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THE PIONEER ANOMALY IN THE CONTEXT OF COSMOLOGY

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ABSTRACT

Studies of radiometric data from the spacecrafts Pioneer 10/11, Galileo and Ulysses have revealed the existence of an anomalous acceleration on all four spacecrafts, inbound to the Sun and with a (constant) magnitude of $(8.5 \pm 1.3) \times 10^{-10}$ m/s^2. The two Pioneer spacecrafts follow approximate opposite hyperbolic trajectories away from the Solar System, while Galileo and Ulysses were moving on bound orbits. Extensive attempts to explain this phenomenon as a result of poor accounting of thermal and mechanical effects as well as errors in the tracking algorithms used, have shown to be unsuccessful.
In this paper we consider the possible influence of the expansion of the universe including a non-vanishing Cosmological Constant. We compare the cosmological perturbation with other approaches to explain the Pioneer anomaly using non-Newtonian gravity. The non-Newtonian contribution coming from the expanding universe to the Newtonian equations of motion in the local Fermi frame (Riemannian normal coordinates) is computed. Using the Gauss-Lagrange perturbation equations we look for changes of the orbital parameters a, e and ω. The cosmological effects on the spacecraft dynamics are very small. But great strides are being made in observational technology in astronomy and fundamental physics and it appears worthwhile to investigate some of the observational consequences of the influence of cosmological expansion including a non-vanishing cosmological constant on local systems.

1. INTRODUCTION

The so called Pioneer 10/11 Anomaly is a small constant Doppler frequency shift which is interpreted as an acceleration towards the sun of $a_p = (8.74 \pm 1.33) \times 10^{-8}$ cm/s² (Anderson et al., 1998, 2002). The effect is only present for light spacecrafts (~ 250 kg) on hyperbolic escape trajectories (Figure 1).

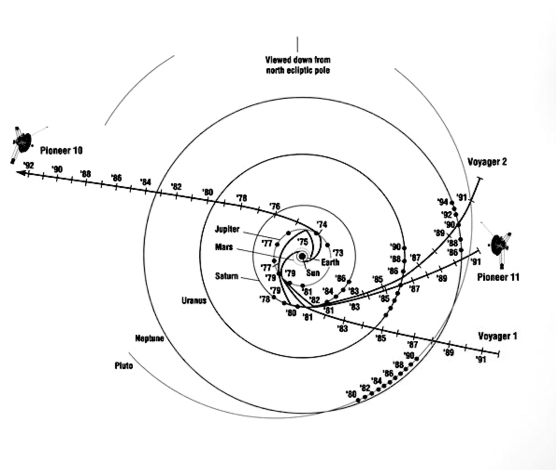

FIG 1. Pioneer 10/11 trajectories

For any other bound object in our solar system this acceleration has not been observed. Up to now it has only been observed during a longer time interval (Figure 2) for the Pioneer 10/11 space probes.

For the Pioneer spacecrafts the anomaly occurs between 20 and 70 AU (Astronomical Units) the data analysis was separated in two parts, Pioneer 11 for the inner and Pioneer 10 for the outer area.

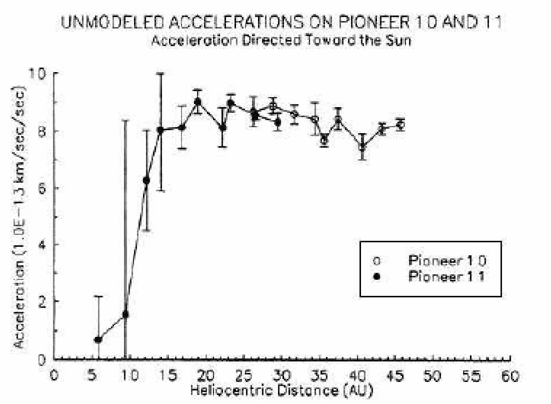

FIG 2. Variation of the acceleration depending on the distance from the sun as measured: 1981 – 1989 and 1977 – 1989.

Because the time – varying Robertson–Walker metric of the standard cosmology implies expansion as a universal phenomenon it is reasonable to assume that the cosmic expansion proceeds on all scales.

