LEVEL HOMOGENEITY VERSUS FREQUENCY IN A REVERBERANT CHAMBER

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OVERVIEW

The acoustic field in a reverberant chamber is of diffuse nature in a given frequency band only if the modal density is sufficiently high, which is not the case at low frequencies. The cut-off frequency is given by the so-called Schroeder formula stating that it is proportional to the square root of the ratio reverberation time / chamber volume. This paper proposes to derive the proportionality factor from the homogeneity of the levels, defined as the ratio between the minima and the maxima at a given frequency, using some assumptions similar to those of the statistical energy approach. Formulation is presented and applied to the INTESPACE reverberant chamber for illustration.

1. INTRODUCTION

It is well known that the acoustic field in a reverberant chamber is of diffuse nature (homogeneous and isotropic) in a given frequency band only if the modal density is sufficiently high. This is not the case at low frequencies where only a few modes are present, depending on the size of the chamber. The frequency under which the acoustic field cannot be considered as diffuse depends mainly on the chamber volume and its attenuation properties which can be represented by the reverberation time.

It can be shown that this cut-off frequency is roughly proportional to the square root of the ratio reverberation time / chamber volume : this is the so-called Schroeder formula. The proportionality factor depends on the criterion selected for level homogeneity and varies in the literature between 1000 and 2000 (SI units).

This paper proposes to define the homogeneity of the level at a given frequency as the ratio between the minima and the maxima in the vicinity of the frequency. Making some assumptions similar to those of the SEA (Statistical Energy Approach), it can be shown that the Schroeder proportionality factor is directly related to this homogeneity. So, it is possible to find the cut-off frequency corresponding to a given homogeneity.

Formulation, including considerations on specimen excitability, is presented in § 2 and applied to the INTESPACE reverberant chamber for illustration in § 3.

2. FORMULATION

2.1. Level homogeneity

It will be assumed for the acoustic modes that the modal spacing (distance between frequencies, inverse of the modal density), the effective parameters and the damping ratios are locally constant and are only function of the frequency, as for SEA. In this case, modal superposition leads to level variations as shown in Figure 1.

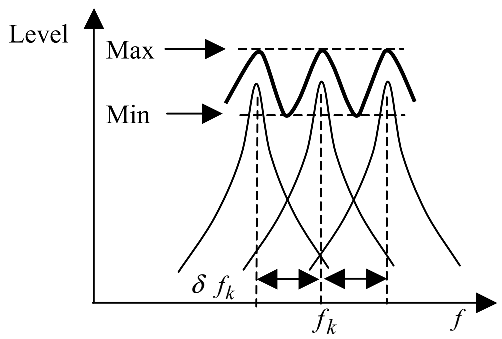

FIG. 1 Level variations from modal superposition

As the effective parameters are locally constant, the contribution of each mode k to the acoustic response at a given frequency f is proportional to its amplification factor given by [1] :

$$(1)\quad H_k(f) = \frac{1}{1-\left(\frac{f}{f_k}\right)^2 + i\,2\,\zeta\,\frac{f}{f_k}}$$

with f_k natural frequency and ζ common viscous damping ratio. If δf_k is the modal spacing, the modal superposition leads to the following maxima and minima for the level :

$$(2)\quad \text{Max} = |H(f_k)| = \left| H_k(f_k) + \sum_n H_{k\pm n}(f_k) \right|$$

$$\approx \frac{1}{2\,\zeta}\left| 1 + \sum_n \frac{1}{\pm n\frac{\delta}{\zeta}i+1} \right| = \frac{1}{2\,\zeta}\left(1 + 2\sum_n \frac{1}{\left(n\frac{\delta}{\zeta}\right)^2+1} \right)$$

$$(3)\quad \text{Min} = \left|\left(H(1 \pm \frac{\delta}{2})\ f_k\right)\right| \approx \frac{1}{2\,\zeta}\left|\sum_n \frac{1}{\pm(n-\frac{1}{2})\frac{\delta}{\zeta}\,i+1}\right|$$

$$= \frac{1}{2\,\zeta}\,2\sum_n \frac{1}{\left((n-\frac{1}{2})\frac{\delta}{\zeta}\right)^2+1}$$

