Direct Optimal Control Methods for a Centralized Approach to Separation Management

Leif Walter
Deutsche Flugsicherung GmbH
ATM Operations & Strategy
leif.walter@googlemail.com

***Abstract*—One key element of future Air Traffic Management concepts are automated support tools that help to separate aircraft from one another in a tactical environment. This work investigates the possibilities of a centralized approach to separation or conflict management, based on optimal control theory and its applications. Therefore we will present an optimization framework that generates conflict-free trajectories for all aircraft within a regarded airspace. The optimization process is based on the direct solution method and uses multiple shooting features with a SQP method to solve the resulting nonlinear problem. Numerical computations were performed with the MUSCOD-II software from the IWR at University of Heidelberg, Germany. The principle advantage of a centralized approach is that the optimization framework has full information of the traffic situation in a sector, so it may generate solutions that incorporate information of all airspace users. This will prevent solutions that could otherwise yield even more severe traffic situations later in time. As an optimality measure we used a function that aims to assess the deviation from a nominal flight path and flight time. This work focuses on en-route traffic scenarios, so we will present some numerical examples of solutions provided by the framework for two-dimensional traffic situations. Numerical use cases will show that the method delivers very promising results — with subsequent research yielding the possibility of further improvement. These results will be validated with real-time simulations at the DFS R&D department in November 2011.**

I. Introduction

Automated support tools are key elements of future *Air Traffic Management* as they will become essential to controller daily work by potentially enabling the increase in capacity that is demanded by future ATM programs (SESAR, NextGen, CARAT). As it was pointed out in the *Performance Review Report 2010*, [1], 5% of all flights in Europe are held on the ground to manage en-route congestion, resulting in more than 50% of all delay minutes within the system. This shows evidence that en-route congestion is a severe problem in Europe, with the situation to become even more severe when we are moving towards the SESAR and NextGen expectations as stated in [2] and [3].

In the future ANSPs (*air navigation service provider*) will need to deliver more capacity, in a cheaper, safer and environmental-friendlier way. This feat will most likely not be achieved without the aid of sophisticated support tools. One of the air traffic management fields that is necessary to provide future capacity is separation management. Separation Management can be understood as a consecutive function to complexity management: while *airspace design* provides the space in which aircraft have to be separated, *flow management* determines the number of aircraft that have to be dealt with in that airspace. So both complexity functions determine the density of a traffic scenario, hence generate the problem that separation management has to solve. A comprehensive overview of current complexity research can be found in [4] or [5].

This paper addresses separation management as it yields a lot of potential when engaged by optimal control methods. Typically one distinguishes between two approaches to separation management, depending on the chosen perspective. In *de-centralized approaches*, airspace users solve conflicts independently from ANSPs by using airborne communication and on-board trajectory computations. Examples for this are projects in the fields of self-separation and articles like [6] and [7]. The disadvantage of a de-central solution is that the whole system is unknown, so that the resolution of a conflict could possibly yield more severe traffic situations. Usually these approaches try to compensate for this by using heuristic rules that are followed by individual solutions.

The *centralized approach* depends on a central controller, i.e. the local ANSP, who has full information of the regarded airspace. Even though the disadvantage of modeling a more complex system is apparent, this approach has the significant advantage of optimizing the global system by incorporating all aircraft in a sector to the optimization process. Until today centralized approaches have usually focused on the above mentioned aspects of flow management or airspace design, most notably in [8] and [9] — but never in a way to generate individual flight trajectories for all users.
In this paper we will investigate the possibilities and the efficiency of a centralized approach to separation management that may be controlled by an ANSP, and could be possibly realized as an automated controller support tool. Therefore we present a framework that is based on optimal control methods. In the scope of this paper we will use the direct multiple shooting method as described in various literature, e.g. in the work of Diehl, [10] or Bock and Plitt, see [11].

Control theory is used to determine controls for a dynamic

system which will drive the system from its initial state to its final state in an optimal way. The optimality of a feasible trajectory is measured by a performance index. Here we use information regarding a particular traffic scenario to derive an optimal control problem of which the solution will yield optimal controls for all aircraft within the sector.
The results presented in this paper mostly originate from the work for the author's diploma thesis, to be submitted to Philipps University Marburg in January 2012, see [12].

