PARAMETER ESTIMATION USING SAGE IN A GNSS-RECEIVER FOR AERONAUTICAL APPLICATIONS

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ABSTRACT

The potential of the SAGE (Space-Alternating Generalized Expectation Maximisation) algorithm for navigation systems in order to distinguish the line-of-sight signal (LOSS) is to be considered. The SAGE algorithm is a low-complexity generalization of the EM (Expectation-Maximisation) algorithm, which iteratively approximates the maximum likelihood estimator (MLE) and has been successfully applied for parameter estimation (relative delay, incident azimuth, incident elevation, Doppler frequency, and complex amplitude of impinging waves) in mobile radio environments. This study discusses receivers using a single antenna or an antenna array. Whereas we estimate the complex amplitudes and relative delays of the impinging waves, in the latter additionally the spatial signature of the impinging waves (incident azimuth and elevation) is estimated. The results of the performed computer simulations and discussion indicate that the SAGE algorithm has the potential to be a very powerful high- resolution method to successfully estimate parameters of impinging waves for navigation systems. SAGE has proven to be a promising method to combat multipath for aircraft navigation applications due to its good performance, fast convergence, and low complexity.

1. INTRODUCTION

Nowadays, both the increasing sizes and velocities of aircrafts and the increase of air traffic lead to stricter requirements for any aircraft navigation aid, such as DME (Distance Measuring Equipment) or ILS (Instrument Landing System), with regards to safety and traffic flow (see [1] for details on aircraft navigation systems). The complexity of the infrastructure and its costs would rise significantly in order to still meet the requirements for aviation navigation concerning accuracy, integrity, availability, and continuity and therefore guarantee safety and efficiency of air traffic. Thus, Global Navigation Satellite Systems (GNSS) as GPS or the upcoming European Galileo will play a very important roll in future aviation navigation; not only in critical situations like approach and landing where very precise positioning is required, but also in the airport ground control of taxiways, holding areas, and transition areas, which can affect the efficiency of the airport operation.

The quality of the data provided by a GNSS-receiver depends largely on the synchronization error with the signal transmitted by the GNSS satellite (navigation signal), that is, on the accuracy in the propagation delay estimation of the direct signal (line-of-sight signal, LOSS). The synchronization of a navigation signal is usually performed by a Delay Lock Loop (DLL), which basically implements an approximation of the maximum likelihood estimator (MLE). The problem which arises is that the order of this estimator (the DLL) is chosen according to the assumption that only the LOSS is present. This means that this estimator tries to estimate the relative propagation delay of only one signal replica. In case the LOSS is corrupted by several superimposed delayed replicas this estimator becomes biased, because of the change of the order of the incident estimation problem. Thus, in order to perform synchronization in the presence of multipath corrupted signals we follow the approach of obtaining the MLE for estimation problems of higher order. Therefore, signal parameters of a number of superimposed delayed replicas have to be estimated jointly. As this leads to a multi-dimensional non-linear optimization problem the reduction of the complexity of this problem is the most important issue to be solved in order to perform precise positioning in a navigation receiver.

Several techniques have been proposed in the literature to solve the multipath problem in GNSS-receivers, (see for instance [2] and [3]). Lately, interesting approaches like [4] and [5] have appeared. The first applies the maximum likelihood principle to the delay estimation in the presence of multipath and unintentional interference in an antenna array receiver, and the latter develops efficient multipath mitigation techniques (with low-complexity) in single antenna and array antenna navigation receivers. In both works, a connection is made between the multipath estimation problem in navigation systems and the same problem in communication systems; further work of the same authors has appeared more recently in [6] and [7], respectively. In this paper, in order to cope with the synchronization problem induced by multipath signals in GNSS-receivers, we explore the potential application of another technique for aircraft critical situations already used in communication systems: the SAGE algorithm (Space Alternating Generalized Expectation- Maximization algorithm). The SAGE algorithm [8] is based on the Expectation Maximization (EM) algorithm [9], [10], which facilitates optimizing maximum likelihood cost functions that arise in statistical estimation problems, but with improvements regarding the trade-off between convergence rate and complexity. The potential of the SAGE algorithm in communication systems has been proven in [11] where it is shown that SAGE is a promising candidate for channel estimation in direct-sequence code division multiple access systems (DS-CDMA). The intention of this paper is to serve as a preliminary study on the application of SAGE for aircraft navigation. The performance of the algorithm is assessed by computer simulations using a simple channel model with very hard multipath conditions.