Using the mathematical formulas :

$$(4)\quad \sum_{n=1}^{\infty} \frac{1}{n^2+x^2} = \frac{1}{2\,x}\left(\pi \coth \pi\, x - \frac{1}{x}\right)$$

$$(5)\quad \sum_{n=1}^{\infty} \frac{1}{(2\,n-1)^2+x^2} = \frac{\pi}{4\,x}\,\text{th}\,\frac{\pi\,x}{2}$$

provides :

$$(6)\quad \text{Max} \approx \frac{1}{2\,\zeta}\,\pi\,\frac{\zeta}{\delta}\coth \pi\,\frac{\zeta}{\delta}$$

$$(7)\quad \text{Min} \approx \frac{1}{2\,\zeta}\,\pi\,\frac{\zeta}{\delta}\,\text{th}\,\pi\,\frac{\zeta}{\delta}$$

leading to :

$$(8)\quad \frac{\text{Max}}{\text{Min}} = \coth^2 \pi\,\frac{\zeta}{\delta}$$

which can be used to quantify the level homogeneity of the acoustic field. It can be expressed in dB (20 $\log_{10}$) with the correspondence :

$\frac{\zeta}{\delta}$	0.931	0.675	0.565	0.390	0.281	0.175
dB	0.1	0.5	1	3	6	12

2.2. Application to reverberant chambers

As the modal spacing is the inverse of modal density $n(f)$, the δ parameter is given by :

$$(9)\quad \delta\,f = \frac{1}{n(f)}$$

For a parallelepipedic reverberant chamber of dimensions L_x, L_y, L_z, giving volume V, wall surface area S and edge length L, the frequencies of the acoustic modes are given by :

$$(10)\quad f(n_x, n_y, n_z) = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2+\left(\frac{n_y}{L_y}\right)^2+\left(\frac{n_z}{L_z}\right)^2}$$

with c sound speed and n_x , n_y , n_z integers ≥ 0 representing the number of half sine waves in each direction. This leads to the theoretical asymptotic formula for the modal density :

$$(11)\quad n(f) = \frac{4\,\pi\,f^2\,V}{c^3} + \frac{\pi\,f\,S}{2\,c^2} + \frac{L}{8\,c}$$

which can be written :

$$(12)\quad n(f) = \frac{4\,\pi\,f^2\,\overline{V}}{c^3}$$

$\overline{V}$ being the corrected volume to account for S and L effects which are negligible at high frequencies but not at low frequencies :

$$(13)\quad \overline{V} = V\left(1 + \frac{c}{8\,f}\,\frac{S}{V} + \frac{c^2}{32\,\pi\,f^2}\,\frac{L}{V}\right)$$

The ζ parameter is related to the reverberation time T_r which corresponds to a sound pressure decrease of 60 dB, so :

$$(14)\quad e^{-2\,\pi\,f\,\zeta\,T_r} = -60\,\text{dB} \approx 10^{-3} \quad \Rightarrow \quad T_r \approx \frac{3\,\ln 10}{2\,\pi\,\zeta\,f}$$

Equations (9), (12) and (14) lead to :

$$(15)\quad f^2 = \frac{c^3}{6\ln 10}\,\frac{\zeta}{\delta}\,\frac{T_r}{\overline{V}}$$

which is coherent with the so-called Schroeder formula stating that it is proportional to the square root of the ratio reverberation time / chamber volume:

$$(16)\quad f = \lambda\sqrt{\frac{T_r}{\overline{V}}} \quad \text{with} \quad \lambda = \sqrt{\frac{c^3}{6\ln 10}\,\frac{\zeta}{\delta}} \approx 1700\sqrt{\frac{\zeta}{\delta}}$$

with c 340m/s

Various values for λ are proposed in the literature. These values can be now associated to the level homogeneity using equation (8), for example (SI units) :