II. Operational Framework

The task for separation management is to resolve conflicts that are projected to occur on a tactical time horizon. In contrast to flow management, here we address individual aircraft on their way through airspace. Currently we are mitigating the risk of conflict by imposing a 5nm (*nautical miles*) lateral minimum separation that could easily be decreased if we had better and more efficient separation methods — and of course more accurate information about the aircraft position, see [13].

Automated ground-systems may assist the controller in separating aircraft so that the available space can be used more effectively. Furthermore, workload is reduced once the controller can rely on automation tools to resolve conflicts automatically. Especially when thinking about future concepts like free flight, where flights are no longer bound to fixed traffic routes, the aspect of separation management will become even more important.

In the scope of this paper we want to investigate initial results for a centralized approach to separation management, where we not only solve a single conflict between two or more aircraft, but rather generate trajectories throughout a given airspace, incorporating information about possible future conflicts or other disadvantageous consequences of a resolution process. That way trajectories can be optimized throughout the whole sector. This optimization process should be synchronized with individual customer preferences, like chosen cost index within the FMS (*Flight Management System*, a key component in the cockpit) etc. This can be realized by deploying a modular approach to system architecture. For the purpose of the scenarios engaged in this paper, we will optimize trajectories so that its deviation from a chosen *nominal trajectory* is minimized.

Notable studies applying mathematical optimization to air traffic management include [14], [15] and [16]. The major difference however is that in this approach we will not only separate aircraft and optimize the whole traffic situation within a sector, we will also compute full control trajectories that might be transmitted via FMS uplink in a future environment. Enabling a ground-to-air data up- and downlink is a key requirement for all future ATM technologies, so assuming this capability is reasonable. Please see [13], [17], [18] and [19] for more information.

A. System Control Loop

We assume that we are facing a traffic situation $\mathscr{T}_0$, from which we can read essential data that describes this particular scenario. This data includes information regarding all airspace users within the airspace (position, speed vector, entry point, destination and so on), as well as geometric information about the airspace itself. The necessary information may be derived by using radar tracking or, in a future environment, data downlink from the aircrafts FMS.

From this information we may build an optimal control problem by looking for control trajectories for each aircraft that will drive the whole system from its initial state (i.e. at the current instant of time t_0) to its final state. What is this final state?

The system is assumed to reach its final state once every regarded aircraft reaches its destination, i.e. its exit point of the sector. By assuming that at the current instant of time t_0, each aircraft has a very different way to go in the sector, we may very well conclude that not all aircraft will reach their destination *at the same time*. Therefore we must introduce individual parameters for each aircraft, indicating the time spent in the sector. By normalizing the dynamic system of each aircraft by that individual time p_i, we obtain a system that does indeed reach its terminal state homogeneously.

Once we have used the data containing all essential information of the traffic situation to formulate an optimal control problem, this problem will be solved by the ground-system in a control center. The resulting optimal control trajectories are distributed among airspace users via data comm uplink to the FMS, which then steers the aircraft accordingly.

This again leads to a new traffic situation $\mathscr{T}_1$ at a later instant of time t_1. The cycle ultimately starts anew to determine whether the dynamic system propagates as projected. Using a nonlinear model predictive control process in such a control framework potentially enables a feedback law that protects the system against possible disturbances.
Figure 1 illustrates this process. It must be noted that even

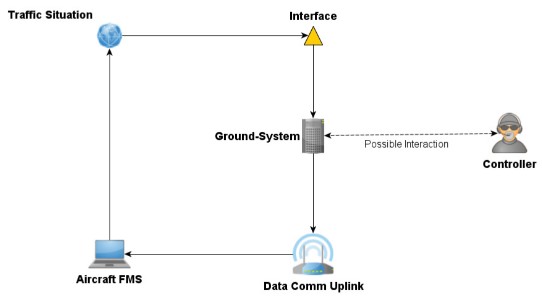

Fig. 1. System framework for centralized separation management.

though this framework is designed to work automatically, there can, and actually should be, a possible controller-machine interaction implemented. This could be realized in a way that the automation suggests solutions for the given traffic situation and the controller may interfere if he does not feel comfortable with the solution.

