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A flight dynamics course based on MATLAB computer assignments

Lars Johansson*

Department of Mechanical Engineering, Linköping University, SE-581 83 Linköping, Sweden

Abstract

In this paper the experience of the author in running a flight dynamics course with MATLAB computer assignments as a large part of the course and the sole means of assessment is discussed.

1. Background

Due to the introduction of aeronautical engineering as one of the specializations available for students taking the mechanical engineering programme at Linköping University some years ago, the author was faced with the task of developing and delivering a flight dynamics course to be given in the fourth year of this programme, mandatory for students taking the aeronautical engineering specialization. The course was in due course given for the first time in 1996 and has subsequently been given each year since. The course is also, with minor adaptions, given for technical physics students.

In the first year, the course was given in what the author regards as a very traditional form, both with regard to the form of delivery and the course content. Thus, the material was covered in lectures and lessons totalling 60 h, with one 6 h computer assignment, which was in fact what is described as computer assignment II below. The grade given on the course was based entirely on a 4 h written exam at the end of the course, with the computer assignment only graded pass no pass.

The contents of the course for the first year was heavily dependent on the format of delivery, i.e. adapted to the lecture format, with some time spent on introductory material such as static stability, some time spent on rehearsing and extending rigid body dynamics from earlier courses.

* Corresponding author. Fax: +46-13-281101.

E-mail address: larjo@ikp.liu.se (L. Johansson).

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and finally the development and solution of the linearized equations of motion, which formed the major new material of the course.

The course was based on the textbook by Nelson [1], which supports a course of the type outlined above well. Possible alternatives, which have recently become available, include the textbooks by Etkin and Reid [2], Russel [3], Schmidt [4] and Cook [5].

2. The new format of the course

It was felt from the outset that it was unsatisfactory to assess the course with a 4 h written exam, with questions mimicking engineering calculations, which is traditional at the engineering school of Linköping University. It is simply too difficult to construct questions which captures the essence of flight dynamics, without being ridiculously easy or ridiculously difficult, and which can be treated in a reasonable way in 4 h.

It was, therefore, decided to change the course by scrapping the written exam completely and instead introduce a considerably extended pensum of computer assignments. After an intermediate year with four assignments, the course now has five computer assignments, which are described below. The assignments are completed by each student individually in MATLAB; individually meaning that each student has distinct aeroplane data and is required to produce his own code and reports, but discussions of the solutions or of the solution methodology with other students are certainly not forbidden. Twenty hours from the time spent on lectures were replaced with scheduled time in the computer lab with the teacher present to help with problems and participating in discussions of the solutions.

The written exam was replaced by reports which each student is required to write on each of the five assignments. The reports are graded from zero to four points, giving a possible maximum of twenty points. A total of nine points are required for the lowest passing grade, 17 for the highest grade and 13 for an intermediate grade. Students handing in reports in very good time, at least 2 weeks before the deadline, will get them back marked before the deadline and has the opportunity to hand in a revised version. Reports handed in after the deadline are still marked, but only counted towards the lowest passing grade. Simple arithmetic shows that it is possible to achieve a passing grade from only three assignments. Experience shows, however, that few students will do only three assignments, but many will skip the fifth, thus, foregoing the possibility of achieving the highest grade. It might seem that these rules for the assessment of the course are a bit on the generous side and that all the students are likely to end up with the highest grade. This has, however, turned out not to be the case: the students are instead distributed rather evenly among the passing grades. The number of students failing the course is, however, small with most dropouts being students for whom the course is not mandatory.

The reactions from the students to the change in assessment method has been very favourable indeed, something which was expected since students are likely to react favourably to anything that means they do not have to do a written exam. Anonymous questionaires revealed some more nuanced opinions. Some recurring opinions and the authors spontaneous reaction to them are as follows:

A simple written exam could be used in addition to the computer assignments. This is perhaps true, but it seems unnecessary to add this burden to the course, especially since the primary reason to introduce the computer assignments in the first place was the difficulty in constructing a good-quality written exam.

Computer assignments are actually more demanding than a written exam. This is probably true to some extent; you are forced to rub your nose into all the details. On the other hand, cramming for a written exam does tend to give you a valuable overview by forcing you to keep all the material simultaneously in your head at one point.