The question whether local phenomena in the surrounding of a star are affected by the expansion of the universe was already addressed by Einstein and Straus (1945) and later by Schücking (1954). The Einstein-Straus model consists of a mass at the centre of an empty spherical region, described by the Schwarzschild metric, which is matched to a Friedmann cosmological metric at an expanding boundary. Since the space-time near the massive star is that of Schwarzschild, the planetary orbits are given by the usual geodesics, and the cosmic expansion seems to exert no influence within a vacuole with radius. According to this approach there is no effect of expansion inside a vacuole of radius

$$r \leq \sqrt[3]{\frac{2GM}{H^2}} = \sqrt[3]{\frac{3M}{\rho_c}} \tag{1}$$

which is approximately $5.2\ \ 10^7$ AU in case of $M = M_{Sun}$ and $H_0 \approx 70$ km/s Mpc. At present the expansion accelerates approaching a Hubble parameter

$$H_\infty = \sqrt{\frac{\Lambda c^2}{3}} \tag{2}$$

where Λ is the Cosmological Constant $\Lambda \approx 10^{-52}\ m^{-2}$ which corresponds to an equivalent density of

$$\rho_\Lambda = \frac{c^4}{8\pi G} \cdot \Lambda \approx 10^{-30} \frac{g}{cm^3} \tag{3}$$

With this the radius of the vacuole is

$$r_\Lambda = \sqrt{\frac{6GM}{\Lambda c^2}} \approx 2\ \ 10^7 \text{ AU} \tag{4}$$

The accelerated expansion of the universe implies that on large scales the universe is not dominated by gravitational attraction, but rather by a long-range repulsive force in terms of Newtonian mechanics. The natural explanation within general relativity is the existence of a non-vanishing Cosmological Constant - but it could also indicate new physics beyond general relativity. All unifying theories or quantum gravity theories predict small modifications from general relativity and deviations from the Newtonian inverse square law either in a Yukawa-like form or by adding a repulsive force term which corresponds to a term which is a consequence of the Newtonian limit of Einstein's field equations of gravitation.

In our approach to investigate the possible influence of cosmic dynamics we will calculate the cosmological correction to the Newtonian equation of motion in a local Fermi frame. The next step is to use the Gauss-Lagrange perturbation equations to look for the influence on the orbital elements a (semi major axis), e (eccentricity) and the precession of the perihelion ω. We compare our results with alternative approaches for the influence of the cosmic background on local dynamics suggested by Milgrom (1983), Rosales, Sánchez-Gomez (1999) and Nottale (2003).
A dramatic influence of cosmic expansion on all bound astronomical objects is expected in the far future of a dark energy dominated universe (Caldwell et al. 2003). The gravitational repulsion and increasing expansion rate will dissociate even gravitationally bound objects.

2. LOCAL DYNAMICS IN CURVED SPACETIME

A curved space-time manifold can locally be approximated by a flat tangent space at every spacetime point.
From a physical point of view the tangent space describes the spacetime as seen by a free falling observer in the local inertial frame (LIF).Using Fermi normal coordinates it is the LIF based on geodesic observer which continues to be locally inertial during the observers course in time.
This is the frame in which astronomical observations and spacecrafts ranging are carried out:

$$\begin{aligned} ds^2 &= -\left(1 + R_{0l0m} x^l x^m\right) dt^2 - \left(\frac{4}{3} R_{oljm} x^l x^m\right) dt dx^j \\ &\quad + \left(\delta_{ij} - \frac{1}{3} R_{iljm} x^l x^m\right) dx^i dx^j \end{aligned} \tag{5}$$

The equation of motion for a two – body problem (m << M) is then:

$$\frac{d^2\vec{r}}{dt^2} + \frac{GM}{r^2}\left(\frac{\vec{r}}{r}\right) + R^i{}_{0j0} x^j = 0 \tag{6}$$

see e.g. Misner et al. (1973). The term $R^i{}_{ojo}\, x^j$ is the non-Newtonian contribution coming from the non-vanishing components of the Riemannian curvature tensor of the cosmic background.

