HIGH ACCURATE POST-MISSION DATA FUSION OF INERTIAL DATA AND DIFFERENTIAL GNSS MEASUREMENTS

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ABSTRACT

This paper addresses the problem of post-processing high accurate differential GNSS and inertial data. Several techniques are discussed which improve the navigation accuracy during GNSS outages or periods with insufficient satellite availability. These techniques are tight coupling with GNSS carrier phase ambiguity resolution and smoothing of the measurement data. Tight coupling improves the aiding of the navigation Kalman filter, especially if less than four satellites are available. The LAMBDA method is used to resolve the GNSS carrier phase ambiguities. Smoothing of the measurement data is used to bridge GNSS outages and to improve the overall accuracy by means of post-processing the data. Measurement results from a navigation system that incorporates all these techniques in a tightly coupled manner are also presented.

1. INTRODUCTION

Continuous availability of GNSS measurements is crucial to an integrated navigation system. If the GNSS signal is lost, errors will increase very fast. This is a serious problem for surveying and remote sensing applications in which continuous satellite availability can not be taken for granted.

High accuracy applications require the use of a differential GNNS system and the resolution of the carrier phase integer ambiguities. Such a system can easily be made of an off-the-shelf real time kinematik receiver. But this system setup permits only loose coupling of the inertial navigation system with GNSS.

Loose coupling has two main disadvantages: It requires at least four available satellites for aiding and the integer estimation and validation process does not benefit from additional knowledge of the dynamics which is provided by the inertial navigation system. This may become a serious problem in urban areas, where an increased multipath level and short times of satellite availability complicate ambiguity resolution and validation. The tightly coupled approach provides better aiding, but has the disadvantage, that carrier phase ambiguity resolution can not be done by a RTK-receiver and has to be implemented as a part of the central navigation filter.

If outages occur frequently the accuracy can be further improved by means of post-processing the measurement data with a smoother.

This paper discusses these techniques which empowers a navigation system to maintain an accurate navigation solution in case of insufficient satellite availability. The following section describes a system which uses tight coupling and smoothing together with differential carrier phase processing and ambiguity resolution. The principles of these techniques are presented, and different types of coupling are discussed. The third section discusses measurement results which were taken from a field trial.

2. GNSS/INS INTEGRATION

The advantages of integrated navigation systems are well known: GNSS, if used as aiding sensor, provides long-term accuracy, but requires a direct line of sight to the satellites. Therefore, outages occur frequently. Inertial navigation is autonomous and provides short-term accuracy, with high update rates.

If both navigation systems are integrated, they compensate the disadvantages of each other. So long-term accurate position, attitude and velocity information can be provided with high update rates.

It is common practice to use a Kalman filter (KF) to accomplish the data fusion. Beside the Kalman filter, other filtering concepts are also known. These often account for the non-linearities of the system model, i.g. sigma point filters. But the nonlinearities are very moderate and, in general, these filters have shown identical performance in integrated navigation systems [1].

Several KF implementations are known. A widely used implementation is the closed loop error state space KF. A strapdown inertial navigation system calculates continuously the position, velocity and attitude. As additional measurements become available, the KF estimates the errors of these quantities. In a closed loop design these errors are fed back to the strapdown mechanization. That is the preferred way of integration with low cost or tactical grade IMUs, because otherwise increasing attitude errors would become a serious problem.

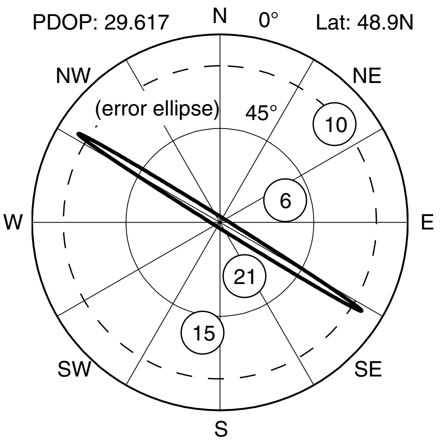

Fig. 1. Skyplot and corresponding position error of a poor satellite geometry.

2.1. GNSS Integration

The main differences arise from the kind of sensor integration. Three concepts of GNSS integration are known: Loose, tight and deep coupling.