The assessment used allows the student to set his own standards in a predictable way. This is hopefully true and was in fact a conscious intention when defining the rules for the assignments.

There should be more of the classical material, like static stability. This is probably true, you tend to think of other things than "what happens when I increase the stabilizer area by 10%", and the like, when you write computer simulation code. This aspect will be carefully considered in the future developments of the course.

3. Unforseen sideeffects

The decision to scrap the written exam and use computer assignments as the sole means of assessment was initially made for the single reason that it was felt that the written exam was a failure, with undue emphasis on material on the fringes of the course contents and being too easy to pass with a high grade. Changes of this kind are, nevertheless, popular among students and administrators alike, since it reduces the pressure on the students and since too many written exams poses a scheduling problem. There were also two other major effects of the change of assessment method which were not foreseen, or at least which did not form part of the motivation for making the change, although, in the opinion of the author, these sideeffects have been beneficial.

The first of these sideeffects is that the content of the course has drifted. Thus, the course changed from being a course focused on the linearized equations of motion to being a course where the nonlinear equations of motions are used as a simulation tool, and the students learn to implement them in MATLAB. It can be noted that the book by Nelson [1] was kept as a text for the course. A possibility would be to use Stevens and Lewis [6] instead, but it was felt that this would be a too decisive step in the direction of a course in computer simulation rather than flight dynamics, and also that the use of this book would require a larger course.

The second major sideeffect is that the rather long hours (20) of scheduled time in the computer lab with rather few students (about 10) working for the most part by themselves gives ample time to explain theory in a one-to-one situation where it is possible to really make sure that things gets through.

4. The computer assignments

The main requirement when developing the computer assignments was that all MATLAB coding should be done by the students themselves, individually. There were to be no files prepared in advance and all instructions and data are given printed on paper only. As an example, the instructions for the first of the computer assignments is given in full, translated into English, in the appendix below.

It can be noted that a linear aerodynamic model is used throughout the computer assignments, despite the fact that a nonlinear model would be more appropriate when combined with the nonlinear equations of motion in assignments I, III and V. The reason for this is that it was not deemed possible to develop and code a better aerodynamic model within the limits of a 66 h 6 weeks course. To compromise the principle that the students does all the coding themselves by supplying a "black box" aerodynamic model was not considered an option. A benefit of this approach is that it is easy to find realistic data for actual aeroplanes, see [1–3].

The contents of the computer assignments are outlined below. The first of the assignments are given in full in the appendix, in a direct translation from the material given to the students. A note on the terminology used might be in order. The phugoid and short period modes are the two typical modes of motion of an aeroplane flying in its plane of symmetry, the so-called longitudinal motion. These modes are readily identified in the homogenous solution to the linearized equations of motion. In the phugoid motion the aeroplane follows a slow lightly damped sinus curve, with its nose pointing in the direction of the velocity vector. In the short-period mode the aeroplane instead makes a quick, strongly damped, angular oscillation about its velocity vector. Further, the elevator is the control surface located on the horizontal tail which is used to pitch the aeroplane about an axis in the direction of the wing.

Computer assignment I. Implement the plane (longitudinal) equations of motion in a body fixed coordinate system with a linear aerodynamic model. Identify the phugoid mode of motion. Identify the short-period mode of motion by changing the force model in order to sabotage the inheritly strong damping of this mode. Test what happens if the force model is changed to give a statically unstable aeroplane. This assignment is given in full in the appendix below.

Computer assignment II. Compute the coefficients in an approximate transfer function from elevator to pitch angle velocity. Try to improve the characteristics of the short-period mode of the aeroplane according to the "thumb-print" criterion using two simple feedbacks. Model the system in the Simulink toolbox and plot the step response with and without the feedback system. Check how the system deteriorates when a simple servo model is added to the model.

Computer assignment III. Implement the feedback system, the elevator and the elevator servo from assignment II into the nonlinear model of assignment I by adding appropriate additional differential equations to the system of ordinary differential equations modelling the aeroplane. Identify the differences in behaviour between this model with the linear, short-period mode only, model of assignment II.

