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Development and Implementation of a Method for Linear Stability Analysis in Natural and Manipulated Boundary-Layer Flows

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Abstract

This paper presents the implementation of a method for modal stability analysis and its application to investigate transitional boundary-layer flows. The underlying physical concepts and a numerical method to solve the governing differential equations are introduced. The method is then applied to generic boundary-layer flows in order to verify the numerical results. In a second part the method is used to compute the stability properties of laminar boundary-layer flow affected by plasma actuators. Dielectric-barrier discharge (DBD) plasma actuators are flow-control devices used to influence the process of boundary-layer transition through the electrohydrodynamic induction of momentum into the surrounding fluid region. The boundary-layer profile and its stability characteristics change due to the actuation. In order to understand the effective mechanisms of these flow-control devices linear stability analysis (LSA) is applied using data from former numerical investigations. The results show a remarkable reduction of disturbance amplification rates for the Tollmien-Schlichting waves (TS-waves) in the linear growth stage, leading to considerable delay of laminar-turbulent transition.

1. Introduction

In times of scarce natural resources, high energy costs and increasing ecological awareness the need for efficient means of transportation has gained particular importance. Especially in the aviation industry aerodynamic friction drag contributes massively to the overall energy consumption. Friction drag in technical flows such as boundary-layer flow past solid bodies is caused by shear stress. Due to the no-slip condition at the body's surface the velocity converges to the freestream velocity in a small region denoted boundary layer. Boundary-layer flow can be either laminar or turbulent. Turbulent boundary-layer flow is characterized by a considerable increase in shear stress compared to the laminar case, thus raising the overall friction drag. Unsurprisingly, it is not desired for many applications in aerodynamics since the dissipation of kinematic energy is significantly higher. Transition in fluid dynamics describes the process in which a flow changes its state from laminar to turbulent. It is common sense that laminar-turbulent transition originates from a stability problem based on the idea that some small disturbances in a laminar base flow grow and eventually lead to a change of the flow regime. If small disturbances attenuate, the flow is considered stable. Otherwise, if the disturbances grow and cause the laminar flow to change into a different state, the flow is unstable. The understanding of the stability problem is crucial since it may initiate the transition to turbulence. It is thus desirable to develop and implement a method for practical analysis of shear flow stability properties. The knowledge of a flow's particular stability properties can be used for the enhancement of laminar flow control applications.

2. Linear Stability Theory

Linear stability theory deals with the response of a laminar flow to disturbances of small amplitude. The linear stability analysis performed in the present work assumes low environmental disturbances and a parallel, two-dimensional, steady base flow $\vec{U} = (U(y), 0, 0)^T$, which is superposed with small wavelike disturbances $\vec{u'}$ and p'. The resulting set of partial differential equations gives rise to the linearized disturbance equations. By taking the divergence of the linearized momentum equations and making use of the linearized continuity equation an expression for the Laplacian operator of the pressure can be obtained. Taking the Laplacian operator of the wall-normal component of the linearized momentum equations and introducing the obtained pressure expression into the resulting equation, a single differential equation for the prescription of the initial development of the wall normal disturbance component v' is obtained.

(1) $$\left[\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) \triangle - \frac{\mathrm{d}^2 U}{\mathrm{d}y^2}\frac{\partial}{\partial x} - \frac{1}{Re}\nabla^4\right] v' = 0$$

In order to describe the evolution of the other disturbance components in the complete three-dimensional flow field this equation is not sufficient. The disturbance evolution in the streamwise and the spanwise direction requires another conservative equation, which is most conveniently given by the wall-normal component of the vorticity equation. Applying the same concepts of superposition with a small disturbance $\Omega' \equiv \Omega'_y = \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x}$ and linearization of the resulting equation gives rise to a second governing differential equation.