Deeply coupled or ultra-tightly coupled systems aim to close the GNSS tracking loops by the central KF. This provides the possibility to improve the signal acquisition and tracking capabilities, especially under rough conditions. That are high accelerations and the presence of jamming or spoo ng signals. As deep coupling requires access to the tracking loops of a GNNS receiver, it is not possible to implement such a system with an off-the-shelf GNSS receiver and it is not applicable in post processing. Therefore deeply coupled systems are beyond the scope of this paper.

Loose coupling is the simplest way of integrating a GNSS receiver into a navigation system. The GNSS receiver is doing all the work here. It calculates a position x and provides a velocity measurement in the the local level frame. The great advantage of loose coupling is the simplicity of its implementation, because no advanced understanding of GNSS details is necessary. The disadvantage of this concept is, that aiding of the navigation system is only possible if four or more satellites are in view. But even if four satellites are in view the concept suffers in present implementations from the fact that the errors of the GNSS position xes are not modeled correctly. Most GPS receivers provide only variances in direction of the axes of the coordinate system, that is either the ECEF-frame or the local level frame. Generally, this is not a big problem if many satellites are in view and the GDOP is low. But in case of a poor satellite constellation, this results in a very conservative estimation of the position errors.

This should become clear with an example taken from a eld test. Fig. 1 shows a very bad satellite geometry with a PDOP greater than 29 and the corresponding position error. The constellation occurred when a car drove along a road with trees at both sides. On the left part of Table 1 are the DOP values with respect to the local level frame. They are very poor. If the DOP values are calculated in a coordinate frame which coincides with the main axes of the error ellipsoid (see left side of Table 1) it can be seen, that the accuracy of only one dimension is bad. The DOP values of the other dimensions are good. That means, that in this case a KF could be aided very well in two dimensions, even though the GDOP is extremely high. But this is only possible, if a full variance-covariance matrix is provided by the GNSS receiver, and not only the variances. But most GNSS receiver don't provide this information. Hence, the aiding is suboptimal with poor satellite geometries which occur very often in urban canyons or on roads with trees at both sides.

local level frame			main axes off error ellipsoid		
north	east	down	x	y	z
15.5	24.7	5.4	0.78	2.4	29.5

Tab. 1. DOP Values, calculated with respect to two different coordinate frames.

The example could also be used to explain the effect of aiding with only three satellites. It can be seen that in Fig. 1 all the satellites are located near a line. If they come closer to the line, the error ellipse will become more and more elongated until it opens at two sides and one DOP value approaches in nity. This would be the same effect if only three satellites are available. Then partial aiding in two dimensions is possible, if tight integration is used.

With regard to high accuracy applications, the big advantage of a loosely coupled system, beside these drawbacks, is, that it can easily bene t from the accuracy of an RTK differential GNSS system. No changes have to be applied to the loosely coupled KF implementation. For these reasons loosely coupled systems are a good choice for aerial applications. The drawbacks are not relevant here.

A tight coupled GNSS/INS system does all the GNSS calculations by itself and does not rely on the GNSS receiver capabilities. Pseudoranges, carrier phase and instantaneous doppler measurements are processed by the KF instead of the position and velocity xes. This involves that the ephemeris data has to be collected from the GNSS receiver and that satellite positions have to be calculated, if not provided directly by the GNSS receiver. The advantage of this approach is, that the position error due to the satellite geometry is inherently correctly evaluated, because of the more detailed measurement model. Even if less then four satellites are available, aiding is possible.

2.2. Tight Coupling and Differential GNSS

As mentioned before the big advantage of a loosely coupled system is, that it can easily be extended to a differential GNSS system. The main dif culties of the implementation of a tightly coupled differential GNSS system arise from the necessity to estimate and resolve the GNSS carrier phase integer ambiguities.

Therefore, the system state of the KF presented in this paper is divided in two parts: The navigation states and the states which represent the integer ambiguities. The navigation states contain elements for the position, velocity, attitude, bias and scale factor errors.

(1)
$$\boldsymbol{x}_{\text{Nav}} = (\Delta\boldsymbol{x}_{\text{position}},\ \Delta\boldsymbol{x}_{\text{velocity}},\ \Delta\boldsymbol{x}_{\text{Euler angle}},\ \Delta\boldsymbol{x}_{\text{bais acc}},\ \Delta\boldsymbol{x}_{\text{bais gyro}},\ \Delta\boldsymbol{x}_{\text{SF acc}},\ \Delta\boldsymbol{x}_{\text{SF gyro}})^T$$

The part for the ambiguities contains the differences of the carrier phase ambiguities of all the satellites in view.