Computer assignment IV. Implement the matrices of the linearized equations of motions, both longitudinal and lateral. Compute the eigenvalues and eigenvectors and evaluate these against the flying quality criteria in the textbook. Use either the symbolic algebra or the control system functions in MATLAB to compute the full transfer function from elevator to pitch angle velocity. Compare the properties of this model (linear, both phugoid and short-period modes) with the models of computer assignments II and III.

Computer assignment V. Implement the full three-dimensional equations of motion, with a linear aerodynamic model. Use the implementation to verify that a longitudinal disturbance will only affect the longitudinal motion, but a lateral disturbance will spill over into lateral motion.

Finally, a few examples of plots similar to those handed in by the students in their reports will be given. The data used for the computations are for the A4–D Skyhawk at Mach 0.4 and at sea level, as given by Schmidt [4].

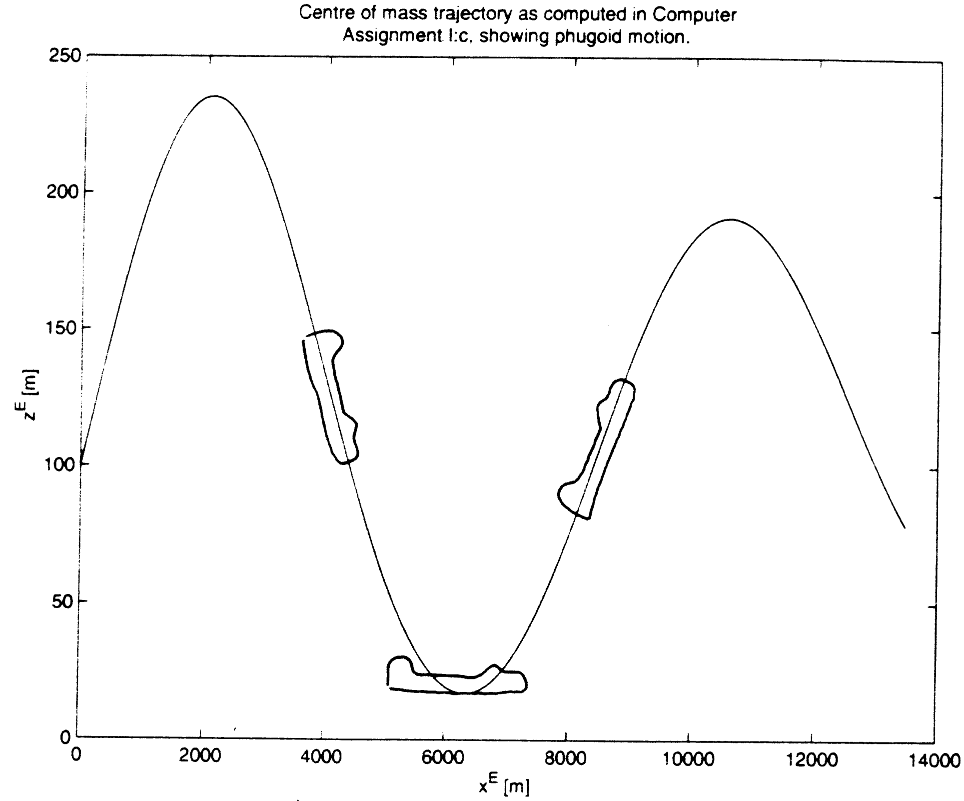

Fig. 1. Centre of mass trajectory from computer assignment I. The aeroplane orientation in the phugoid mode is indicated.

The first plot, Fig. 1, shows the result of the simulation in assignment I:c, see the appendix below. This is the first computation with a resonably complete aeroplane model that the students will perform, as the assignments I:a and I:b are included mainly to force the students to develop their implementations step by step. The plot shows the trajectory of the centre of mass of the aeroplane, and what is observed is essentially the phugoid mode. In this assignment, the students are required to deduce what the phugoid motion looks like, using the clue that they should compare the magnitude of the pitch angle with that of the angle of attack, and draw the orientation of the aeroplane into the figure as shown. One difficulty here is that the students often do not reflect over the fact that the scale of the x^E and z^E axes are different: some will just draw the aeroplane at the calculated angle of attack without adjusting for the difference in scale and end up with an almost horizontal aeroplane in all positions.