B. Generating the Control Problem

In order to generate a control problem for a specific traffic situation, we need to formulate a dynamic system describing the propagation of all aircraft within a sector. We have to determine control variables that are then evaluated by a chosen objective function — and finally we have to specify constraints and boundary conditions that have to be satisfied.

a) Dynamic System: The system of ordinary differential equations for the numerical examples that will be investigated in section IV was chosen to best meet the BADA (Base of Aircraft Data, [20]) model that is used within the AFS (*Advanced Flight Simulator* by DFS) real-time simulation. Since we only inspect 2-dimensional en-route scenarios without level changes, the dynamic system is quite simple:

$$\begin{aligned}\dot{\theta}(t) &= \frac{g}{V(t)}\tan(\mu(t))\\ \dot{x}(t) &= V(t)\cos(\theta(t)),\\ \dot{y}(t) &= V(t)\sin(\theta(t)).\end{aligned}$$

Here $\theta(t)$ is the aircraft heading, $V(t)$ the true airspeed, $\mu(t)$ the bank angle, $x(t)$ the lateral and $y(t)$ the longitudinal position. Control variables are chosen to be the aircraft velocity and its bank angle, so $u_0(t) = \mu(t)$ and $u_1(t) = V(t)$.

From now on we assume that the traffic vector $\mathscr{T}(t_0)$ for the initial instant of time t_0, $t_0 = 0$ without loss of generality, yields the following initial state information:

$$\begin{aligned}\theta^i(0) &= \theta_0^i,\\ x^i(0) &= x_0^i, \\ y^i(0) &= y_0^i,\end{aligned} \tag{1}$$

for all aircraft $i = 1, \ldots, m$, with m being the total number of aircraft. The nominal heading θ_0^i is assumed to be the direct heading between entry and exit point. Note that for the sake of simplicity, we will assume direct routes from extry to exit point - one could define more complex routes throughout the sector without having to modify the general approach.

For the remainder of this paper all three differential states are incorporated in a vector ω, i.e. $\omega := (\theta(t)^T, x(t)^T, y(t)^T)^T$. We can very well assume that the systems initial state $\omega(0) = \omega_0$ is known (either from radar tracking or FMS downlink), and the desired state at the final instant of time T can be formulated as a terminal condition

$$\omega(T) = \omega_T.$$

Depending on the actual traffic scenario, ω_T contains information about the corresponding exit point of the aircraft, i.e. information about $x(T)$ and $y(T)$.

An important feature of this approach is the introduction of an individual aircraft time p^i, that represents the time spent by that aircraft in the regarded airspace. This time may be very different for various aircraft. As a consequence we set the global time to be fixed $T = 1$. By using the vector $p = ((p^i, p^i, p^i)^T)_{i=1,\ldots,m} \in \mathbb{R}^{3m}$, and the identity matrix $I \in \mathbb{R}^{3m\times 3m}$, we may obtain a diagonal matrix $p \cdot I =: P \in \mathbb{R}^{3m\times 3m}$. This matrix has to be multiplied with the right-hand side of the ODE system, so that the dynamic system finally becomes:

$$\dot{\omega}(t) = P \cdot f(\omega(t), u(t)),$$

where f is the right-hand side of the ODE system 1. Beside the dynamic system we also define variables that represent the distance between two individual aircraft. For every pair of aircraft (i, j) we define an algebraic variable

$$a_{(i,j)}(\omega(t)) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{2}$$

which determines the separation between aircraft i and j. In the remainder we will choose the index $k = i + j$, so that $k = 1, \ldots, (m-1)!$ and $a = (a_k)_{k=1,\ldots,(m-1)!}$. For the scenarios investigated in section IV we assume a minimum separation of 5nm, so that

$$h_{(i,j)}(\omega(t)) = a_{(i,j)} - 5 \geq 0.$$

Note that these algebraic variables were introduced for the purpose of monitoring the separation between aircraft during the optimization process. They can be considered as an auxiliary variable, as it does not interfere with the dynamic system itself.

b) Objective Function: Formulation of the objective function is not a straightforward task. Here we suggest a performance index that assesses the deviation from individual flight trajectories to their nominal ones. This contains the difference of sector times, as well as deviations in velocity or heading angles.

First we want to identify the difference between the actual time that the aircraft spent within the sector and its nominal sector time, i.e. the time that was filed in its flight plan. Let $\hat{p}^i$ be the nominal sector time by aircraft i, then we want to minimize the difference $\hat{p}^i - p^i$. By defining the function

$$\psi(p) = \sum_{i=1}^{m} \sqrt{(\hat{p}^i - p^i)^2}, \tag{3}$$

we obtain a measure to minimize the difference of actual and filed sector time.