This paper is organized as follows. The signal model is outlined in section 2. A short introduction to maximum likelihood estimation of superimposed signals is given in section 3. The SAGE algorithm is described in section 4, and the initialization method for SAGE in section 5. In section 6 we show the results of our simulations. Finally, in section 7 we discuss our conclusions.

2. DATA MODEL

We assume that L narrowband planar wavefronts of wavelength λ, complex amplitude α_ℓ, delay τ_ℓ, incident azimuth ϕ_ℓ, and incident elevation ϑ_ℓ, $1 \leq \ell \leq L$ are impinging on an array of M, $1 \leq m \leq M$ isotropic sensor elements. The transmission medium is considered linear such that the noise corrupted baseband signal at the antenna output $\mathbf{y}(t) \in \mathbb{C}^{M\times 1}$ can be modelled as a superposition of L wavefronts generated by L point sources and additional complex white Gaussian noise $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$, $\mathcal{N}(0, \sigma_n^2)$. The L point sources are located far from the array such that the direction of propagation is nearly equal at each sensor and the wavefronts are approximately planar (far-field approximation). Thus, the propagation field within the array aperture consists of plane waves and we can write

$$\mathbf{y}(t) = \sum_{\ell=1}^{L} \mathbf{s}_\ell(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}_\ell(t)$ is given by

$$\mathbf{s}_\ell(t) = \mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right)\ \alpha_\ell(t)\ c(t-\tau_\ell). \tag{2}$$

Here, $\mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right)$ denotes the steering vector of an uniform rectangular array (URA) of $M = M_x \cdot M_y$ omnidirectional sensors with M_x elements being displaced by Δ_x along the x-axis and M_y elements displaced by Δ_y along the y-axis as depicted in Fig. 1. Assuming $\Delta_x = \Delta_y = \frac{\lambda}{2}$ (λ is the wavelength corresponding to the carrier frequency), i. e.

$$\begin{aligned} \mathbf{a}_\ell\left(\phi_\ell(t), \vartheta_\ell(t)\right) = [1, \ldots, \mathrm{e}^{\mathrm{j}\pi\ (m_x\ u_\ell\ +\ m_y\ v_\ell)}, \\ \ldots, \mathrm{e}^{\mathrm{j}\pi\ ((M_x-1)\ u_\ell\ +\ (M_y-1)\ v_\ell)}]^{\mathrm{T}} \end{aligned} \tag{3}$$

$$0 \leq m_x \leq M_x - 1 \ \text{ and } \ 0 \leq m_y \leq M_y - 1$$

where

$$u_\ell = \cos\phi_\ell(t)\cos\vartheta_\ell(t) \ \text{ and } \ v_\ell = \sin\phi_\ell(t)\cos\vartheta_\ell(t).$$

Fig. 1 gives the definitions of azimuth and elevation for this URA.

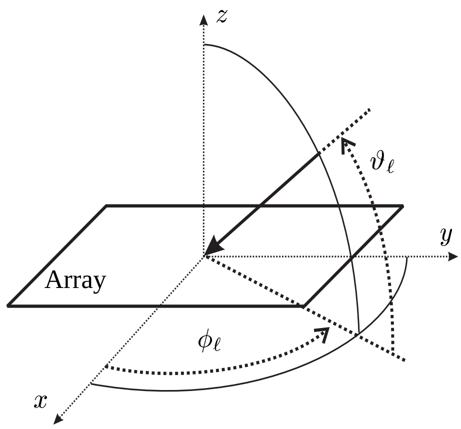

Fig. 1. Definitions of Azimuth $(-180° < \phi_\ell \leq 180°)$ and Elevation $(0° \leq \vartheta_\ell \leq 90°)$.

In Eq. (2) $c(t - \tau_\ell)$ denotes the pseudo-random-noise (PN) sequence with delay τ_ℓ. In this work we apply Gold codes [12] as used for the GPS C/A code with code period $T = 1$ ms, 1023 chips per code period each with a time duration T_c, and raised cosine (RC) pulse shape with single-sided bandwidth $B = (1+\beta)\cdot\frac{1}{2T_c} = 0.624$ MHz. The roll-off factor is $\beta = 0.22$. We use the RC pulse shape due to the reason that the calculation of the Cramer Rao bound (CRB) is more manageable (cf. section 6). Without loss of generality, all conclusions drawn from the simulations in this work are also valid for other pulse types.