(2)
$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_{\text{Nav}} \\ \hline \Delta\boldsymbol{N} \end{pmatrix}$$

The states of the ambiguities must frequently be adjusted, to account for changes in the amount of available satellites. Satellite differences are used to eliminate the receiver clock error from the measurements. Therefor a base satellite is chosen to carry out the differences. All differences are calculated with that satellite. Assume that N_{ik} is the difference of the carrier phase measurements of the satellites i and k and k is the base satellite.

(3)
$$N_{ik} := N_i - N_k$$

If the base satellite vanishes behind the horizon or is covered, a new base satellite must be selected. If the new base satellite was visible before, it is possible to calculate new ambiguities with respect to the new base satellite:

(4)
$$N_{ij} = N_{ik} - N_{jk}$$

Now, j is the new base satellite. The variances and covariances must be recalculated appropriately. If the new base satellite was not visible before, all ambiguities must be reinitialized.

The pseudoranges and carrier phase measurements are differentially corrected, and therefore should be free of common mode errors, if the base line is not too long.

If a new satellite becomes available, the state vector is extended with a new ambiguity state. The new value, its variance and covariances must be initialized carefully to prevent the KF from loosing time with adjusting these values.

2.3. Ambiguity Resoluiton

Determination of the position with centimeter accuracy is only possible if the integer ambiguities are resolved. Today, the LAMBDA (least-square ambiguity decorrelation adjustment) [2] [3] method is the state of the art method for carrier phase ambiguity resolution. Its advantages are high reliability, fastness and a moderate computational burden which enables real time applications.

The KF oating estimation of the integer ambiguity vector and its variance-covariance matrix are used as input for the LAMBDA method. The main principle of the LAMBDA method is to nd the most likely integer ambiguity vector. The probability density function of the Gaussian distributed double differenced ambiguity vector is

(5)
$$f(\bar{\boldsymbol{N}}) \sim e^{-\left(\bar{\boldsymbol{N}}-\hat{\boldsymbol{N}}\right)^T \boldsymbol{Q}^{-1}\left(\bar{\boldsymbol{N}}-\hat{\boldsymbol{N}}\right)}$$

with

(6) $\hat{\boldsymbol{N}} =$ oat estimation

(7) $\boldsymbol{Q} =$ variance-covariance matrix of $\hat{\boldsymbol{N}}$

The LAMBDA method maximizes the probability function, by means of minimizing the quadratic norm of residuals:

(8)
$$\bar{\boldsymbol{N}} = \max_{\bar{\boldsymbol{N}}} f(\boldsymbol{N}),\ \bar{\boldsymbol{N}} \in \mathbb{Z}^n$$

(9)
$$= \min_{\bar{\boldsymbol{N}}} \left(\bar{\boldsymbol{N}} - \hat{\boldsymbol{N}}\right)^T \boldsymbol{Q}^{-1} \left(\bar{\boldsymbol{N}} - \hat{\boldsymbol{N}}\right),\ \bar{\boldsymbol{N}} \in \mathbb{Z}^n$$

The LAMBDA method provides an algorithm which decorrelates the ambiguities prior to the search. Hence, the computational burden is reduced. Once a solution is found, it is compared to the second best solution. Only if the ratio test in (10) is passed, the ambiguities are accepted.

(10)
$$\frac{\chi^2_{\text{best}}}{\chi^2_{\text{2nd best}}} \geq c$$

with

(11)
$$\chi^2 = \left(\bar{\boldsymbol{N}} - \hat{\boldsymbol{N}}\right)^T \boldsymbol{Q}^{-1} \left(\bar{\boldsymbol{N}} - \hat{\boldsymbol{N}}\right)$$

Otherwise, the solution is rejected and the ambiguities remain part of the KF state vector.