As a second optimality measure we need to identify the deviation from the aircraft's nominal path. As we have mentioned above, we here assume that these nominal flight paths are direct routes from entry to exit point. We want to control aircraft in a way that even though we manage conflicts, we also minimize the deviation from its planned flight track as chosen by the user himself. Therefore, we compute a nominal position $\bar{\omega}(\hat{t})$ for the instant of time $\hat{t} \in [0,1]$ and for aircraft i by setting

$$\begin{aligned}\bar{x}^i(\hat{t}) &= x_0^i + \bar{V}^i \cos(\theta_0^i)\hat{t}p^i,\\ \bar{y}^i(\hat{t}) &= y_0^i + \bar{V}^i \sin(\theta_0^i)\hat{t}p^i,\end{aligned}$$

where $\bar{V}^i$ is the nominal, i.e. filed, cruise speed of aircraft i. By integrating over the difference of nominal position and actual position, we obtain a measure for the deviation from an aircraft's nominal path. Hence we define

$$L_1^i(\omega(\hat{t})) = \sqrt{(\bar{x}^i(\hat{t}) - x^i(\hat{t}))^2 + (\bar{y}^i(\hat{t}) - y^i(\hat{t}))^2}, \quad (4)$$

for all aircraft i. This measure is important because we want the airspace to look like it was projected in strategic applications in terms of workload distribution and flow management. Huge deviations from the nominal flight track should be avoided so that we do not impact the macroscopic system.

Furthermore, we want to minimize the total interference with the aircraft, hence we want to minimize the magnitude of control changes. This applies to both change of velocity and change of bank angle. In other words, we need to minimize the total deviation of controlled velocity and the aircraft's nominal velocity, as well as the total bank angle flown. Recalling from above that $u_0^i(t)$ is the bank angle and $u_1^i(t)$ the velocity, we may define

$$\begin{aligned}L_2^i(u_0^i(t)) &= (u_0^i)^2, &(5)\\ L_3^i(u_1^i(t)) &= \|u_1^i - \bar{V}^i\|. &(6)\end{aligned}$$

Taking together equations (4), (5) and (6), and by integrating over the whole time horizon, we obtain the Lagrange term of the objective as

$$\begin{aligned}\int_0^1 L(\alpha,\omega(t),u(t))dt &= \int_0^1 \sum_{i=1}^m [\alpha_1 L_1^i(\omega(t),u(t)) \quad (7)\\ &\quad +\alpha_2 L_2^i(\omega(t),u(t))\\ &\quad +\alpha_3 L_3^i(\omega(t),u(t))]dt,\end{aligned}$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)^T$ is a vector composed of weights assigned to the different terms within the objective function. Using equations (3) and (7), we finally obtain the performance index as

$$J(\alpha,\beta) := \int_0^1 L(\alpha,\omega(t),u(t))dt + \beta\psi(p), \quad (8)$$

with β being an additional weight. Note that all individual measures in (8) are indeed dependend on each other. For example the deviation from a nominal flight track naturally also influences the sector time. Furthermore, in reality we need to introduce weights to conform for objective components of different magnitudes (e.g. bank angle in radians and sector times in seconds). It should be noted that even though we address multiple indicators within our objective function, we are not trying to optimize all of them at the same time. Instead we optimize one scalar value $J(\alpha,\beta)$, for which a trade-off between multiple indicators (depending on the weights) is expected.

From the considerations above we may now formulate an optimal control problem in correspondence with a standard form of problems with fixed final time. The problem is as follows:

$$\min_{u(t),\omega(t)} \int_0^1 L(\omega(t),u(t))dt + \psi(p) \quad (9)$$

subject to

$$\begin{aligned}\dot{\omega}(t) - P \cdot f(\omega(t),u(t)) &= 0,\\ g(\omega(t)) &= 0,\\ h(a(t)) &\geq 0,\\ \omega(0) - \omega_0 &= 0,\\ u(0) - u_0 &= 0,\\ r^e(\omega(T),u(T)) &= 0,\end{aligned}$$

where $r^e(\omega(T),u(T))$ contains information about the aircraft's position and its controls at the sector exit point as equality constraints. Since we want aircraft to be transfered from one sector to another in a steady flight state, we demand terminal controls to be nominal, i.e.