(17) λ 1000 $\Rightarrow \zeta / \delta$ 0.35 $\Rightarrow$ Max/Min $\approx$ 3 dB
λ 2000 $\Rightarrow \zeta / \delta$ 1.4 $\Rightarrow$ Max/Min $\approx$ 0.026 dB

For a reverberant chamber with a given volume and reverberation time, equation (16) provides the frequency corresponding to a given level homogeneity.

2.3. Specimen excitability

With a specimen in the reverberant chamber for qualification purpose, the excitability of a given structural mode with frequency f_s and viscous damping ratio ζ_s depends on the acoustic level homogeneity but also the modal spacing, as illustrated in Figure 2. The structural mode will be appropriately excited with a good homogeneity or a small modal spacing compared to the half power bandwidth : if the homogeneity is not good at f_s, the modal spacing can compensate. Using equations (9) and (12), a ratio $\delta/\zeta_s < 1$ (more than 2 acoustic modes within the halfpower bandwidth) leads to :

$$(18)\quad \zeta_s > \frac{c^3}{4\,\pi\, f^3\, \overline{V}}$$

At low frequencies, the viscous damping ratio of a structural mode must be sufficiently high to have a good excitability in case of bad homogeneity.

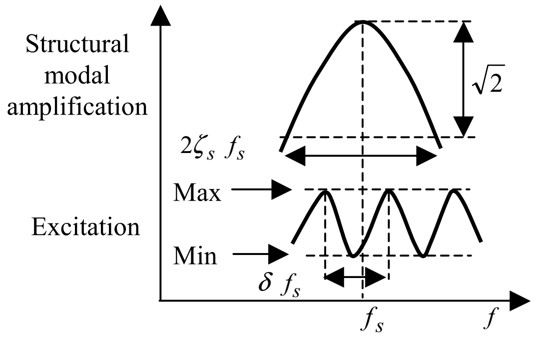

FIG. 2 Excitability of a specimen structural mode

3. THE INTESPACE REVERBERANT CHAMBER

3.1. Properties

The INTESPACE reverberant chamber is parallelepipedic with the dimensions : 10.3 m x 8.2 m x 13 m, giving :

V 1098 m^3
S 650 m^2
L 126 m

According to equation (10), the first acoustic mode is at about 13 Hz and the distribution of modal frequencies is illustrated by Figure 3.

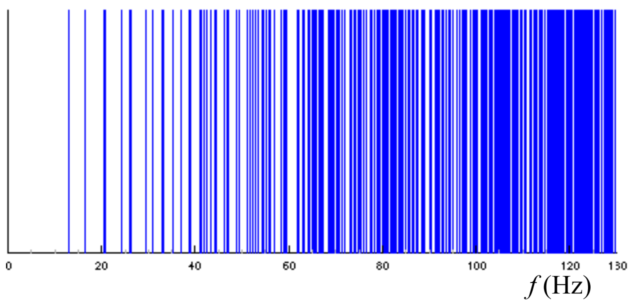

FIG. 3 Modal frequencies giving modal spacing

The corrected volume $\overline{V}$ is given by equation (12) :

$$(19)\quad \overline{V} \approx V\left(1+\frac{25.2}{f}+\frac{132}{f^2}\right)$$

The correction is significant at low frequencies where the level homogeneity is interesting to know : for example at 100 Hz, the factor is 1.27. A correction of less than 10 % requires to be higher than 260 Hz.

The reverberation time versus frequency was measured using 3 distinct sources, giving the results of Figure 4. Above 130 Hz, it varies slowly, about 7-9 s up to 2500 Hz, before decreasing significantly. At lower frequencies, the variations are higher, between 10 and 20 s.