The following Figs. 2–4, show step responses as calculated with the different models in computer assignemnts II, III and IV, respectively. The students are required to explain the differences between the plots in terms of the differences in the models used. Thus, Fig. 2 shows the step response when a transfer function based on the short-period approximation is used, whereas

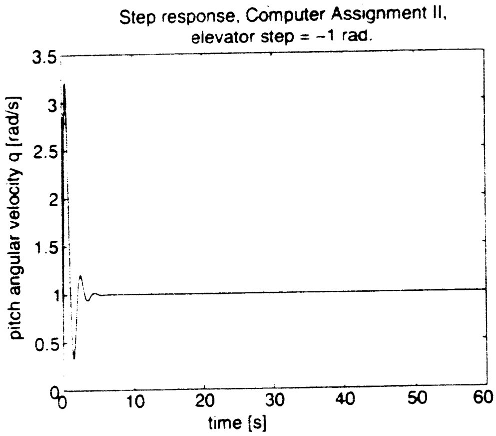

Fig. 2. Step response from computer assignment II, where a linear model (transfer function) modelling the short-period mode only is used.

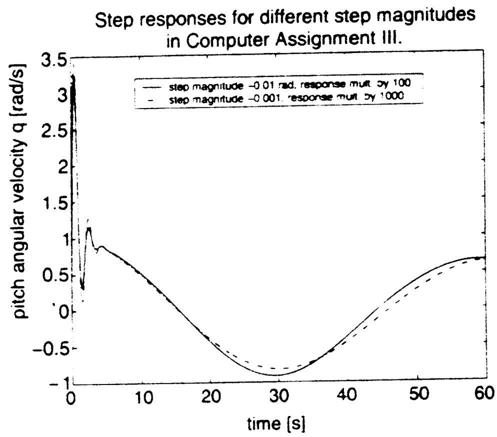

Fig. 3. Step response from computer assignment III where the full nonlinear equations of motions is used (but a linear aerodynamic model).

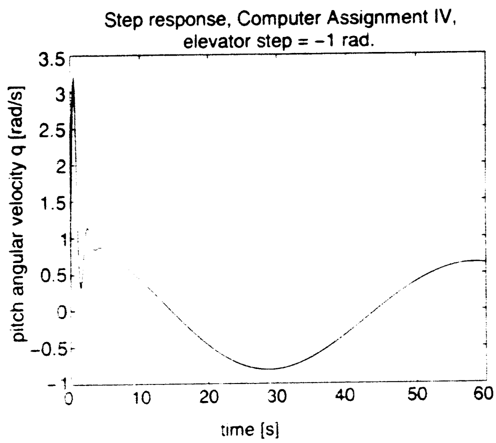

Fig. 4. Step response from computer assignment IV where a linear model incorporating both the short period and phugoid modes is used.

Fig. 4 shows the step response using the complete transfer function incorporating both the short period and phugid modes. Apart from the difference in the models it should be observed here that making an elevator step and holding for 60 s is not a very useful input in normal flight. Fig. 3, finally, shows the step response obtained in assignment III where the full nonlinear equations of motion are used. It is seen in the plot that the nonlinearity of the model means that different magnitudes of the step used will result in qualitatively different responses, not just a scaling. This point is among the more difficult for the students to grasp and usually requires a certain amount of clues from the teacher. It should also be observed here that a too large step will result in flight conditions where the aerodynamic model used is no longer valid.

Appendix A. Computer assignment I

The motion of an aeroplane moving in the xz-plane is to be studied, i.e. the so-called longitudinal degrees of freedom are included. We introduce one coodinate system $Oxyz$ attached to the aeropane and one coordinate system $Ox^Ey^Ez^E$ fixed to the ground and assumed to be inertial (see Fig. A1).

A.1. Equations of motion and kinematical equations

The longitudinal equations of motion for an aeroplane, together with kinematical relations for the orientation and position of the aeroplane can be written

$$
\begin{aligned}
\dot{u} &= -qw - g\sin\theta + X/m,\\
\dot{w} &= qu + g\cos\theta + Z/m,\\
\dot{q} &= M/I_{yy},\\
\dot{\theta} &= q,\\
\dot{x}^E &= u\cos\theta + w\sin\theta,\\
\dot{z}^E &= (-u\sin\theta + w\cos\theta)(-1),
\end{aligned}
\qquad \text{(A.1)}
$$