$$\begin{aligned}u_0^i(T) &= 0,\\ u_1^i(T) &= \hat{V}^i.\end{aligned}$$

III. Optimal Control

Typically control problems are solved by either indirect or direct methods. While indirect methods use optimality conditions derived from *calculus of variations* to obtain a multi-point boundary value problem that can later be solved by numerical methods, direct solution methods first discretize the problem and then solve it by techniques of nonlinear programming. The indirect method has the disadvantage of introducing so-called *adjoint variables* that become part of the MPBVP — and for which initial values are very difficult to determine. More information on the indirect method can be found in [21], [22] or [23].
In this paper we use the direct method to obtain solutions for problem types of (9), described as *first discretize, then optimize*. We perform a multiple shooting discretization that transforms the ∞-dimensional control problem into a finite-dimensional nonlinear optimization problem which is finally solved with the SQP method. More detailed information on the direct method and its variations can be found in [11], [24], [25] and [26].

A. Discretization

Optimal control problems are typically infinite-dimensional problems since they yield an infinite number of unknowns. Even if you are dealing with a finite time horizon (as in problem (9)), the control function that we are looking for still has an infinite number of unknown values along its continuous trajectory.

As a first step we introduce grid nodes τ_i to divide the regarded time horizon $[0,1]$ in multiple intervals. By choosing a number of N grid nodes, including initial and terminal points, we have

$$0 = \tau_0 < \tau_1 < \cdots < \tau_{N-2} < \tau_{N-1} = 1, \qquad (10)$$

as a grid of multiple shooting nodes.

For each single interval $[\tau_j, \tau_{j+1}]$ we obtain an initial value problem based on the differential equations that define the dynamic system. The solution of each IVPs only depend on the initial value itself, i.e. at the state value for a particular grid node $s_j = (\omega(\tau_j), a(\tau_j))$. This way the state values at grid nodes remain as the only unknowns with regard to the states. All values between grid nodes are determined by the differential equations and the defining algebraic equations respectively.

Consequently, the differential state trajectories between grid nodes are given by $\omega_j(t) = \omega(t; s_j)$ for $t \in [\tau_j, \tau_{j+1}]$. The algebraic states on the other hand may be directly computed from the given differential states, according to equation (2).

Furthermore, we will parameterize the control trajectory in a way that we assume it to be piece-wise linear and continious between grid nodes. Other discretizations are possible as well, including piece-wise constant or cubic controls. Choosing the type of discretization is usually a trade-off between computation time, feasibility and stability on one side, and realism on the other. Here we have chosen piece-wise linear controls as this approach already delivers very promising results without yielding too expensive computation times. By parameterizing the control trajectory as

$$u(t) = q_j \cdot t + v_j, \qquad (11)$$

for $t \in [\tau_j, \tau_{j+1}]$, we have reduced the total number of unknown variables in our examples (considering four aircraft, i.e. $m = 4$) from ∞ down to 448, with 192 state unknowns and 256 control unknowns.

Finally, we also need to discretize the objective function, which is easy since we simply evaluate the objective at the grid nodes, obtaining a straightforward sum approximation of the integral, yielding

$$J(\alpha, \beta) = \sum_{j=0}^{N-1} L_i(\alpha, \omega_j, a_j, u_j) + \beta\psi(p). \qquad (12)$$

By parameterizing the state variables, along with (11) and (12), we have transformed the infinite-dimensional control problem (9) in a finite-dimensional problem. This problem can be formulated as

$$\min_{q,v,s} \sum_{j=1}^{N-1} L_j(\alpha, q_j, r_j, \omega_j) + \beta\psi(p) \qquad (13)$$

subject to

$$\begin{aligned}
s(\tau_{j+1}) - \omega_j(\tau_{j+1}; s_j, q_j, r_j) &= 0, j = 0, \ldots, N-1,\\
g(s_j) &= 0, j = 0, \ldots, N-1,\\
h(s_j) &\geq 0, j = 0, \ldots, N-1,\\
\omega(0) - \omega_0 &= 0,\\
q(0) - q_0 &= 0,\\
v(0) - v_0 &= 0,\\
r^e(\omega_N, s_N) &= 0,
\end{aligned}$$

which can then be engaged by nonlinear optimization methods. Here

$$\begin{aligned}
q &= (q_j)_{j=0,\ldots,N-1},\\
v &= (v_j)_{j=0,\ldots,N-1},\\
s &= (s_j)_{j=0,\ldots,N}.
\end{aligned}$$

