Innovative controller design for systems with parameter variations

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1 INTRODUCTION

Aeronautical control systems cover in general a wide range of operating points and can therefore often not be controlled with one single control law, even though for certain cases this might be possible ([1], [2], [3]). An adaption of the controller to the actual operating conditions (e.g. flight attitude) has to be performed. This adaption procedure is called *gain scheduling*. The classical gain scheduling approach consists of synthesizing a set of controllers according to different operating points and performing an interpolation between these points for which the controllers where calculated. It is often necessary to calculate several hundreds or even thousands of controllers to guarantee acceptable performance in the whole flight domain. This procedure has the following disadvantages:

- memory requirements may be costly,
- interpolation can become difficult with increasing number of parameters,
- stability analysis at the nodes can cause problems.

In this paper, a gain scheduling technique is presented and applied, which does not involve the described interpolation procedure, and thus does not incorporate the mentioned disadvantages. The principal of the two different approaches are shown schematically in figure 1. The method is applied to two descriptive examples of aeronautical control problems:

- the longitudinal short period motion (SPO) of a military fighter aircraft,
- the formation flight of two satellites on an elliptic orbit.

The method is based on a certain representation of the systems equations called *L*inear *F*ractional *T*ransformation (LFT). This representation is required for many applications in modern control theory - both, synthesis and analysis - , mainly μ-analysis. The presented work uses this representation and demonstrates the feasibility and advantages of LFT scheduled gains.

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2 THEORY

In this section a brief introduction of the theory on LFTs and the used modal control technique is given.

2.1 LFT representation of variable systems

A linear fractional transformation is a function of a (possibly complex) variable δ of the form:

$$f(\delta) = \frac{a + b \cdot \delta}{c + d \cdot \delta} \tag{1}$$

E.g. the transfer matrix of a linearized system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

connecting the output y with the input u reads:

$$G(s) = C\left[sI - A\right]^{-1} B + D \tag{3}$$

is in a linear fractional form, where the variable is s. Considering the transfer matrix of a linearized system depending on a set of parameters δ_i summarized by a matrix Δ

$$G(s, \Delta) = C(\Delta)\left[sI - A(\Delta)\right]^{-1} B(\Delta) + D(\Delta) \tag{4}$$

a linear fractional form of this matrix reads:

$$\begin{aligned} F_u(M, \Delta) &= M_{21}\Delta\left[I - M_{11}\Delta\right]^{-1} M_{12} + M_{22} \\ &\text{with} \\ G(\Delta, s) &= F_u(M, \Delta) \end{aligned}$$

where the matrix Δ contains the parametric variations δ_i as well as the Laplace variable s (frequency).

Depending on the purpose of the LFT representation of the system (stability analysis, synthesis purposes ...)

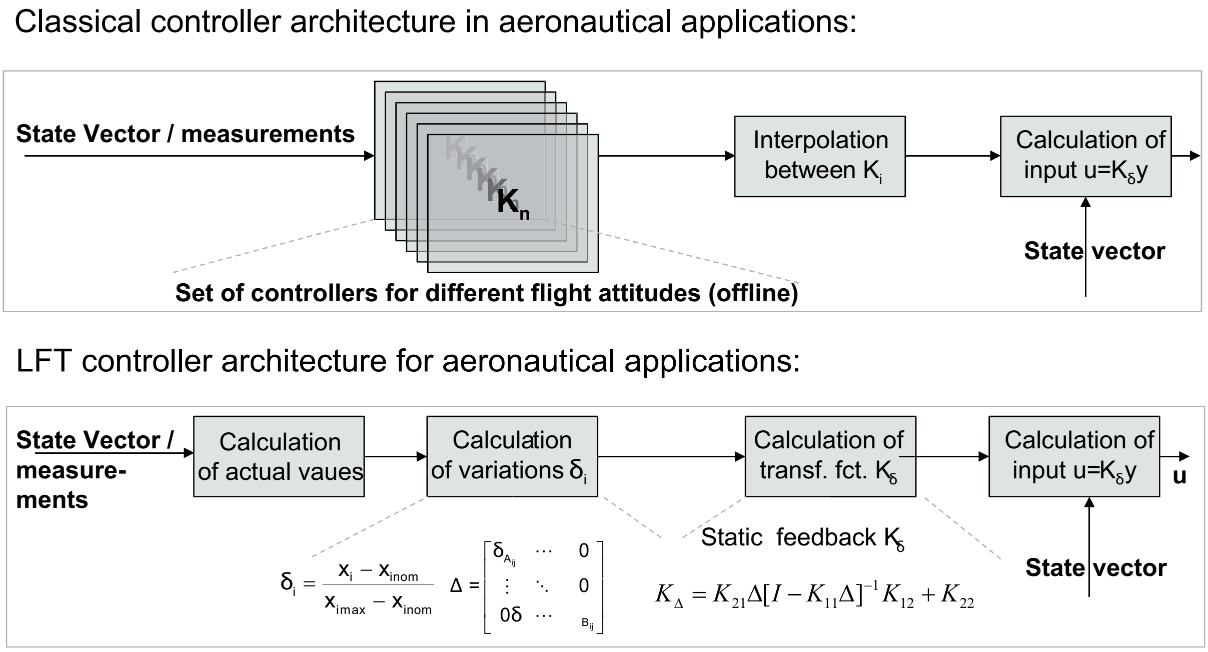

Figure 1: Possible controller structures: classical interpolated controller and LFT scheduled gain.