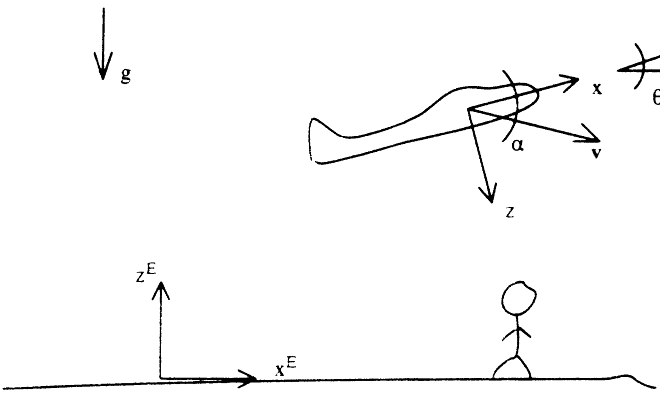

Fig. A1. Aeroplane in plane motion.

where X, Z and M are forces and moments on the aeroplane not including the force of gravity, i.e. aerodynamical forces and moment and forces and moment from the engine. The factor (-1) in the last equation serves to rotate the ground–fixed system so that the z^E–axis points upwards. The last two equations are the relations between the aeroplane velocity in moving (aeroplane fixed) and ground-fixed coordinates, respectively. The velocity vector $\mathbf{v}$ is always the time derivative of the position vector relative to a fixed reference, but can be represented in either fixed or moving coordinates. The first two equations are the equations of motion expressed in the aeroplane fixed coordinate system so that u and w are the components of the velocity vector in a rotating coordinate system and $\dot{u}$ and $\dot{w}$ are components of the derivative relative to a rotating reference of the velocity vector. The acceleration of the aeroplane is the time derivative relative to a fixed reference of the velocity vector; its components in the rotating system are $\dot{u} + qw$ and $\dot{w} - qu$.

The above six equations shall first be integrated using the following initial conditions:

$$
\begin{aligned}
u_i &= u_0, \\
w_i &= 0, \\
q_i &= 0, \\
\theta_i &= 0, \\
x_i^E &= 0, \\
z_i^E &= h_0,
\end{aligned}
\tag{A.2}
$$

where u_0 and h_0 are the velocity and height of the reference condition of your data. If the reference condition is sea-level flight, the height is set to $h_0 = 100\,\text{m}$, to avoid underground flight.

A.1.1. Assignment I:a

Implement Eqs. (A.1) above in MATLAB with $X = Z = M = 0$. Calculate the motion for 100 s with the initial conditions (A.2) above and plot z^E as a function of x^E. In the absence of aerodynamical forces and engine forces the correct solution is, of course, a parabola. Verify that the endpoint of your numerical solution agrees with the analytical solution.

A.2. Trimmed flight

In trimmed flight the forces are, by definition, in balance with no acceleration. From (A.1) it is found that

$$
\begin{aligned}
X/m &= g \sin \theta_0, \\
Z/m &= -g \cos \theta_0, \\
M/I_{yy} &= 0,
\end{aligned}
\tag{A.3}
$$

where index "0" denotes the trimmed condition.

A.2.1. Assignment I:b

Implement the forces (A.3) and perform a simulation with $\theta_0 = \theta_i$. Verify that the motion is now along a straight line parallell to the ground.

A.3. Force model

We assume, with a drastic simplification, that the aerodynamic forces on the aeroplane can be written as

$$
\begin{aligned}
X/m &= g\sin\theta_0 + X_u(u-u_0) + X_w(w-w_0),\\
Z/m &= -g\cos\theta_0 + Z_u(u-u_0) + Z_w(w-w_0),\\
M/I_{yy} &= M_w(w-w_0) + M_{\dot{w}}(\dot{w}-\dot{w}_0) + M_q(q-q_0).
\end{aligned}
\tag{A.4}
$$

This is a linear model, which is assumed to be valid close to a reference condition, which is taken as the trimmed condition introduced above (index "0").

Note that (A.4) does not contain any terms for changes in throttle or elevator settings, which are, thus, assumed to be constant.

Also not the difference between "initial condition" and "reference condition". The initial condition is the condition of the aeroplane at the starting time of the computation; the reference condition is the condition about which the force model has been linearized. If we had had a better force model, there would not have been a reference condition.

