PRESSURE RESTRAINT DESIGN FOR INFLATABLE SPACE HABITATS

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1. ABSTRACT

A large, inflatable space structure can be launched in its collapsed configuration to a destination in space where the structure is subsequently deployed to full size. Besides offering the attractive potential of smaller launch vehicle requirements and reduced payload fairing dimensions, inflatable space structures offer significant corollary advantages which have been clearly demonstrated by numerous gossamer spacecraft missions. The primary challenge of deployable space inflatable design is to demonstrate robust predictability of the inflatable's integrated gas barrier and pressure restraining structures. Gossamer spacecraft generally employ highly isotropic films simultaneously fulfilling structural and gas barrier roles. While these membranes adequately support the low shell loads of gossamer craft, their structural performance falls far short of the requirements for containment of the large volumes of life support atmosphere associated with habitat architecture.

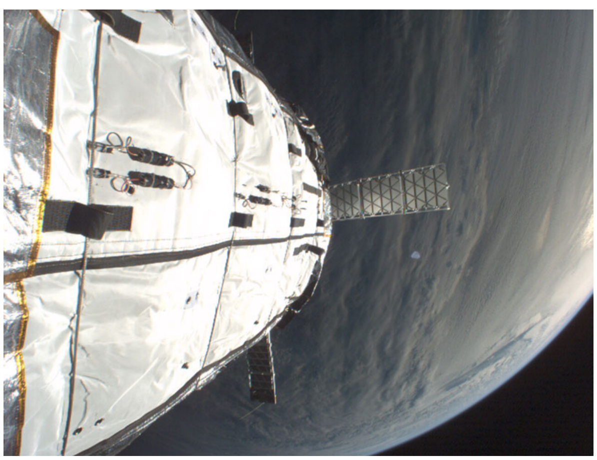

FIG 1. Genesis-1 inflatable spacecraft (Bigelow Aerospace photo).

Bigelow Aerospace Genesis (FIG 1) and NASA TransHab (FIG 8) are projects that set the current standard for inflatable space habitat technology. Genesis-1 and Genesis-2, launched in July 2006 and June 2007 respectively, feature restraint structures primarily designed, engineered and manufactured by Thin Red Line Aerospace, and are the first spacecraft on orbit to successfully incorporate flexible, high-stress, pressure shells. This paper uses Genesis project experience to present a developmental methodology for inflatable space habitat pressure restraint design with emphasis on the difficulty of structural and geometric reproducibility. Mathematical models for analysis reinforce the design process but meaningful analysis may only be selectively and cautiously applied; models attempting to prioritize the large number of variables characteristic of flexible, anisotropic structural constituents quickly become unwieldy and often unusable. Modelling assumptions must be rigorously matched to physical reality, and its limitations acknowledged to ensure timely transition to hardware testing.

2. INTRODUCTION

Based on Thin Red Line Aerospace methodology applied to the Genesis project, four successive phases in the development of high pressure restraint structures for space applications are represented in this paper by means of the following areas of discussion:

1) Preliminary analysis of stresses (Section 3).
2) Architectural options (Section 4).
3) Structural predictability (Section 5).
4) Assembly considerations (Section 6).

3. PRELIMINARY ANALYSIS OF STRESSES

The definition of requirements provides the initial roadmap for designers involved in any space hardware project. Based on mission requirements as far as they have been defined, a highly simplified, predominantly algebraic analysis is applied to assess the habitat vessel's structural geometry in terms of the maximum anticipated shell stresses for the vessel's constituent geometries.

3.1. Nomenclature

σ_1	meridional stress
σ_2	circumferential stress
R	radius of curvature of a circumference
R_1	radius of curvature of a meridian
R_2	length of the normal between a point on the shell and the axis of revolution.
r	distance from a torus' axis of revolution to a point on its shell
r_1	torus' radius from its axis of revolution to the centre of its circular axial section
r_2	radius of a torus' circular axial section

t	shell thickness
E	Young's (elastic) modulus
ν	Poisson's ratio
p	internal pressure

3.2. Background

Subjected to distributed internal pressure loading, a flexible shell attempts to assume the default geometry of a spherical surface of revolution. (A surface of revolution is a three dimensional surface generated by sweeping a two dimensional curve about an axis of revolution.) While such a vessel's axial profile can forcefully be constrained to deviate from its circular profile, deviation from the vessel's axial symmetry is much more difficult to attain, and fortunately, offers no conspicuous benefit. Therefore, for the purpose of stress analysis in the development of space habitat restraint shells we limit our attention to geometries presented by surfaces of revolution.

Meridians are curves generated along a vessel's surface where it is intersected by an axial plane, i.e. a plane comprising the vessel's axis of revolution. Circumferences are generated where a vessel's surface is intersected by a plane perpendicular to the vessel's axis of revolution. Spheroids, ellipsoids, toroids and the like offer the benefit of continuously smooth, doubly curved surfaces capable of completely enclosing a vessel's internal space, as opposed to singly curved cylindrical and conical surfaces which do not offer this benefit.

Singly curved surfaces however, are readily developed from planar geometry—an attribute which simplifies engineering and manufacture immensely. Conversely, doubly curved surfaces invariably present significant obstacles in this regard. It is worth noting that the concepts introduced thus far are very much the same as those encountered in the design of conventionally rigid pressure vessels. For ease of manufacture, a conventional pressure vessel's geometry is typically dominated by a central cylindrical constant section. Doubly curved shell geometry is then used to close the ends of the cylinder, and is only begrudgingly applied for lack of reasonable alternative.