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} - \frac{1}{Re}\triangle\right)\Omega' = -\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial v'}{\partial z} \tag{2}$$

The description of small three-dimensional disturbances can thus be reduced to a system of these two linear disturbance equations (see Reeh [1] for further explanation) in the wall-normal velocity component v' and the wall-normal vorticity component Ω'. This coupled initial-boundary-value problem obeys to the homogeneous boundary conditions $\hat{v} = \mathcal{D}\hat{v} = \hat{\Omega} = 0$ at the wall and in the free stream. The initial growth of the small disturbances may be described by the wave modes (Fourier modes)

$$v'(x,y,z,t) = \hat{v}(y)e^{i(\alpha x+\beta z-\omega t)} \tag{3}$$

$$\Omega'(x,y,z,t) = \hat{\Omega}(y)e^{i(\alpha x+\beta z-\omega t)}. \tag{4}$$

Introduction of this normal-mode approach into the two disturbance equations gives rise to the well-known dimensionless Orr-Sommerfeld and Squire equations

$$\left[(-i\omega + i\alpha U)\left(\mathcal{D}^2 - k^2\right) - i\alpha\frac{\mathrm{d}^2U}{\mathrm{d}y^2} - \frac{1}{Re}\left(\mathcal{D}^2 - k^2\right)^2\right]\hat{v} = 0 \tag{5}$$

$$\left[-i\omega + i\alpha U - \frac{1}{Re}\left(\mathcal{D}^2 - k^2\right)\right]\hat{\Omega} = -i\beta\frac{\mathrm{d}U}{\mathrm{d}y}\hat{v}, \tag{6}$$

where $\mathcal{D} = \frac{\partial}{\partial y}$. The resulting relation between the disturbance behavior in time (expressed by the angular frequency ω) and space (characterized by the wave number $k = |\vec{k}| = \sqrt{\alpha^2 + \beta^2}$) physically represents a dispersion relation of the type

$$D(\alpha, \beta, \omega, Re_{\delta_1}) = 0, \tag{7}$$

which can only be solved numerically. In general the quantities α, β and ω are complex. In order to enable eigenvalue analysis two of the three quantities must be known. In the present work the resulting eigenvalue problem is solved for the spatial framework providing the complex streamwise wave number $\alpha = \alpha_r + i\alpha_i$. The angular disturbance frequency ω and the spanwise wave number β are assumed to beprescribed. The spatial approach is appropriate to describe the downstream evolution of instabilities since TS-waves grow spatially in proportion to $\vec{u_r}' \propto e^{-\alpha_i x}$. However, the eigenvalue α appears up to fourth order in equation (5). To reduce the nonlinear eigenvalue problem to a linear one, two consecutive transformations are necessary. The first transformation is a variable transformation of the independent variable y, which eliminates one power of α in the $\left(\mathcal{D}^2 - k^2\right)$ terms by the introduction the an exponential approach

$$\begin{pmatrix} \hat{v} \\ \hat{\Omega} \end{pmatrix} = \begin{pmatrix} \hat{V} \\ \hat{E} \end{pmatrix} e^{-\alpha y} \tag{8}$$

into the differential equations. The second transformation removes the remaining second power of the eigenvalue by the companion matrix method at the cost of larger matrices Eventually a generalized eigenvalue problem of the form

$$\mathcal{L}_{spatial}\vec{\hat{e}} = \alpha\mathcal{M}_{spatial}\vec{\hat{e}} \tag{9}$$

is obtained, where

$$\vec{\hat{e}} = \begin{pmatrix} \alpha\hat{V} \\ \hat{V} \\ \hat{E} \end{pmatrix}. \tag{10}$$

