Hochschule für Angewandte Wissenschaften

Seminar „Flugzeugkabine und Kabinensysteme“
im Rahmen der DGLR Jahrestagung 2004

Modul: Faserverbund- und Sandwichtechnologie

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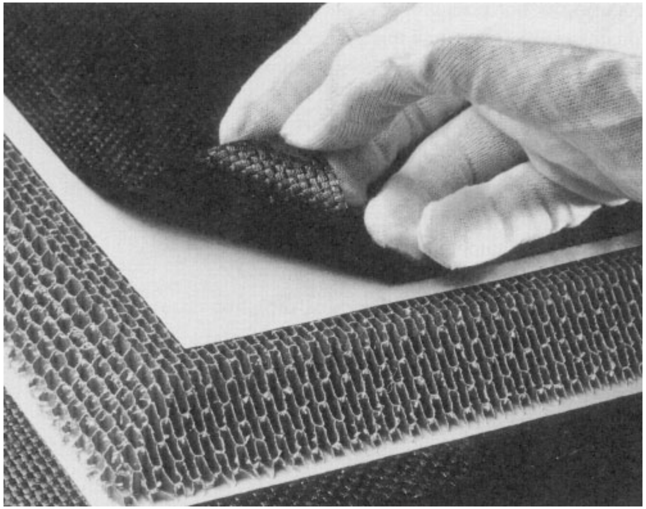

Modul: Faserverbund- und Sandwichtechnologie

Inhaltsübersicht

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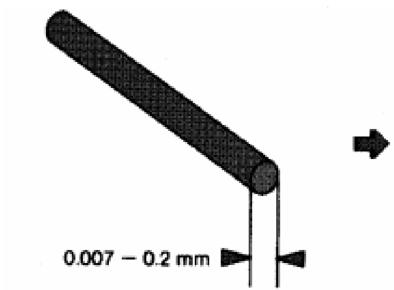
Inhaltsübersicht