Shell theory studies the stress-strain relationships in curved surfaces. To be flexible, a shell wall would need to be relatively thin - certainly less than ten percent of its smallest radius of curvature (the applicable shell thickness for Genesis is 0.17 percent). When such a thin-walled structure is subjected to internal pressure loads, the predominant stresses are membrane stresses, i.e. stresses constant throughout the thickness of the wall whereby the shell's radial stress σ_3 and any bending stresses tend towards zero. Membrane stress is imparted in two principle directions: a meridional membrane stress acting parallel to the meridian, and a circumferential (commonly referred to as hoop) membrane stress acting parallel to the circumference.

3.3. Constant Section Stresses

Considering the open-ended, cylindrical constant section of a pressure vessel, the principle shell stresses are

$$(1) \quad \sigma_1 = \frac{pR}{2t} \quad \text{and} \quad \sigma_2 = \frac{pR}{t}$$

However, we need only consider membrane stresses, in which case shell thickness can be ignored. The equations in (1) then become

$$(2) \quad \sigma_1 = \frac{pR}{2} \quad \text{and} \quad \sigma_2 = pR$$

3.4. End Cap Stresses

3.4.1. Smooth Surfaces of Revolution

Equilibrium between pressure and membrane stresses in a generalized smooth surface of revolution gives

$$(3) \quad \sigma_1 = \frac{pR_2}{2t} \quad \text{and} \quad \sigma_2 = \frac{pR_2}{2t}\left(2 - \frac{R_2}{R_1}\right)$$

Ignoring shell thickness as before reduces (3) to

$$(4) \quad \sigma_1 = pR_2 \quad \text{and} \quad \sigma_2 = pR_2\left(2 - \frac{R_2}{R_1}\right)$$

Being doubly curved we see that the maximum stress in a smooth surface of revolution is half of the maximum stress generated in the singly curved, cylindrical surface. Alternately stated, a doubly curve surface offers twice the mass efficiency of its singly curved counterpart.

3.4.2. Spherical Sections

An end cap formed by a spherical section is the specific case of 3.4.1 above for which $R_2 = R_1 = R$, leading to the combination and simplification of the equations in (4):

$$(5) \quad \sigma_1 = \sigma_2 = \frac{pR}{2}$$

3.4.3. Toroidal Sections

Genesis and TransHab [1] both use a toroidal section for their end cap geometries. While TransHab end caps could essentially have been combined to form a full, classic torus (i.e. complete with centre opening), r_1 and r_2 of Genesis' end caps are almost equal, giving the vessel a rather more hemispherical impression in its polar regions. Toroidal membrane stresses are given by:

$$(6) \quad \sigma_1 = \frac{pr_2}{2}\left(\frac{r + r_1}{r}\right) \quad \text{and} \quad \sigma_2 = \frac{pr_2}{2}$$

The maximum meridional stress in a torus occurs where its surface comes nearest to the axis of rotation, i.e. along the torus' inner equatorial circle ($r = r_2$). Along this circle we have:

$$(7) \quad \sigma_{1,\max} = \frac{pr_2}{2t}\left(\frac{2r_1 - r_2}{r_1 - r}\right)$$

The torus has the interesting property that for large values of r_1

the maximum hoop stress becomes independent of r_1:

(8) $\lim_{r_1\to\infty}\left(\frac{2r_1-r_2}{r_1-r}\right)=2$

thereby giving the relationship

(9) $\lim_{r_1\to\infty}\sigma_{1,\max}=pr_2$

Effectively, the stresses in a torus depend primarily on r_2 and are therefore independent of its global diameter. As such it may appear conceptually enticing to develop toroidal space architecture of great overall diameter to so provide commensurately substantial enclosed volume without appreciably affecting the structure's shell stress. However, increases in surface area and manufacturing complexity largely offset this benefit. This is nonetheless a potentially worthy attribute to bear in mind, especially in context of the other geometries described above for which enclosed volume and shell stress are proportionately correlated.

3.5. Geometric Discontinuity

Proven continuity in structural capability where differing geometries meet is one of the more critical challenges in the design of any pressure vessel. Considering a typical vessel with cylindrical section closed by two doubly curved, polar end caps, reconciliation between the geometries could be achieved by endowing the constant section with twice the wall thickness of the end caps. While certain conventional vessel types may be adapted to such physical discontinuity, this is a scenario in which flexible vessels markedly depart from the characteristics of their conventional counterparts. Section 5, "Structural Predictability", addresses discontinuity in greater detail.

3.6. General Membrane Equation

If the habitat vessel comprises more complex geometry, or similarly, to clarify stress distribution in areas of geometric transition as encountered in section 3.5, it is helpful to consider a general approach to defining membrane equilibrium. As we have seen in section 3.2, membrane theory assumes that the bending stiffness is zero. Following Flügge [2], we consider equilibrium of a generalized, infinitesimal element of membrane using the notation shown in FIG 2 [2]. The hoop edges of the element are defined by ϕ and $\phi+d\phi$, and the meridional edges are defined by θ and $\theta+d\theta$. The meridional and hoop stress resultants for the aforementioned edges are N_ϕ and N_θ respectively, and the normal load on the membrane element is the differential pressure p_r.