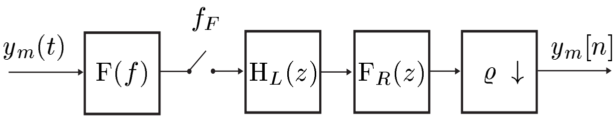

Fig. 2. Receive Filtering

For a strictly band-limited signal corrupted by additive white Gaussian noise, a sufficient statistic can be obtained by pre-filtering $y_m(t)$ with an ideal analog lowpass filter and subsequent sampling at Nyquist rate $\frac{1}{T_s} = 2\ B$ [13]. Here, $y_m(t)$ denotes the signal received by antenna m of the URA. This is a solution of no practical interest since such a filter can not be realized (or even closely approximated) with sufficient accuracy at reasonable complexity. Thus, we apply the realization [13] and [14], which is shown in Fig. 2 in order to get a sufficient statistic $y_m[n]$ using a realizable analog filter $\mathrm{F}(f)$ with bandwidth B_F, while $B_F \geq B$ as the analog filter has not a sharp cutoff. After this pre-filtering with $\mathrm{F}(f)$ the signal is sampled with a sampling frequency of $f_F = \frac{\varrho}{T_s} = 2\ \varrho\ B \geq 2\ B_F$. Here, ϱ denotes the oversampling ratio. The sampled signal then is filter by a digital lowpass filter $\mathrm{H}_L(z)$ with bandwidth $B_L = B$ and by a digital filter

$$\mathrm{F}_R\left(\mathrm{e}^{\mathrm{j}2\pi f T_s}\right) = \frac{1}{\mathrm{F}(f)} \quad |f| < B \tag{4}$$

which reverses the effect of $\mathrm{F}(f)$ in the passband B of the useful signal. Finally, downsampling by the ratio ϱ is performed. Thus, we get a sufficient statistic $\mathbf{y}[n]$ where the spatial observations of the URA are collected at N time instances, whereas $\mathbf{y}[n] = \mathbf{y}(n \cdot T_s)$ with $n = 1, 2, \ldots, N$.

Generally, there is a trade-off between oversampling rate and analog pre-filtering complexity. Oversampling relaxes the requirements on the analog pre-filter regarding a sharp-cutoff, but in order to obtain a sufficient statistic with uncorrelated noise samples downsampling to the Nyquist rate has to be done afterwards. Thus, all sharp-cutoff filtering is done by a discrete-time system, and only nominal continuous-time filtering is required. Since discrete-time FIR filters can have an exactly linear phase, it is possible using this oversampling and reverse filtering approach to implement antialiasing pre-filtering with virtually no phase distortion. This is especially important for this work where it is critical to preserve not only the frequency spectrum, but also the waveshape of the signal as well [14].

The channel parameters are assumed constant during the observation interval $N \cdot T_s$ since the coherence time of the assumed channel is $T_{coh} \geq T_s \cdot N$. Collecting the snapshots of the observation interval leads to

$$\begin{aligned}
\mathbf{Y} &= [\mathbf{y}[1], \mathbf{y}[2], \ldots, \mathbf{y}[N]] \in \mathbb{C}^{M \times N}, && (5)\\
\mathbf{N} &= [\mathbf{n}[1], \mathbf{n}[2], \ldots, \mathbf{n}[N]] \in \mathbb{C}^{M \times N}, && (6)\\
\mathbf{S}_\ell &= [\mathbf{s}_\ell[1], \mathbf{s}_\ell[2], \ldots, \mathbf{s}_\ell[N]] \in \mathbb{C}^{M \times N}, && (7)\\
\boldsymbol{\theta} &= \left[\boldsymbol{\theta}_1^{\mathrm{T}}, \boldsymbol{\theta}_2^{\mathrm{T}}, \ldots, \boldsymbol{\theta}_L^{\mathrm{T}}\right]^{\mathrm{T}}, && (8)\\
\boldsymbol{\theta}_\ell &= [\alpha_\ell, \tau_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}. && (9)
\end{aligned}$$

Thus, the signal model can be written in matrix notation

$$\mathbf{Y} = \mathbf{S}(\boldsymbol{\theta}) + \mathbf{N} = \sum_{\ell=1}^{L} \mathbf{S}_\ell(\boldsymbol{\theta}_\ell) + \mathbf{N}. \tag{10}$$

3. MAXIMUM LIKELIHOOD ESTIMATION OF SUPERIMPOSED SIGNALS

The problem at hand is to estimate $\boldsymbol{\theta}_\ell = [\alpha_\ell, \tau_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}$, $\ell = 1, 2, \ldots, L$. The estimation of L is not taken care of in this work. In the following we presume L is given.
As we assume spatially and temporally uncorrelated elements in $\mathbf{N}$, the covariance of the noise is $\sigma_n^2 \cdot \mathbf{I}$. Thus, the likelihood function for our signal model is given by the conditional probability density function (pdf)

$$\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta}) = \frac{1}{(\pi\sigma_n^2)^{M \cdot N}} \exp\left(-\frac{\|\mathbf{Y} - \mathbf{S}(\boldsymbol{\theta})\|_{\mathrm{F}}^2}{\sigma_n^2}\right). \tag{11}$$