In order to discretize this boundary-value problem for a numerical solution a spectral Chebyshev collocation method is used. The dependent variables $\hat{V}$ and $\hat{E}$ are represented by truncated series of Chebyshev polynomials and a Gauss-Lobatto grid is used for the discrete presentation of the independent variable y. Since the Gauss-Lobatto points are only defined on the finite domain $\xi \in [-1, 1]$ the algebraic function $y(\xi) = \frac{\delta}{2}(1 + \xi)$ maps the grid points into the physical boundary-layer domain $y \in [0, \delta]$. The resulting matrix equations are solved with a built-in MATLAB routine. A more detailed description of the implementation of the numerical method can be found in the works of Reeh [1]. All quantities in equation (9) are non-dimensionalized by using the displacement thickness δ_1 and the velocity at the boundary layer edge U_δ in the case of boundary-layer flow. Note that the figures presented in the following depict dimensional quantities when used. The verification of the numerical method was accomplished by comparing the obtained results to similarity solutions of the boundary-layer equations. Boundary-layer profiles of the Falkner-Skan family were computed with a shooting procedure based on a Runge-Kutta initial-value problem solver. The Falkner-Skan profiles physically represent solutions to the boundary layer past wedges and around edges. Depending on the wedge or corner angle, analogies can be drawn to the important cases of accelerated ($\frac{\partial P}{\partial x} < 0$) and decelerated

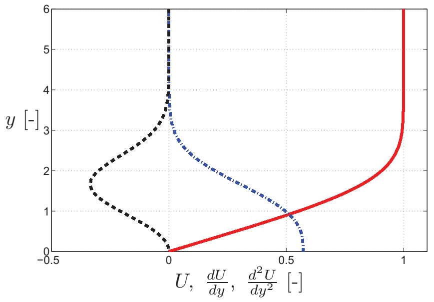

Figure 1: Blasius profile and derivatives.

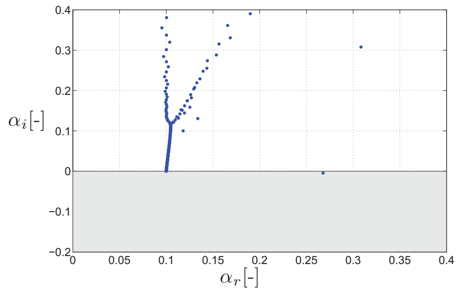

Figure 2: Numerically computed eigenvalue spectrum for the Blasius case at $Re_{\delta_1} = 800$, $\beta = 0.1$ and $\omega = 0.1$.

($\frac{\partial P}{\partial x} > 0$) boundary-layer flows. In addition, the Blasius solution as the most important reference case is included. It physically represents a fully developed laminar boundary-layer flow on a flat plate geometry at zero incidence angle. The generic profiles were then used to verify the modal stability results with references such as Drazin and Reid [2] or Schmid and Henningson [3]. Figure 1 and Figure 2 show the Blasius boundary-layer profile and an eigenvalue spectrum for a particular disturbance in the imaginary plane. In the spatial framework a small disturbance grows exponentially when at least one of the eigenvalues has a negative imaginary part.

3. Application to Laminar Flow Control using Plasma Actuators

Laminar flow control (LFC) is a topic of increasing importance in fluid dynamics. Delaying the laminar-turbulent transition is much more energy effective than relaminarizing separated boundary-layer flow [4]. Therefore, it is highly desirable to influence the flow in an early transition stage, where disturbances are still small, by means of stabilization. In the past, boundary-layer blowing and suction have been extensively investigated (see Frannson [5]). This work presents a relatively new method of affecting Tollmien-Schlichting wave dominated boundary-layer transition by means of Dielectric-barrier discharge actuators. DBD actuators consist of at least two electrodes separated by a dielectric material. If a high AC voltage is applied between the electrodes, a periodic charge buildup on the dielectric surface occurs and causes ionization of surrounding fluid molecules. The charged molecules are accelerated in the electromagnetic field of the weakly ionized plasma and by collision with neutral molecules transfer momentum into the fluid. In quiescent air above a solid surface, this leads to a flow tangential to the wall. If applied in a boundary-layer flow, the actuator adds momentum in the proximity of the wall. The setup and working principle of the actuator in quiescent air is illustrated in Figure 3. As the actuator represents a zero-net mass-flux device, the tangential acceleration is accompanied by a smaller induced wall-normal velocity component. Since the physical mechanisms involved are not yet thoroughly understood, ongoing investigations concentrate on formulating a phenomenological model for the effect of DBD actuators on flow phenomena [7]. Although the plasma is excited by an alternating power supply at frequencies of several kHz, in steady operation mode a quasi-steady body force generates a constant wall jet with an effective origin close to the physical actuator. Duchmann [6] showed that this wall jet can be used to stabilize a laminar boundary layer and delay the transition to turbulence. By controlled modulation of the supply-voltage frequency, a pulsed operation mode of the DBD actuator is obtained. If the actuator is operated at the frequency of incoming disturbances and a correct phase relation is adjusted, TS-waves can be attenuated by destructive interference and transition can be significantly delayed [8],[9]. Additionally, like in the case of steady operation, the mean boundary-layer profiles are affected, resulting in altered stability characteristics.