A general equation relating N_ϕ, N_θ and p_r can be found by examining the components of the pressure and stress resultants in the direction of the outward normal to the membrane surface. These components are:

(1) hoop component $= N_\theta r_1 d\theta d\phi \sin\phi$

(2) meridional component $= N_\phi r d\theta d\phi$

(3) pressure component $= -p_r r r_1 d\theta d\phi$

Assembling these equations gives the equation for the equilibrium of the membrane element:

(4) $N_\theta r_1 \sin\phi + N_\phi - p_r r r_1 = 0$

Radius *r* from this equation can also be written as

(5) $r = r_2 \sin\phi$

Divide by $r\, r_1$ and substitute for *r* from Equation 5 to get

(6) $\frac{N_\phi}{r_1}+\frac{N_\theta}{r_2}=p_r$

Equation 6 gives us a generalized equation of equilibrium for an infinitesimal, three dimensional surface under membrane loads. Further details regarding stress relationships in shells can be found in a standard text such as Roark's [3], and more advanced discussion of shell theory can be found in specialised texts such as those by Flügge [2] and Calladine [4].

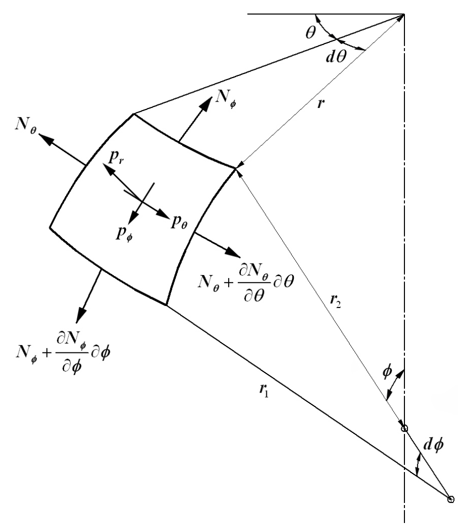

FIG 2. Membrane element (after Flügge).

3.7. Conclusion

Through this preliminary geometric analysis we are provided with a working knowledge of the shell stresses which can be anticipated for a structure fulfilling the initial statement of a habitat's geometric requirements. Reflecting upon the specifics of this preliminary analysis we readily observe that incorporation of only doubly curved geometry provides a vessel of higher specific strength, and therefore lower structural mass for same volume, than a vessel incorporating constant section geometry. Mission requirements permitting, we conclude that doubly curved geometry is preferable if this option is supported by verification of its architectural feasibility.

4. ARCHITECTURAL OPTIONS

It is important for the designer to develop a keen insight into the full spectrum of flexible, pressure confining architectures: particular attributes of architectures initially dismissed as inappropriate will invariably shed light on impasses encountered later in the design process. For that reason we provide an overview of the design rationale for the major categories of flexible vessel:

1) Monofilm
2) Woven fabric
3) Webbing
4) Hybrid

The ultimate application of chosen architecture is, of course, contingent upon manufacturing capability, the details of which largely fall outside the scope of the developmental methodology contemplated by this paper. Manufacturing comments will only be introduced where appropriate in the context of the discussion, particularly in section 6.

4.1. Introduction

A surface of revolution is symmetrical about the axis of rotation which passes through its poles. FIG 3 illustrates a spherical surface of revolution having meridional and circumferential grid lines. The meridians and circumferences are perpendicular to one another; however, while the circumferences are parallel to one another, the meridians are non-parallel and converge at the poles.

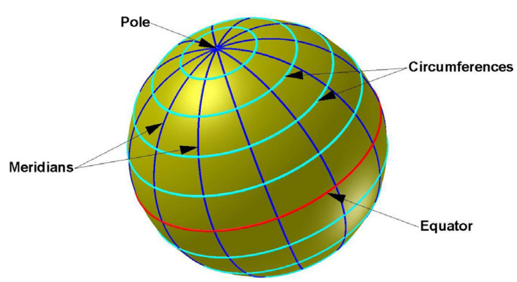

FIG 3. Spherical surface of revolution.

If a spherical vessel's shell is disassembled along its meridians, it would be as if we had peeled an orange: the resulting wedges are three dimensional, doubly curved shell segments, which when recombined, would give us the original closed shell. In the context of flexible vessels this is important to remark upon since meridians can be used to define the edges of meridional gores which are the two dimensional approximations (ignoring thickness) of the shell segments described above. Since gores are per definition two dimensional, they can be cut from planar sheets of material, and when connected edge-to-edge, reproduce a close facsimile of the fully closed, three dimensional shell. The primary discrepancy is that the 'manufactured' shell is undesirably facetted between its meridians. This effect can be reduced by increasing the number of meridians and the ensuing number of gores.

4.2. Monofilm Vessels

An example of one of the simplest flexible pressure vessels is the spherical toy beach ball. The beach ball is illustrative of the use of two dimensional meridional gores of thermoplastic film heat welded to one another along their edges to produce an approximated spherical shell. Few gores are needed for this approximation because the outward distortion of gore material under pressure seeks the default spherical geometry inherent to the distributed internal loading described earlier. Simultaneous to offering good specific strength, many thermoplastics such as Mylar are also acceptable barrier films for a vessel's pressurized contents. For the purpose of comparison, thermoplastic films are here considered to be essentially isotropic, i.e., having the same physical properties measured in different directions of the film's structural membrane surface. While making them good candidates for applications requiring low membrane stress, such as balloons, the isotropic nature of thermoplastic films is not suited to higher membrane stress, since increasing thickness to increase strength is an option of diminishing returns due to the corresponding dramatic loss of film flexibility. Typical field-allowable membrane stresses for thermoplastic film gores would be less than 2 kg/cm. Monofilm vessel may be considered a flexible counterpart of conventional metal pressure vessels.