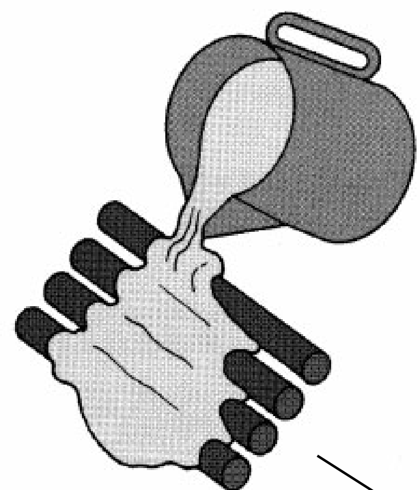
What is a polymer composite

Fiber

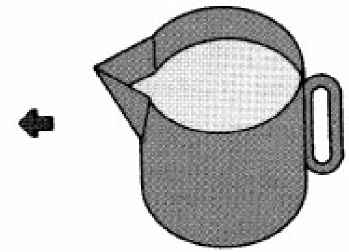


Typical fibers:
- glass
- carbon
- aramid

Laminate



Matrix

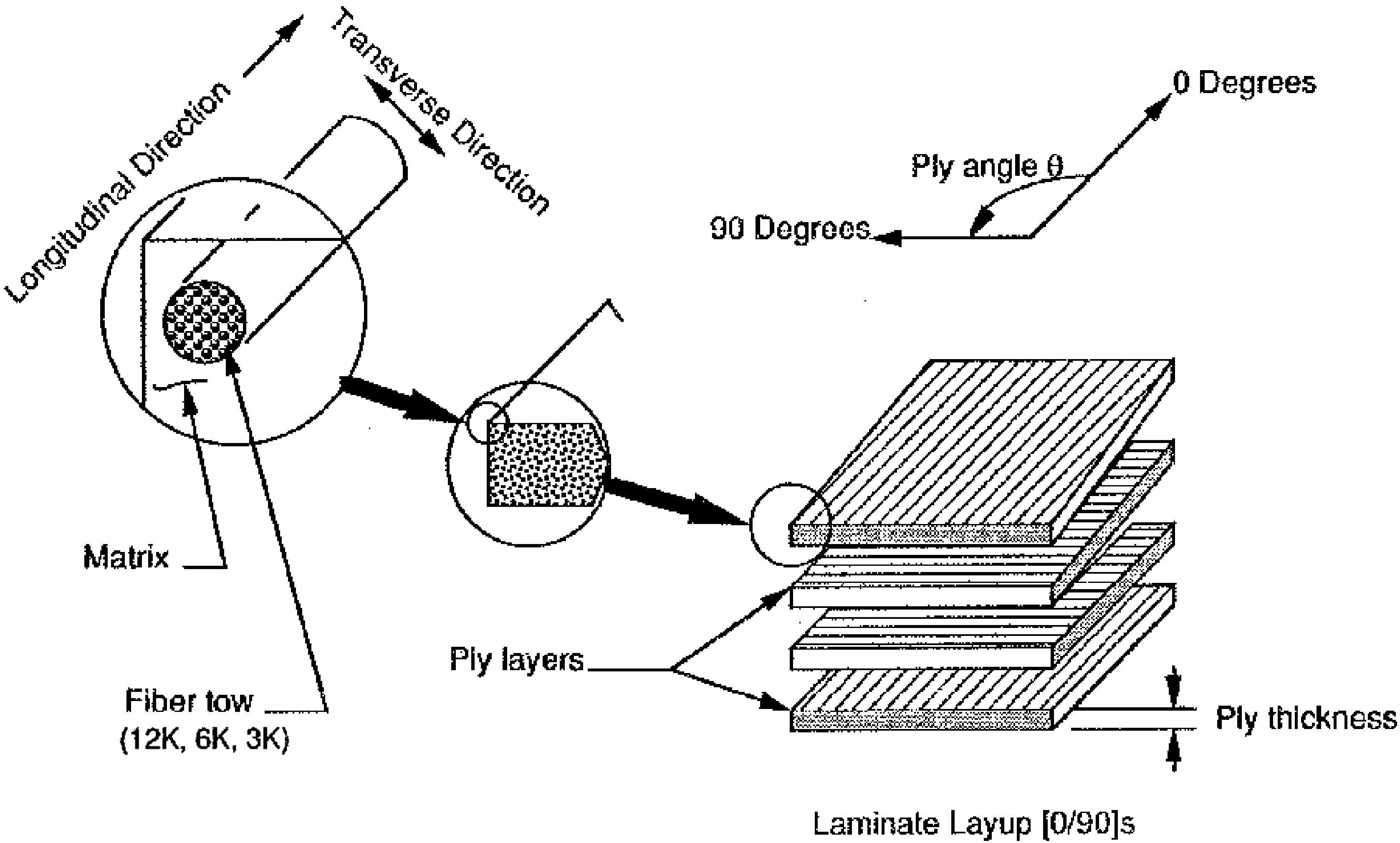


Typical matrices:
- polyester
- epoxy
- vinylester
- thermoplast

Role of the fiber:
- provides high specific strength and stiffness

Role of the matrix:
- keeps fibers together, protects the fibers
- transfers loads between fibers, compensates peak stresses
- provides minimum distance between fibers

Composite Anatomy

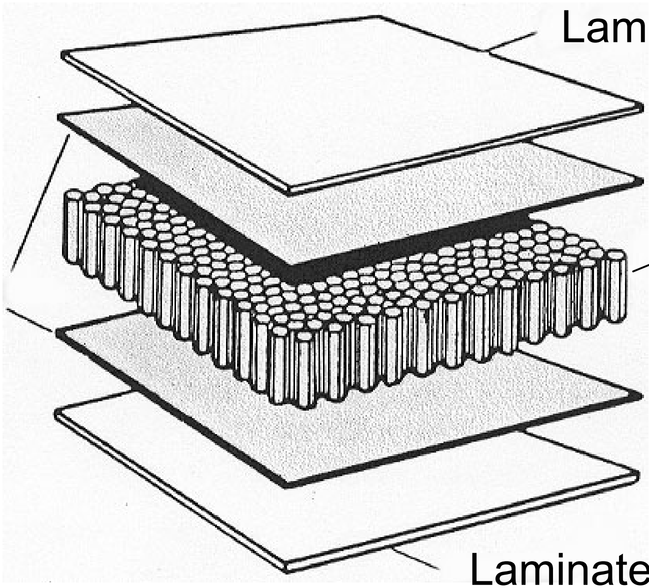


Sandwich Structures

General set-up

- 2 relatively thin facesheets (providing high stiffness and strength)
- Thick, but light core

→ Distance between facesheets provides a high area moment of inertia, whereas the additional weight –necessary for the core– is almost neglectible