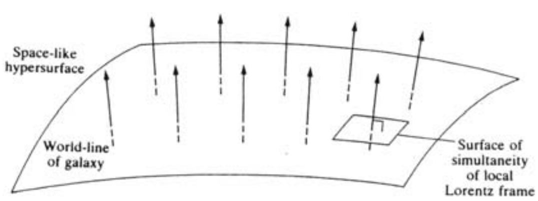

FIG 3. Universal coordinate system

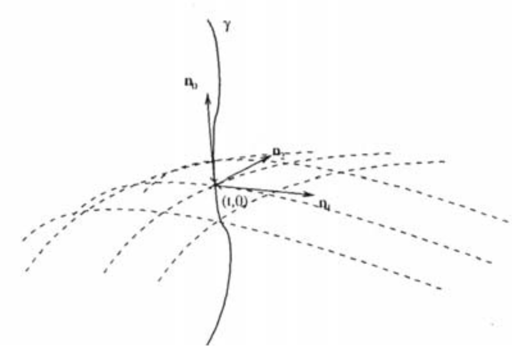

FIG 4. Fermi coordinate system

3. COSMOLOGICAL PERTURBATIONS

Current cosmological models that take the observed accelerated expansion into account involve a cosmological constant Λ. The expansion dynamics R(t) for a universe with Euclidean 3-space ("flat" universe) is described by a solution of the Friedmann-Lemaitre equations:

$$\frac{R}{R_0} = \sqrt[3]{\frac{\rho_{m,0}}{2\rho_\Lambda}\left(\cosh\left(\frac{t}{\tau}\right)-1\right)} = \sqrt[3]{\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\left(\sinh\frac{t}{2\tau}\right)^2} \qquad (7)$$

$$\tau = \frac{1}{3H_0\sqrt{\lambda_0}} = \frac{1}{\sqrt{24\pi G\cdot\rho_\Lambda}} = \frac{14.13}{\sqrt{\rho_\Lambda/(10^{-30}\,g\,cm^{-3})}}\cdot 10^9 \text{ Jahre} \qquad (8)$$

Eq. (7) include the following parameters: $\rho_{m,0}$ – density of matter, ρ_Λ – density of the vacuum, $\Omega_{m,0}$ – scaled density of matter, $\Omega_{\Lambda,0}$ – scaled vacuum density parameter.

The present "standard model" (Figure 4) of cosmology is model 6 with $\Omega_{\Lambda,0} = \frac{\rho_\Lambda}{\rho_c} = \frac{c^2\Lambda}{3H_0^2} = 0.7$ and $\Omega_{m0} = \frac{\rho_{m0}}{\rho c} = 0.3$ known as ΛCDM (Lambda Cold Dark Matter) model. Recent observations put a strong limit on the cosmological constant: $\Lambda > 0$ but $\Lambda \le 10^{-52}\ m^{-2}$ (Blome et. al., 2003).

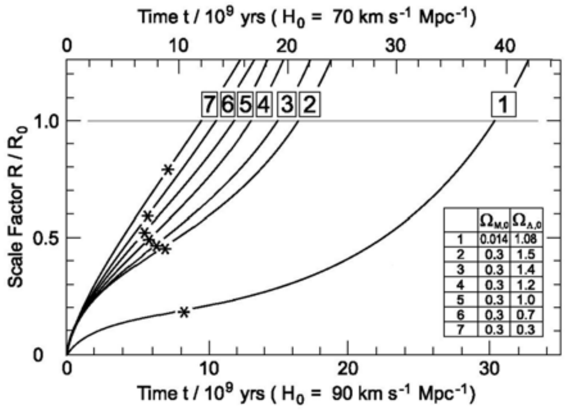

FIG 5. Evolution of the cosmological scale factor $R(t)/R_0$ as a function of time in several models with $\Lambda > 0$ (Overduin, Priester, 2002)

The transition from slowing down to an accelerated expansion occurs at the point of inflection $t_* = \tau\cdot\text{arcosh } 2 = 1.137\cdot\tau$ corresponding to a redshift of:

$$z_* = \left(\frac{2\cdot\Omega_{\Lambda 0}}{\Omega_{m0}}\right)^{1/3} - 1 \qquad (9)$$

The universe approaches for $t >> t_*$ to a de Sitter like expansion:

$$R \sim \exp(H_\infty t) \qquad (10)$$

with $H_\infty = \left(\frac{\Lambda c^2}{3}\right)^{1/2}$.

