagation essentially depends on the particular realization of stochastic field even at the same average parameters (fig. 1a).

Fig. 1b demonstrates the decrease of influence of temperature turbulence on the acoustic wave as the parameter T'_{rms}/T_0 is reduced.

In fig. 2 the traces of three eigenrays in random temperature field are shown. In addition the distributions of the parameter A along these traces are shown too. This parameter characterizes the elementary ray tube area. It is seen that $|A| \to 0$ if the ray traces are intersected.

In fig. 3 the ray traces of acoustic wave propagating through the random velocity field with zero mean component (without wind) are shown, $v'_{rms} = \sqrt{\overline{v'^2}}$. The field of vector $\mathbf{M}_t = \mathbf{v}'/c$ is shown by arrows. The streamlines of velocity fluctuations field are selective traced.

Fig. 4 illustrate the wind influence on the eigenrays traces of acoustic wave in random field. There is “non-classical” profile [9] of one-component surface wind with the distribution of $M_{2W}/M_{2W\,\max}$ along x_1/L was chosen for numerical modeling (fig. 4a). Here $M_{2\text{w}} = V_{2\text{w}}/c$, $M_{2\text{w}\,\max} = V_{2\text{w}\,\max}/c$, $V_{2\text{w}}$ is the wind velocity component in x_2-direction, $V_{2\text{w}\,\max}$ is the maximum value of wind velocity.

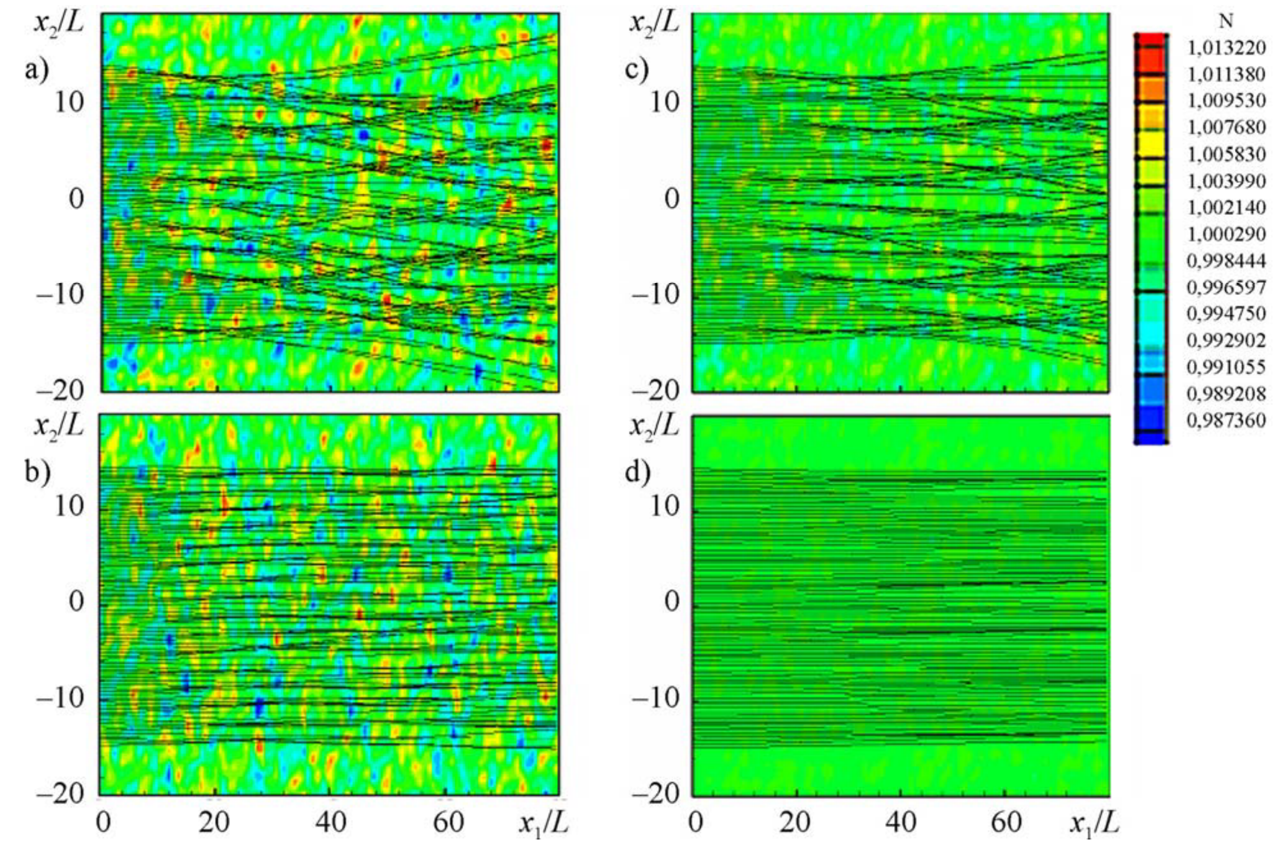

Fig 1. Propagation of the acoustic rays through various realizations of random temperature field: a) L=0,1 m ; $\frac{T'_{rms}}{T_0} = 1{,}172 \cdot 10^{-2}$; two distinct realizations of temperature field; b) L=0,1 m ; $\frac{T'_{rms}}{T_0} = 0{,}586 \cdot 10^{-2}$ (overhead) and $\frac{T'_{rms}}{T_0} = 0{,}1465 \cdot 10^{-2}$ (beneath)