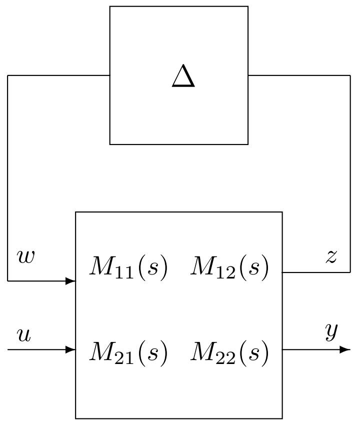

Figure 2: Linear fractional ($M - \Delta$) form

the matrix Δ can be built up by different sets of parameters δ_i. It is important to note that the matrix Δ can contain those parameters, that determine the actual operating point. By using these scheduling parameters it is possible to obtain an LFT representation of the system which covers the whole domain of dependence, parameterized by Δ. Any controller design technique that involves only operations that can be applied to LFTs and that return LFTs is a potential method for LFT gain scheduling. In this case the controller K can be obtained in a linear fractional form parameterized by Δ in the same way as the corresponding system:

(5) $$K(\Delta) \quad = \quad K_{21}\Delta\left[I - K_{11}\Delta\right]^{-1} K_{12} + K_{22}$$

One control technique that fulfills these requirements is the modal control technique. Furthermore, this technique is widely used in aeronautical applications because of the possibility to determine the dynamic properties of the closed loop system in terms of pole placement and decoupling constraints. For the presented framework of LFT modeling and gain scheduling using LFTs a toolbox under MATLAB/SIMULINK has been developed at ONERA in the Flight Dynamics ans System Control Department (DCSD), see [10].

2.2 Modal control theory and gain scheduling using linear fractional transformations

In the following the well-known design procedure (well-known in the matrix case) of the modal control technique is shortly recalled. It can be directly applied to LFT models as it only requires matrix operations that are defined for LFTs and whose operations result in LFTs. Details about the theoretical background can be found in [10].

Design procedure for eigenstructure assignment. Eigenstructure assignment by output feedback for (1) can be performed as follows :

1. Choose p desired closed-loop eigenvalues λ_i and the corresponding matrices $e_i \in \mathbb{R}^{n_u}$, $e_i \neq 0$, $E_i \in \mathbb{R}^{n_u \times n_x}$ and $F_i \in \mathbb{R}^{n_u \times n_u}$.

2. Compute (v_i, w_i), $i = 1, \ldots, n_y$ satisfying:

$$\begin{bmatrix} v_i \\ w_i \end{bmatrix} = \begin{bmatrix} A - \lambda_i I & B \\ E_i & F_i \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ e_i \end{bmatrix} \tag{6}$$

3. Using the following notation for Eigenvectors, Eigenvalues ...

$$\begin{aligned} V &= [v_1 \ldots v_{n_y}] \\ W &= [w_1 \ldots w_{n_y}] \\ &\text{and} \\ \Lambda &= \text{diag}[\lambda_1 \ldots \lambda_{n_y}] \end{aligned}$$

   Compute K satisfying:

$$K = W(C\,V + D\,W)^{-1} \tag{7}$$

Usually, e_i, E_i and F_i are chosen in order to satisfy some decoupling properties.

In the above design procedure, all matrices or scalars can be replaced by LFTs ($A \longrightarrow A(\Delta), B \longrightarrow B(\Delta), \ldots, \lambda_i \longrightarrow \lambda_i(\Delta), V \longrightarrow V(\Delta), \ldots$). Therefore, the feedback gain can be written as $K(\Delta)$ corresponding to the form given by equation (5).
The introduced procedure will be used for the controller synthesis and implementation in LFT form of the two treated examples.

3 APPLICATION 1: CONTROLLER FOR THE SPO OF A GENERIC MILITARY AIRCRAFT

The first example treats the design of a controller in LFT form for the longitudinal inner loop (short period oscillation, SPO). These type of controllers are designed to ameliorate handling qualities and reduce pilot work load.