The LAMBDA method uses only the oat estimation of the ambiguities and the information in the variance-covariance matrix. But the carrier phase ambiguities are strongly correlated with each other and with the position error, because the pseudorange and carrier phase measurements depend on the position of the GNSS antenna. Therefore ambiguity resolution bene ts from a small position error, which helps to pass the ratio test. An integrated navigation systems can bridge short outages, i.e. caused by traf c signs or bridges, by inertial navigation. A GNSS

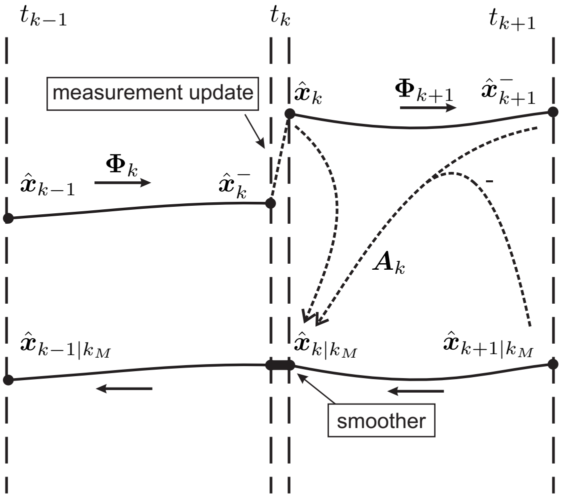

Fig. 2. Forward and reverse run of a RTS smoother.

only system must reinitialize the position with pseudoranges, but they may suffer from strong multipath errors, because carrier smoothing is not effective immediately after an outage.

Under good conditions the ratio test is passed after a few seconds. But there are many in uences that can prolongate the time to x or even inhibit resolution of the carrier phase ambiguities. These are a small amount of satellites, a poor GDOP or multipath errors. Such problems are typical for urban environments.

The aim of a post-processing system is rst to resolve as many ambiguities as possible and then to calculate an optimized trajectory. For that, resolved ambiguities are saved to le. The KF used here has the ability to process the measurement data forward and backward in time.

If a backward run is initiated after a forward run, the resolved ambiguities are obtained from the le a soon as a new satellite becomes visible. Every epoch, known ambiguities are written to the le. Thus the ambiguities already resolved in the forward run are lled up backward in time.

2.4. RTS - Smoother

The Rauch-Tung-Striebel smoother (RTS) [4] was rst presented in 1965 a few years after R. E. Kalman presented his lter in 1960 [5]. The Kalman lter is a recursive lter that optimally, by means of maximum-likelihood, estimates the state vector of a dynamic system on the condition that a linear (or linearized) system model, past and current noisy measurements are known. In comparison to the KF, the RTS smoother calculates the most likely estimation of the state vector of a linear dynamic system on the condition that past, current and future measurements are known. Therefore it is only applicable in post processing, but promises improved accuracy, especially during GNSS outages.

The implementation of a RTS smoother may be regarded as an add-on to the Kalman lter. The data processing can be divided in two steps. In the rst step the RTS smoother acts as an Kalman lter, but every time the system state vector changes, it and its variance-covariance matrix is logged into a le. If the system state changes in the propagation step of the KF, the state vector and the propagation matrix are logged, too. But if the state changes due to a measurement update, no measurement data is logged, because it is not needed by the algorithm.

In the second step, only the logged data is used by the algorithm. It is processed backward in time. The algorithm recursively estimates a new system state, by means of maximum likelihood. Fig. 2 illustrates how the data is used. The upper part shows the forward run and the lower part the reverse run and the involved variables. The meanings of the variables are as follows:

t_k	Time of measurement update or system state update, if the system state is updated between measurements.
$\hat{\boldsymbol{x}}_k^-$	Estimation of the system state at t_k on the condition that the measurements of t_1, t_2, ..., t_{k-1} are known.
$\hat{\boldsymbol{x}}_k$	Estimation of the system state at t_k on the condition that the measurements of t_1, t_2, ..., t_k are known.
$\hat{\boldsymbol{x}}_{k\|k_M}$	Estimation of the system state at t_k on the condition that the measurements of t_1, t_2, ..., t_{k_M} are known. $k_M \geq k$
$\boldsymbol{\Phi}_k$	Propagation matrix: $\hat{\boldsymbol{x}}_k^- = \Phi_k\, \hat{\boldsymbol{x}}_{k-1}$.
$\boldsymbol{A}_k$	Weighting matrix.

Between two GNSS measurements the KF predicts the system state $\hat{\boldsymbol{x}}_k^-$, by means of inertial navigation. When a new GNSS measurement becomes available, the KF calculates a new corrected prediction $\hat{\boldsymbol{x}}_k$. But the longer the time between the measurements is, the greater the errors will become. Hence the KF trajectory contains great discontinuities. The smoother has the ability to level these discontinuities, by means of a new weighting of the previously calculated system states $\hat{\boldsymbol{x}}_k^-$, $\hat{\boldsymbol{x}}_k$ and $\hat{\boldsymbol{x}}_{k|k_M}$. The nal trajectory therefore is smooth and contains no discontinuities.