Note that by defining functions F, G and H according to the functions in problem (13), we easily obtain a nonlinear optimization problem in standard form, as

$$\min_{\zeta \in \mathbb{R}^{n_\zeta}} F(\zeta) \qquad (14)$$

subject to

$$\begin{aligned}
G(\zeta) &= 0,\\
H(\zeta) &\geq 0,
\end{aligned}$$

where $\zeta = (q^T, v^T, s^T)^T$.
We have now formed a nonlinear problem of finite dimension that originates in the control problem (9) introduced above. Finding a solution of these nonlinear problems is a common task in optimization and we will present one solution method in the next section.

B. Nonlinear Programming

Nonlinear optimization or nonlinear programming comprises methods that search for an optimal vector instead of an optimal function as it would be necessary for generic control problems. However, we have transformed the problem by the discretization process above, so that methods of nonlinear programming may be applied. A comprehensive overview of methods in this field can be found in either [27] or [28].

In the scope of this paper we will only summarize the methods that are used and refer to further sourcess whenever more detailed study could be valuable. The author is providing a closer look on nonlinear optimization in his diploma thesis, [12].

Model	**Velocity/[nm/s]** (TAS)	**Cruise/[nm/s]** (TAS)
B737-800	0.0894-0.1336	0.12
Cessna C550	0.0694-0.1117	0.09

TABLE I
AIRCRAFT TYPES

In general nonlinear problems as (14) are typically solved by using the Lagrangian function

$$L(\zeta, \lambda, \mu) = F(\zeta) - \lambda^T G(\zeta) - \mu^T H(\zeta), \tag{15}$$

with λ,μ as Lagrange multipliers, for which we try to meet certain optimality conditions, see [27]. To achieve conditions for optimality one applies iterative descent algorithms that generate successive iterates ζ^k by using

$$\zeta^{k+1} = \zeta^k + t^k d^k, \tag{16}$$

where t^k is the step length and d^k is the search direction. In this approach we have used a SQP (*sequential quadratic programming*) algorithm that has a special way to obtain d^k. In each iteration the search direction is computed by solving a quadratic sub-problem (hence the name), which corresponds to a second-order Taylor approximation of the Lagrangian, with linearized constraints. The SQP algorithm itself can be adapted by choosing different strategies for approximating the Hessian of the Lagrangian, and by choosing one specific globalization technique. Here we have configured *MUSCOD-II* such that its SQP module uses a BFGS update on the Hessian approximation of the Lagrangian contained in the SQP, as well as a VMCWD (*Variable Metric Constrained WatchDog*) technique to determine the step length t_k (*globalization*), see [29].

IV. NUMERICAL RESULTS

In this section we will finally look at some numerical use cases that have been solved, using the methods presented above. Every scenario is assumed to take place at an altitude of 35000ft, i.e. FL350 - which is the typical environment for en-route flight phases. Furthermore, each scenario runs in a generic airspace of 100nm $\times$ 100nm.

Our use cases include two types of aircraft, see table I for more detailed information. The artificial controller provided by this method is able to influence the aircrafts bank angle $u_0^i(t)$ and its velocity $u_1^i(t)$. Both controls are assumed to be piece-wise linear as outlined above. Each use case is solved multiple times, each time with different weights assigned to the objective function. The different elements of the objective were introduced in paragraph II-B0b. Table II shows the different weights that were assigned to individual indicators. As the numbers show they were primarily chosen to represent different orders of magnitude — and to dis- or enable certain indicators within the objective.

Indicator	Set A	Set B	Set D	Set E
Sector Time	0	10	10	0
Flight Track	0	0	0	10
Cruise Speed	100	0	100	0
Bank Angle	100	0	100	0

TABLE II
WEIGHTS SETS USED TO SOLVE USE CASES.