3.1 Longitudinal aircraft modeling

In this chapter a description of the aircraft model is given (see also [4]). The aerodynamic model and the flight mechanical model are introduced. For the aerodynamic model representative data of a military fighter aircraft during subsonic flight is used. For the flight mechanical model the general rigid body equations of motion are considered, whereas the flexibility of the aircraft structure is only taken into account in the dependency of some aerodynamic parameters on the dynamic pressure (change of effectiveness of the control surfaces). Similar model derivation can be found in [7].

3.1.1 The aerodynamic model of the aircraft

The aerodynamic model is based on data taken from a military fighter aircraft and includes compressibility effects in terms of the dependency of the aerodynamic coefficients on the Mach number. Furthermore, their dependency on the flight attitude (angle of incidence α, side-slip angle β,...) and the aerodynamic inputs, i.e. the deflection of the flight control surfaces is considered.

Force and moment coefficients A brief presentation of the aerodynamic force and moment coefficients is given. As well established, these coefficients are defined in such a manner, that the resulting aerodynamic forces and moments acting in the directions of the body-fixed coordinate system can be calculated by multiplying the dynamic pressure q_{dyn} with the reference area S and the corresponding force or moment coefficient $C_{x,y,z}$ (and the reference length $\bar{l}$ for the calculation of moments respectively) by

$$\begin{aligned} F_{x,y,z} &= q_{dyn} S C_{x,y,z} \\ L, M, N &= q_{dyn} S C_{L,M,N} \bar{l} \end{aligned}$$

For example the aerodynamic force F_x in the x-direction reads:

$$F_x = q_{dyn} S C_x$$

and for the aerodynamic moment around the x-axis, respectively:

$$L = q_{dyn} S C_L \bar{l}$$

Force coefficients The aerodynamic coefficients are split up into terms of aerodynamic derivatives $c_{x\ldots\alpha\ldots}$, whose dependencies on the Mach number are given in tabulated form as discrete data in an aerodynamic database (apart from C_x, which is directly given tabulated as a set of values depending on Ma and α). For the three force coefficients C_x, C_y and C_z the following dependencies are considered:

$$\begin{aligned} C_x &= C_x(\alpha, Ma) \\ C_y &= C_y(\beta, Ma, q_{dyn}, \delta n) \\ C_z &= C_z(\alpha, Ma, \delta m) \end{aligned}$$

For an isolated examination of the longitudinal motion only C_x and C_z have to be considered. The force coefficient C_z is split up as follows:

$$C_z = c_{z\alpha} \cdot \alpha + c_{z\delta m} \cdot \delta m$$

In this equation $c_{z\alpha}$ depends on the Mach number, and is given as a 1-d table. The principle of the calculation of the other aerodynamic coefficients is the same. The appearing discrete data have to be interpolated in order to obtain closed form expressions. This was realized by polynomial interpolation. For $c_{z\alpha}$ this interpolation formula reads:

$$c_{z\alpha} = c_{cz\alpha 1} + c_{cz\alpha 2}Ma + c_{cz\alpha 3}Ma^2 + c_{cz\alpha 4}Ma^3$$

Other coefficients had to be interpolated with significantly higher order to guarantee small errors.

Moment coefficients For the three moment coefficients C_L, C_M and C_N the following form is considered:

$$\begin{aligned} C_L &= C_L(\alpha, \beta, Ma, q_{dyn}, p, r, V, \delta l, \delta n,) \\ C_M &= C_M(\alpha, Ma, q_{dyn}, q, V, \delta m) \\ C_N &= C_N(\beta, Ma, q_{dyn}, p, r, V, \delta l, \delta n) \end{aligned} \tag{8}$$

They are split up into terms of aerodynamic derivatives in the same way as the force coefficients.

3.1.2 The considered equations of motion

As well established, the four degrees of freedom equations of motion for the longitudinal movement are written in terms of the angle of attack and pitch-rate (α, q) for the short period oscillation (SPO) and in terms of the velocity and altitude (V, H) for the phygoid. In this paper only the SPO will be studied. The simplified two degrees of freedom equations of motion for this part of the longitudinal motion read:

$$\begin{aligned} \dot{\alpha} &= \frac{1}{V}\left(g\cos\gamma\cos\Phi + \frac{1}{m}\left[\frac{1}{2}\rho S V^2 \right.\right. \\ &\quad \left.\left. \cdot \left(C_x \sin\alpha - C_z\cos\alpha\right) - F\sin\alpha\right]\right) + q \\ \dot{q} &= \frac{1}{2B}\rho S V^2 \bar{l}\left[C_M + \left(x_{cg} - x_{cg,nom}\right)C_z\right] \end{aligned} \tag{9}$$

where B is the inertia moment around the y-axis, F the engine force, g the gravitational acceleration, $\bar{l}$ the reference length, m the mass, S the reference area, $x_{cg}, x_{cg,nom}$ the actual and nominal center of gravity position, respectively and ρ the air density.
The linearization of the SPO equations yields in a system of the form shown in equation (1).