4.3. Woven Fabric Vessels

For flexible pressure vessels subjected to higher membrane stresses, or where specific strength is required, fibre-based materials replace films as the fundamental load bearing element of the shell. The fibres woven to form a fabric run at right angles to one-another in a basket weave fashion largely allowing fibre orientation to correspond to the two directions of membrane stress. Warp fibres, which are aligned with the length of a fabric, are interlaced with fill fibres spanning the width of the fabric. Woven fabrics remain more pliable than isotropic films when thickness is increased because of the flexibility of individual fibres and their freedom, under load, to shift their relative positions within the weave. The manufacture of woven fibre products is a very mature, extremely diverse, technologically advanced and commercially competitive industry offering extensive off-the-shelf product selection. Fabrics may be constructed of a great diversity and combination of fibre types. The fabric weave, or warp and fill components independently, can be precisely tailored to balance performance prerequisites such as strength, flexibility, abrasion resistance, elongation, surface finish and post-manufacture processing attributes. The primary trade-off for the strength advantage offered by suitable wovens is firstly, more difficult assembly involving high-strength stitched seams as opposed to welded seams for films, and secondly, the permeability of wovens to the fluids confined by the vessel. The use of woven structures for a pressure vessel requires an elaborate coating process to line the fabric with barrier substances (such as polyurethane or silicone) or, in most instances more suitably, the use of a separate barrier liner structure thereby relegating the woven assembly to restraint structure. Both alternatives introduce substantial and costly engineering challenges, however the benefit of segregating restraint and impermeable barrier structures offers significant advantage and is now the preferred approach for most high strength flexible pressure vessels. A good example of state-of-the-art collapsible flexible pressure vessel engineering is the airbag system shown in FIG 4 which cushioned the landing of NASA's exploration rovers on Mars: Built of fabric gores, the airbags were engineered for approximately 200lb/in (36kg/cm) membrane stress while needing to be of extremely high specific strength construction, packaged very tightly, and exposed to an outlandish variety of environments.

FIG 4. NASA Mars Exploration Rover landing airbags (NASA photo).

In general, only products comprised of high modulus fibres are considered viable for construction of high performance flexible pressure vessels since these fibres exhibit a tensile strength two to four times greater than the ubiquitous Nylon or Polyester. Furthermore, as a rough frame of reference, common high modulus fibres such as Kevlar®[1], Spectra®[2] and Vectran®[3] have a breaking elongation of only three percent while Nylon's is closer to thirty percent. While the low elongation is a great benefit for maintaining vessel geometry, it quickly becomes an Achilles' heel if the vessel is not meticulously engineered: Due to their high modulus it is difficult to guarantee proper load sharing between individual fibres within a woven product, thereby making any interface, joint or seam susceptible to point load induced failure (described in more detail in section 6). This does not ultimately bode well for 'broad' fabrics since, the greater the width of the weave, the more difficult it becomes to precisely balance the load sharing between individual fibres. The ultimate strength limitation for fabric gore vessels stems from the aforementioned inefficiency of fibre load sharing in broad fabrics combined with the necessity of individual meridional gores to be cut in a lens shape to allow for the assembly of the intended three dimensional pressure shell. When such a tapered gore profile is cut from a planar fabric, the load bearing fibres corresponding to the vessel shell's circumferential stress direction are severed, as are the meridional fibres where they intersect the gore's rapidly tapering edges near the poles.

Consequently, the degree of preservation of the structural integrity of the vessel's shell relies almost entirely on how well load carrying pathways between adjacent gores are maintained across the meridional seams connecting the neighbouring gores. Unfortunately, broad fabric cross-seam strength loss is always substantial—often over forty percent for high performance fabric woven of high modulus fibre. Moreover, if higher membrane stresses require application of thicker fabric, the seam will carry a yet lower percentage of the base fabric strength due to the diminished load sharing precision amongst the greater number of fibres. Multiple layers of fabric are sometimes used to avoid thick monolayers, however load sharing between fabrics constructed of high modulus fibre becomes a very significant integration challenge and source of stress distribution uncertainty due to manifestation of indeterminate load pathways. A further limitation posed by heavier fabric structures is the increased seam sewing difficulty, most dramatically manifesting itself in the polar areas of meridional seam convergence—and often the ultimate limiting factor in the performance of a vessel constructed of fabric gores. Meridional convergence is one of the great recurring challenges in flexible vessel design.

4.4. Webbing Vessels

A method which has been successfully applied to circumvent the dual limitations of fabric gore strength is the use narrow fabric, more commonly known as webbing (FIG 5b), which by virtue of its reduced width, greatly improves load sharing between fibres. As the name implies, narrow fabrics present the same basic construction as broad fabric (FIG 5a), with the distinction that webbing is intended for unidirectional strength.

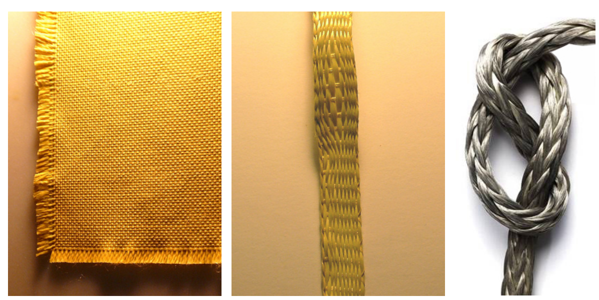

FIG 5. From left to right: (a) broad fabric, (b) webbing, and (c) cordage.