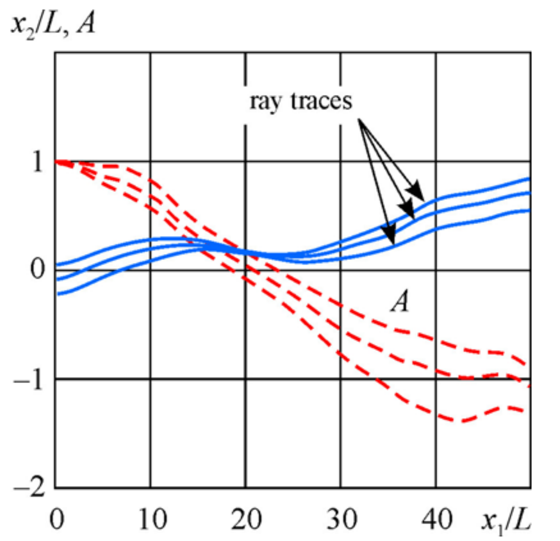

Fig. 2. Acoustic rays focusing in the random temperature field and distributions of parameter A along the rays traces

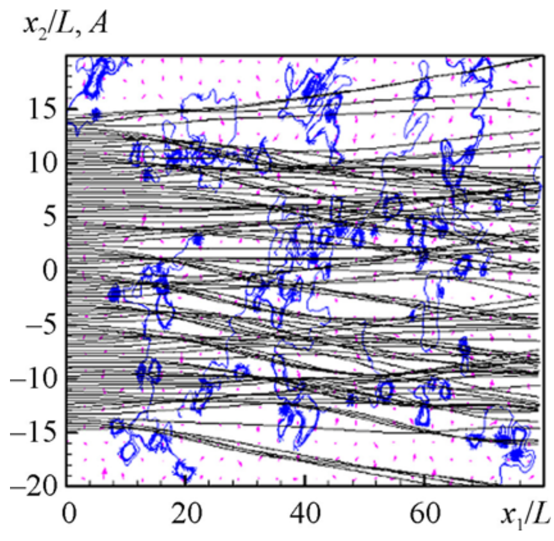

Fig. 3. Propagation of the acoustic rays through the random velocity fluctuations field - L= 0,1 m; $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$. The field of vector $\mathbf{M}_t = \mathbf{v}'/c$ is shown by arrows. The streamlines of velocity fluctuations field are selective traced.