3.2 Controller synthesis

The controller synthesis is performed according to given requirements detailed in [6], [5]. These requirements are expressed in minimum damping and frequency properties of the closed loop system (α, q) as a function of the flight phase (velocity). In order to simulate maneuvers consisting of a given profile of α, a controller structure shown in figure 3 was chosen. Using this structure one additional pole can be placed, since the introduction of the integrator augments the number of states by one. Since the SPO equations are modeled as a single input system, the Eigenstructure assignment detailed in 2.2 reduces to a pole placement. The poles were placed as a function of the velocity (Mach number) at:

$$\lambda_i = \begin{cases} (-1+j)\cdot(3+3Ma), & i=1 \\ (-1-j)\cdot(3+3Ma), & i=2 \\ -(1.75+1.75Ma), & i=3 \end{cases} \tag{10}$$
$$j = \sqrt{-1}$$

Increasing velocity leads to linearly increasing frequency of the closed loop system, whereas the damping stays constant.

The scheduling variables, with respect to which the controller is adapted, are the Mach number Ma, the altitude h and the angle of attack α. Without entering into details it must be mentioned that the matrix Δ of the implemented controller does not consist of the scheduling variables Ma, α, h but of artificial summarized variables. This was done in order to minimize the complexity of the involved LFT. Details about the performed LFT modeling can be found in [8].

3.3 Implementation and simulation results

The introduced aircraft model with the corresponding LFT controller was implemented under MATLAB/SIMULINK in a non-linear flight simulator. The adaption of the controller to the actual flight attitude can be performed in two different ways. The first way consists of calculating the transfer function matrix $K(\Delta)$ according to equation (5). This calculation requires the inversion of a matrix, which can be disadvantageous with regard to required calculation times. The second way of adapting the controller consists of solving the corresponding linear system of the term containing the matrix inversion. The solution of the linear system can be done iteratively with several algorithms. In the treated example, an implementation with the Richardson method was chosen. The Richardson method uses a fixed point algorithm to solve a linear system of equations (see [9] for details). The simulation results of an extreme maneuver covering the whole flight domain in α are shown in figure 4. The number of iterations required for convergence stays below 10.

4 APPLICATION 2: SATELLITE FORMATION FLIGHT

In the following, the second treated example for the controller design in LFT form is presented. First, in section 4.1 the general mission scenario is introduced. Secondly, in section 4.2 the equations of the relative dynamics of the system are presented.