The full smoother equations are:

(12) $$\boldsymbol{A}_k = \boldsymbol{P}_k \boldsymbol{\Phi}_k^T \left(\boldsymbol{P}_{k+1}^-\right)^{-1}$$

(13) $$\hat{\boldsymbol{x}}_{k|k_M} = \hat{\boldsymbol{x}}_k + \boldsymbol{A}_k \left(\hat{\boldsymbol{x}}_{k+1|k_M} - \hat{\boldsymbol{x}}_{k+1}^-\right)$$

were $\boldsymbol{P}$ denotes the covariance matrix of $\vec{x}$. $\boldsymbol{P}_{k|k_M}$ can be calculated as follows, but is not needed for the algorithm:

(14) $$\boldsymbol{P}_{k|k_M} = \boldsymbol{P}_k + \boldsymbol{A}_k \left(\boldsymbol{P}_{k+1|k_M} - \boldsymbol{P}_{k+1}^-\right) \boldsymbol{A}_k^T$$

The implementation of an GNSS/INS smoother must be adjusted to t the requirements of an error state space KF. The lter propagates the position using the inertial data between measurement updates. After each update, the error state vector is applied to the absolute values and afterwards set to zero. Therefore not the state vector itself is logged, but the absolute values of the position, velocity, attitude, biases and scale factors. In a smoother step, the system state must also be back propagated, by means of inertial measurement data. If the point of a measurement update is reached, all absolute state variables have to be transformed into error values with respect to the absolute values corresponding to $\hat{\boldsymbol{x}}_{k+1|k_M}$. Then (13) can be used to calculate $\hat{\boldsymbol{x}}_{k|k_M}$.

2.5. System description

Fig. 3 shows a ow chart of the overall system. If it acts as an KF, the logged measurement data can be processed forward or reverse in time, while resolved ambiguities are loaded from or saved to a le. All the state and covariance data of the permanent state variables is also written to a le for further use in smoother mode. The states of the ambiguities are neglected, because they are of no use in the smoother pass. The inertial data is also logged with the state and covariance data for an easier usage by the smoother. If the system acts as an smoother, only the logged data is used. The number of passes and the direction in KF mode, can be chosen arbitrary. After a KF pass, it is always possible to invoke a smoother pass.

3. MEASUREMENT RESULTS

To test the algorithm, a eld test was conducted, with an Ashtech Z-FX dual frequency GPS receiver and a tactical grade IMU mounted in a car. The test trial lasted approximately one and a half hour. It started and stopped at the fairground of Karlsruhe. The fairground has an open view, and allows to conduct alignment maneuvers. Because of the ability to process the measurement data forward and reverse, alignment maneuvers were conducted at the beginning and the end of the trial. The test drive went through Karlsruhe, highways and the foothills of the northern Black Forest. The GPS measurements were differentially corrected with a base station at the University of Karlsruhe. The maximum antenna separation was about 15km.

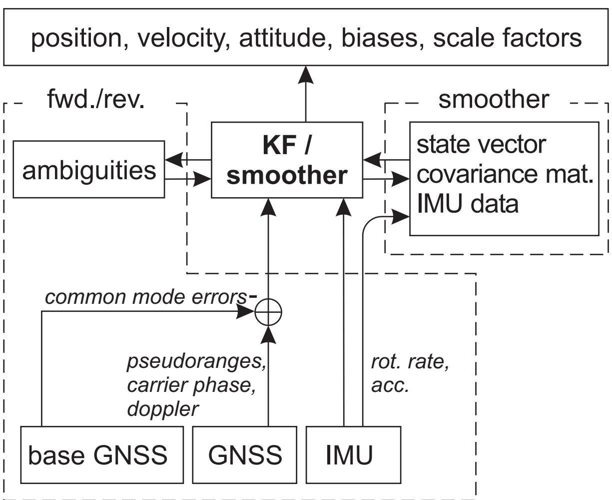

Fig. 3. Flow chart of a GNSS/INS smoother.