Here, $\|\ \cdot\ \|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix. The maximum likelihood estimator (MLE) is given by

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \left\{\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta})\right\}. \tag{12}$$

Estimation of $\boldsymbol{\theta}$ is a computationally prohibitive task since there is no analytical solution for the global maximum, $\mathrm{p}(\mathbf{Y}; \boldsymbol{\theta})$ generally is not a concave function of $\boldsymbol{\theta}$, and $\boldsymbol{\theta}$ and L usually have high dimension. Since the values for maximization of the complex amplitude α_ℓ can be given in closed form as a function of the other parameters [11], the computation of the MLE for $\boldsymbol{\theta}_\ell = [\alpha_\ell, \tau_\ell, \phi_\ell, \vartheta_\ell]^{\mathrm{T}}$ is a $3 \cdot L$-dimensional non-linear optimization procedure.

4. THE SAGE ALGORITHM

Whereas no closed form expression can be found for the MLE, a numerical approach employs either a grid search or an iterative maximization of the likelihood function. In this work we try an iterative approach, the SAGE algorithm. Instead of performing the above mentioned high dimensional non-linear optimization procedure directly, the SAGE algorithm gives a sequential approximation of the MLE given in Eq. (12). The method can be considered as an extension of the well known Expectation-Maximization (EM) algorithm. In the following we sketch the fundamental ideas of the SAGE algorithm.

The basic concept of the SAGE algorithm is the hidden data space [8]. Instead of estimating the parameters of all waves $\boldsymbol{\theta}$ in parallel in one iteration step as done by the EM algorithm the SAGE algorithm estimates the parameters of each wave $\boldsymbol{\theta}_\ell$ sequentially. Also, instead of estimating the complete $\boldsymbol{\theta}_\ell$, SAGE breaks down the multi-dimensional optimization problem into several smaller problems by conditioning sequentially on a subspace of parameters $\boldsymbol{\theta}_S$ while keeping the parameters of the complement subspace $\boldsymbol{\theta}_{\bar{S}}$ fixed. We choose the hidden data space as one noisy wave $\mathbf{X}_\ell = \mathbf{S}_\ell + \mathbf{N}_\ell$ where $\mathbf{N}_\ell$ is white Gaussian noise with variance $\nu_\ell\ \sigma_n^2$. This choice of the hidden data space leads to a fast convergence rate and low complexity due to sequential updating and one-dimensional optimization procedures. The stochastic mapping of the hidden data space to the observable signal is $\mathbf{Y} = \mathbf{X}_\ell + \sum_{\substack{\ell'=1\\ \ell'\neq\ell}}^{L} \mathbf{S}_{\ell'} + \mathbf{N}_{\ell'}$. We choose the sequence of parameter vector $\boldsymbol{\theta}_S(\mu)$ (μ is the parameter update index) as $\boldsymbol{\theta}_S(1) = [\tau_1], \boldsymbol{\theta}_S(2) = [\phi_1], \boldsymbol{\theta}_S(3) = [\vartheta_1], \boldsymbol{\theta}_S(4) = [\alpha_1], \boldsymbol{\theta}_S(5) = [\tau_2], \boldsymbol{\theta}_S(6) = [\phi_2], \ldots$. For fast convergence we set $\nu_\ell = 1$ [11].
Thus, after estimating the hidden data space in the so-called expectation step (E-step) with

$$\hat{\mathbf{X}}_\ell = \mathbf{Y} - \sum_{\substack{\ell'=1\\ \ell'\neq\ell}}^{L} \mathbf{S}_{\ell'}(\hat{\boldsymbol{\theta}}_{\ell'}), \tag{13}$$

we get the so-called maximization step (M-step) with

$$\hat{\tau}_\ell = \arg\max_{\tau_\ell} \left\{ \frac{|\mathbf{a}(\hat{\phi}_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\ \hat{\mathbf{X}}_\ell\ \mathbf{c}(\tau_\ell)|^2}{MN\nu_\ell\sigma_n^2} \right\}, \tag{14}$$

$$\hat{\phi}_\ell = \arg\max_{\phi_\ell} \left\{ \frac{|\mathbf{a}(\phi_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\ \hat{\mathbf{X}}_\ell\ \mathbf{c}(\hat{\tau}_\ell)|^2}{MN\nu_\ell\sigma_n^2} \right\}, \tag{15}$$