Currently, researchers at TU Darmstadt are engaged with experimental, numerical and theoretical investigations for a better understanding of the plasma

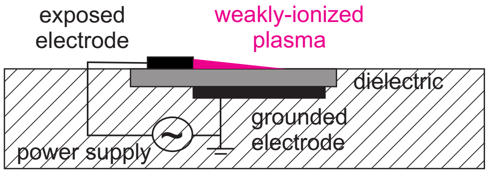

Figure 3: DBD plasma actuator setup and working principle, from Duchmann [6].

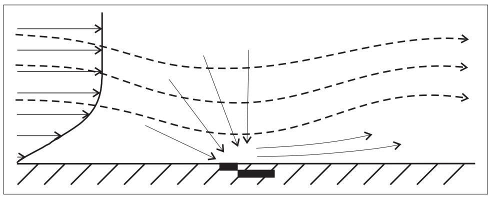

Figure 4: Influence of the plasma actuator on the boundary-layer flow, from Grundmann [8].

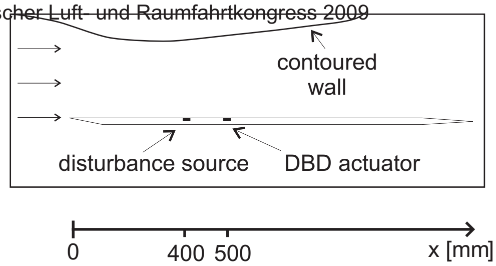

Figure 5: Investigated flow region, from Duchmann [6].

actuator's active principle and an extension of the flow-control applications. The major aim of the present work is to investigate the changes of the boundary-layer stability properties related to the different operation modes, enabling deeper insight into the flow-control mechanisms. In the past, controlled boundary-layer stability experiments were conducted to investigate the actuators ability to delay laminar-turbulent transition. The setup chosen for both the experimental and the numerical investigations is a flat-plate centered in a wind tunnel. The shape of the wind-tunnel walls are chosen to obtain a small adverse pressure gradient (APG) in order to accelerate the process of disturbance amplification. The APG has a destabilizing effect on the boundary-layer flow such that all consecutive stages of transition can be observed within the limited length of the wind-tunnel test section. In the numerical simulation the freestream velocity $U_\infty = 8$m/s at the inlet is constant. 400mm behind the elliptical leading edge of the flat plate, a pulsed disturbance source induces two-dimensional waves ($\beta = 0$) in the base flow. The generated disturbances are similar in amplitude and frequency ($f = \frac{\omega}{2\pi} = 110$Hz) to natural primary instabilities. These so-called Tollmien-Schlichting waves amplify in the boundary-layer initiating the laminar-turbulent transition process. 100mm further downstream, a DBD actuator is situated on the surface of the flat-plate. A principle sketch of the investigated setup relevant for all results presented here is outlined in Figure 5. Details about the experimental and numerical transition investigations conducted with the described setup can be reviewed in the works of Grundmann [8] and Duchmann [6]. In this work the boundary-layer data obtained from the numerical simulations of Quadros [9] are used as an input for the LSA. In order to investigate the plasma actuator's influence on flow stability and to verify existing experimental and numerical results, modal linear stability analysis is conducted. The local manipulation of the boundary-layer profiles by the plasma actuator is expected to have a stabilizing effect similar to a decreased pressure gradient. The stability calculations are intended to show the flow's reduced affinity to become unstable. Active wave cancelation (AWC) by means of DBD actuators has been investigated numerically by Quadros [9]. He used a finite volume method to describe the flow domain. The CFD-code FASTEST (Flow Analysis Solving Transport Equations with Simulated Turbulence) enables large eddy simulations (LES) based on the Germano method. A semi-empirical model of the body force is used to describe the influence of an DBD actuator operated in the pulsed mode. Numerically, the actuator operation can be modeled as an additional body force term in the Navier-Stokes equations. The simulations proved the positive effects observed in Grundmann's experiments [8]. Active wave cancelation could be effectively used to delay Tollmien-Schlichting wave dominated transition. However, the mean boundary-layer profiles showed slight changes compared to the unaffected flow, which could not be attributed to the small fluctuations introduced by the disturbance source. The changes are assumed to originate from an offset body force of the actuator operated in pulsed mode, giving rise to a stabilty analysis decoupled from the wave cancelation mechanism. Since the disturbance evolution in the considered region is still amplitude-independent (and hence linear), the decoupling is justified although the additional fluctuating component induced by the plasma actuator is neglected. The investigations performed in this work utilize the mean boundary-layer profiles derived from Quadros' numerical simulations. The boundary-layer profiles are investigated in the streamwise region 0.4m $< x < 0.6$m. The stability calculations are performed for two cases. In the first case only the disturbance source is active. The plasma actuator is switched off in order to obtain the flow's stability behavior with uncontrolled Tollmien-Schlichting wave growth as reference. In the other case, the DBD actuator is operated to induce momentum in opposite phase to the arriving waves, thereby controlling transition. The total transition delay emerges from two effects, the active wave cancelation and the stabilization of the mean base-flow profiles. However, only the latter effect can be investigated by the presented LSA.