Historical Development

Approx. 1000 B.C., Egypt

Mummy caskets made of agglutinated parchment leaves

Approx. 800 B.C., Egypt

Bricks reinforced with chaffed straw

Approx. 500 B.C., Inka und Maya

Pottery reinforced with plant fibers

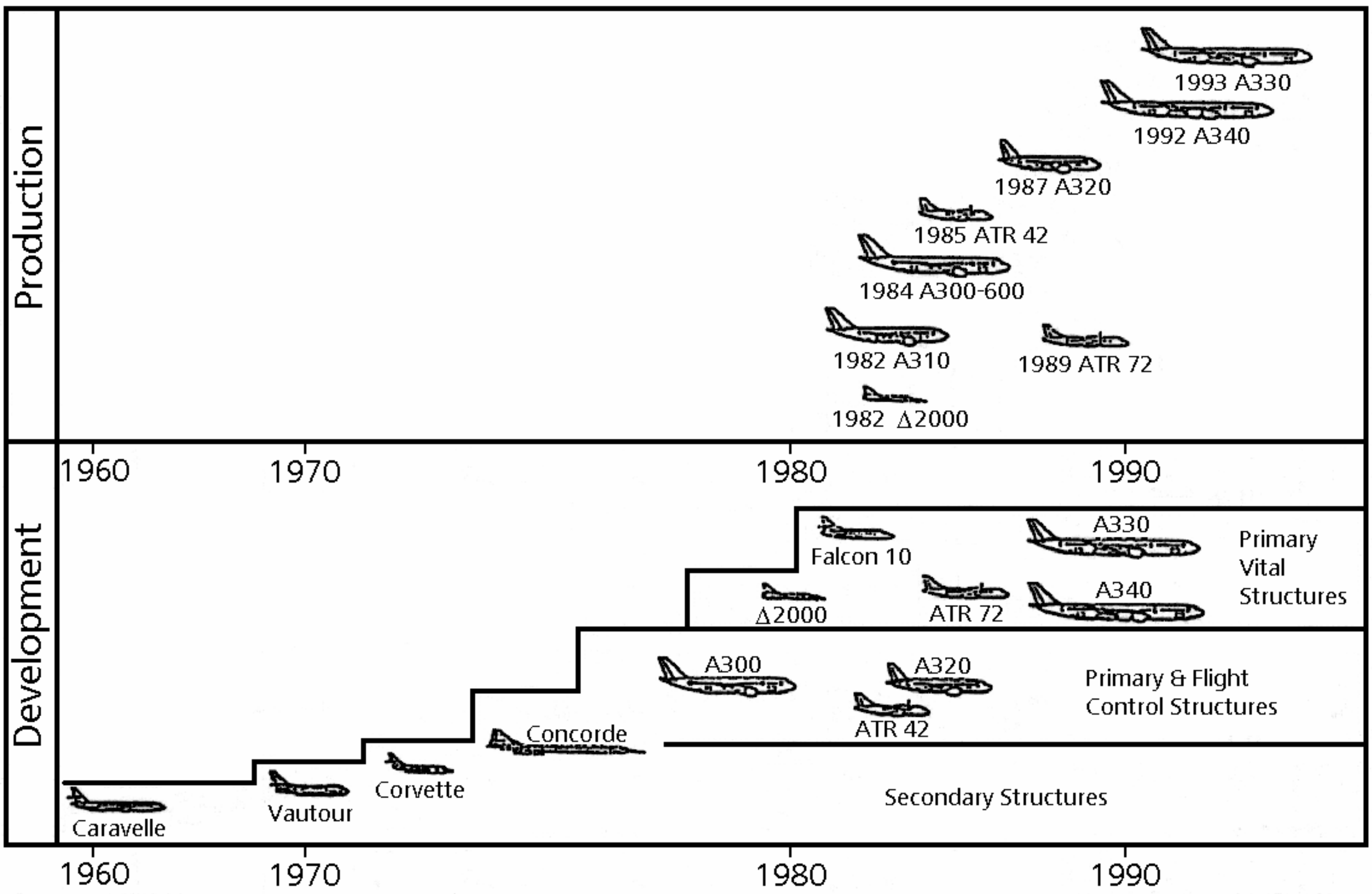
Later…

Damascene swords with alternating layers of hard and soft steel, gypsum and plaster with cattle hair, asbestos fibers in phosphate cement

Historical Development

Approx. 1920	Laminates made of cellulose fibers with phenolic resin
Approx. 1930	Polymer matrix with glass fiber reinforcement
End of 1940	Development of highly stiff fibers (Boron fibers, USA)
1945	Honeycomb Sandwich
1964	Carbon fibers (USA)
1966	Aramid fibers (organic polymers)

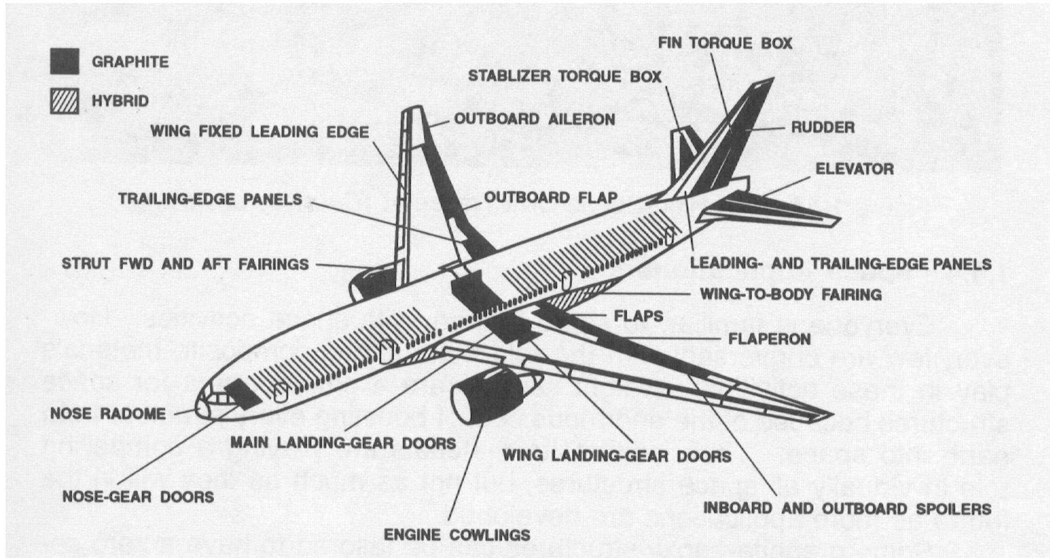
Aerospace Applications of Composites



Aeronautical Applications - Airbus

Fraction of Composites w.r.t. structural weight:

- Airbus A300-600 5%
- Airbus A310-300 8%
- Dornier 228 12%
- Boeing 777 10%
- Airbus A320 16%
- Airbus A340 13%
- Airbus A380 22%



Boeing 777

Advantages and Disadvantages

Advantages:

- high mass specific strength and stiffness
- “tailored” materials and structures
  - adaptation to load path
  - use of an-isotropic properties
  - design flexibility/complex shapes
  - multifunctional design
- good damping and fatigue
- low corrosion and good chemical resistance
- high electric resistance
- low thermal expansion
- not magnetic

Disadvantages:

- special experience needed
  - design
  - pre- and final analysis
  - production technology
- limited knowledge in life cycle behaviour and operational use
- quality control and assurance
- material properties are generated during production process
- certification problems
- limited temperature resistance of several fibers and matrices
- low failure strains/ no plasticity
- relative high cost of several fibers and pre-forms
- relative low experience in automation