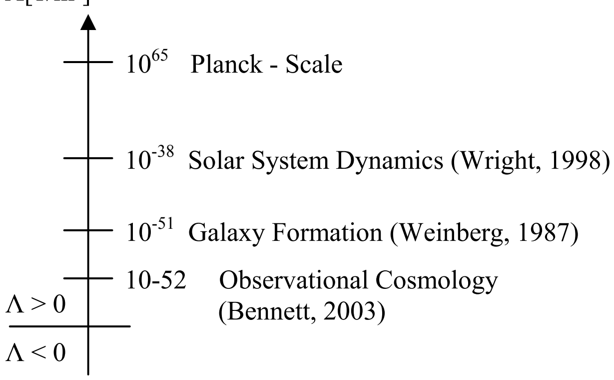

FIG 6. Assumption of the cosmological constant Λ.

4. INFLUENCE OF EXPANSION ON NEWTONIAN DYNAMICS

Consider a local geodesic reference frame whose origin is commoving with an expanding universe. A particle of mass m is at a distance r from the origin. The gravitational field of the universe exerts a radial force

$$\left|\vec{F}\right| = m\cdot H^2\cdot q\cdot\left|\vec{r}\right| = m\cdot\frac{\ddot{R}}{R}\cdot\left|\vec{r}\right| \qquad (11)$$

see e.g. Ohanian, Ruffini (1994). If we only consider the cosmological constant - an approximation if $t >> t_*$ - the force term reduces to

$$\left|\vec{F}_\Lambda\right| = \frac{m\Lambda c^2}{3}r \qquad (12)$$

The equations of motion for a two body system interacting by Newtonian gravitation in an expanding cosmic environment read:

$$\frac{d^2\vec{r}}{dt^2}+\frac{GM}{r^2}\frac{\vec{r}}{r}+R^i_{0j0}x^j=0 \tag{13}$$

or with the cosmic scale factor R(t):

$$\frac{d^2\vec{r}}{dt^2}+\omega_0^2\vec{r}+\left(-\frac{\ddot{R}}{R}\right)\cdot\vec{r}=0 \tag{14}$$

Which means for $\Lambda = 0$:

$$\frac{d^2\vec{r}}{dt^2}+\omega_0^2\vec{r}-\frac{2}{9t^2}\vec{r}=0 \tag{15}$$

where we have assumed an expansion of a flat universe for $t << t_*$.

In the other extreme we have a late-time universe ($t >> t_*$) with an expansion dominated by a cosmological constant $\Lambda > 0$ with an equation of motion

$$\frac{d^2\vec{r}}{dt^2}+\omega_0^2\vec{r}-\frac{\Lambda c^2}{3}\vec{r}=0 \tag{16}$$

Without perturbation by cosmic expansion and for vanishing Λ the solutions are for a bound orbit:

$$e<1: \quad r=a(1-e\ \cos\varphi) \tag{17}$$

$$t=\sqrt{\frac{ma^3}{GMm}}(\varphi-e\ \sin\varphi) \tag{18}$$

and for an unbound hyperbolic trajectory:

$$e>1: \quad r=a(e\ \cosh(\varphi+1)) \tag{19}$$

$$t=\sqrt{\frac{ma^3}{GMm}}(e\ \sinh(\varphi-1)) \tag{20}$$

The perturbing acceleration coming from expansion without the Λ-term can be read off from eq. (15):

$$a_{\exp}=\frac{\ddot{R}}{R}r=\frac{2r}{9t^2}\approx\frac{2\cdot1.5x10^{12}}{9\cdot(6.3x10^{17})^2}\approx\frac{1}{3}10^{-23}\left[\frac{m}{s^2}\right] \tag{21}$$

If expansion is dominated by Λ – according present observations - we get from eq. (15) r = 80 AU:

$$a_\Lambda=\frac{\Lambda c^2}{3}r\approx\frac{\Lambda(3\cdot10^8)^2(1.2\cdot10^{13})}{3}\approx10^{-23}\left[\frac{m}{s^2}\right] \tag{22}$$

This formulae can also be expressed by the cosmological parameters Ω_Λ and H_0:

$$a_\Lambda=\Omega_{\Lambda_0}\cdot H_0^2\cdot|\vec{r}| \tag{23}$$

Because the perturbing accelerations are very small we can use the canonical force components approach (Gaussian method) to calculate the consequences for the orbital parameters.