Webbing's fill fibres are functionally de-emphasized to merely permit the basket weave method of manufacture characteristic of fabric. The improved load sharing and reduced fill fibre content lead to a much improved specific strength, albeit unidirectional, over broad fabrics. FIG 5b displays a Kevlar® webbing weave optimized for high specific strength, showing exposed, widely-spaced fill fibre bundles. A great benefit of webbing is that it is readily connected to rigid structures or secured to like material with less than ten percent loss of base material strength, whereas, as described earlier, the seam

[1] Kevlar® is a registered trademark of DuPont Corp.
[2] Spectra® is a registered trademark of Honeywell International Inc.
[3] Vectran® is a registered trademark of Kuraray Co. Ltd.

connection of fabric gores often results in loss of over forty percent of base fabric strength.

Webbing is most typically used to manufacture straps for connection purposes, such as truck tie-downs and safety belts. Webbing's 'flatness' allows convenient application of stitching or adhesive for incorporation of connectors such as buckles, and to provide load dissipation as provided by the carrying strap of a heavy shoulder bag. If load dissipation is not needed, cordage becomes the next step in improved specific strength: In shunning the width requirement of webbing and its associated transverse fill fibre structure, cordage is able to pursue uncompromised unidirectional strength by switching construction method from a weave to a braid (FIG 5c).

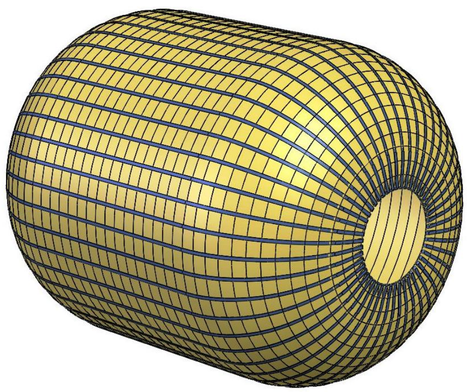

FIG 6. Thin Red Line Genesis Project restraint design.

Because of its 'unidirectionality', there is a drawback in webbing's need to be aligned bidirectionally, i.e. in both meridional and circumferential senses, to accommodate the shell stresses imparted in those directions. This drawback manifests itself primarily in the following three problem areas which, it is important to note, recur to varying degrees in almost all flexible pressure vessel designs.

The first problem involves the convergence of members of the meridional structure upon the vessel's poles (an example of which is shown in FIG 7) leading, if un-remedied, to an intolerable accumulation of material. The second problem occurs where the vessel tapers towards its poles: to maintain circumferential strap hoops indexed to the position in which they carry their correct share of hoop load – the problem is analogous to tying a ribbon around the tapered end of a rugby ball. The third problem is to functionally index the meridional and circumferential structures to one another to minimize the restraint's sensitivity to manufacturing, handling and operational trauma, all the while keeping their independent load carrying tasks segregated to minimize the manifestation of indeterminate load pathways.

The flexible pressure restraints of both Genesis and TransHab comprise two primary structures, each aligned with the meridional and circumferential global stress directions. TransHab's meridional straps are interwoven with the circumferential straps in basket fashion. Genesis incorporates an open array of meridional straps confining an uninterrupted circumferential structure of hoop straps connected edge-to-edge to achieve full coverage of the life support atmosphere retaining internal bladders. With its continuously smooth hoop structure Genesis offers somewhat lower mass and more predictable load pathways. The first problem, of meridional convergence, is solved in both designs by terminating the meridional straps at very large, rigid polar end structures—of sufficient diameter to accommodate side-by-side attachment of all meridional straps. While this solution serves the intended application of allowing the end structure to function as bulkhead or docking port, it precludes development of an entirely flexible vessel, or even a vessel with bulkheads much smaller in diameter than twenty-five percent of the vessel's deployed diameter. TransHab solves the second problem, of end cap hoop strap placement, with its basket weave as positioning matrix for all straps, further reinforced by intermittent indexing stitching connecting meridional and circumferential straps. As shown in FIG 6 Genesis solves the problem with its continuous circumferential structure of hoop straps sewn edge-to-edge. The third problem, of indeterminate load pathways, appears to be unavoidably characteristic of bidirectional webbing restraints and remains unresolved in these designs. This problem is discussed in greater depth in section 5.

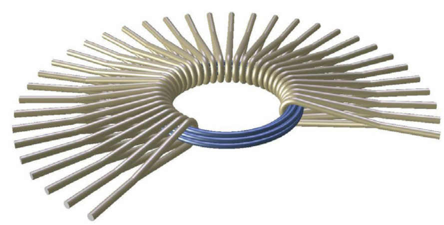

FIG 7. Polar meridional convergence.

4.5. Hybrid Vessels

In the evolutionary quest for high specific strength it may have seemed intuitive to forego broad fabric to pursue webbing-based designs. Nonetheless, for reasons described at the beginning of section 4, it is prudent to look farther afield for design inspiration in light of a vast history of flexible structure innovation reaching to the dawn of mankind. Another motivation to look to history stems from the realization that the basic manufacturing technology embraced for Genesis and TransHab is, perhaps embarrassingly, also quite ancient.

Hybrid restraints comprise a generally impervious barrier structure reinforced with an external grid of webbing or cordage. Hybrid design is based on the premise that materials providing the surface coverage and impermeability necessary for the containment of the vessel's fluid contents are ill-suited to simultaneously bear the vessel's global pressure and mass loads—and vice versa. Segregating material roles provides greater design flexibility, allowing the structure to be more precisely tailored to application demands. Furthermore, the replacement of a single specialized 'do it all' material with a variety of constituents, each chosen to perform a specific function in optimum fashion, facilitates off-the-shelf component availability. Unrelated fields offer myriad examples of the hybrid design approach: Most wooden buildings exemplify the seemingly age-old wisdom of segregating surface coverage and load bearing material roles: beams and joists provide global structural load bearing, while sheathing provides full surface coverage to enclose walls and cover floors. Here, as in hybrid flexible vessels, the design intent is to de-emphasized the structural roll of the surface coverage to the extent it need only provide local load carrying capability for the short spans between the components of the global load bearing structure.