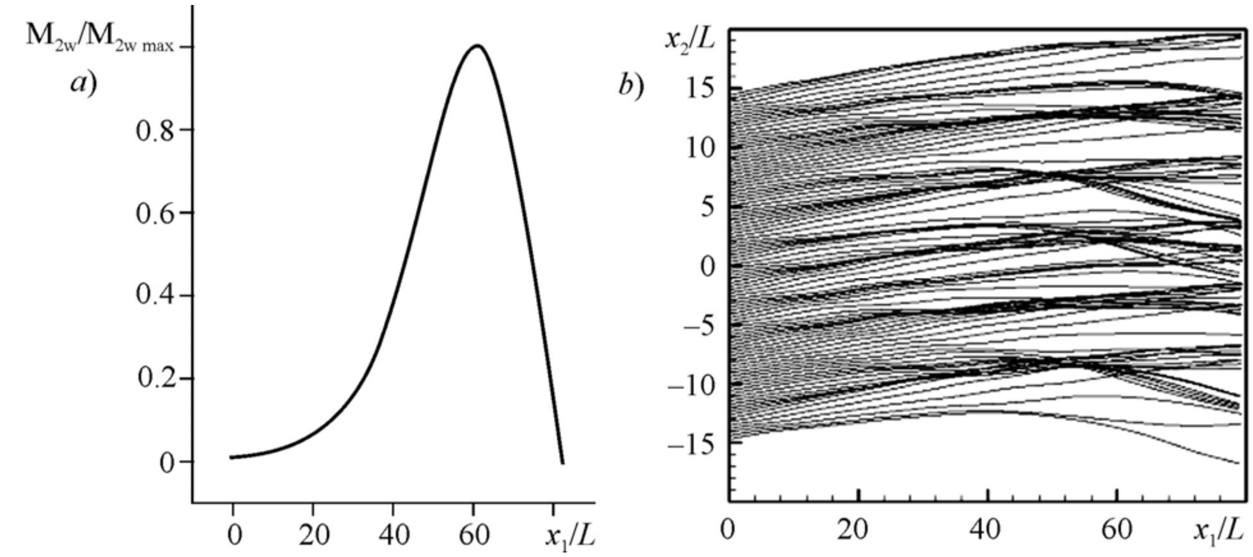

Fig. 4. Wind influence on the propagation of acoustic rays in the random velocity field; there is the same realization of the field of vector $\mathbf{M_t}$ as shown in fig. 3: a) dimensionless wind velocity distribution; b) acoustic rays traces

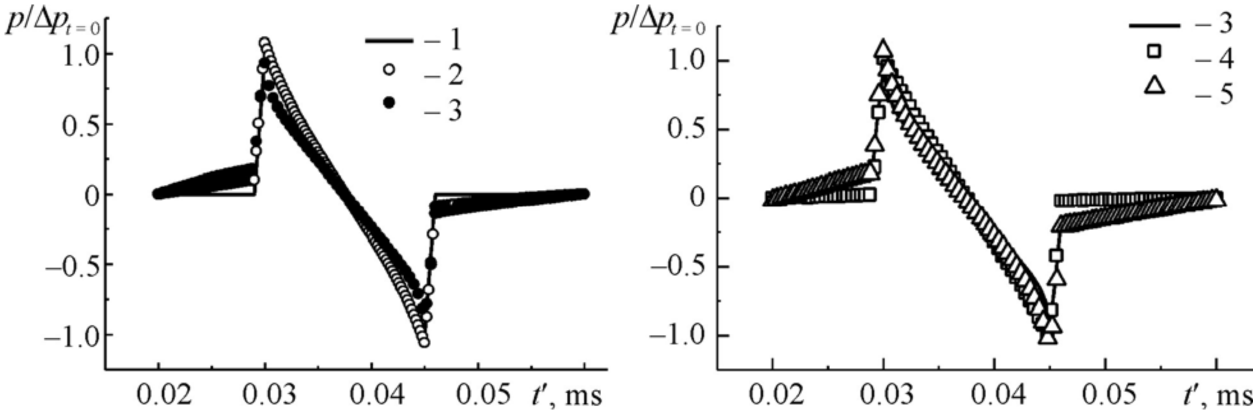

Fig. 5. Evolution of the N-wave propagating through stochastic field: 1 – initial signature; at x_1=50L : 2 – $T'_{rms}/T_0 = 1{,}172 \cdot 10^{-2}$, $v'_{rms}/c_0 = 0$, $M_{2w\,max} = 0$; 3 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$, $M_{2w\,max} = 0$; 4 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$, $M_{2w\,max} = 0{,}0302$; 5 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$, $M_{2w\,max} = 0{,}0201$.

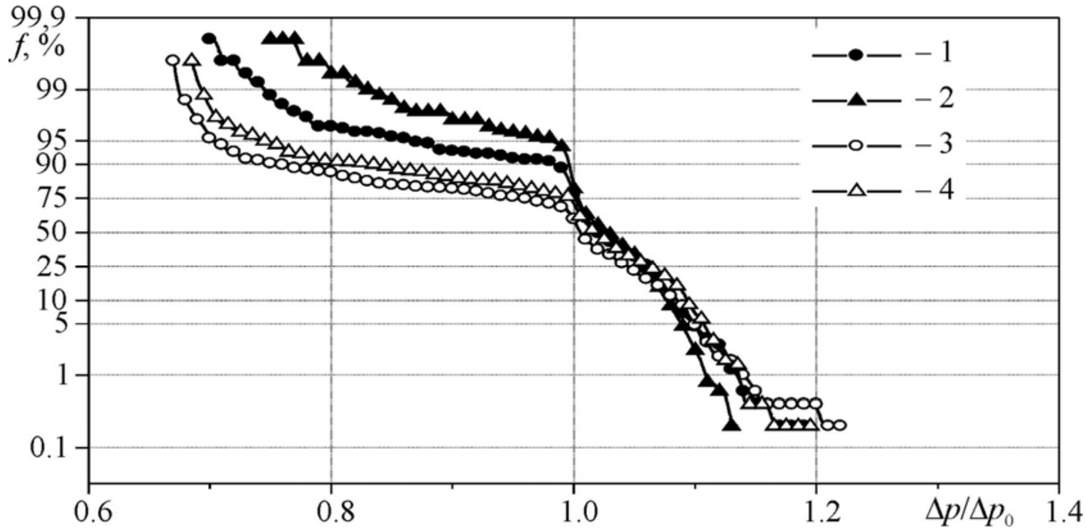

Fig. 6. Cumulative probability f of $\Delta p/\Delta p_0$ in the sonic boom wave propagating through stochastic field: x_1=25L : 1 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$; 2 – $T'_{rms}/T_0 = 1{,}172 \cdot 10^{-2}$, $v'_{rms}/c_0 = 0$; x_1=50L : 3 – $T'_{rms}/T_0 = 0$, $v'_{rms}/c_0 = 0{,}586 \cdot 10^{-2}$; 4 – $T'_{rms}/T_0 = 1{,}172 \cdot 10^{-2}$, $v'_{rms}/c_0 = 0$