4.1 Change of orbital parameters

A non-vanishing Λ leads to a modified Newton-Poisson equation for the gravitational potential V

$$\Delta V=4\pi G\rho+\frac{c^2}{2}\Lambda \tag{24}$$

The right side of the equation can be written as $4\pi G(\rho+\rho_{vac})$ with

$$\rho_{vac}=\frac{c^2}{8\pi G}\Lambda \quad \text{or} \quad \Lambda=\frac{8\pi G}{c^2}\rho_{vac}\,. \tag{25}$$

For the Newton-Lambda – Potential (Figure 3) we get:

$$V(r)=-\frac{GM}{r}-\frac{\Lambda\cdot c^2}{6}r^2 \tag{26}$$

This generalized Newtonian potential leads to a gravitational force with a repulsive term in case that $\Lambda > 0$.

$$f(r)=-\frac{GM}{r^2}+\frac{\Lambda c^2}{3}r \tag{27}$$

The additional Λ-term gives rise to a non-Newtonian equation of motion for the spacecraft dynamics:

$$\ddot{r}-r\dot{\varphi}^2+\frac{GM}{r^2}=-\frac{\Lambda c^2}{3}r \tag{28}$$

$$\ddot{\varphi}+2\frac{\dot{r}\dot{\varphi}}{r}=-\frac{\Lambda c^2}{3} \tag{29}$$

In the context of General Relativity the motion of planets and spaceprobes follow geodesic lines of the Schwarzschild-de-Sitter metric:

$$ds^2=A(r)\cdot c^2dt^2-\frac{1}{A(r)}\cdot dr^2-r^2\cdot(d\theta^2+\sin^2\theta\cdot d\varphi^2) \tag{30}$$

In the weak field limit we have:

$$g_{00}=A(r)=1+\frac{2\cdot V(r)}{c^2} \tag{31}$$

The Newton-Binet equation for the orbit in curved space is given:

$$\frac{d^2u}{d\varphi^2}+u=\frac{c^2\cdot r_s}{l^2}+3\cdot r_s\cdot u^2+\frac{\Lambda\cdot c^2}{3\cdot l^2\cdot u^3} \tag{32}$$

with $u = 1/r$ and $r_S = GM/c^2$. The first term comes from ordinary Newtonian dynamics the second and third term on the left take curvature and a non-vanishing Λ into account.

These terms lead to a perihelion shift for a bound motion

$$\left(\frac{\Delta\varphi}{2\pi}\right)_{SSM}=\frac{6\pi\cdot GM}{c^2\cdot a\left(1-e^2\right)} \tag{33}$$

$$\left(\frac{\Delta\varphi}{2\pi}\right)_{\Lambda}=-\frac{a^3\left(1-e^2\right)^3\cdot c^2}{2GM}\cdot\Lambda \tag{34}$$

For planet Mercury the perihelion shift is determined with an accuracy better than 5 x 10^{-3}. This means that $\Lambda \leq 10^{-38}$ m^{-2}. The cosmological constant would have to be about 12 orders of magnitude bigger than the upper bound deduced from cosmological observation.

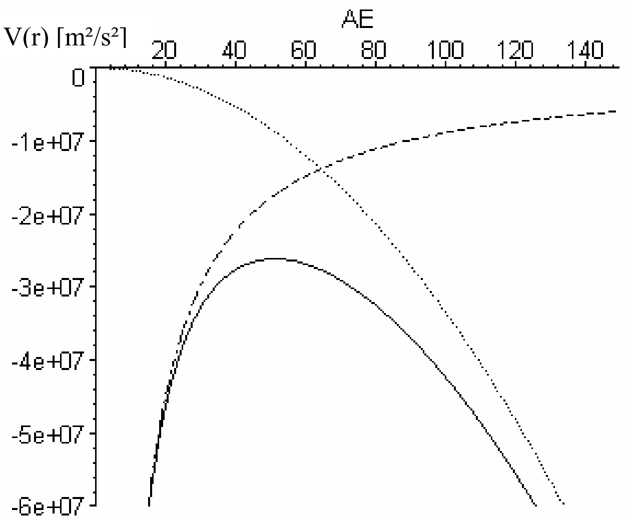

FIG 7. Newton - Lambda Potential (toy model): dash = $-\Lambda c^2 r^2/6$,dot = $-GM/r$ with $\Lambda = 10^{-39}$ $1/cm^2$ as an example.