In the case of a flexible hybrid pressure vessel, the restraint comprises a grid of reinforcing tendons restraining an oversized bladder which encloses the vessel's internal volume. The restraint would typically comprise a meridional array of tendons, converging upon and structurally connecting polar end structures, and a circumferential set of parallel tendon hoops disposed between the poles perpendicular to the meridional tendons. Since the membrane stress induced by distributed internal pressurization is proportional to the radius of the distended membrane, the bladder material is only subjected to the local forces where it bulges outwards between tendons.

The drawback of hybrid vessels is that no predictably effective method has been found to exploit the hybrid's tendon grid for bidirectional load-carrying duty on steep end cap surfaces. This problem has only been circumvented by replacing the aspired-to flexible end cap with rigid versions—in a fashion which is somewhat analogous to the application-limiting 'solution' presented by the webbing vessel architecture described earlier. A further concern which appears to have delayed flexible hybrid research, especially in the context of habitation systems, pertains to the perception of bladder vulnerability.

5. STRUCTURAL PREDICTABILITY

5.1. Indeterminate Load Pathways

The problem of indeterminate load pathways manifests itself at all structural levels of a flexible vessel. In the case of Genesis and TransHab (FIG 8) perhaps the most disconcerting manifestation of performance uncertainty occurs at the global structural level: The flexible meridional restraint structure, the flexible circumferential restraint structure, and the rigid core are each optimized for a balanced combination of maximum strength and geometric-dimensional accuracy. Each of these three structures is subjected to differing structural demands, and each displays significantly different mechanical properties. The meridional and circumferential restraint components must be structurally indexed to one another to preserve load-bearing alignment throughout packaging and deployment; and the restraint structure as a whole induces large tensile loads on the core to which the restraint is anchored at both its poles.

Due to the aforementioned interconnectivity, the smallest variation in operational environment leads to significant load shifting between the three structures. It should be noted that load sharing between restraint and internal core is more effective in TransHab where the polar extremities of the end caps are axially oriented extensions of the internal core; Genesis end caps attach tangentially to radially oriented polar bulkheads anchored to opposite ends of the core, whereby the slightest dimensional mismatch between restraint and core immediately leads to substantial load shifting between the structures. It is inherently impossible to definitively model performance of the described restraint structures. As such, achieving acceptable 'performance comfort' for man-rated inflatable architecture will, for the time being, remain mired in the obstructive structural mass required by a four-over factor of safety dating back to the Hindenburg era.

FIG 8. NASA TransHab (NASA photo).

5.2. Non-linear Material Behaviour

The mechanical properties of wovens are affected by a multitude of variables pertaining to, among others, variability in manufacturing equipment and processes, yarn twist, yarn diameter and cross sectional geometry, yarn friction, yarn lateral compression, and yarn crimp angle. (Crimp is the ratio of yarn length over milled fabric length.) As such, fabrics are generally highly variable in nature and do not behave as continuum materials. Instead, they perform as a system of discrete constituents, most of which themselves exhibit significant behavioural non-linearity.

Webbing exhibits non-linear response under tensile loads in its axial direction. At lower loads there is virtually no elastic deformation in the yarn; elongation occurs due to realignment of warp fibres to increasingly optimize an axial orientation. At higher loads, elongation is primarily due to warp and fill yarn crimp interaction leading to significant reduction in webbing width and thickness. Elongation is due to crimp angle reduction and increasing realignment of the warp yarns combined with the first tangible signs of yarn extension. At yet higher loads the weave increasingly binds and the constituent yarns are forced to elongate to accommodate the increasing load. Yarn properties govern mechanical response when weave binding is complete as is evidenced by stability of the webbing's cross-sectional dimensions.

FIG. 9 shows warp yarns (green) aligned in the direction of a fabric as it comes off a roll, and fill yarns (orange and red) aligned in transverse fashion across the width of the fabric. The red arrows indicate axial loading which results in de-crimping of the warp yarns.

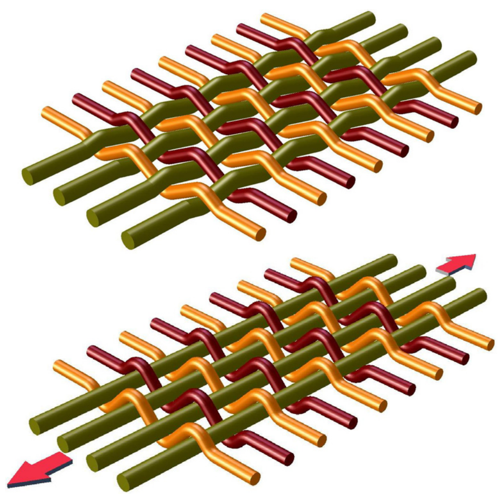

FIG 9. Warp yarn de-crimping under axial load.

5.3. Structural Analysis

5.3.1. Membrane Theory

For convenience we follow the approach of Jenkins et al. [5] to offer a short overview of the basic constitutive relationships for membrane mechanics which are relevant to design of an inflatable space habitat. Specialised texts providing more in-depth review in areas such as tensor calculus and non-linear mechanics are readily available. Membrane theory studies the stress-strain relationships in shell structures, specifically when the shell walls are thin enough to display no stress variation throughout their thickness and, as a result, are not subjected to bending stresses. Real membrane materials exhibit high levels of complexity rendering closed-form analysis impractical.