Modul: Faserverbund- und Sandwichtechnologie

Inhaltsübersicht

Classification of Fiber Materials

Possible classification of fiber materials:

- Natural organic fibers extracted from plants
  like jute or sisal fibers

- Natural anorganic fibers of mineral origin
  e.g. asbestos fibers

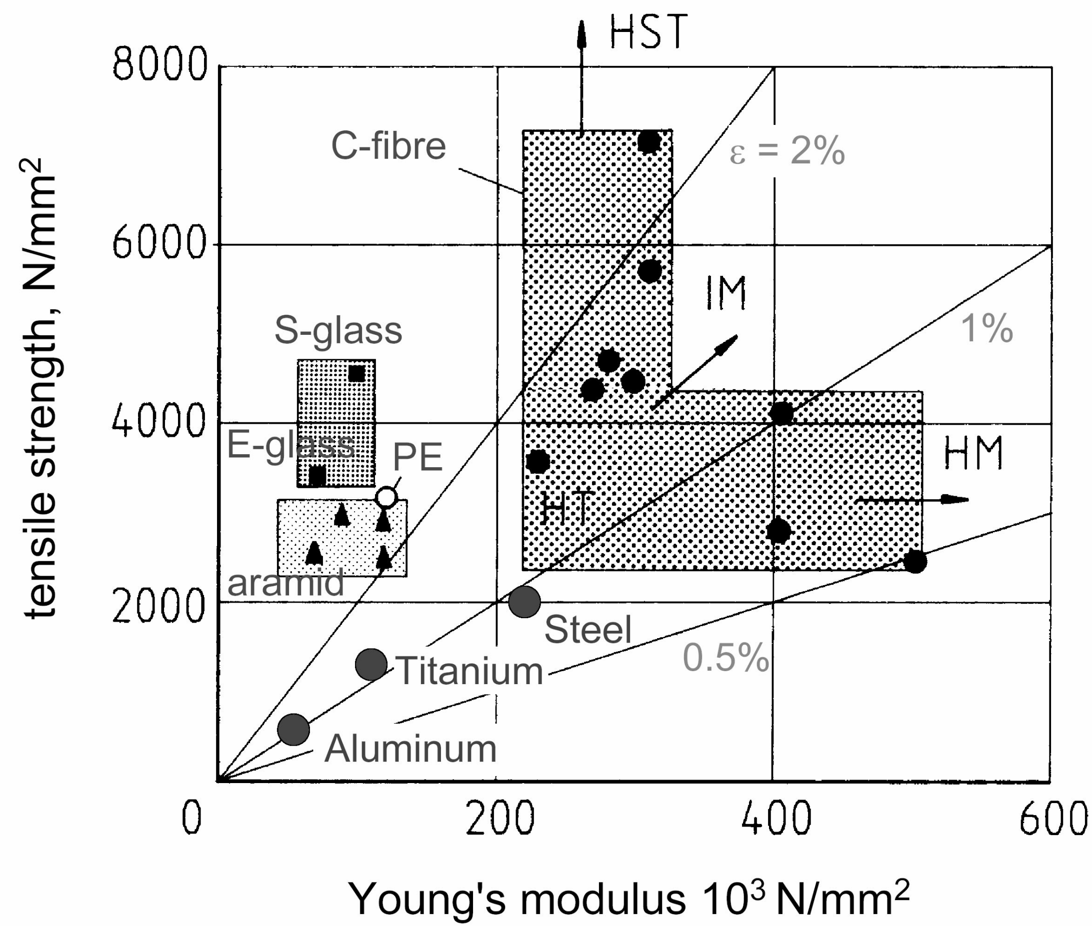
- Synthetic organic fibers
  e.g. aramid, polyester, polypropylene fibers

- Synthetic anorganic fibers
  e.g. glass, metallic, ceramic fibers

Fiber materials of main interest: $\rightarrow$ Glass, Carbon, Aramid

Comparison of Fiber Materials

Comparison of tensile strength and Young's modulus



Types of Fibers:

HT	High Tension
HS	High Strain
IM	Intermediate Modulus
HM	High Modulus
UHM	Ultra High Modulus

Matrix Materials / Resins

Typical thermosetting matrix materials:

- Unsaturated polyester resins (UP)
  Mediocre mechanical properties, high cure shrinkage
- Phenolic resins
  Mediocre mechanical properties, thermal resistance, processing problems
- Polyimide
  Thermal resistance, difficult to process, complicated manufacturing
- Bismaleinimides
  Thermal resistance, absorption of moisture, good mech. Properties
- Epoxy resins

Sandwich Cores

Type of Cores

"Designed" Cores

- Aluminum Cores
- Paper Cores
- Nomex Cores
- Glass Fiber Cores
- Carbon Fiber Cores
- Corrugated Shape Cores
- …

"Natural" Cores

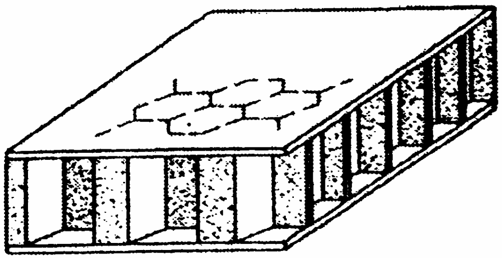
- Balsa-Wood (longitudinal)
- Balsa-Wood (transverse)
- PVC Foam (e.g. Herex C70)
- PMI Foam (e.g. Rohacell)
- PEI Foam (e.g. Airex R82)
- …

Closed cell foams are to be preferred, due to less moisture absorption.