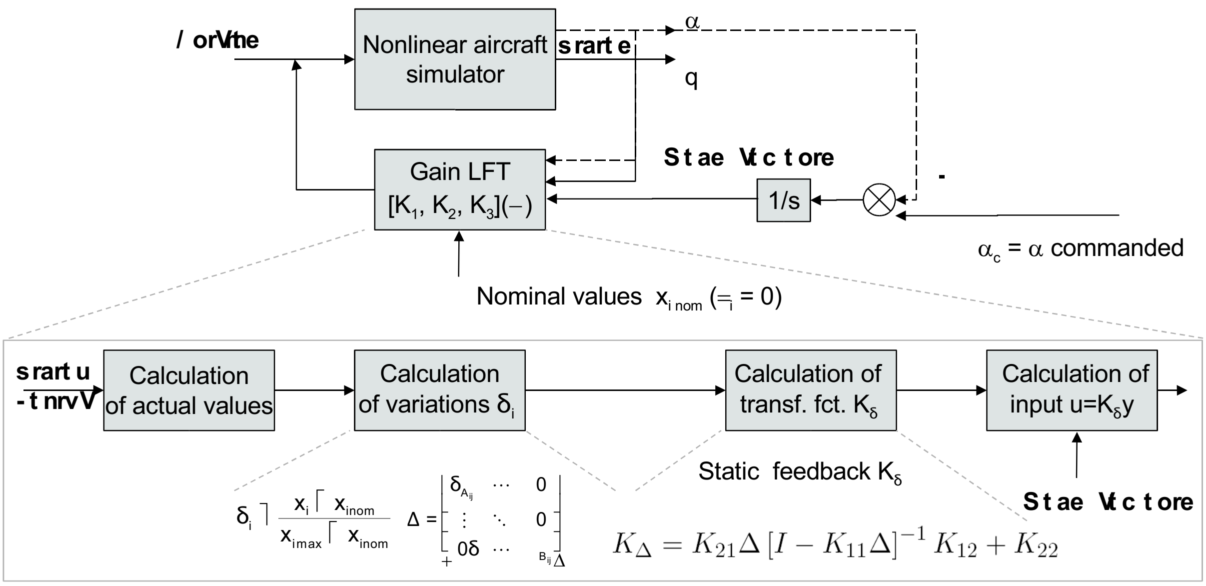

Figure 3: Chosen controller structure with integrator

4.1 Mission Scenario

Formation flying environments range from low earth orbits (LEO, e.g. Techsat 21) to libration point orbits (e.g. Darwin) or heliocentric earth-trailing orbits (e.g. LISA, Laser Interferometer Space Antenna), thus yielding a large spectrum of different perturbations like earth oblateness, solar radiation pressure and atmospheric drag. Elliptic and highly elliptic earth orbits are candidates in several formation flying missions. The mission scenario considered in this paper is a geostationary transfer orbit of two satellites (GTO) as a prototype of earth-centered highly elliptic orbits. In fact, GTOs are suitable orbits for a number of formation flying missions, e.g. MAX. Table 1 indicates the approximate orbital elements a (semi-major axis) and e (eccentricity) of a GTO. As part of the mission, the two satellites are assumed to perform certain maneuvers like changing distance and relative attitude. Formation flying satellites in GTO travel through a variety of different gravitational environments. For instance, the relative position dynamics at the perigee are close to the dynamics of a formation in LEO, whereas the apogee dynamics are somewhat similar to the quiet gravitational environment (almost double integrator behavior) encountered in the vicinity of a libration point. This change of gravitational environment strongly influences the formation dynamics. Details can be found in [11]. This diversity represents a challenge for control design and allows at the same time to demonstrate the applicability of the proposed control design technique to formation flying mission scenarios.

Table 1: GTO orbital elements

Orbital element	Symbol	Value	Unit
semi-major axis	a	24200	km
eccentricity	e	0.72	–

4.2 Formation flying modeling

In formation flying GNC (guidance, navigation, and control) design, the main concern lies in the relative motion between two satellites. In order to derive the equations of the relative dynamics, it is useful to introduce a so-called local vertical-local horizontal (LVLH) coordinate system (cf. Fig. 5).

Here, the vector $\vec{r}$, expressed in the ECI (earth-centered inertial) coordinate system $x-y-z$, describes the position of a reference point supposed to follow a Keplerian orbit. By introducing the true anomaly ν, the semi-major axis a, the eccentricity e, and the earth gravitational parameter $\mu_\oplus$, the identities (11a) to (11d) are obtained. A detailed derivation of the equations of motion can be found in [11].

(11a) $$n = \sqrt{\frac{\mu_\oplus}{a^3}}$$

(11b) $$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

(11c) $$\dot{\nu} = \frac{n(1+e\cos\nu)^2}{(1-e^2)^{3/2}}$$

(11d) $$\ddot{\nu} = -\frac{2n^2 e\sin\nu(1+e\cos\nu)^3}{(1-e^2)^3}$$

The instantaneous position $\vec{r}$ of this reference point defines the origin of the rotating LVLH coordinate system $(r-c-o)$ in which the inter-satellite distance

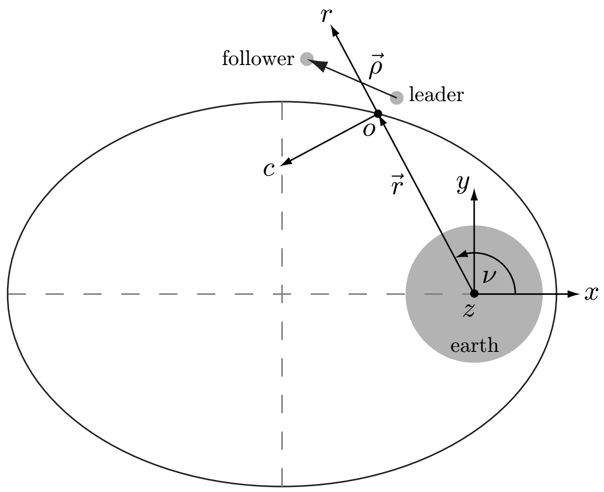

Figure 5: LVLH coordinate system

$\vec{\rho}$ is expressed. Here, r, c and o are the radial, in-track and cross-track distances, respectively. The term "in-plane" motion is used in order to refer to r and c together. o will be called "out-of-plane" motion. This non-inertial frame introduces additional acceleration terms, e.g. Coriolis acceleration. The dynamics can be linearized if the inter-satellite distances are small compared to the semi-major axis. Using the introduced notations, the equations of the relative motion of the two satellites can be derived. Details about the made simplifications and assumptions can be found in [11]. Here just the used equations, that are known as the Lawden equations, are stated. The Lawden equations describe the relative motion between two satellites while the reference point is moving on an elliptic Keplerian orbit:

$$\begin{pmatrix} \ddot{r} \\ \ddot{c} \\ \ddot{o} \end{pmatrix} = \begin{pmatrix} f_r \\ f_c \\ f_o \end{pmatrix} - \frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}} n \cdot \left[\begin{pmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{r} \\ \dot{c} \\ \dot{o} \end{pmatrix} + \frac{(1+e\cos\nu)}{(1-e^2)^{3/2}} n \cdot \begin{pmatrix} -(3+e\cos\nu) & 2e\sin\nu & 0 \\ -2e\sin\nu & -e\cos\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r \\ c \\ o \end{pmatrix} \right] \tag{12}$$

As this system of differential equations features the time-varying parameter ν, it belongs to the class of LPV (linear parameter-varying) systems.