$$\hat{\vartheta}_\ell = \arg\max_{\vartheta_\ell} \left\{ \frac{|\mathbf{a}(\hat{\phi}_\ell, \vartheta_\ell)^{\mathrm{H}}\ \hat{\mathbf{X}}_\ell\ \mathbf{c}(\hat{\tau}_\ell)|^2}{MN\nu_\ell\sigma_n^2} \right\}, \tag{16}$$

$$\hat{\alpha}_\ell = \frac{\mathbf{a}(\hat{\phi}_\ell, \hat{\vartheta}_\ell)^{\mathrm{H}}\ \hat{\mathbf{X}}_\ell\ \mathbf{c}(\hat{\tau}_\ell)}{MN}. \tag{17}$$

Here, $\mathbf{c}(\tau_\ell)$ denotes the sampled reference PN sequence delayed by τ_ℓ (generated in the receiver). The E-step and the M-step are performed iteratively until the algorithm converges. Eq. (14) is often called rake searcher, as delay estimates $\hat{\tau}_\ell$ correspond to the maximum cross-correlation values. Equivalently, the estimates of azimuth $\hat{\phi}_\ell$ of Eq. (15) and elevation $\hat{\vartheta}_\ell$ of Eq. (16) are given by the maximum spatial correlation. Various choices of the hidden data space could be made which would individually influence the convergence rate. Less informative hidden data spaces yield faster convergence, but more informative hidden data spaces yield an easier M-step [8]. Although the sequence of log-likelihood values form a monotonically non-decreasing sequence of parameter estimates, convergence to even a local maximum is not guaranteed. Therefore, the SAGE algorithm has to be initialised in a region which is close enough to a local (at best the global) maximum. Then the sequence of estimates will converge in norm to it [8].

5. INITIALIZATION OF THE SAGE ALGORITHM

Two methods are proposed in [11] of initialising the SAGE algorithm for channel estimation in mobile radio environments. The first one is a technique which commonly is referred to as successive interference cancellation. This method starts with the pre-initial setting $\hat{\boldsymbol{\theta}}_\ell = [0, 0, 0, 0]^{\mathrm{T}}$ for each $\ell = 1, 2, \ldots, L$. It successively performs a maximum search of correlation processes for the delay and jointly for azimuth and elevation:

$$\hat{\tau}_\ell = \arg\max_{\tau_\ell} \left\{ |\hat{\mathbf{X}}_\ell\ \mathbf{c}(\tau_\ell)|^2 \right\}, \tag{18}$$

$$(\hat{\phi}_\ell, \hat{\vartheta}_\ell) = \arg\max_{\phi_\ell} \left\{ |\mathbf{a}(\phi_\ell, \vartheta_\ell)^{\mathrm{H}} \hat{\mathbf{X}}_\ell\ \mathbf{c}(\hat{\tau}_\ell)|^2 \right\} \tag{19}$$

In order to perform the E-step, it subtracts signal estimates of waves whose parameter estimates already have been initialised from the observed data $\mathbf{Y}$. Since in Eq. (18) ϕ_ℓ and ϑ_ℓ are unknown, $\hat{\mathbf{X}}_\ell$ is summed incoherently over all sensors to provide the initial delay estimate.

In the second method all initial estimates of the delays are first computed with the MUSIC (MUltiple SIgnal Classification) algorithm [15]. The remaining parameter estimates are calculated during an initialization cycle of the SAGE algorithm. This method might fail for navigation signals as they could be highly correlated, if multipaths with short relative delays are present.

Since for navigation systems it is vital to have accurate delay estimates to get a low pseudorange error, initialization of the SAGE algorithm is a profound task. In order to ensure fast convergence and good estimation results, the initialization of the SAGE algorithm in a GNSS-receiver could be augmented by appropriate *a priori* knowledge of the parameters of the impinging waves. For example, we could take the estimates of the delays provided through acquisition and additionally could use almanac data to get initial estimates for the spatial signature of the LOSS. Furthermore, a good initialization or *a priori* knowledge of the parameters to be estimated will also reduce the search space of the parameters $\boldsymbol{\theta}$ and therefore reduce complexity of the optimization procedures.

Since this work should preliminary assess the potential of the SAGE algorithm for navigation systems, we use a simple method. For the following simulations we selected the first method explained in this section. We assume that there is no knowledge of almanac data or of acquisition estimates available for initialization.

