RELIABILITY OF WING STRUCTURES CONSIDERING STOCHASTIC PARAMETERS IN FLUID-STRUCTURE-INTERACTION

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OVERVIEW

Today numerical methods for both, the structural and the aerodynamic problem are reaching highly versatile and reliable levels. Therefore, the coupled problem of static aeroelasticity can be solved at a high standard. But in real aircraft the structure may differ from the original design in many parameters, e.g. skin thicknesses or Young's modulus. This in turn alters the stiffness of the structure and the structural and aerodynamic response.

The current paper investigates the influence of stochastic parameters in thicknesses and Young's modulus on the coupled response of the structure and aerodynamics. It uses Gaussian normal distributions for the a.m. parameters in a finite element model for a given wing geometry. Aerodynamic data is calculated by means of a panel method. This is of course not the most sophisticated method, but due to the high number of necessary calculations, it is a reasonable choice. An in-house software (ifls) is used for the coupling of the fluid-structure problem. Other in-house software provides the method for the assessment of the probabilistic issue. This is First Order Reliability (FORM) method, depending on the type of reliability question. For this purpose relevant limit state functions have been defined and the influence of stochastic input parameters has been investigated. The results show that realistic variances in some parameters have an essential influence on the aerodynamic performance, while others are of less importance.

1. FINITE ELEMENT STRUCTURE MODEL OF TEST WING

For the purpose of stochastic investigations a code developed at the Institute of Aircraft Design and Lightweight Structures (IFL) was used to generate a generic finite element model of the structure. It is based on the parametric description of an airplane wing geometry and a layout of the load-bearing structure [1], [2]. The code is written in Patran Command Language (PCL) which enables an automated generation of finite element wing models by the preprocessor MSC Patran®. For the integration of finite element models in a stochastic simulation environment the program routines were extended to a flexibly assign material and structural parameters.

A HIRENASD wind tunnel model [3] scaled down from 58 m of span was used as a test structure for investigations carried out in the context of the MUNA project. The wing box structural layout as well as the arrangement of engines were taken on from the predecessor project [4] and are similar to the wing of an AIRBUS A340 aircraft (see fig.1, on the left).

The geometry data is imported from an ASCII input file and were used to generate a finite element shell model of the wing. The stiffening components of the structure, like stringers, spar caps and rib stiffeners can be modelled as bar or rod elements or taken into account by smearing in the wall thickness of neighbouring structure areas in a simplified manner. For this study a simplified modelling of the skin structure was chosen to reduce the number of degrees of freedom of the numeric structure model.

A transonic transport aircraft design was used with weighs given in table 1 for the calculation of the target lift for the aerodynamic and static inertial loads.

Gross weight	**m_{TOW}**	**to**	**256**
Fuselage and empennage unit structure +payload	m_{RF} $+m_N$	to	95
Wing structure	m_W	to	35
Total fuel mass	m_F	to	106
Propulsion group	m_{PG}	to	20

TAB 1. Weights for the transonic transport aircraft design used in this study

2. LOAD CASES AND STRUCTURAL SIZING

2.1. Loads calculation

Due to a high number of static aeroelastic calculations required for the probabilistic analysis instead of an Euler or RANS code a high order panel method HISSS [5] was used to calculate the discrete aerodynamic nodal loads. The lack of accuracy by generating a load distribution over the wing surface at higher Mach numbers had to be accepted so that the numeric costs could be kept in limits. A finite-element solver NASTRAN was used to calculate the nodal displacements of the finite-element structure model.