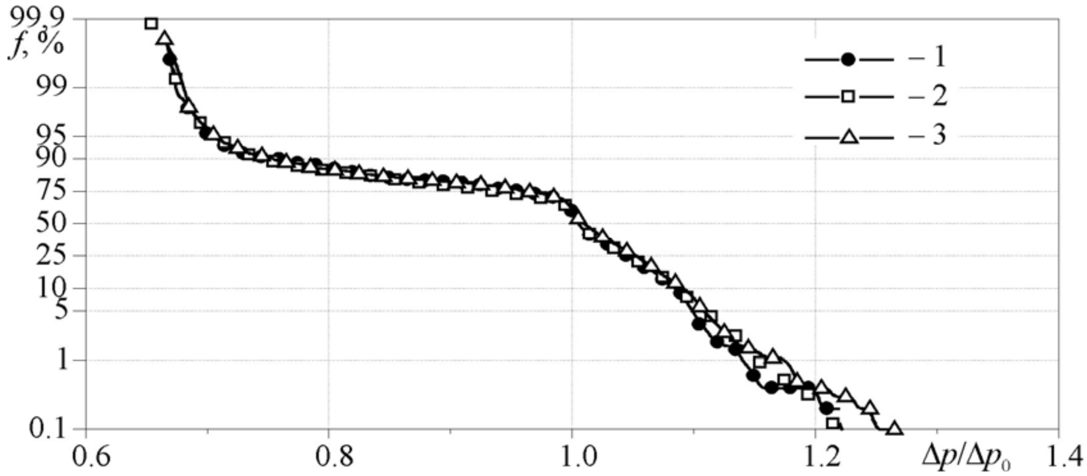

Fig. 7. Cumulative probability f of $\Delta p/\Delta p_0$ in the sonic boom wave propagating through stochastic velocity fluctuations field without and with lateral wind (x_1=50L ; $v'_{rms}/c_0 = 0{,}586\cdot 10^{-2}$): 1 – $M_{2w\,max} = 0$; 2 – $M_{2w\,max} = 0{,}0302$; 3 – $M_{2w\,max} = 0{,}0201$.

The values of T'_{rms}/T_0 and v'_{rms}/c_0 are specified in such a way that fluctuating part $\mu = -\frac{T'_{rms}}{2T_0} - \frac{v'_{rms}}{c_0}$ of the refraction index $n = 1 + \mu$ was the same both for temperature fluctuation field and for velocity fluctuation field.

The evolution of the N-waves propagating through stochastic temperature and velocity fields (with and without wind) at the distance of x_1=50L from a source are shown in fig. 5. In fig. 5 p' is the overpressure and p'_0 is the maximum value of p' in the initial wave.

The form of received signal essentially depends on the specific realization of random field as well as on the receiver location. In the present case the wind (curves 4 and 5) reduces the velocity fluctuations effect. While for another realization the receiver located at the same point may records opposite situation. In order to get information on the turbulence influence on one or another characteristic of sonic boom wave the statistical ensemble averaging is necessary.

In fig. 6 and 7 the results of calculations of cumulative probability f of $\Delta p/\Delta p_0$ in the sonic boom wave propagating through stochastic temperature and velocity fluctuations fields at the distances of x_1=25L and x_1=50L from source are presented. Δp is the maximum overpressure and Δp_0 is the calculated value of Δp without turbulence. The results are generalized on the base of calculation about 50 eigenrays and 100 various realizations for each curve. It is seen than lateral wind has not noticeably effect on cumulative probability, in spite of the fact that wind influence for distinct ray at distinct realization is essential(fig. 5).

The nominal value τ_0 is rise time value τ_0 calculated in absence of turbulence. The curves have an nonrealistic stepped character at 0,5 < τ/τ_0 < 1. It is not unlikely that this behavior is the consequence of the fact that no molecular absorption effects were included in proposed model.

As a whole, one can see that it takes place qualitative agreement between results calculated with the use of suggested method and experiment. The implementation of noncommercial code elaborated in the frame of this subtask in a routine sonic boom software, for example «BOOM» [10], will be the next step of investigations of atmosphere turbulence effects.

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