The initial alignment is derived by means of the RTS smoother. First the lter is initialized with a coarse estimate of the heading and the position. The other values are set to zero, e.g. roll and pitch angles. After processing the rst minutes which include the alignment maneuvers, the RTS smoother is invoked. After that accurate attitude and position values of the beginning of the trial are available, which were further used as initialization values for the following analysis.

3.1. Ambiguity resolution

To resolve most of the ambiguities it is necessary to process the data with the KF at least twice. One time forward and a second time reverse to ll up known ambiguities. The second run can already be used to save the data for the RTS smoothing algorithm. Hence, at least three passes are necessary to resolve most of the ambiguities and calculate a smoothed solution. Table 2 shows the amount of resolved ambiguities. The trial consists of 4739 GPS epochs with a total of 16442 double differenced ambiguities. In the rst pass 61.7 percent of the ambiguities were resolved. The second pass resolved another 25.7 percent of the ambiguities. But if a third run was invoked, still another 0.9 percent of the ambiguities were resolved. That are 145 ambiguities. The forth run resolved no further ambiguities.

An explanation for this behavior is the in uence of the position accuracy on the ambiguity resolution process. Because of the frequent outages the trial can be divided into separated periods with at least four available satellites. The carrier phase ambiguities are constant at the time a satellite is continuously available. Hence, if the LAMBDA algorithm succeeds once, all the ambiguities in such a period can be resolved. If a period is very short ambiguity resolution may fail, especially if the outage before the period lasted a longer time and the strapdown position therefore becomes inaccurate. But if the outage after the period is very short, it may be possible to resolve the ambiguities in the reverse pass, because the inertial system then provides an accurate position, especially if it was possible to resolve the ambiguities of the following period. If the ambiguities of such a period are resolved in the reverse pass, it is necessary to invoke a third pass to ll up the ambiguities in the forward direction.

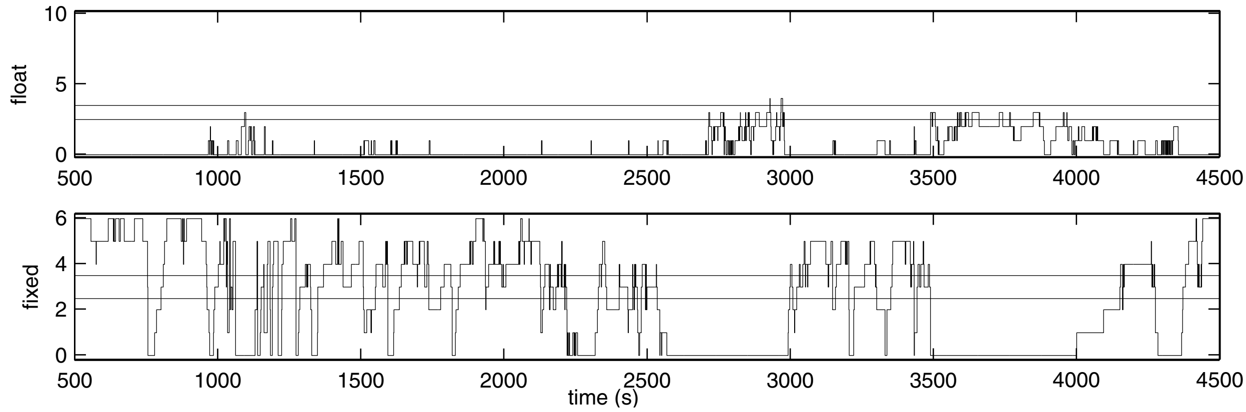

Fig. 4. Float and xed double differenced ambiguities after three passes.