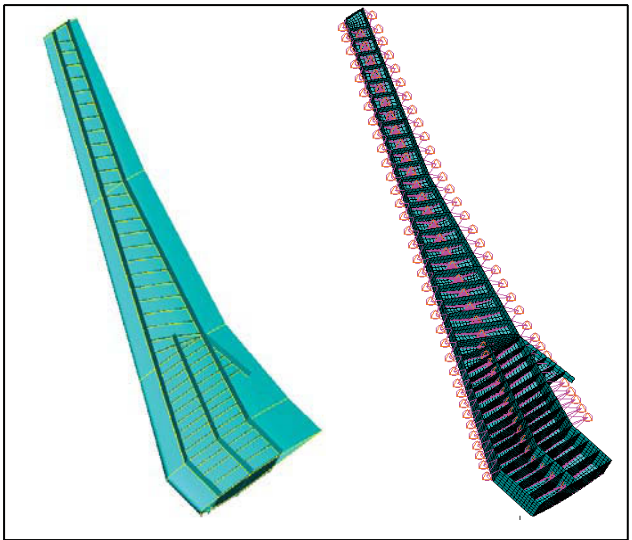

FIG. 1. HIRENASD wing geometry and structural layout

The in-house code-coupling library ifls [6] was used to perform the fluid-structure interaction. The code handles the load and displacement transfer between nonconforming grids by using a three-field approach in combination with Lagrange multipliers. The structure of the coupling routines was laid out to allow the interaction between different established numerical programs.

Nodal loads calculated on the aerodynamic surface were applied to the nodes of the structural grid by means of conservative interpolation in the region of the wing box. In the region of the flap and slat structure the aerodynamic nodal loads were applied to structural nodes which were created additionally and tied to the wing box by multi point constraints of RBE3 type.

The static inertial loads including fuel weight, engine loads and the weight of the flap structure had also to be taken into account to generate realistic load cases. The flap and slat structure were idealized as point masses and tied to the spar structure by multi point constraints in the same way as the aerodynamic forces. The masses of the high lift devices needed for this simplified approach were estimated by handbook methods [7]. Tank loads were also modelled with point masses and RBE3s (see fig.1, right-hand side). The tank mass was estimated for each wing bay by calculation of the volume taken by the fuel for a given degree of refuelling.

2.2. Structure sizing and design loads

The wing box structure was sized with respect to strength requirements and stability constraints. The strength sizing was carried out by a fully stressed design approach, using stress distribution computed for a limit load and a yield-stress criterion. The design against buckling failure was carried out by handbook methods [8] using optimum design curves and semi-empirical formulas for estimation of minimum skin thickness, stiffener spacing and cross-section geometry.

2,5g maneuver			
Altitude	H	km	11
Mach number	Ma		0,82
Gross weight	m_{TOW}	to	256
landing impact			
Altitude	H	km	0
Mach number	Ma		0,2
Gross weight	m_L	to	182

TAB 2. Design loads

Two load cases were selected for the sizing process: a 2,5g maneuver and the landing impact (see table 1). Stress distributions resulting from the load cases were used for strength and stability sizing of thin walled and stiffening structure.

Due to constraints defining the highest permitted distortion given in [4], the wing box was sized under consideration of stiffness requirements. The contribution of the structural members to the predefined deformation had to be calculated following the pattern of the modified fully utilized design method (MFUD) proposed by Patnaik et al [9]. For the constrained degree of freedom (in this case it is a bending displacement) the sensitivity factors had to be calculated for each component of the structure. These factors are defined as $\partial w/\partial m$ where ∂w is a partial change of displacement and ∂m is a change of structural mass. Both components are computed by attaching an additional material (by increasing wing thickness or stiffener cross-section) to each structural member and calculating the displacement w of the modified structure for a reference load. Sensitivity factors are used within the MFUD to weight the increase of wall thickness until the displacement constraint is achieved. This method permits to attach an additional structural mass only in the areas of the wing structure, whose stiffness influences the given deformation mostly. As has been shown in [9] the weight of the structure sized using this approach is very close to those obtained by very time-consuming optimization procedures.

3. STOCHASTIC SIMULATIONS

3.1. First order reliability method

In the present work, the probability of failure P_f of the wing structure is computed. It describes the probability that the structure does not to comply with the predefined requirements. Thus, the term failure has to be distinguished from other terms, like e.g. crash or disaster. Since the coupled fluid-structure analyses are very time consuming, the first order reliability method (FORM) was implemented to calculate the stochastic characteristics of the wing [10]. FORM introduces the reliability index β to describe the reliability of the structure. The main input to the method is the limit state function $G(\boldsymbol{X})$, where $\boldsymbol{X}$ is the vector of stochastic variables that influence the structure. By definition, the limit state function is positive, if the structure fulfils its requirements. Negative values are returned, if at least one requirement is violated.