	Set A	Set B	Set C	Set D
Time				
Scenario A	00:43.324	00:28.384	00:40.958	00:04.454
Scenario B	01:57.170	01:19.553	07:52.416	00:04.325
Scenario C	04:14.832	00:32.181	04:54.172	04:12.262
Iterations				
Scenario A	121	79	111	12
Scenario B	288	192	1119	9
Scenario C	662	89	647	493

TABLE III
COMPUTATION TIMES FOR ALL SCENARIOS.

Every solution was started with an heuristic pre-optimization to obtain initial control values. This heuristic approach included simple rules, as *aircraft will turn left to solve conflicts*. Indeed all initial state values were extrapolated from an entry-exit linearization.

Solutions are visualized by a two-dimensional top view on the sector, showing flight tracks for all regarded aircraft. For every scenario we will show statistics, including total delay, maximum delay for an individual flight, total flight track excess and the magnitude of speed control (*SC Magnitude*). Table III shows computational times and number of iterations for all three scenarios as well as for the different weight sets in the objective. Computations were performed on a 2GHz Intel Core 2 Duo machine.

A. Scenario A: Rogue

In this scenario four B737-800 aircraft need to pass the center of a sector in order to reach their destination at the opposite of their entry point. While heuristically it is not difficult to obtain a feasible solution that is a conflict-free trajectory for all aircraft, the determination of an optimal solution is not straightforward.

Table IV shows the results obtained for each objective function. It clearly shows that a ground-system can determine a solution within reasonable time. As table III indicates, computation time never exceeded the one minute mark for any feasible solution. Figure 2 shows the solution using weight set A. It is interesting to see that even without any heuristic information within the algorithm, the mathematical method finds a very similar solution to what is taught to controllers: to resolve conflicts as soon as possible so that flight track excess is minimized.

	Total Delay/[s]	Max. Delay/[s]	Track Excess	SC Magnitude
A	11.12	2.91	0.32%	0.004
B	0.01	<0.01	1.33%	5.384
C	0.01	<0.01	0.76%	4.420
D	42.18	11.21	1.51%	1.613

TABLE IV
STATISTICS FOR SCENARIO A.

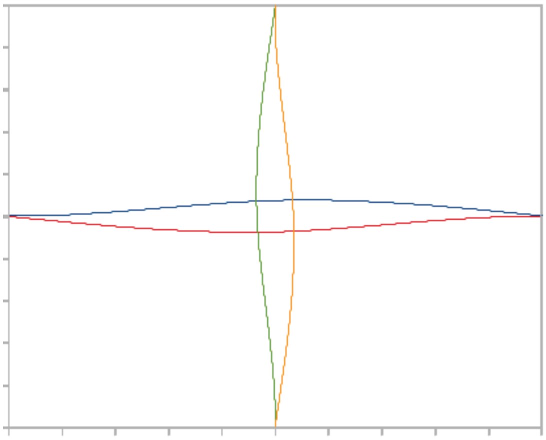

Fig. 2. 2D top-view, showing the solution for scenario A (weight set A).

B. Scenario B: Velocity Rogue

In this scenario we have two aircraft following each other, with the leading one being the slower aircraft as shown in table I. Both aircraft start on the left side of the sector, at points $(0, 50)$ and $(10, 50)$ respectively, with both aircraft heading for the same destination $(100, 50)$. Consequently, the faster aircraft has to overtake the slower one at some point of the sector — or the system could choose to slow down the faster one so significantly, that it may follow the slower one.

A third aircraft (red color) approaches them from the right, starting at $(100, 50)$ and a fourth strikes through the center as well, starting at $(50, 0)$ and heading for its destination at the opposite side of the sector. Table V shows the results that were obtained. In figure 3 we can see the solution provided by weight set C. It shows the compensation between a reduced delay as well as minimized influence to the aircraft. The trajectories shown are conflict-free.

C. Scenario C: Switching Lanes

In scenario C all four aircraft are of the same type. They start on the left side of the sector and head for the opposite position again. Two of these vessels need to cross their flight paths as aircraft 1 is heading from $(0, 40)$ to $(100, 60)$, and aircraft 2 heads from $(0, 60)$ to $(100, 40)$. Table VI shows numerical results for scenario C. It must be noted that for weight set D, the algorithm did not deliver an operationally feasible solution. While minimum separation was satisfied on all grid nodes, it was violated between two of them.