6. SIMULATION RESULTS

We tested the SAGE algorithm for a static channel under severe multipath conditions via Monte Carlo simulations. Receivers using a single antenna (M=1) and an array antenna (M=9) are simulated. We simulated an URA with $M_x = 3$, $M_y = 3$, and $\Delta_x = \Delta_y = \frac{\lambda}{2}$. We assume that the channel parameters are constant during the observation interval.
Signal-to-noise ratio (SNR) denotes the LOSS-to- noise ratio. We choose for all simulations SNR=-19 dB which in our case is equal to $\frac{C}{N_0} = 42$ dB-Hz. We consider the reflected multipath and the LOSS to be in-phase, which corresponds to the worst possible case [3]. In this work we only consider reflective multipath. The signal-to- multipath ratio (SMR) is 5 dB for all reflections (cf. [3] and [6]). We coherently average correlation over 100 code periods. Thus, the observation interval is 100 ms. A non-coherent estimation is performed which means that no *a priori* knowledge of the phases or the complex amplitudes is available. We only used the mentioned simple initialization method and no acquisition was done. In order to describe the behavior and to assess the performance of the SAGE algorithm we apply the root mean square error (RMSE) and the Cramer Rao bound (CRB), which is derived following [16] and [4]. In the following CRB_1 denotes the CRB for the case where only the LOSS present ($L = 1$). Although the estimator shown is biased we think the comparison to the CRB is still meaningful. In the following parameters with the subscript 1 stand for the LOSS and parameters with the subscript 2 stand for the reflection.

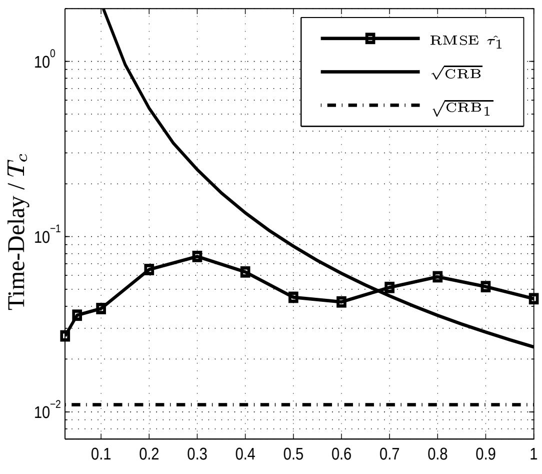

Fig. 3. RMSE of the time-delay of the LOSS / T_c versus the relative time-delay of the multipath / T_c. Parameters: $L = 2$, $M = 1$.

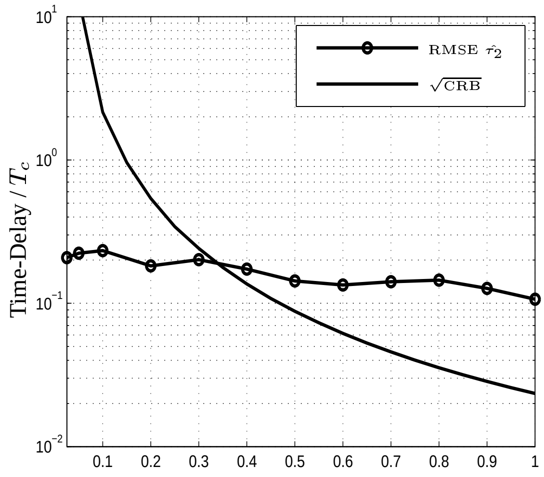

Fig. 4. RMSE of the time-delay of the multipath / T_c versus its relative time-delay / T_c. Parameters: $L = 2$, $M = 1$.

Fig. 3 and Fig. 4 depict the results of the estimates $\hat{\tau}_1$ and $\hat{\tau}_2$ for the single antenna ($M = 1$) case with $L = 2$. We assume an isotropic sensor element with $\mathbf{a}_\ell = 1$ and the parameter vector reduces to $\boldsymbol{\theta}_\ell = [\alpha_\ell, \tau_\ell]^{\mathrm{T}}$. The estimator is biased for relative delays shorter than $0.7 \cdot T_c$ as the ability of SAGE to distinguish the two waves is slowly dwindling away towards shorter relative delays. Simulation results have shown that SAGE more and more often converges to either two equal delay estimates ($\hat{\tau}_1 = \hat{\tau}_2$) or to the true LOSS delay estimate and a noisy multipath delay, which mostly has a large difference to the true value. Thus, the performance of SAGE slowly tends to the case where only the LOSS is present which is given by CRB_1. The RMSE for $\hat{\tau}_1$ is approximately between 8 and 23 meters, as the length of one chip resembles 293 meters. In order to improve the estimation of the LOSS one could apply additional filtering in time domain based on the channel estimation performed by SAGE and afterwards perform another estimation.