In order to generate unique results for every problem, the vector of stochastic variables is transformed into a vector of standard normal random variables $\boldsymbol{X'}$. This leads to a limit state function $G(\boldsymbol{X'})$ which is analysed using the FORM routine. The FORM is a gradient based optimization procedure which calculates the minimum distance β between the limit state function defined by $G(\boldsymbol{X'}) = 0$ and the origin of the standard normal variable space spanned by the normalised stochastic variables.

At the beginning of the FORM algorithm, a $\beta_{initial}$ has to be estimated. The better the estimation of this initial value factor the fewer iterations are needed in the algorithm to get the final β. Haldar and Mahadevan [11] give a value of 5.0 as to be appropriate. With the $\beta_{initial}$ and the limit state function value, all parameters are defined to start the main iteration of the FORM algorithm consisting of three main steps: (cp. [11])

1. Transformation of stochastic variables into standard normal variable space. In order to get unique results, all non-standard normal variables have to be transformed. For normal variables, a general conversion can be applied, for other variables, the Method of Rackwitz and Fiessler [12] has to be used.
2. Generation of derivatives of the limit state function with respect to the standard normal variables. The coupled fluid-structure model can not be solved algebraically. Thus, the derivatives have to be estimated by finite differences in the neighbourhood of the design point.
3. Calculation of the direction, where the steepest trend in the limit state function occurs and estimation of a new design point and the corresponding β value

This iteration is repeated until the limit state function value is zero and the β value converges. The resulting β value is then transferred to the fitness value calculation routine of the optimization.

3.2. Combination of the FORM-routine with the fluid-structure interaction code library

To simulate the impact of the variation of structural parameters on the aeroelastic response of the wing, the ifls-code-library was embedded into the routines performing the FORM algorithm. A NASTRAN input file of the finite element wing model was created with the ability to vary the structural properties during the stochastic process. Two input parameters were defined to vary within the wing box structure: namely the thickness t of the thin-walled structural members and their elastic modulus E. A normal distribution was assumed for stochastic input parameters. The form of the normal distribution and therefore the extent of the deviation in the input parameters are characterized by the coefficient of variation (COV) $V = \sigma / \mu$. V is defined as a ratio of the standard deviation σ to the mean value μ, where μ is expressed by the value of the reference structure. For a random variable with a coefficient of variation of 0.1 the probability is 31,7 % that the deviation of this variable is more than ±10% of the mean value.

3.3. Definition of the limit state function

To apply the FORM analysis to the coupled fluid-structure problem a realistic failure criterion had to be defined to describe the performance of the simulated wing structure. For this kind of problem the random input is given by a variation in structural parameters. This variation alters the torsion and bending stiffness of the lift generating structure causing a change in the lift distribution compared to the reference structure. Under conditions of stationary cruise flight the lift change must be corrected by a change in the geometric angle of attack until target lift will be achieved.

The static aeroelastic equilibrium state was estimated by ifls iteratively for given lift and flow conditions by a variation of an overall (geometric) angle of attack α^g of the wing. The change in the resulting angle of attack α_{EqSt} for the equilibrium state was used to asses the influence of the random input parameters on the aerodynamic properties of the analyzed wing structure. The deviation in equilibrium state angle of attack $\Delta\alpha_{EqSt}$ can be investigated in both positive and negative direction. The greater values of α_{EqSt} caused by a lower elastic modulus resp. by reduction in wall thickness, respectively, were assessed to be more critical than smaller ones, caused by a stiffer wing structure.

The probability of deviation in equilibrium state angle of attack was investigated for different values of $\frac{\Delta\alpha_{EqSt}}{\alpha_{EqSt}}$ varying between 0.5% and 1.0%. Each value corresponds to a limit state function in the normal variable space:

$$G(X) = \Delta\alpha_{EqSt} - \Delta\alpha_{EqSt,req} = 0 \quad (1)$$

For a given limit state function and distribution in random parameters the FORM algorithm calculates a combination of these parameters, for which the reliability index β becomes minimum. In the inversion of the argument, the probability of the aeroelastic response represented by the limit state function becomes maximal.