4.2 Gauss-Lagrange perturbation equation

For bound orbits ($e < 1$) we have the Gauss-Lagrange equations for the perturbed orbital parameters in case of a radial perturbation as

$$\frac{da}{dt}=\frac{2e\sin\varphi}{n\cdot\sqrt{1-e^2}}\cdot F_r \tag{35}$$

$$\frac{de}{dt}=\frac{\sqrt{1-e^2}\cdot\sin\varphi}{n\cdot a}\cdot F_r \tag{36}$$

$$\frac{d\omega}{dt}=-\frac{\sqrt{1-e^2}\cdot\cos\varphi}{n\cdot a\cdot e}\cdot F_r \tag{37}$$

With $n=\sqrt{\frac{GM}{a^3}}$, after substitution of $F_r=\frac{\Lambda c^2}{3}\cdot r$, $l=r^2\dot{\varphi}$, $\mu=GM$, we get

$$\frac{da}{dt}=0 \tag{38}$$

$$\frac{de}{dt}=0 \tag{39}$$

$$\frac{d\omega}{dt}\neq 0 \tag{40}$$

which leads over a complete orbit ($t = 2\pi$) to

$$\frac{\dot{\omega}}{2\pi}=\frac{\Lambda c^2 a^{3/2}\left(1-e^2\right)}{2\sqrt{\mu}} \tag{41}$$

For unbound, i.e. hyperbolic trajectories (Figure 4)

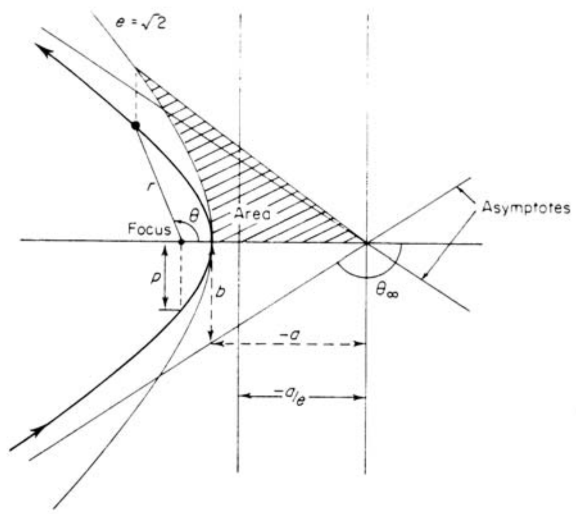

FIG 8. Hyperbolic trajectory ($e > 1$). The angle θ is the true anomaly φ.

we have:

$$\frac{da}{dt}=-\frac{2a^{3/2}}{\sqrt{GM\left(e^2-1\right)}}F_r e\sin\varphi \tag{42}$$

$$\frac{de}{dt}=\sqrt{\frac{a\left(e^2-1\right)}{GM}}F_r\sin\varphi \tag{43}$$

$$\frac{d\omega}{dt} = -\frac{1}{e}\sqrt{\frac{a(e^2-1)}{GM}}F_r \cos\varphi \qquad (44)$$

We continue as seen for the bound orbits with the substitution and get with $0 \leq \varphi = \varphi * < \varphi_\infty = \arccos(-1/e)$:

$$\left.\frac{da}{dt}\right|_T = -\frac{2a^{3/2}}{\sqrt{\mu(e^2-1)}}\frac{\Lambda c^2}{3}\left(\frac{l^2}{\mu}\right)^3 \frac{1}{l}\int_0^{\varphi *}\frac{e\sin\varphi}{(1+e\cos\varphi)^3}d\varphi \neq 0 \qquad (45)$$

$$\left.\frac{de}{dt}\right|_T = \sqrt{\frac{a(e^2-1)}{\mu}}\frac{\Lambda c^2}{3}\left(\frac{l^2}{\mu}\right)^3 \frac{1}{l}\int_0^{\varphi *}\frac{\sin\varphi}{(1+e\cos\varphi)^3}d\varphi \neq 0 \qquad (46)$$

$$\left.\frac{d\omega}{dt}\right|_T = -\frac{\Lambda c^2\sqrt{e^2-1}}{3n\cdot a\cdot e\cdot l}\left(\frac{l^2}{\mu}\right)^3 \int_0^{\varphi *} - \frac{\cos\varphi}{(1-e\cos\varphi)^3}d\varphi \neq 0 \qquad (47)$$