5.3.2. Hooke's Law

The simple relationship for stress and strain is given by Hooke's Law:

(1) $\sigma = E\varepsilon$

where

σ = stress
E = Young's (elastic) modulus
ε = strain

This relationship is unidirectional. We can however generalise it to three dimensions by including particulars for the other directions. Doing so we write a more general form as

(2) $\sigma_{ij} = D_{ijkl}\varepsilon_{kl}$

or inversely:

(3) $\varepsilon_{ij} = C_{ijkl}\sigma_{kl}$

where

σ_{ij} = stress in direction ij
ε_{kl} = strain in direction kl
$\mathbf{D}$ = stiffness tensor
$\mathbf{C}$ = inverse of $\mathbf{D}$ ($\mathbf{C} = \mathbf{D}^{-1}$)

In tensor notation we write

(4) $\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$

where

$\mathbf{D}$ = stiffness tensor with components D_{ijkl}
$\boldsymbol{\sigma}$ = Cauchy stress tensor with components σ_{ij}
$\boldsymbol{\varepsilon}$ = strain tensor with components ε_{kl}

We can also write

$\mathbf{C}$ = inverse of $\mathbf{D}$ ($\mathbf{C} = \mathbf{D}^{-1}$)

It should be noted that stress tensor $\boldsymbol{\sigma}$ and strain tensor $\boldsymbol{\varepsilon}$ are symmetrical in their subscripts ($\sigma_{ij} = \sigma_{ji}$).

5.3.3. Linearly Elastic, Isotropic Membrane

Considering flexible vessels, our analysis interest lies predominantly in the definition of plane stress—to which we can appropriately apply membrane theory. Initially we consider a vessel constructed of the simplest material: linearly elastic and isotropic in any planar direction, i.e. a material with uniform Young's modulus (represented by the constant E). As a further result of this homogeneity we can likewise present Poisson's ratio as the constant ν. Equation (3) now becomes:

(5) $$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ 0 & 0 & -2(1+\nu)E \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}$$

In engineering notation we typically write

$\varepsilon_{11} = \gamma_1$; $\varepsilon_{22} = \gamma_2$; $\varepsilon_{12} = \gamma_6$
$\sigma_{11} = \gamma_1$; $\sigma_{22} = \gamma_2$; $\sigma_{12} = \gamma_6$

5.3.4. Linearly Elastic, Anisotropic Membrane

Isotropic membrane models however, inadequately accommodate and represent the complexity of materials suitable for construction of the high shell load structure required by an inflatable habitat. Substituting an anisotropic model for the isotropic model adds a step along the path to a model that aspires to capture the requisite complexity. Equation (2) now becomes:

(6) $$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}E_1/(1-\nu_{12}\nu_{21}) & \nu_{12}E_1/(1-\nu_{12}\nu_{21}) & 0\\ \nu_{12}E_2/(1-\nu_{12}\nu_{21}) & E_2/(1-\nu_{12}\nu_{21}) & 0\\ 0 & 0 & G\end{bmatrix}\begin{Bmatrix}\gamma_1\\ \gamma_2\\ \gamma_6\end{Bmatrix}$$

where

G = shear modulus

The value of G relates shear stress to shear strain. For the anisotropic case, there is no mathematical expression relating shear modulus G to Young's modulus E and Poisson's ratio ν. (As the degree of anisotropy increases, the number of constants or moduli required to describe the material increases to a maximum of 21.)

5.3.5. Non-Linear, Elastic Membrane

The models for both isotropic and orthotropic membranes took advantage of linear elasticity. Woven materials however, particularly for high-load shells such as the TransHab and Genesis restraint structures, exhibit significant non-linear stress-strain behaviour. Stress-strain nonlinearity of a Genesis constant section circumferential strap is shown in FIG. 10.

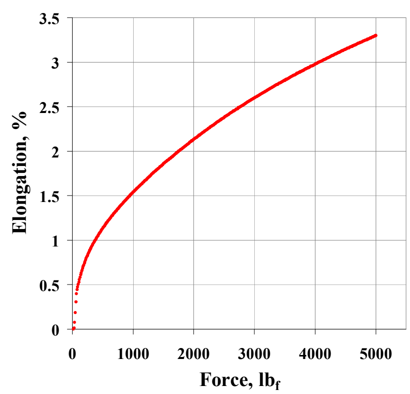

FIG 10. Stress-strain nonlinearity displayed by a Genesis constant section circumferential strap.

The stiffness tensor **D** does not have constant components. Instead, the components are functions of the strain:

(7) $$D_{ijkl}=\text{fn}(\varepsilon_{kl})$$

where fn( ) represents a function of the variables inside the brackets.

We are not attempting to define the precise nature of the relationship between D_{ijkl} and ε_{kl} since it depends on numerous factors beyond our immediate insight. Rather, we are simply stating that the relationship is non-linear, and that the components of D_{ijkl} cannot be represented by a unique set of values.

Structural analysis of interaction between materials exhibiting non-linear properties is a challenging field of research requiring computational solutions involving sophisticated numerical methods and iteration. The methodology generally involves incrementally increasing either the load or the displacement, after which the stiffness matrix is adjusted accordingly [6].

5.3.6. Non-Elastic Membranes

Further complexity is added to the stress models by considering materials that do not exhibit an elastic relationship between stress and strain. Adding this level of intricacy brings the material model closer to the behaviour of real materials.