An example problem for two random variables X'_1 and X'_2 and two limit state functions $G_1(\mathbf{X'})$ and $G_2(\mathbf{X'})$ is depicted in fig. 2. Corresponding to the definition of the reliability index β given in part 3.1 the probability of $G_1(\mathbf{X'})$ is greater then of $G_2(\mathbf{X'})$.

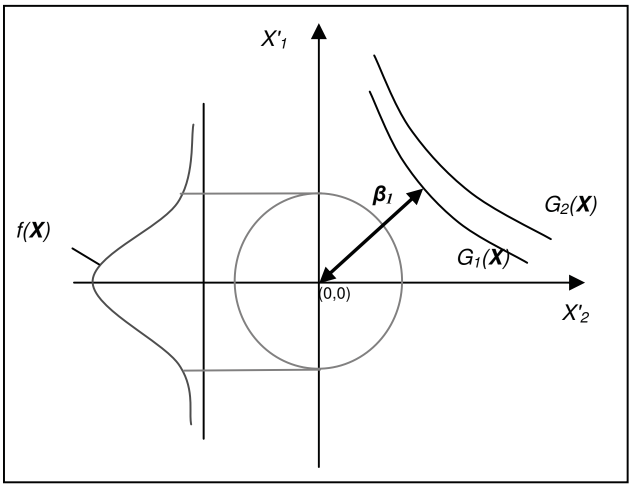

FIG. 2. Random input parameter distribution and limit state functions in the normal variable space

3.4. Sensitivity analysis by a global variation in structural parameters

As already mentioned, the variation of the wall thickness and elastic modulus causes a deviation in stiffness qualities of the wing structure. Due to the manipulation of these qualities the tendency of the wing is affected to exceed its shape under a certain load. To examine the influence of the structural stiffness on the wing aerodynamics, a well-known concept of the elastic angle of attack α^{el} was used. This kinematical term describes the local change in the geometric angle of attack α^g in flight direction due to elastic deformation of the wing. For wings with a sweepback α^{el} depends on the torsion deformation Θ as well as on the bending angle w':

$$\alpha^{el} = \Theta\cos\varphi - w'\sin\varphi \quad (2)$$

From the kinematical interrelationship in equation (2) follows that for a wing with positive sweep angle φ the torsion and bending components of elastic angle of attack are influenced mutually. For common transonic transport aircraft wing structures the angle α^{el} is dominated by the bending deformation and for this reason negative. Deviation in torsion deformation Θ as well as in the bending angle w' forms the change in elastic angle of attack, expressed by the term $\frac{\Delta\alpha^{el}}{\alpha^{el}}$. The a.m. tendency can be turned into its opposite, if the change in torsion deformation is much higher, than in bending angle.

The variation of structural parameters in skin, spars, or ribs influences the torsion and bending distortions in different ways. Reduction of the wall thickness as well as of the Young's modulus in the skin parts has the greatest effect on the bending and shear stiffness of the wing reducing the bending moment of inertia and shear coefficient of a local wing box cross-section. Torsional stiffness is also reduced, depending on the ratio of the height and depth of the wing box and thickness ratio of the skin and spar webs. Reduction of both parameters in the spar webs influences mostly the torsional and shear stiffness having only a secondary effect on the bending moment of inertia. Due to the lowest contribution of the ribs to the bending and torsional stiffness of a wing box structure the variation of the input parameters in this component has a negligible effect on the deformation behaviour of the wing also.

To estimate the effect of the variation of torsion and bending distortions on the deviance in the elastic angle the propagation of uncertainty law was applied on equation (2). For a relative deviation $\frac{\Delta\alpha^{el}}{\alpha^{el}}$ in elastic angle of attack a mathematical connection (3) follows:

$$(3)\quad \frac{\Delta\alpha^{el}}{\alpha^{el}} = \frac{\Delta\Theta}{\Theta}\left(\frac{\Theta}{\alpha^{el}}\right)\cos\varphi - \frac{\Delta w'}{w'}\left(\frac{w'}{\alpha^{el}}\right)\sin\varphi$$

The terms $\frac{\Delta\Theta}{\Theta}$ and $\frac{\Delta w'}{w'}$ are the relative deviation of torsion and bending distortions due to variation in structural input parameters. The terms $\left(\frac{\Theta}{\alpha^{el}}\right)\cos\varphi$ and $\left(\frac{w'}{\alpha^{el}}\right)\sin\varphi$ in equation (3) are ratios of the torsion and bending angles relative to the elastic angle of attack. They depend on the sweep angle φ, the load distribution in chord and span wise directions as well as on the ratio of the torsional stiffness *GJ* relative to the bending stiffness *EI*.