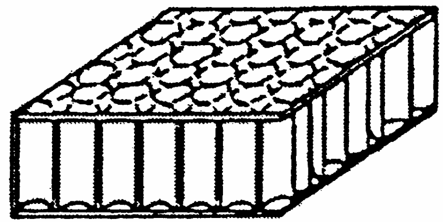
In case of using aluminum cores in conjunction with CFRP facesheets, special care shall be taken concerning corrosion effects (aluminum core).

Sandwich Cores

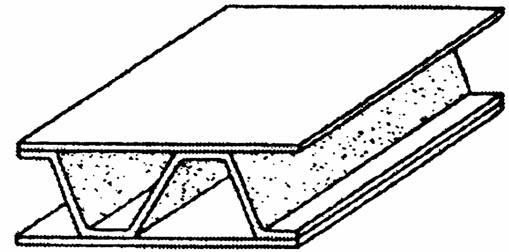
„Designed“ Cores



Honeycomb

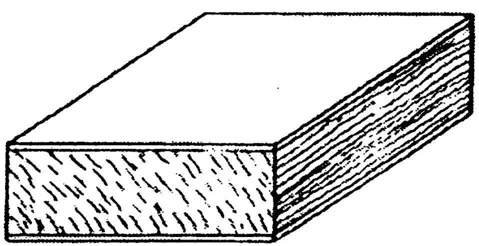


Tubes

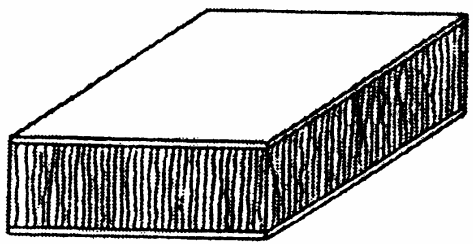


Corrugated Shape

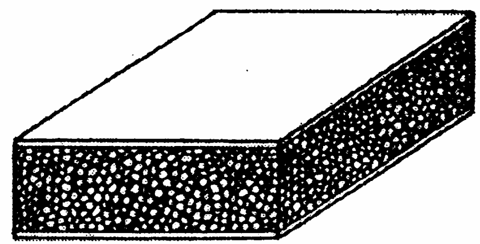
„Natural“ Cores



Balsa-Wood (long.)



Balsa-Wood (trans.)



Foam

Sandwich Cores

Typical material properties of sandwich core materials

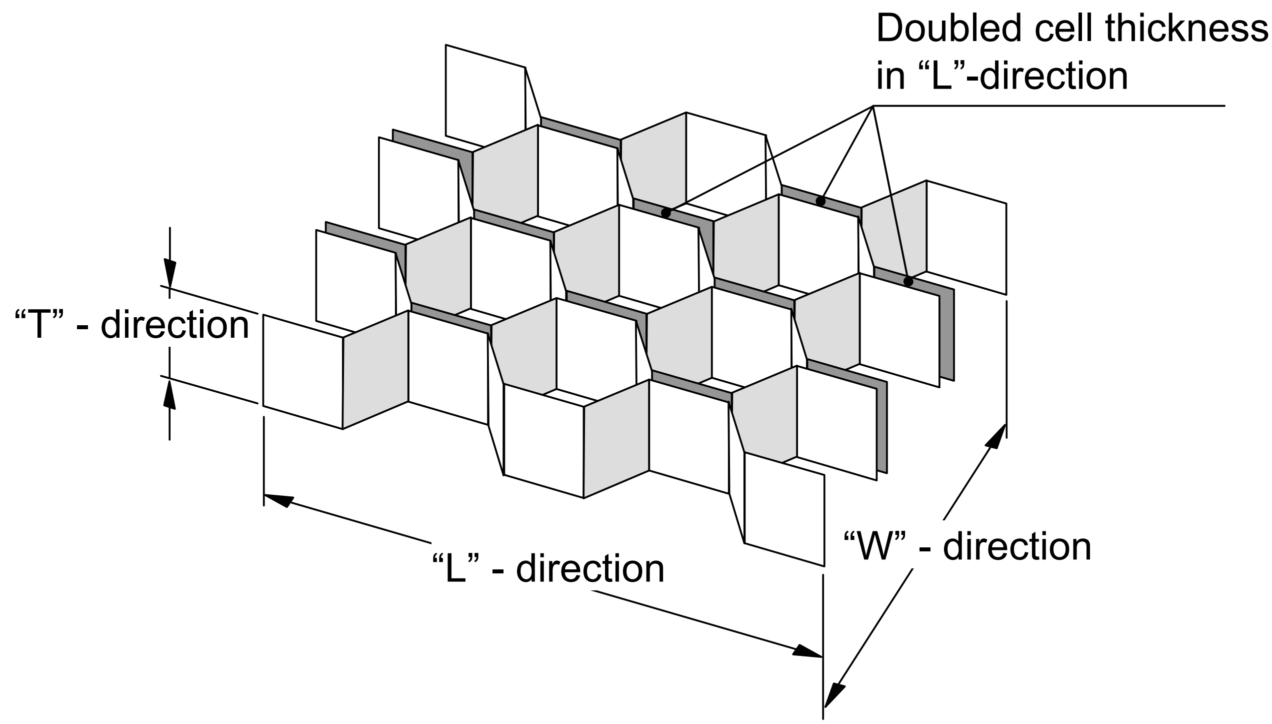
Core Material	Density ρ [kg/m^3]	Compression Strength σ_z [MPa]	Compression Stiffness E_z [MPa]	Maximum Shear *) Strength τ_K [MPa]	Maximum Shear *) Stiffness G_K [MPa]	Rel. Cost [-]
Nomex Honeycomb (cell size c=3.2mm)	48.0	2.24	137.9	1.21	41.4	3.3
Aluminum Honeycomb (5052, cell size c=3.2mm)	49.6	2.07	517.2	1.45	310.3	1.0
Glass Fabric Honeyc. (±45°, cell size c=3.2mm)	48.0	2.83	158.6	1.34	131.0	5.4
PVC Foam (e.g. Herex)	60.0	0.80	58.0	0.80	22.0	1.3
PMI Foam (e.g. Rohacell)	52.0	0.90	69.0	0.80	22.0	3.8
PEI Foam (e.g. Airex)	60.0	0.60	33.0	0.70	14.0	5.0