Of further interest is the loading history of the material in non-elastic models. The stress-strain relationship depends on the number of cycles and the stress/strain level associated with these cycles. We can write

(8) $$\mathbf{N}=\mathrm{F}\left[\text{history}(\mathbf{E})\right]$$

where

N = tensor of membrane stresses
E = strain tensor
history = function of the loading history
F = constitutive law

Note that the constitutive law F might not be in a form that can be represented by a closed-form analytical expression. The constitutive law will cover elasto-plastic behaviour, strain hardening, and other effects that cause non-recoverable and loading history dependent deformation whereby F will be a function of a multitude of often interdependent variables.

5.3.7. Additional Considerations

Sections 5.3.3 to 5.3.6 incorporate various aspects of material behaviour into the membrane model. Besides materials, manufacturing, and handling variables, a large number of mission dictated operational and environmental effects will demand further consideration. Examples of such factors include time-dependent material degradation, exposure to radiation and orbital debris trauma, and temperature-dependant variation in stiffness and elastic limits. Each of these effects will independently be defined as a function of a sub-set of other variables.

5.3.8. Combined Effects

Returning to Equations (2) and (4) we define the components of the stiffness tensor as

(9) $$D_{ijkl}=Fn\left((x_m),\varepsilon_{kl}\right)=\text{fn}\left(\text{fn}(x_m),\varepsilon_{kl}\right)$$

where

Fn = functional (function of functions)
x = variables affecting the components D_{ijkl} of the stiffness tensor
m = number of x variables
n = number of dependent effects

Functions typically include history effects and non-linear relationships involving a multitude of variables. The number of variables m to capture all input effects for complex materials such as fabrics can be very large. These variables will provide the input for a number of complex, interrelated functions that determine material properties.

5.3.9. Summary

We see that the simple relationships in Equations (1) to (5) become excessively complex as we add realistic material effects. The inclusion of anisotropic materials introduces a shear modulus which cannot be calculated in general form from Young's modulus and Poisson's ratio. Adding non-linear elastic stress-strain relationships removes the feasibility of obtaining closed form definitions, thereby requiring numerical methods as alternative. Any requirement to move away from elastic stress-strain relations in the direction of permanent deformation significantly increases the complexity of any base model. These non-recoverable stress-strains signify that the stress-strain relationship becomes dependent on the history of the stress-strain loading. The effect of operational and environmental variables dictates that the components of the stiffness tensor can only be written as non-linear functions.

The great variability in wovens compounded by the performance nonlinearity of the large number of constituent variables renders conventional load and deflection analysis unreasonably challenging. Writing the constitutive equations would require a series of tensors where the components are functions that depend on load-deflection and on load cycling.

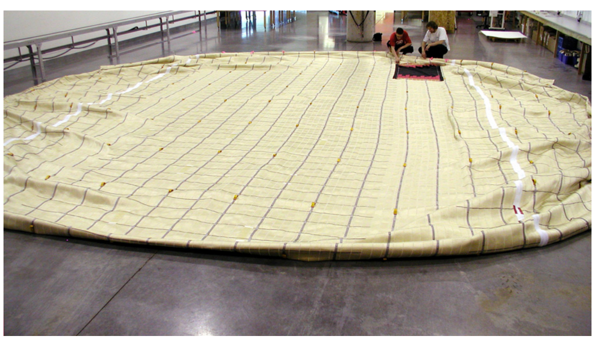

FIG 11. Thin Red Line 320 square meter circumferential restraint structure (Thin Red Line photo).

6. ASSEMBLY CONSIDERATIONS

In consideration of conclusions drawn from the preceding section, Thin Red Line Aerospace incorporated a broad set of developmental protocols adapted to Genesis restraint development. Two of these protocols, which are described below, are illustrative of the adaptive methods needed for effective flexible space habitat development.

Webbing straps form the fundamental building blocks of the Genesis and TransHab restraint structures. Each Genesis strap was individually subjected to cycles of high loading to relieve stress dependant mechanical effects to a degree which was deemed representative for the ultimate operational environment. The load-cycling was performed by automated equipment running a programmable algorithm which, after cycling, maintained a prescribed position under load while non-contact indexing marks were automatically applied. Almost ten thousand of these strap integration marks were applied to each Genesis restraint to optimize uniform shell stress distribution. It is noteworthy that every strap was considered unique, whereby the elongation data obtained during the cycling was kept on file as a 'fingerprint' which could be consulted on later occasion in case of an observed anomaly in the integrated structure.

Edge-to-edge connection of circumferential straps (using the aforementioned indexing marks) was logically required to match or exceed the webbing's fill fibre strength. The sewn connection needed also to impart a minimum of distortion on the webbing and furthermore be applied with great uniformity. Uniformity was accomplished using sophisticated, digitally controlled equipment. However, the width of the webbing was altered by the integration process leading to difficulty in dimensional prediction of the circumferential structure's axial profile. Acceptable predictability was ultimately provided through transverse and bi-directional load testing of edge-connected strap coupons. Such non-standard, contextual tests ultimately provided the requisite material definitions needed for assembly.

7. CONCLUSION

Analysis provides useful results to assist in the initial selection of space habitat geometry, and for determination of maximum anticipated shell stresses for the vessel's constituent geometries. Difficulty arises if attempts are made to base analysis and modelling on material constituents in anticipation of meaningful results. If unacceptable results then demand inputs to be structured in attempts to match experimental observation, the risk associated with application of this modelling data becomes unreasonable in the opinion of the authors. It is the authors' opinion that transition to physical testing with emphasis on contextual methods is most appropriate immediately after the preliminary geometric analysis presented in section 3. The design of the Genesis pressure restraint structures adopted this approach, for which the successful launch and deployment of two of these spacecraft is seen as considerable validation.

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