After integration up to $\varphi * = 0.7$ we get approximately:

$$\left.\frac{da}{dt}\right|_{\varphi *} = 1.7\left(-\frac{2a^{3/2}}{\sqrt{\mu(e^2-1)}}\frac{\Lambda c^2}{3}\left(\frac{l^2}{\mu}\right)^3 \frac{1}{l}\right) \approx 10^{-18}\left[\frac{m}{s^2}\right] \qquad (48)$$

$$\left.\frac{de}{dt}\right|_{\varphi *} = 1.03\sqrt{\frac{a(e^2-1)}{\mu}}\frac{\Lambda c^2}{3}\left(\frac{l^2}{\mu}\right)^3 \frac{1}{l} \approx 10^{-30}\left[\frac{1}{s}\right] \qquad (49)$$

$$\left.\frac{d\omega}{dt}\right|_{\varphi *} = 0.101\left(-\frac{\Lambda c^2\sqrt{e^2-1}}{3n\cdot a\cdot e\cdot l}\left(\frac{l^2}{\mu}\right)^3\right) \approx 10^{-40}\left[\frac{rad}{s}\right] \qquad (50)$$

where we have used the specific angular momentum l and the gravitational parameter $\mu = G\,M$.

The main result is that in case of a hyperbolic trajectory the a, e and ω show a very small time variation whereas in case of a bound orbit only ω changes.

5. NON-NEWTONIAN GRAVITY

There has beenrenewed interest in recent years in the possibility of deviations from Newton's "inverse-square ($1/r^2$) law of universal gravitation. Within the attempts to unify gravitation and in order to solve the Dark Matter problem in astronomy several theories have been suggested which always include a modification of Newtons law of gravity. We would like to mention only two of these modifications.

5.1 Newton-Yukawa – Potential

$$V(r) = -\frac{GMm}{(1+\alpha)r}\left(1+\alpha\, e^{-\frac{r}{\lambda}}\right) \qquad (51)$$

This potential would lead to an additional acceleration:

$$a_P = a_Y = -\frac{\alpha\, a_1}{2(1+\alpha)}\frac{r_1^2}{\lambda^2} \qquad (52)$$

with:
α = coupling strength relative to Newtonian gravity
λ = Yukawa - force range
a_1 = Newtonian acceleration at a distance r_1 = 1 AU

If the Newton-Yukawa Potential is used for the calculation of the Pioneer anomaly a force range of λ = 200 AU and $\alpha = -10^{-3}$ is necessary to reach the measured acceleration but out to 65 AU there is no observed indication of an r term in the acceleration (Anderson, 2002).

5.2 Modified Newtonian Dynamics (MOND)

Milgrom (1983) and Beckenstein (2005) have put forward the suggestion that for low accelerations Newtonian Dynamics is modified. In their theory the equation of motion is:

$$m\mu\left(\frac{a}{a_0}\right)\vec{a} = \vec{F}\begin{cases}1 & |\vec{a}| >> a_0 \\ \sim \vec{a}/a_0 & |\vec{a}| << a_0\end{cases} \qquad (53)$$

with $a_0 = 2 \times 10^{-10}$ [m/s²].

For low acceleration a non – Newtonian equation of motion results:

$$a - a_0 = \frac{GM}{r^2} \qquad (54)$$

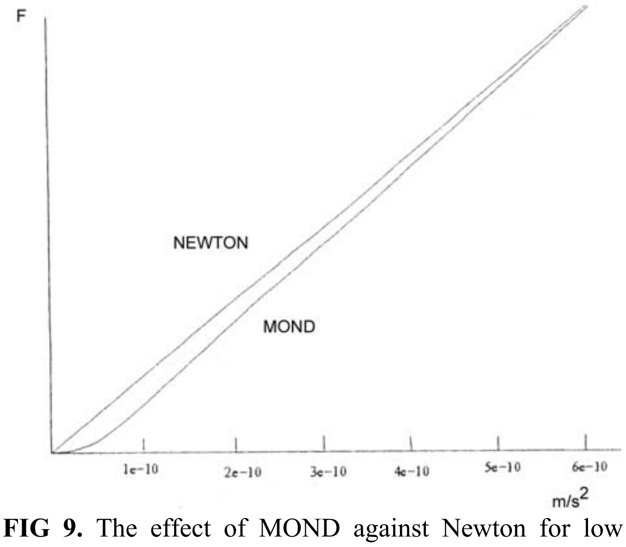

FIG 9. The effect of MOND against Newton for low accelerations.