*) Explanation see next slide …

Honeycomb Cores

Honeycomb Cores: L-Direction and W-Direction

The shear related properties of honeycomb cores are in W-direction lower (approximately by a factor of 3/5) due to the manufacturing process.



Honeycomb Materials

Examples of honeycomb materials

- Aluminum (various alloys)
- Fiberglass fabric reinforced honeycomb
  e.g. dipped in a heat-resistant phenolic resin
- Aramid-fiber reinforced honeycomb
  e.g. Dupont's Nomex aramid-fiber paper dipped in a heat-resistant phenolic resin
- Special honeycombs
  e.g. bias weave carbon fabric reinforced honeycomb dipped in either a heat-resistant phenolic resin or a polyimide resin or
  honeycomb filled with sound absorbing fiberglass batting („Acousti-Core")

Classification of the Manufacturing Methods

… according to …

Raw Material / Semi Finished Products

- Wet in Wet
- Prepreg (Tapes)
- Pultrusion

Tools

- Hand laminate
- Filament winding
- Compression mould
- Autoclave
- Fiber spatter
- Resin Transfer Molding (RTM)
- Vacuum saturation

Curing Temperature

- Cold curing $\leq 60^\circ C$
- Warm curing $60^\circ C \ldots 100^\circ C$
- Hot curing $\geq 100^\circ C$

Curing Pressure

- Pressure less
- Low pressure
- High pressure

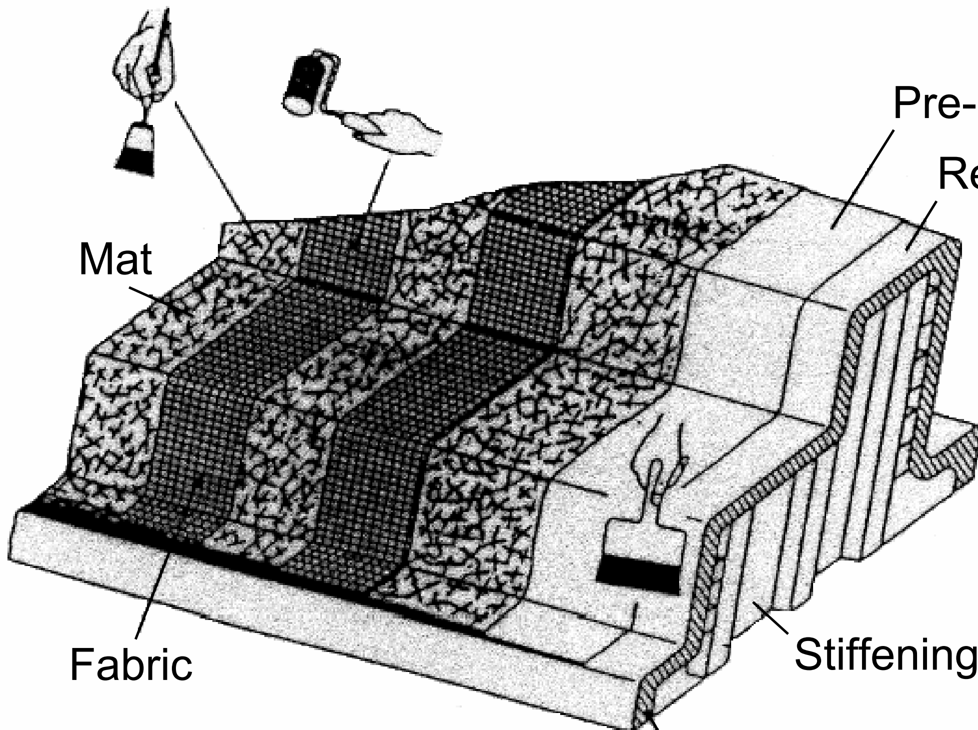
Manufacturing Methods and related Performance

- Quality of shape contour (e.g. surface quality)
- Reproducibility (accuracy, precision)
- Reliability
- Safety
- Fatigue behaviour
- Efforts related to finishing / post processing
- Weight
- Expenditures for maintenance (inspections, repairs, etc.)
- Expenditures for quality assurance (tests, scrap rate)

Costs

- Raw materials
- Jigs, rigs, tools
- Personnel (qualification)
- Quality assurance
- Maintenance for complete life cycle
- Inspections (and related methods, equipment)
- Repairs