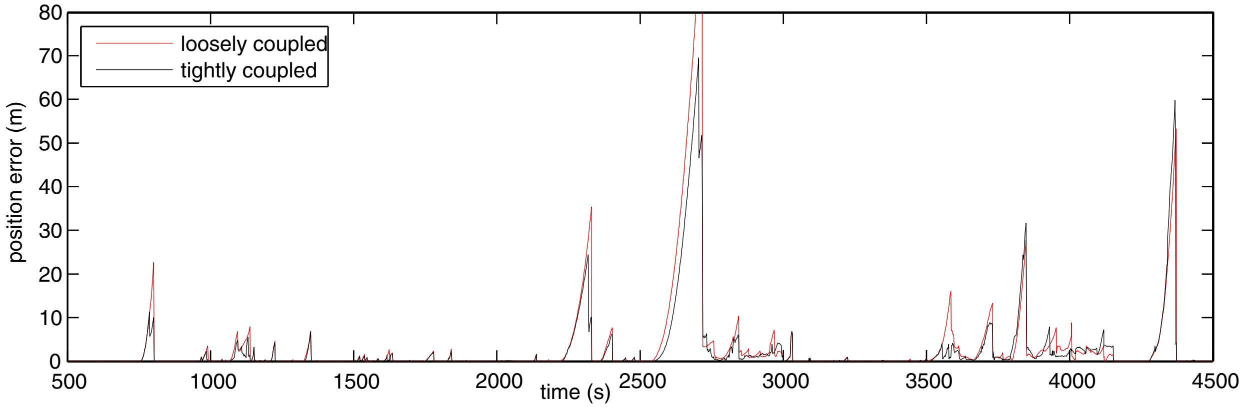

Fig. 5. Comparison of position error.

run	fixed ambiguity epochs percentage	total
1st	61.7%	10154
2nd (reverse)	87.4%	14372
3rd	88.3%	14517
4th (reverse)	88.3%	14517

Tab. 2. Fix Rate

Fig. 4 shows the number of oating and xed double differenced ambiguities. Because of the fact that double differenced ambiguities are used, three ambiguities are necessary to calculate a position x with carrier phase measurements. Four oating ambiguities are necessary to invoke the LAMBDA search, because redundancy is required. It can be seen from Fig. 4, that almost all periods with more than four available satellites, that are more than three ambiguities, have been resolved. There are only seven remaining epochs spread over two periods with four oating ambiguities. The rest of the unresolved ambiguities belong to periods were no ambiguity resolution is possible.

3.2. Integration results

To compare the position accuracy of loosely and tightly coupled systems, a loosely coupled system was simulated with the tightly coupled KF. Therefore, the tightly integrated lter was modi ed to process measurements only if four or more satellites are available. Preprocessed ambiguities have been used for this comparison, to make sure the differences arise only from the type of integration. This loosely coupled simulation differs from a real loosely system, because the GPS position accuracy is correctly modeled and no external KF is used for carrier phase ambiguity resolution. Hence, the results are supposed to be slightly better, than those of a real loosely coupled system, using position xes of an RTK GPS receiver.

Table 3 shows the RMS error of the tightly and loosely coupled system with respect to the smoothed tightly coupled solution. The difference between the smoothed loosely and the smoothed tightly coupled solution is also presented here. But there is no absolute reference available and no decision can be made, which of these solutions is better. Regarding the more accurate tightly coupled KF solution, the smoothed tightly coupled solution is also supposed to be more accurate.

Big differences in the position errors are present

	pos.	horiz.	height
loosely	9.96m	5.46m	8.33m
tightly	7.65m	4.52m	6.18m
Difference between loosely and tightly smoothed results:			
	1.14m	0.50m	1.02m

Tab. 3. RMS position error of loosely and tightly coupled results during 2h test drive with respect to the tight coupled smoothed trajectory.

only during GPS outages. If four or more satellites are available and the carrier phase ambiguities have been resolved, the position results are almost identical. This can be seen in Fig. 5. But in most cases the tightly coupled system behaves better during outages, especially if only a few number of satellites becomes visible at the end of an outage. Then the tightly coupled system starts earlier to correct the large accumulated position errors.

3.3. RTS smoother results

To verify the results of the RTS smoother, an orthophoto with an resolution of 25cm per pixel was used. The run of the road was digitized and included in Fig. 6. For copyright reasons the picture itself is not included in the gure. The accuracy of the picture is about 1-3m. So again no decision can be made if the tightly or loosely smoothed trajectory is more accurate. But it can be seen that both trajectories remain on the road during the complete outage. Hence, the horizontal position error is supposed to be only about a few meters. The outage takes place form $t = 2540$ till $t = 2718$. From Fig. 4 it can be seen that after the outage four satellites have been available only a few seconds. It takes another 4 minutes until continuously at least four satellites are available again and the carrier phase ambiguities were resolved. Fig. 6 also shows clearly the different behavior of the loosely and tightly coupled system in the KF pass. The tightly coupled system starts earlier to correct the position. But the differences of the smoothed trajectories remain far behind those of the KF trajectories. A maximum position difference of four meters was measured during this outage.