4.3 Controller synthesis

The introduced equations of the relative motion of the satellite formation are parameterized by the time varying parameter ν, the true anomaly. The dynamical properties of the system depend strongly on this parameter. Furthermore, it can be seen that the out-of-plane motion is decoupled from the in-plane motion. In the following only the out-of-plane motion will be considered. In the implemented model, the controller is scheduled with respect to the true anomaly ν.
The dynamical properties of the closed loop system are taken to be constant, the chosen poles read:

$$\lambda_i = \begin{cases} (-8+6j)\cdot 10^{-4}, & i=1 \\ (-8-6j)\cdot 10^{-4}, & i=2 \end{cases} \qquad j=\sqrt{-1} \tag{13}$$

This pole placement is equivalent to a damping ratio of 0.8 and a frequency of 10^{-3}. The size and the structure of the implemented controller in LFT form for of the out-of-plane motion is shown in table 2.

Table 2: Size of the implemented controller in LFT form

LFT	States	Size of Uncertainty Matrix
System	2	6×6
Controller	0 (static gain)	12×12

4.4 Implementation and simulation results

The controller in LFT form was implemented into a simulator under MATLAB/SIMULINK to verify the proper performance. The maneuver that was realized consist of an approach of the two satellites to a final distance of 10 m from an initial distance of 100 m (initial condition). The simulation results are shown in figure 6. The commanded input consists of a step and is marked in red, whereas the time response of the system is marked in blue.

In figure 7 the results of the same simulations are shown, this time expressed in terms of the error $x_{commanded} - x_{actual}$. Furthermore, the results of the same commanded maneuver obtained with controllers synthesized with H_∞ are plotted in figure 7. The results obtained with the H_∞ controllers where obtained by interpolating the gains between the nodes, see also [11]. The number of interpolation nodes was varied between 1 and 100. The results are plotted for these two values. While the LFT gain obtained with the modal control technique is static, the controllers obtained with the H_∞ are dynamic. Furthermore, the feedback connection of the system in LFT form with the corresponding controller yields in an LTI system with respect to the input-output behaviour, whereas the controller and system seperately are each LPV systems. Using the appropiate pregain, the static bahaviour is constant with

an error that tends to zero. In contrast to that, the connection of the LPV system with a set of interpolated gains (no matter with which method they are synthesised) does not yield into an LTI system. By this, the oscillations seen in the curves in figure 7 obtained with interpolated H_∞ controllers can be explained.
The two methods basically differ in the way how an adaption of the controller to the actual operating point is obtained. The clasical technique requires an interpolation of the different calculated gains, whereas the LFT scheduled gains are adapted via the calculation of either the transfer matrix involving a matrix inversion or via the solution of a linear system (iteratively via a fixed point algorithm).
In the treated example, the controllers obtained with the H_∞ technique could also take into acount modal spefications of the closed loop system by weighting the "error" between the outputs of the system and the outputs of a defined benchmark system.
It can be seen that the LFT gain performs best with respect to the errors encountered within one orbit. However, it is not proven that the LFT controller also performs optimal with respect to other requirements, like e.g. fuel consumption.

5 CONCLUSION AND OUTLOOK

In this paper a design method for controller was introduced, that allows for covering the whole domain of dependence of control systems with one single controller without performing any interpolation. The method was applied to two realistic and complex examples of aeronautical and space system control. In the first example of the military fighter aircraft, the proper performance of the controller was shown in non-linear simulations covering the whole domain of dependence.
In the second example of a satellite formation flight control problem, the performance of the implemented controller in LFT form was compared to results obtained with the classical interpolation approach. It was shown that one single controller in LFT form performs better than a set of 100 interpolated controllers.
The advantages of the method were detailed and illustrated by extensive simulations covering the whole operating domains.