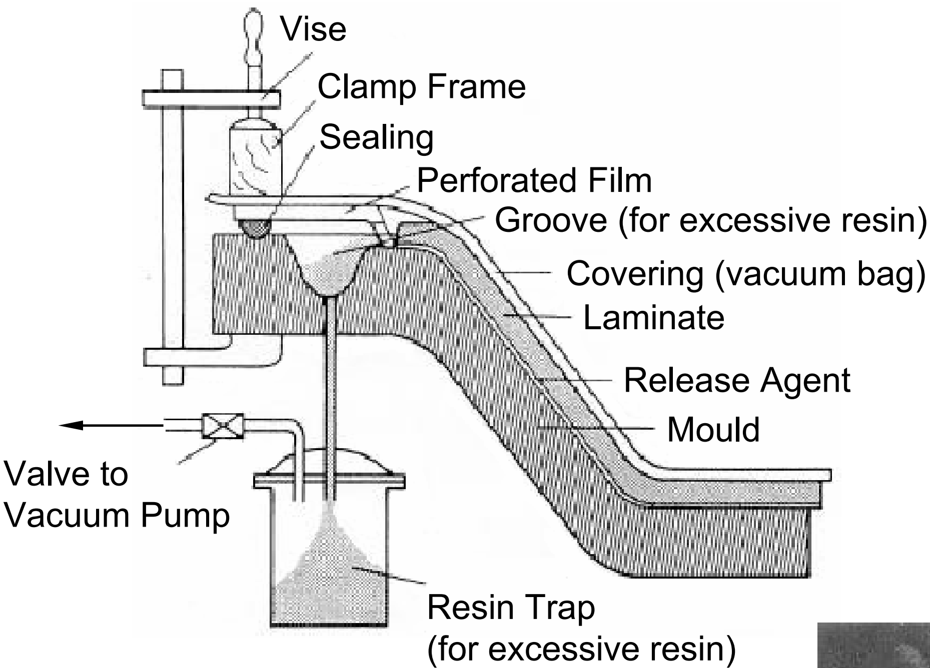
Hand Lay-Up

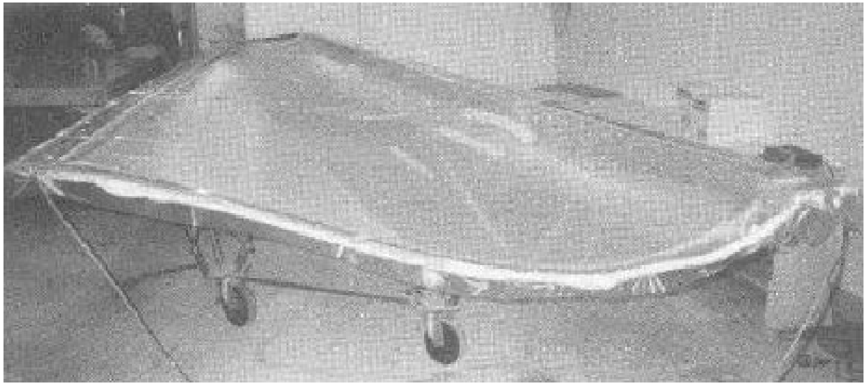


- Epoxy- and Polyester-Matrices
- Glass, Carbon, Aramid Fibers
- Achievable Fiber-Volume-Fractions:
  - Mats: 12 - 20%
  - Fabrics: 40 - 50%

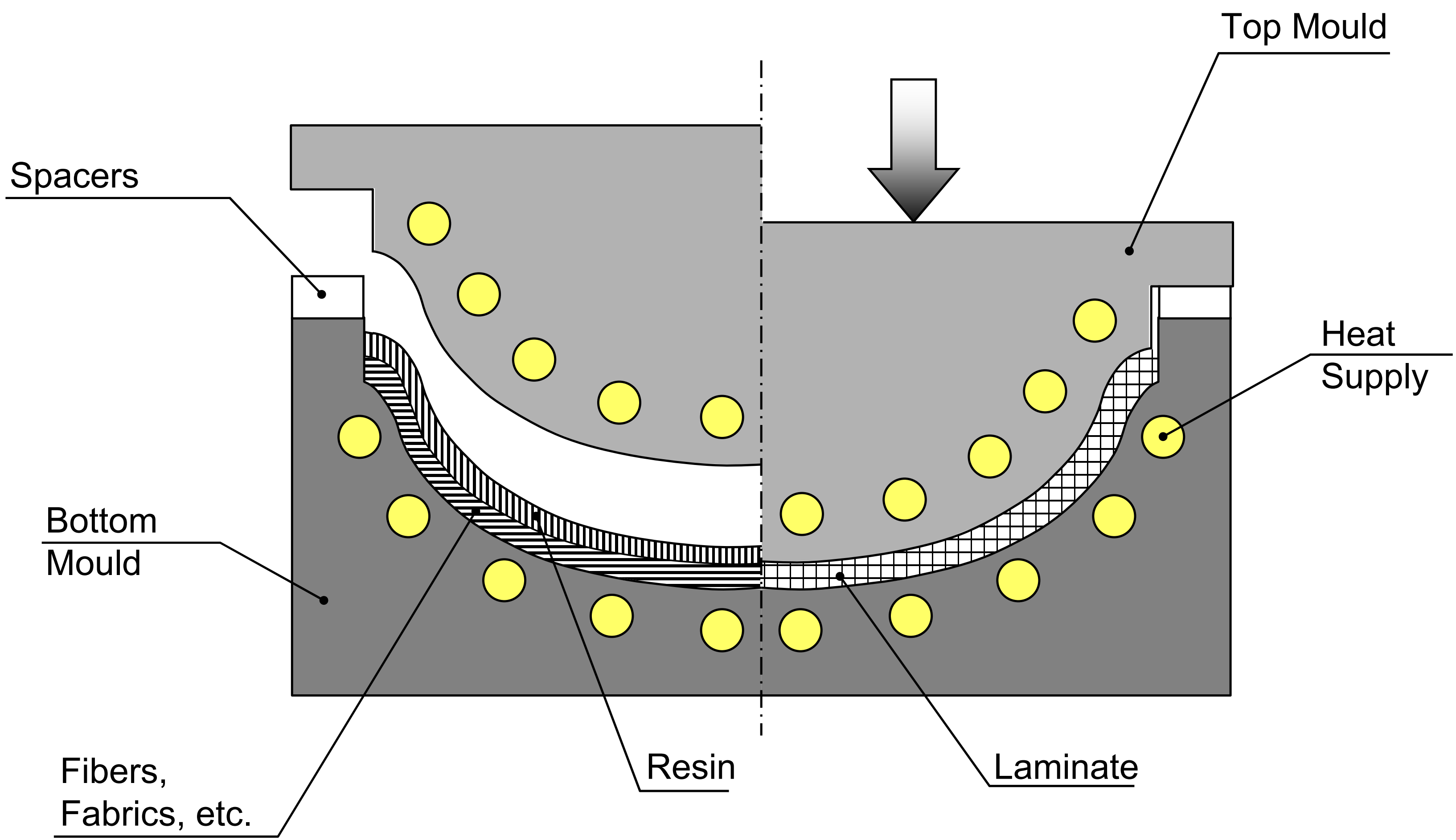
- Low mould and tooling costs; low investments
- Small and medium sized series