A parameter study was carried out to estimate the sensitivity of the structural and thus of the static aeroelastic response relative to the components of the wing structure affected by uncertain input parameters. The influence of each component was estimated by changing successively the wall thickness and elastic modulus in the skin, spar webs and ribs. To avoid local effects both input parameters were varied simultaneously by ±10% in the whole area of the wing. A structural response of a modified structure was determined for a reference load corresponding to the 1g load case. From this response, the over-all deviation in torsion and bending deformation was calculated.

The wing box investigated in this simple study with components varied separately and in the same manner should not represent a real case. The real wing structure is assembled by many different parts in which the dimensions and material properties varies independently from each other. The intent of this simple approach was only to estimate the main trend of the deviation within the structural and static aeroelastic response depending on the component of the structure in which the variation of input parameter occurs.

The results for the change in elastic angle of attack $\frac{\Delta\alpha^{el}}{\alpha^{el}}$ and the components $\frac{\Delta w'}{w'}\left(\frac{w'}{\alpha^{\Theta}}\right)\sin\varphi$ and $\frac{\Delta\Theta}{\Theta}\left(\frac{\Theta}{\alpha^{el}}\right)\cos\varphi$ of equation (3) are given in fig. 3. For representation reasons, the components of eq. (3) are labelled simply by terms α^{el}, Θ and w'. At this point, a remark on the algebraic sign convention has to be made: the signs of deviations of α^{el} and w' are positive in the positive direction of the ordinate. The sign of deviation of torsion component is positive in the negative direction of the ordinate due to negative angle ratio of torsion angle an elastic angle of attack $\left(\frac{\Theta}{\alpha^{el}}\right)$.

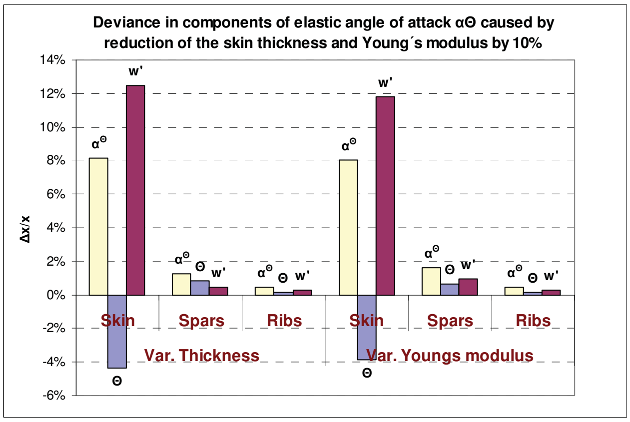

FIG. 3. Change in elastic angle of attack α^{el} and change in bending and torsion components of this angle caused by reduction of structural parameters by 10%

A variation of the structural parameters shows as expected the greatest effect on the structure's stiffness and therewith on the change in the angle of attack in the skin areas. With a 79% of the wing structural mass the skin forms the main component of the wing box structure. The results of the structural response show that, in spite of a relatively high ratio of the torsion angle to the elastic angle of attack, the latter is still be dominated by an angle of bending deformation. It is also remarkable that the impact on α^{el} is nearly identical for the variation of the wall thickness as well as of the elastic modulus in the skin areas.

The contribution of deviation in both deformation components to α^{el} is somewhere different for the variation of structural parameters in spar webs and rib surfaces. The change in the torsion angle is negative with respect to the sign convention showing therefore a stiffer torsional behaviour. This tendency is due to the skewed root rib of a swept wing which influences the warping moment of inertia and thus the torsional behaviour of the wing box.

For the reduction of wall thickness as well as of the elastic modulus in each structural member, the resulting angle of attack has to be increased to produce the target lift. To estimate the tendency of the change in the equilibrium state angle of attack α_{EqSt} caused by the input variation in structural components a static aeroelastic response was calculated for each modified structure model already described. The relative deviance in this angle is depicted in fig. 4 for each case.