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References

[1] J.-M. Biannic, C. Roos, A. Knauf
Design and Robustness analysis of Fighter Aircraft Control Laws
European Journal of Control - special issue on LMI techniques in Control, Vol. 12, No. 1, January 2006, pp. 71-85

[2] A. P. Feuersänger
Design of a Robust Back-Up Controller for an Aircraft with Reduced Stability
Proceedings of the AIAA Guidance, Navigation and Control Conference and Exhibit, Keystone, CO, USA, 2006

[3] U. Shaked
An LPD approach to Robust H_2 and H_∞ Static Output Feedback Design
IEEE Transactions on Automatic Control, Vol. 48, No. 5, 2003, pp. 866-872

[4] R. Brockhaus
Flugregelung
Springer, 1998

[5] FAR Part 25
Airworthiness Standards, Transport Category Airplanes
Federal Airworthiness Regulations, FAA, USA

[6] MIL-F-8785C
Military Specification - Flying qualities of piloted airplanes
Federal Airworthiness Regulations, FAA, USA

[7] A. P. Feuersänger
Control of an aircraft with reduced stability
Technical report, ONERA-DCSD, Toulouse, France, 2005

[8] A. Knauf, C. Döll, and J.-F. Magni
LFT-Modelling of the Longitudinal Motion of a Generic Military Aircraft
European Conference for Aero-Space Sciences, July 4-7, 2005, Moscow, Russia

[9] J.-F. Magni, A. Knauf, and C. Döll
Practical Implementation of LFT Scheduled Gains
6th Asian Control Conference, July 18-21, 2006, Bali, Indonesia, 2006

[10] J.-F. Magni
Linear Fractional Transformation Toolbox - Modelling, Order Reduction, Gain Scheduling
Technical report TR 6/08162, ONERA-DCSD, Toulouse, France, July 2004

[11] S. Gaulocher, J.-P. Chrétien, C. Pittet, M. Delpech, and D. Alazard
Closed-Loop Control of Formation Flying Satellites: Time and Parameter Varying Framework
2nd Int. Symposium on Formation Flying Missions & Technologies, September 14-16, 2004, Washington, DC, USA