Fig. 5, Fig. 6, Fig. 7, and Fig. 8 depict the results of the estimates $\hat{\tau}_1$, $\hat{\tau}_2$, $\hat{\phi}_1$, $\hat{\phi}_2$, $\hat{\vartheta}_1$, and $\hat{\vartheta}_2$ for using an URA with $M = 9$ for $L = 2$. For the directions of arrival (DOAs) of the two waves we choose $\phi_1 = -24°$, $\vartheta_1 = 30°$, $\phi_2 = 10°$, and $\vartheta_2 = 20°$. The estimator is biased for relative delays shorter than $0.4 \cdot T_c$ due to the same reason as already explained for the single antenna case. The performance of SAGE using an array antenna for the estimation of the delay of the multipath is remarkably good as well as for the estimation of the delay of the LOSS. The RMSE of $\hat{\tau}_1$ is approximately between 2 and 3 meters and the RMSE of $\hat{\tau}_2$ is between 3 and 6 meters.

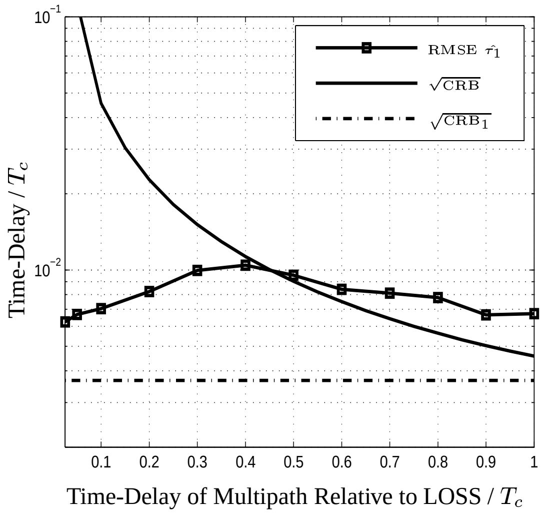

Fig. 5. RMSE of the time-delay of the LOSS / T_c versus the relative time-delay of the multipath / T_c. Parameters: $L = 2$, $M = 9$, $\phi_1 = -24°$, $\vartheta_1 = 30°$, $\phi_2 = 10°$, $\vartheta_2 = 20°$.

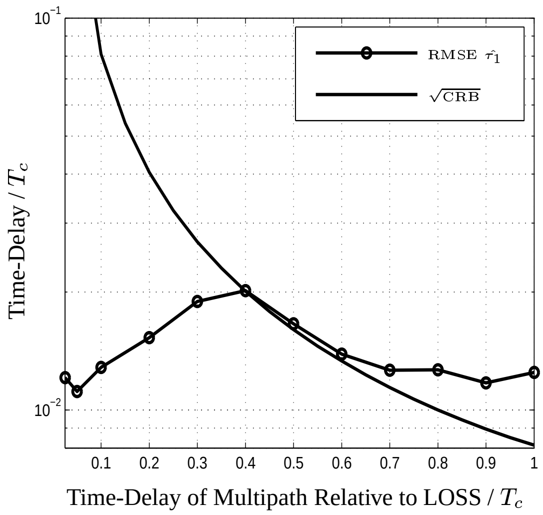

Fig. 6. RMSE of the time-delay of the multipath / T_c versus its relative time-delay / T_c. Parameters: $L = 2$, $M = 9$, $\phi_1 = -24°$, $\vartheta_1 = 30°$, $\phi_2 = 10°$, $\vartheta_2 = 20°$.

Fig. 7 and Fig. 8 show the results of the DOA estimation for the azimuth and the elevation for the two impinging waves. For the chosen scenario SAGE is able to give a reliable DOA estimation. DOA estimation is a very important issue when it comes to additional use of adaptive spatial filtering (adaptive beam forming) in order to mitigate interference. One can easily detect whether an impinging wavefront is a LOSS from a satellite or interference (e.g. multipath, spoofing), as the DOAs of the LOSS of the satellites are known from the almanac together with a sufficient good position solution. With good DOA estimation then one could enhance the estimation of synchronization parameters by appropriate spatial filtering. However, further assessment of the abilities of SAGE concerning DOA estimation have to be performed in the future.