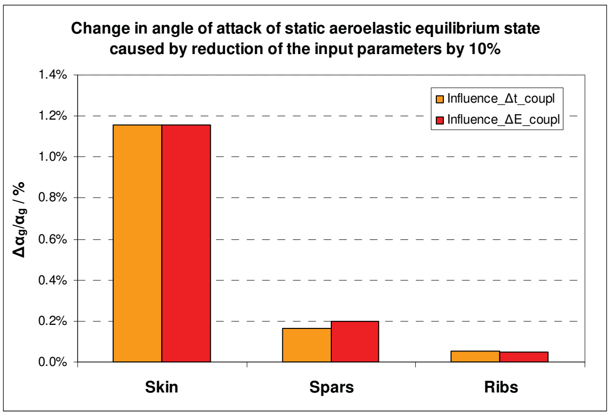

FIG. 4. Deviation in equilibrium state angle of attack for different structural parameters reduced by 10 %

The deviations of equilibrium state angle of attack α_{EqSt} resulting from static aeroelastic analysis shows a good agreement with the trend predicted by the change within elastic angle of attack α^{el} depicted in fig. 3. The almost identical values for $\frac{\Delta\alpha_{EqSt}}{\alpha_{EqSt}}$ for variation of both parameters in skin parts should be treated as a special case taking into account the global character of the applied variations.

4. RESULTS AND DISCUSSION

Within the stochastic analysis, the impact of random input parameters on the static aeroelastic response of the transport aircraft wing was investigated. Based on the results of the sensitivity analysis the investigation was carried out at first only for skin areas due to the most critical impact on the wing aerodynamics. The wing structure was divided into four areas in which the input parameters were independently varied. The division of the areas is given in table 3 as a function of the span co-ordinate.

Area	η_i	η_o
1	0.0	0.22
2	0.22	0.44
3	0.44	0.72
4	0.72	1.0

TAB 3. Areas of parameter variation

In each area, the structural parameters were varied simultaneously in the top and bottom skin parts. By this simplification, the number of random variables *X'i* decreased to a total of four that in turn led to significant reduction of numerical expenditure.

The Gaussian normal distribution for random input parameters was assumed. To estimate the coefficient of variance for the thickness distribution manufacturing data sheets for maximum thickness deviation were analysed. A coefficient of variance, which lies between 0.02 and 0.04, seems to be realistic. Results presented in the following part were calculated for the COV=0.05 also to show the effect of greater scatter within the input parameters. For the variation of the elastic modulus the same coefficients were used to guarantee the comparability of the results.

The allowed relative deviation $\frac{\Delta\alpha_{EqSt}}{\alpha_{EqSt}}$ in the elastic angle of attack compared to the reference structure was analysed in the range between 0.4% and 1.5% for different coefficients of variation. Each value of this deviation defines a limit state function *G(**X'**)*. For a given value of *G(**X'**)* the FORM routine calculates a combination of random variables, for which the reliability index converges. With regard to the investigated problem a combination of relative deviation of the structural input parameters in each area was found, for which the probability of a given deviation in the angle of attack becomes a maximum.

The results of the variation in wall thickness and elastic modulus in skin areas are presented in figures 5 and 6.

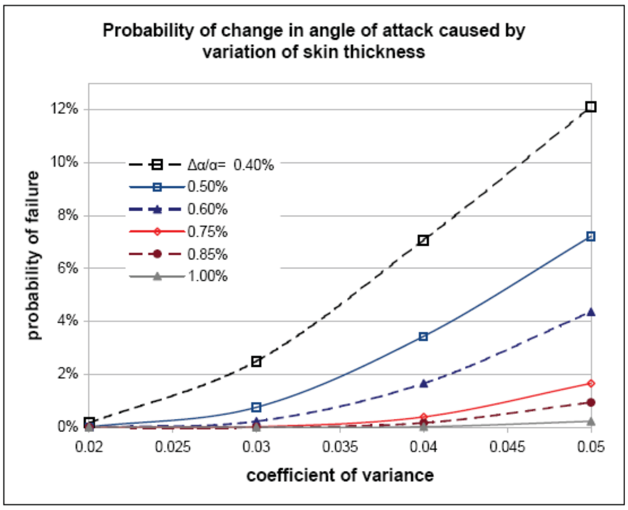

FIG. 5. Probability of change in angle of attack caused by variation of skin thickness for different performance criteria