It is assumed that all six phase variables except u_0 and h_0 are zero in the reference condition, and that $\dot{w}_0$ is also zero. Thus, if you used a nonzero value for θ_0 in assignment I:a, correct this.

We shall study the aeroplane motion for the following initial conditions:

$$
\begin{aligned}
u_i &= u_0\cos(\theta_i) \approx u_0,\\
w_i &= u_0\sin(\theta_i) \approx 0,\\
q_i &= 0,\\
\theta_i &= 0.1\ \text{rad},\\
x_i^E &= 0,\\
z_i^E &= h_0.
\end{aligned}
\tag{A.5}
$$

The aeroplane is, thus, trimmed with the stick fixed when suddenly (at time $t = 0$) there is a disturbance in pitch angle.

A.3.1. Assignment I:c

Implement the force model (A.4). Perform a simulation with the initial conditions (A.5) for 100 s and plot the six phase variables as functions of time as well as $z^E(x^E)$ and the angle of attack as a function of time. If the curves tremble suspiciously, then increase the precision in the MATLAB function ode45.

The computed motion should consist of an oscillation with a period of the order of tenths of seconds, which is called the phugoid mode. This motion is normally slightly damped, but can be slightly growing.

Find the period time of the motion by measuring in one of the plots. Compare with the rough approximation

$$\tau_{phugoid} = \sqrt{2}\pi u_0/g.$$

In the phugoid motion, u usually reaches its maximum about 90° before θ. Measure this phase difference in your plots.

Finally, try to figure out what the aeroplane motion through the air looks like and describe it in words. Draw the aeroplane orientation at a few places in the $z^E(x^E)$ plot. Clue: compare the magnitude of the angle of attack with that of the pitch angle.

A.4. *Static stability*

The concept "static stability" means that the aerodynamic forces should give a restoring moment if there is a disturbance in the angle of attack. This gives a rough idea of the stability of the aeroplane. The pitching moment as a function of angle of attack should look something like this for static stability (see Fig. A2).

The conditions for static stability are written as

$$\frac{dM}{d\alpha} < 0,$$

$$M(\alpha = 0) > 0.$$

A.4.1. *Assignment 1:d*

Put $C_{m_1} = 0.1$ in the force model. Perform a computation with the initial conditions (A.5) for 100 s, and plot the six phase variables as functions of time as well as $z^E(x^E)$ and the angle of attack

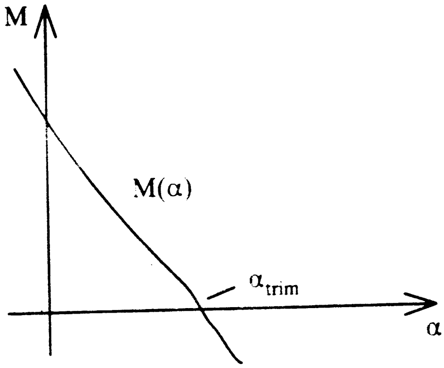

Fig. A2. Moment curve for a statically stable aeroplane.

as a function of time. Comment on the result. It might be necessary to increase the coefficient more to $C_{m_t} = 0.5$ say, to obtain the desired effect for large and heavy transports.

A.5. The short-period mode

The longitudinal motion of an aeroplane typically has two different modes, the phugoid mode studied in assignment I:c above and the short-period mode. The short-period mode has a much shorter period than the phugoid mode and is also strongly damped, which is a desirable property since an aeroplane with a growing short-period mode is completely uncontrollable. This strong damping, however, makes the mode difficult to see in the plots.

A.5.1. Assignment I:e

Restore the static stability that was sabotaged in assignment I:d. Instead, sabotage the damping of the short-period mode by putting $M_q = Z_{\dot{w}} = M_{\dot{w}} = 0$ in the force model. Change the initial conditions (A.5) such that $\theta_i = 0$ and instead put $q_i = 0.1$ rad/s. Perform a calculation with these initial conditions for a time corresponding to one fifth of the phugoid oscillation period, and plot the six phase variables as functions of time as well as $z^E(x^E)$ and the angle of attack as a function of time. Compare the angle of attack with the pitch angle for both this oscillation and the phugoid oscillation and comment on the difference.

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