	Total Delay/[s]	Max. Delay/[s]	Track Excess	SC Magnitude
A	20.89	13.93	0.52%	<0.001
B	0.10	<0.01	2.70%	19.998
C	0.01	<0.01	0.45%	1.848
D	45.90	17.05	2.67%	1.867

TABLE V
STATISTICS FOR SCENARIO B.

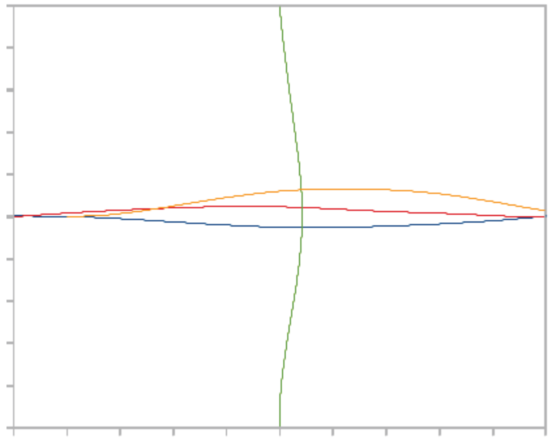

Fig. 3. 2D top-view, showing the solution for scenario B (weight set C).

	Total Delay/[s]	Max. Delay/[s]	Track Excess	SC Magnitude
A	47.19	39.22	0.77%	1.594
B	<0.01	<0.01	3.62%	48.713
C	<0.01	<0.01	0.30%	32.559
D*	263.51	198.19	1.26%	74.521

TABLE VI
STATISTICS FOR SCENARIO C.

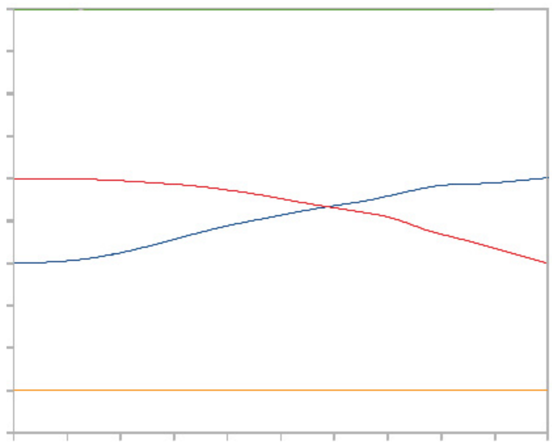

Fig. 4. 2D top-view, showing the solution for scenario C (weight set C).

Figure 4 shows the results for weight set C. Here, both the red and blue aircraft change their velocity before their tracks cross each other to guarantee separation minima.

All three use cases have shown the strong potential of this method to automatically generate solutions. Except for weight set D in scenario C, minimum separation was always ensured and it was possible to find no-delay trajectories within the range of aircraft performance. Furthermore, it was shown that with an objective function that tries to minimize the influence of a centralized control system (set A), the algorithm will deliver promising results as well. Weight set C was chosen to combine all factors - however, results indicate that this performance index will lead to high computational costs, as shown in table III.

V. Conclusion

We presented a framework for a centralized approach to separation management, using an optimal control element that automatically computes conflict-free trajectories for all aircraft within a regarded airspace. As an optimality measure we used a function comprising a set of four indicators that aim to assess the deviation from a nominal flight track and time and we investigated the results for different weight sets within this function.

Numerical results showed that this method can achieve very promising results, even when faced with complicated traffic scenarios that yield multiple conflicts. This method could indeed lead to a significant improvement in terms of separation management in order to enable more capacity and throughput.
These results need to be validated by real-time simulations as it will be done in November 2011 at DFS, where human controllers will be confronted with a set of traffic scenarios (including the ones presented above). The results of these simulations will be compared to numerical simulations. This will bring more insight to the task of identifying the *best* objective function, as well as information about which objective function is reproducing controller behaviour most accurately. This is considered to be a valuable input as a possible future support tool should deliver solutions that can easily be followed by the controller to approve.

Moreover we saw that even though the results are promising, they are sometimes complicated and expensive to achieve. Initial research showed that there is a vital necessity for pre-optimization modules that prepare the optimal control algorithm with useful initial control and state trajectories. As we showed the necessary computation times to obtain certain solutions are beyond the scope of a future real-time environment. Furthermore the determination of scales within the solution algorithm is an important issue.

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