4. CONCLUSION

The measurement results have shown that a smoother is a powerful tool to overcome the problems of poor satellite availability. Therefore it perfectly matches surveying applications in which a high accurate and steady position and attitude information is more important than real-time capabilities. In case of long lasting outages of several minutes it was possible to reduce the position error to only a few meters and the nal navigation solution contained no discontinuities, like a real-time Kalman lter solution.

It was shown that a tightly coupled approach utilizes the measurement information better, especially during periods with large DOP values or insuf cient satellite availability. In comparison to a loosely coupled system the results have demonstrated that a tightly coupled system reduces the position error of the navigation solution.

Further, the measurement results have shown that the tightly coupled approach affects the ambiguity resolution process in a positive manner. It bene ts from the bridging capabilities of the inertial navigation system and, as a result, it is possible to resolve ambiguities even during short periods of satellite availability.

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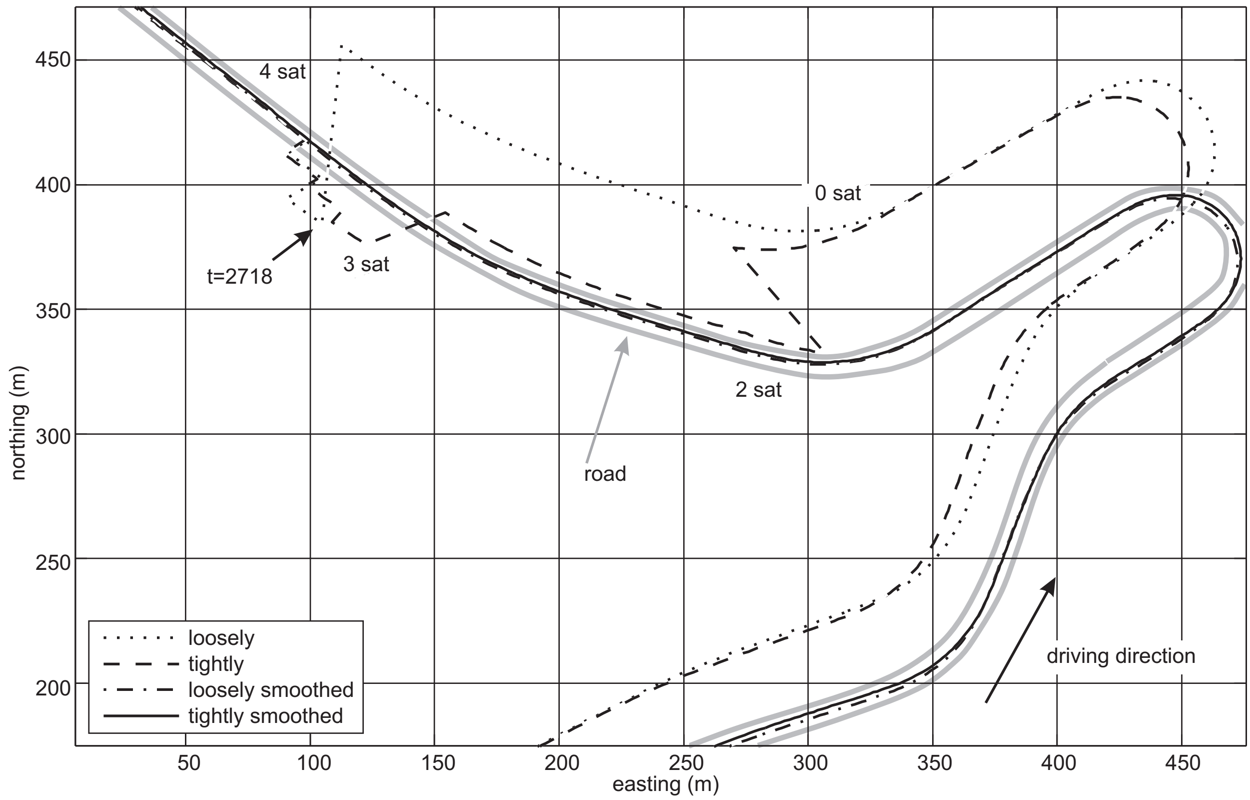

Fig. 6. Comparison of navigation results of several lters after a GPS outage of approx. 3 minutes. The run of the road was extracted from an orthophoto which is not included for copyright reasons.