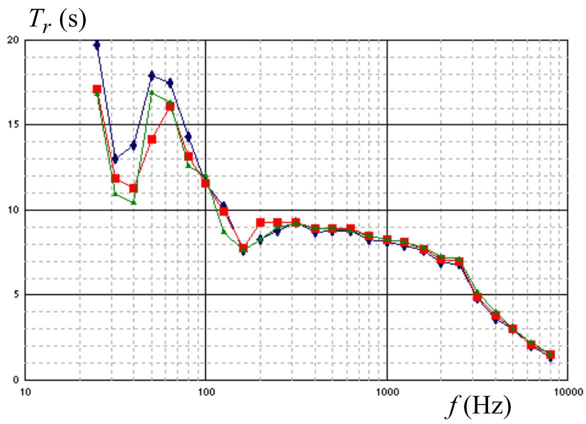

FIG. 4 Reverberation time versus frequency

This leads to the following chamber properties in the normalized 1/3 octave frequency bands :

f (Hz)	31.5	40	50	63	80	100
δ (%)	4.7	2.6	1.5	0.80	0.42	0.23
ζ (%)	0.29	0.23	0.13	0.11	0.10	0.09

3.2. Level homogeneity

The relatively high values of the reverberation time, closely related to the performance of the chamber, provides very small values for the damping parameter ζ, penalizing its homogeneity at low frequencies where the modal density is low and gives high values for δ.

Combining equations (8) and (16) leads to the following results concerning the frequency versus homogeneity in dB (iterative process) :

dB	0.1	0.5	1	3	6	12
f (Hz)	130	120	110	100	85	70

So, at 130 Hz, the homogeneity of the sound pressure field is about 0.1 dB, which is quite good. Then it increases rapidly to reach 3 dB at 100 Hz. Below 100 Hz, the variations become very large. However, the assumptions are less and less valid : see Figure 5 for the acoustic response measured up to 50 Hz, which shows that the levels are generated by individual modes.

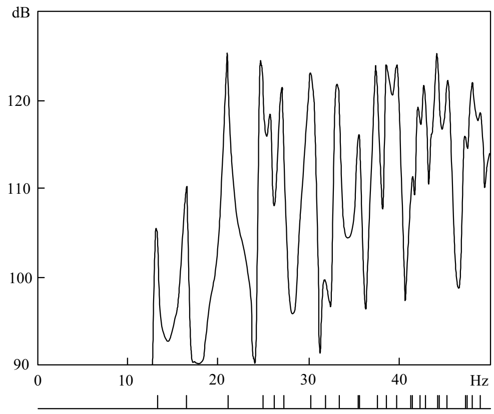

FIG. 5 Acoustic response at low frequencies

Concerning mode excitability, equation (18) leads to the following results for the minimum modal damping ratio :

f_s (Hz)	30	40	50	60	70	80
ζ_s (%)	5.3	2.6	1.5	0.9	0.6	0.4

At these frequencies, the acoustic level homogeneity is bad because of very low damping of the acoustic modes, but the damping of the specimen, which is usually higher, may compensate by including several acoustic modes in the half power frequency bandwidth : for example a modal damping of 1.5 % is sufficient to excite adequately a mode of the specimen at 50 Hz.

4. CONCLUSIONS

The previous considerations lead to estimate the level homogeneity versus frequency in a reverberant chamber knowing its equivalent volume (with respect to modal density) and its reverberation time.

The application to the INTESPACE chamber shows that this homogeneity is rapidly decreasing at low frequencies where the average variations are for example higher than 3 dB below 100 Hz. This calls into question the diffuse nature of the acoustic field in this frequency band. However, for acoustic qualification of a specimen, a bad homogeneity can be compensated by several acoustic modes in the half power bandwith of each considered structural mode, according to its viscous damping ratio. These considerations can be applied to any reverberant chamber according to its characteristics.

REFERENCES

[1] Girard A., Roy N., "*Structural Dynamics in Industry*", ISTE 2007.