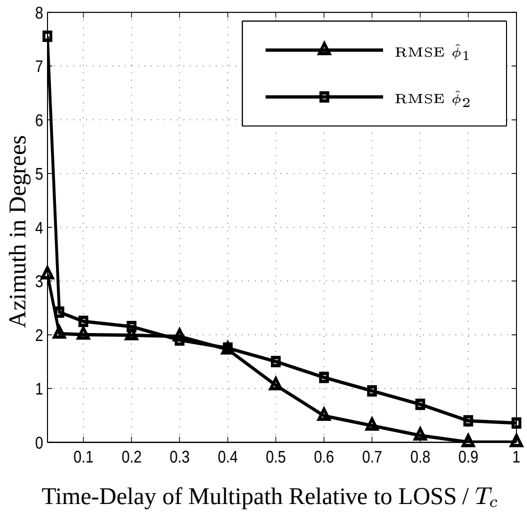

Fig. 7. RMSE of the azimuth of the LOSS and the multipath versus its relative time-delay / T_c. Parameters: $L = 2$, $M = 9$, $\phi_1 = -24°$, $\vartheta_1 = 30°$, $\phi_2 = 10°$, $\vartheta_2 = 20°$.

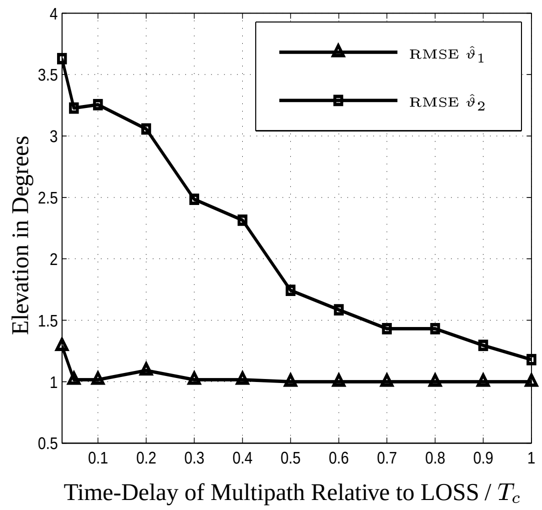

Fig. 8. RMSE of the elevation of the LOSS and the multipath versus the relative time-delay of the multipath / T_c. Parameters: $L = 2$, $M = 9$, $\phi_1 = -24°$, $\vartheta_1 = 30°$, $\phi_2 = 10°$, $\vartheta_2 = 20°$.

7. CONCLUSION

In this work we have addressed the problem of estimating the propagation time-delay of the LOSS in a GNSS-receiver under the presence of severe multipath in a static channel. The tested method in order to reduce the complexity in solving the MLE is the SAGE algorithm. The SAGE algorithm iteratively approximates the MLE and significantly reduces the complexity by breaking down the multi-dimensional non-linear optimization problem of the MLE, which was addressed into a number of one-dimensional ones. Using a single antenna ($M = 1$) or an array antenna ($M = 9$) SAGE is giving very good performance under severe multipath conditions and a small bandwidth of $B = 0.624$ MHz. Thus, we think that SAGE has proven to be a promising method to combat multipath for navigation applications due to its good performance and low complexity.

Future work will deal with further assessment of the DOA estimation ability of SAGE, additional space-time processing in order to enhance the estimation, finding appropriate methods to estimate the number of impinging waves L, and deriving sufficient optimization methods to solve the remaining one-dimensional non-linear optimization processes.

As for implementation in an single antenna receiver, standard techniques like a DLL for the delay estimation in the M-step could be implemented. Further, a Phase-Lock Loop (PLL) could be used to augment the computation of the complex amplitude in the M-step. This would lead to a coherent estimation as depicted in Fig. 9. Here, $\hat{\varphi}_\ell^{(k)}$ denotes the phase estimate of the ℓ-th wave for the k-th iteration step.

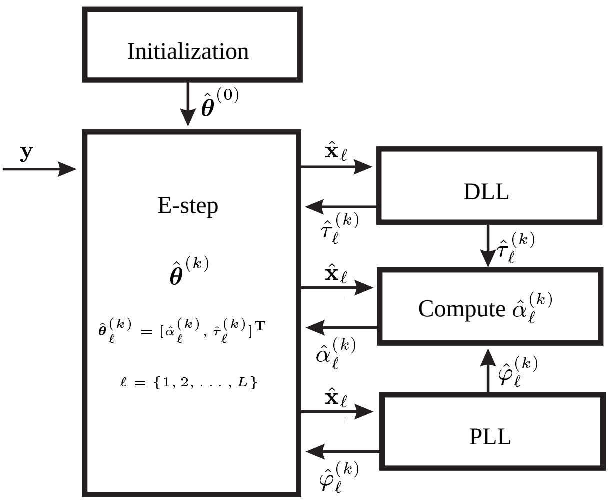

Fig. 9. Implementation Approach

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