4. Results

Modal analysis investigates the stability properties locally for single boundary-layer profiles. An example for such a local modal solution is given in Figure 6 showing slight shape changes in the streamwise eigenfunctions caused by the actuator. However, these eigenfunctions do not allow quantitative statements since they are normalized with their maximum values. In general the shape of the mean-flow profile changes as the boundary layer develops in the flow domain. This affects the local displacement thickness δ_1 and the edge velocity U_δ. Thus, the local Reynolds number $Re_{\delta_1} = \frac{U_\delta \delta_1}{\nu}$ and the dimensionless angular

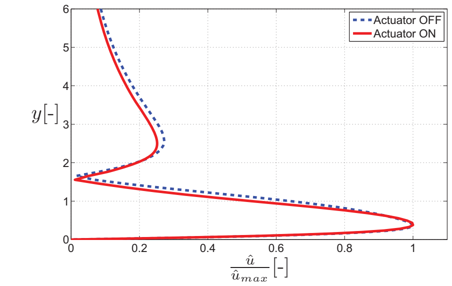

Figure 6: Streamwise eigenfunctions normalized with their maximum values at $x = 540$mm.

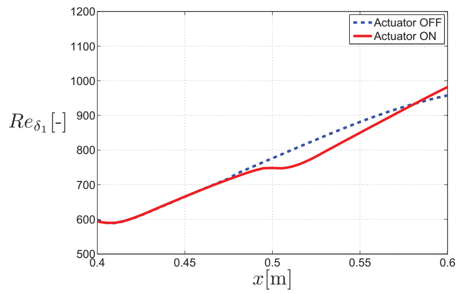

Figure 7: Local Reynolds number evolution for the investigated flow region.

disturbance frequency $\omega = \frac{2\pi f \delta_1}{U_\delta}$ vary. Both serve as input parameters for the dispersion relation (7). Figure 7 demonstrates the streamwise evolution of the local Reynolds number.