$$a_{MOND} = cH_0 = c^2\sqrt{\frac{\Lambda}{3}} \approx \left(6{,}2x10^{-10} \div 7.77x10^{-10}\right)\left[\frac{m}{s^2}\right] \qquad (55)$$

where we have used H_0 = (72 ±8) km/s/Mpc. The calculated cosmological perturbation acceleration due to MOND is with in the limits of the Pioneer anomaly but there are some fundamental problems to integrate this theory into the framework of Einsteins theory of General Relativity.

6. SPACECRAFT PERTURBATION: Non-Newtonian Gravity and / or the influence of cosmic expansion

The question wether local physical phenomena are affected by the expansion of the universe has a long history. Already McVittie (1933) asked: do planetary orbits expand with the Universe. In 2001 George F.R. Ellis discussed more general the relationship between cosmology and local physics. Recently there has been a revival of interest (J.L. Anderson 1995, Bonnor 1996) in this question in the context of the observed Pioneer-anomaly and in relation to the far future of the universe (Caldwell et al. 2003).

In the following we summarize those ideas concerning the influence of expansion to the anomalous Pioneer deacceleration which are beyond the perturabation which occurs naturally in a local Fermi frame as discussed in section 4 of this paper.

Nottale (2003) relates in an approach which implements Mach's principle the cosmological constant to the Pineer acceleration:

$$a_p = c^2\sqrt{\frac{\Lambda}{3}} = \frac{c^2}{\sqrt{3}L_U} = \Omega_\Lambda^{1/2} H_0 c \tag{56}$$

using the cosmological parameters Ω_Λ = 0.73 ± 0.05 and H_0 = 71 ± 4 km/s.Mpc he finds the length scale L_U = 2.85 ± 0.25 Mpc which also implies Λ = (1.29 ± 0.23) x 10^{-56} cm^{-2} in agreement with observation.

Rosales and Sánchez-Gomez (1999) propose that the acceleration could be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. The additional non-Newtonian acceleration is

$$a_P = \frac{H_0}{c} \approx 8.5\cdot 10^{-8}\ cm/s^2 \tag{57}$$

if the present Hubble parameter is H_0 = 85 $km\cdot s^{-1}\cdot Mpc^{-1}$ a value not supported by observation.

J.P. Mbelek (2003) suggests that the anomalous acceleration of Pioneer comes from the interaction of the spacecraft with a scalar field (Quintessence). In this case the equation of motion can be written

$$\frac{d\vec{V}}{dt} = -\nabla V_N + \frac{1}{c^2}\cdot\frac{dV(\Phi)}{dt}\cdot\frac{d\vec{r}}{dt} \tag{58}$$

with

$$a_P = \frac{1}{c^2}\cdot\frac{dV(\Phi(t))}{dt}\cdot\vec{V}$$

The perturbation depends on the equation of state of dark energy – represented by a scalar field – and the time evolution of its potential V(Φ).

7. CONCLUSION

We pointed out the possible influence of cosmic expansion on a spacecraft on a hyperbolic trajectory in the solar system, i.e. of the Pioneer spaceprobe. To have an acceleration in the order of magnitude of the Pioneer anomaly $a_{Pio} \approx a_\Lambda \approx 10^{-8}\ cm/s^2$ we need a cosmological constant of Λ = 10^{-38} 1/m² >> $\Lambda_{cosmology}$ which is not compatible with present observations.

In the following table (Table 1) we summarise the different approaches by non-Newtonian gravity – coming from standard and non-standard cosmology) for the Pioneer anomaly:.

	Lambda	Expansion	Rosales	Milgrom
a_i [m/s²]	2.5x10^{-23}	1/3x10^{-23}	8.5x10^{-10}	7.7x10^{-10}

TAB 1. Acceleration due to non-Netwtonian terms.

In addition to a possible non-Newtonian acceleration there is for a hyperbolic orbit a change of orbital size, a contribution to the precession of the perihel added to the well-known general relativistic effect and the eccentricity can change. But all this effect are too small in order to explain the observed Pioneer anomaly and are beyond present measurement possibilities.

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