In the diagrams a probability of failure is plotted for a series of limit state functions over the coefficient of variance *V*. Due to almost linear correlation between the reliability index and limit state functions for a given *V*, some curves could be extrapolated from the calculated results. These curves are plotted by dashed lines in fig. 5 and 6. For the investigated coefficients of variance the results for a relative deviation of an angle of attack of 1.5% were calculated for the local variation in the skin thickness of 10% and more. This degree of variation within the wing structure seems not very realistic to be considered further.

For each limit state function the probability of the failure arises with the scatter in the input parameter expressed by the COV. The lower the allowed difference in the angle of attack, expressed by a failure function the higher is the probability to violate the requirements.

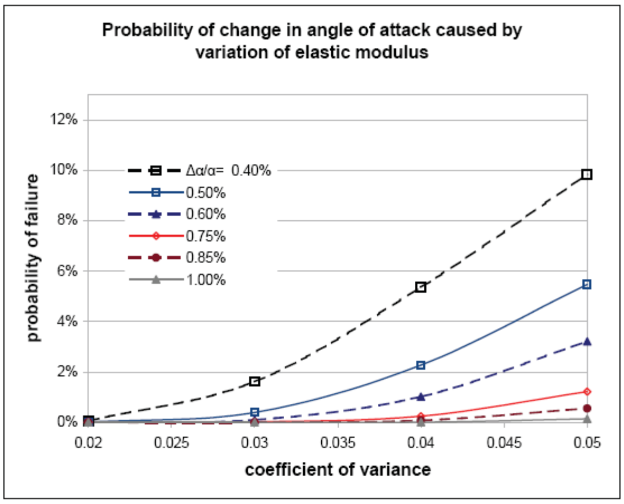

FIG. 6. Probability of change in angle of attack caused by variation of elastic modulus for different performance criteria

The comparison of the results for variation in skin thicknesses and elastic modulus shows very similar probability curves for both input parameters. The probability of failure obtained for the variation of Young's modulus is somewhat smaller as for a variation of skin thicknesses. This tendency shows a good agreement with the predictions made within the sensibility study carried out in chapter 3.4.

From the results of the stochastic analysis depicted in figures 5 and 6 it can be seen that the probability of higher deviations (>1%) within the global aerodynamic properties of the wing still be very small even for a greater variance of structural parameters. This demonstrate a high robustness of the coupled fluid structure system affected by the considered type of uncertainty.

5. CONCLUSION

In the present work, the influence of random structural parameters on the aerodynamic performance of a metallic test wing structure was investigated. The results of the FORM analyses have shown the capability of good predictions of the general tendencies on coupled aeroelastic systems.

The investigations demonstrate the suitability of the FORM analysis to handle some classes of stochastic uncertainties effecting the aeroelastic response of a wing structure. Due to the gradient based optimization procedure which forms the basis of the FORM the main requirement relative to the investigated problem is the existence of only one minimum solution for the reliability index β. To handle problems which violate this requirement as they are the uncertainties within fibre orientation angles of composite materials, another stochastic analysis methods like Latin hypercube sampling should be used instead of the FORM.

To reduce the numeric costs of stochastic simulation some simplifications had to be made within the analysis process. The influence of the weight reduction on the target lift caused by reduction in the wall thicknesses was neglected. The simultaneous variation of structural parameters within the top and bottom skin in only four areas represents a highly idealized test case compared to the real structure (c.p. the remarks in part 3.4). Considering these simplifications the results obtained in the present work should represent a conservative trend.

The variation of the input parameters in top and bottom skin parts as well as in spar webs for a higher number of independent areas of variation is a part of actual work as well as the consideration of weight reduction in the target lift. The another effect which could be considered is the tendency of the skin areas to buckle if the local bending stiffness of the panes is reduced by a variation of structural parameters having a significant influence on the aerodynamic drag.

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