Differences between the mean-velocity profiles with and without DBD actuation are small and modifications become apparent in the wall-near part of the derivatives. However the LSA is highly sensitive to those shape changes. For a global demonstration of the plasma actuator's stabilizing effect, neutral curves (curves for parameter combinations which yield zero amplification) of the two considered cases are compared in the ω-Re-parameter space at three different streamwise positions in Figure 8. The observed decline of the instability area downstream of the actuator is similar to the effect of a strong decrease in the streamwise pressure gradient $\frac{\partial P}{\partial x}$. In order to access the stability properties over a certain streamwise distance, a stability calculation for each profile is necessary. Figure 9 clearly demonstrates the positive effect of the plasma actuator over the investigated region with remarkably reduced growth rates for the primary instabilities. The instabilities amplify much less intensely during DBD actuation and thus transition is delayed. Figure 10 shows the N-factor development for a relatively small streamwise region, integrating the dimensionalized instability growth rates obtained from the numerical data, starting from the point of neutral stability. The figure depicts the reduction of streamwise amplitude growth (normalized by the disturbance amplitude at the point of neutral stability) of

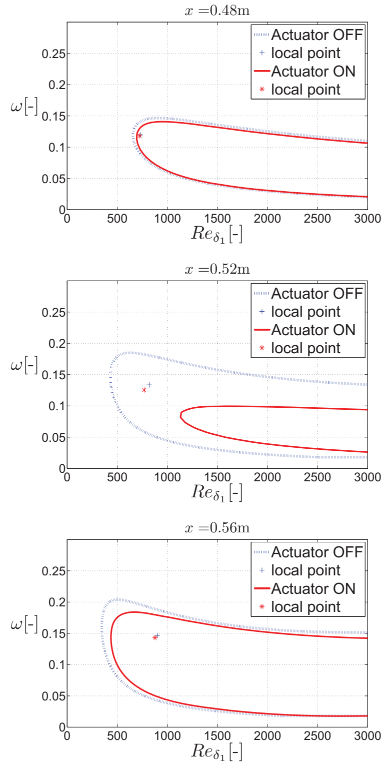

Figure 8: Neutral curves at different streamwise positions.

a an incoming TS-wave subjected to the decoupled effect of the mean boundary-layer manipulation.

5. Discussion

The linear stability method was successfully applied on numerical boundary-layer data yielding insight into the mechanism of transition delay through DBD actuation. The manipulations of the mean boundary-layer profiles caused by the actuation have been investigated locally, not taking into account the wave canceling mechanism. It was clearly shown that the stabilizing effect of the plasma actuator on the mean boundary-layer profiles is an important factor in transition manipulation even when the actuator is operated

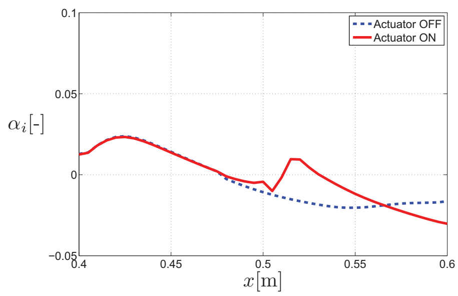

Figure 9: Disturbance growth rates for the investigated flow region

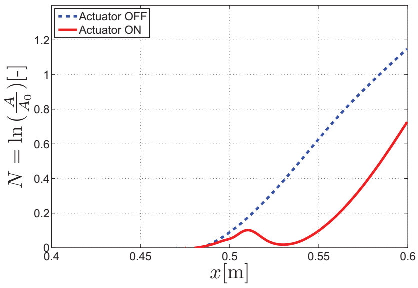

Figure 10: N-factor evolution

in pulsed mode. Wave damping through destructive interference is an additional superimposed and effective mechanism. This suggests a steady manipulation of the base flow with a similar stabilizing effect as a decrease in the streamwise pressure gradient $\frac{\partial P}{\partial x}$. When dealing with stability calculations affected by plasma actuators special consideration must be paid to the flow in the direct electrode proximity. The wall-normal component induced by the actuation is relatively small in the present case compared to the streamwise effect ($V < 0.01 U_\delta$) such that the assumption of wall-parallel flow for LSA is not remarkably violated. The induced pulsation from the plasma actuator was not taken into account as a superposed disturbance. The exposed electrode may also represent a roughness strip possibly leading to the generation of new disturbances through a receptivity process in real experiments. Regarding these discussion points further stability analysis is desirable to improve the understanding of the effects of plasma actuators on laminar flow.

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