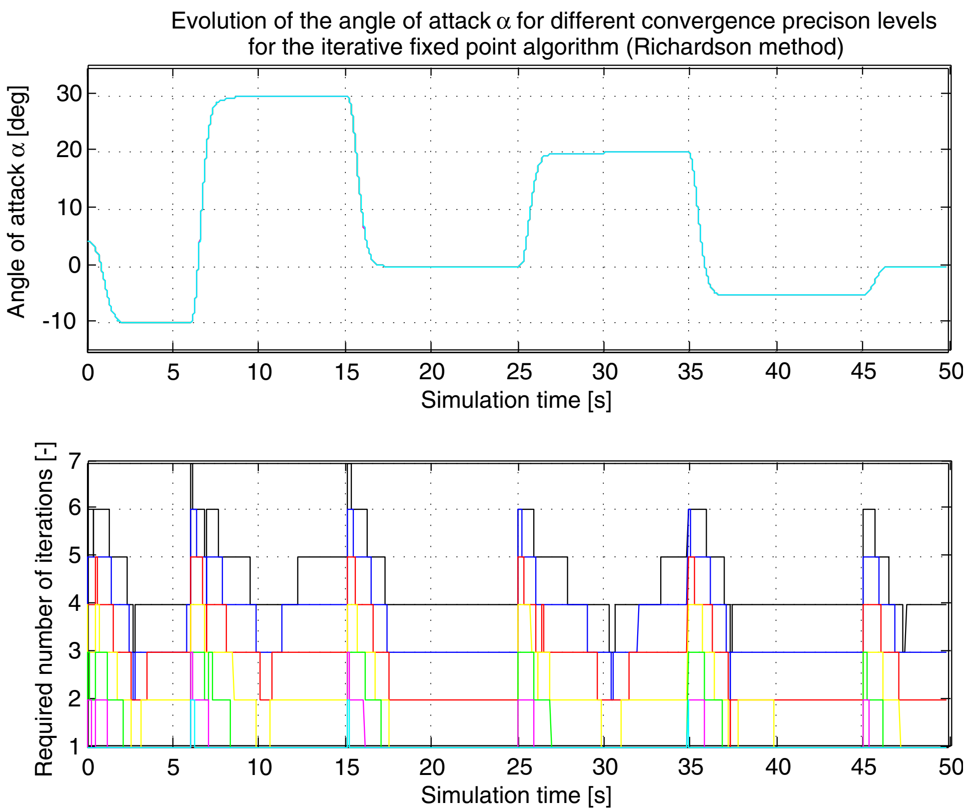

Figure 4: Time response on a predefined maneuver

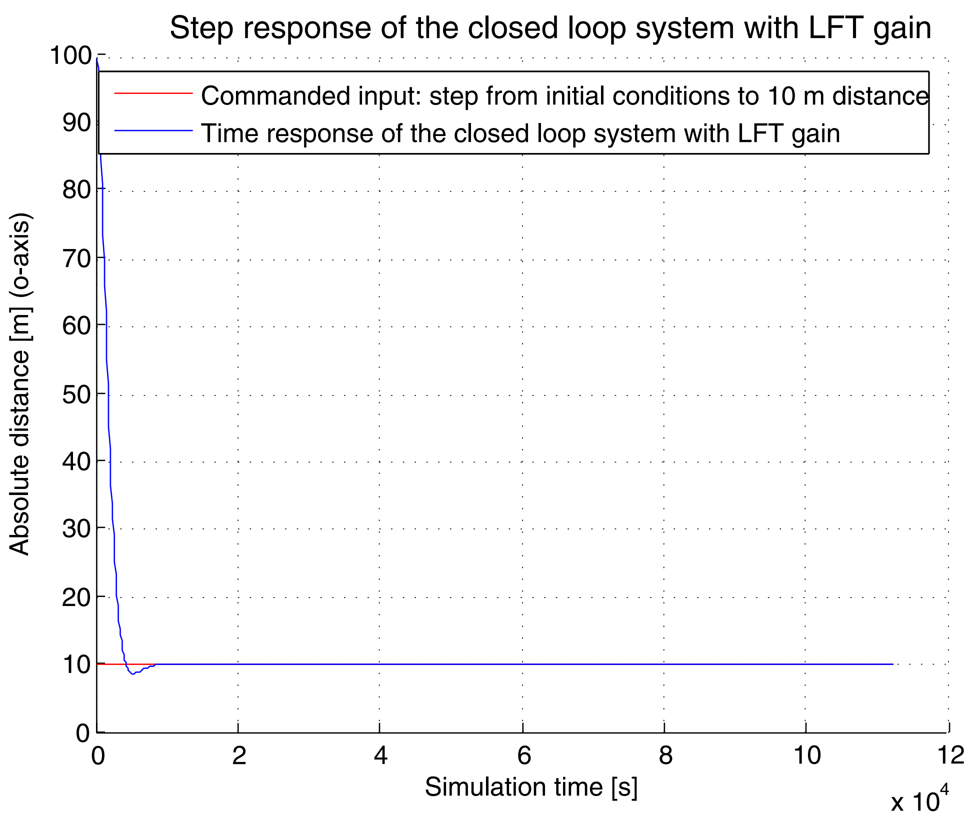

Figure 6: Time response on a predefined maneuver

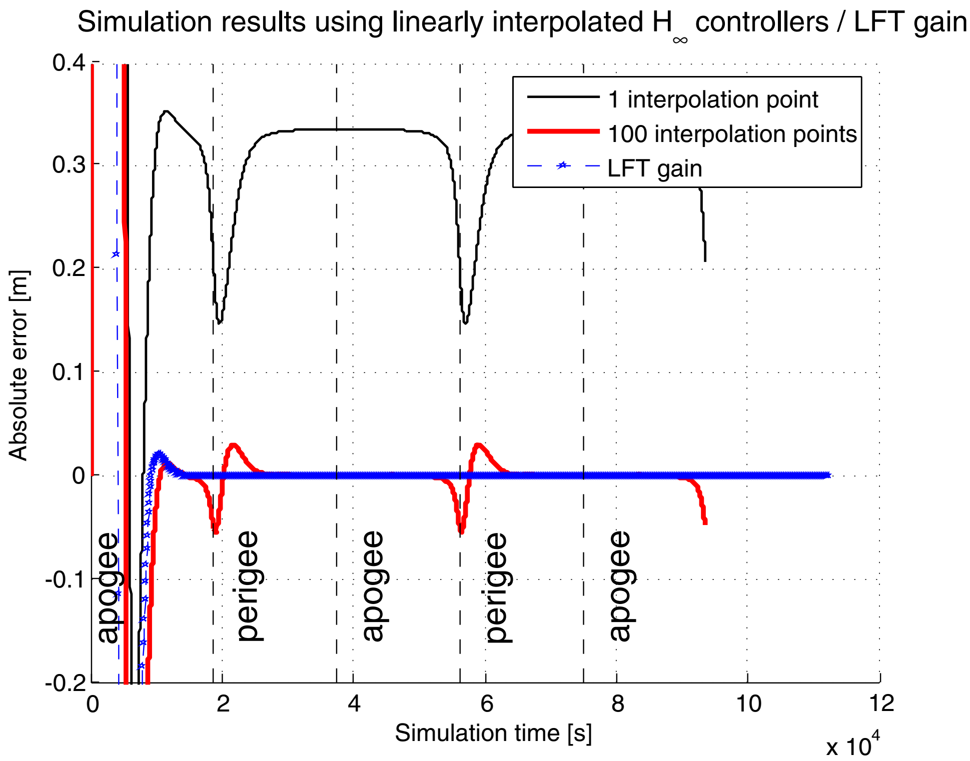

Figure 7: Comparison of results obtained with interpolated H_∞ controllers and with LFT scheduled gain