Vacuum / Pressure – Bag

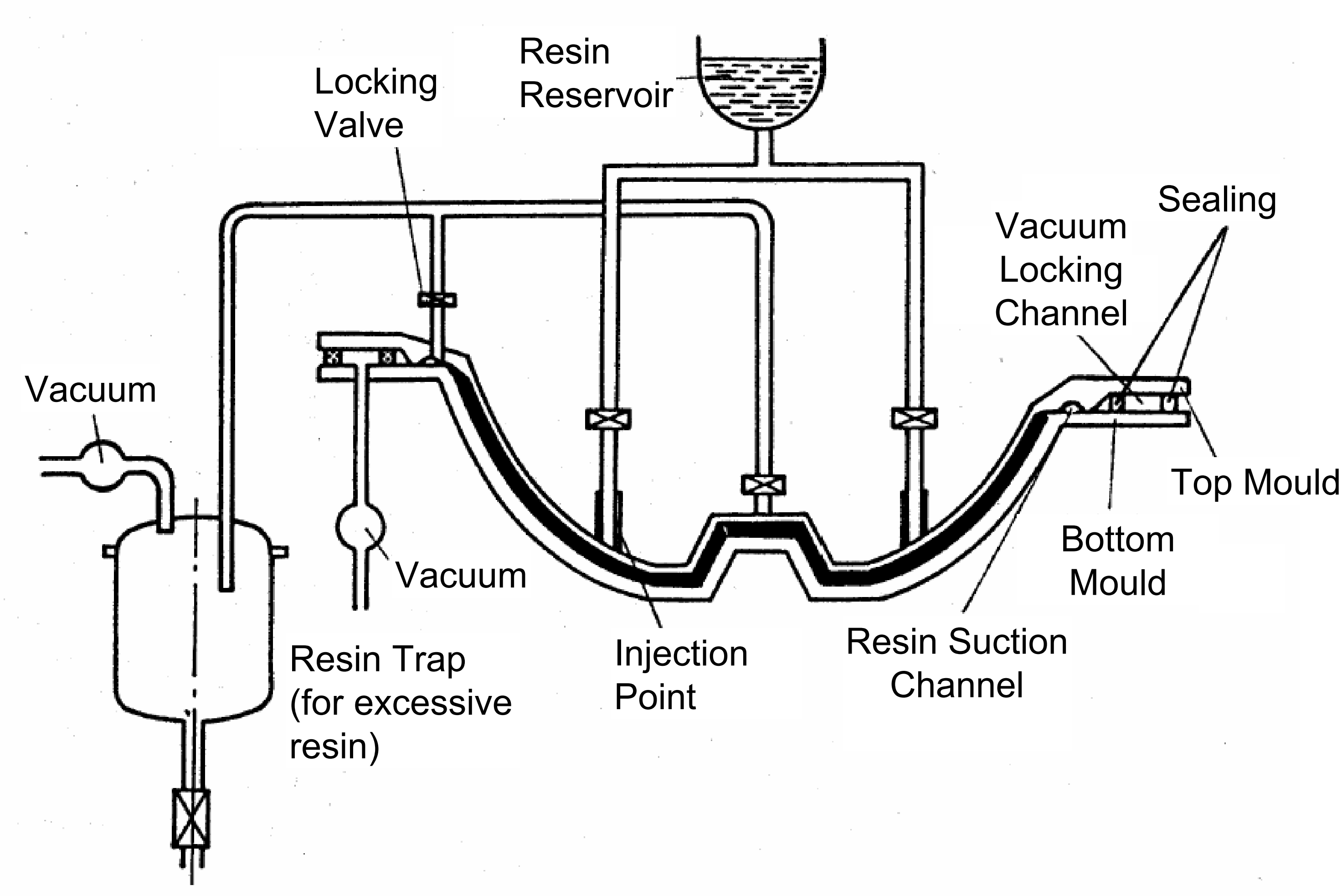




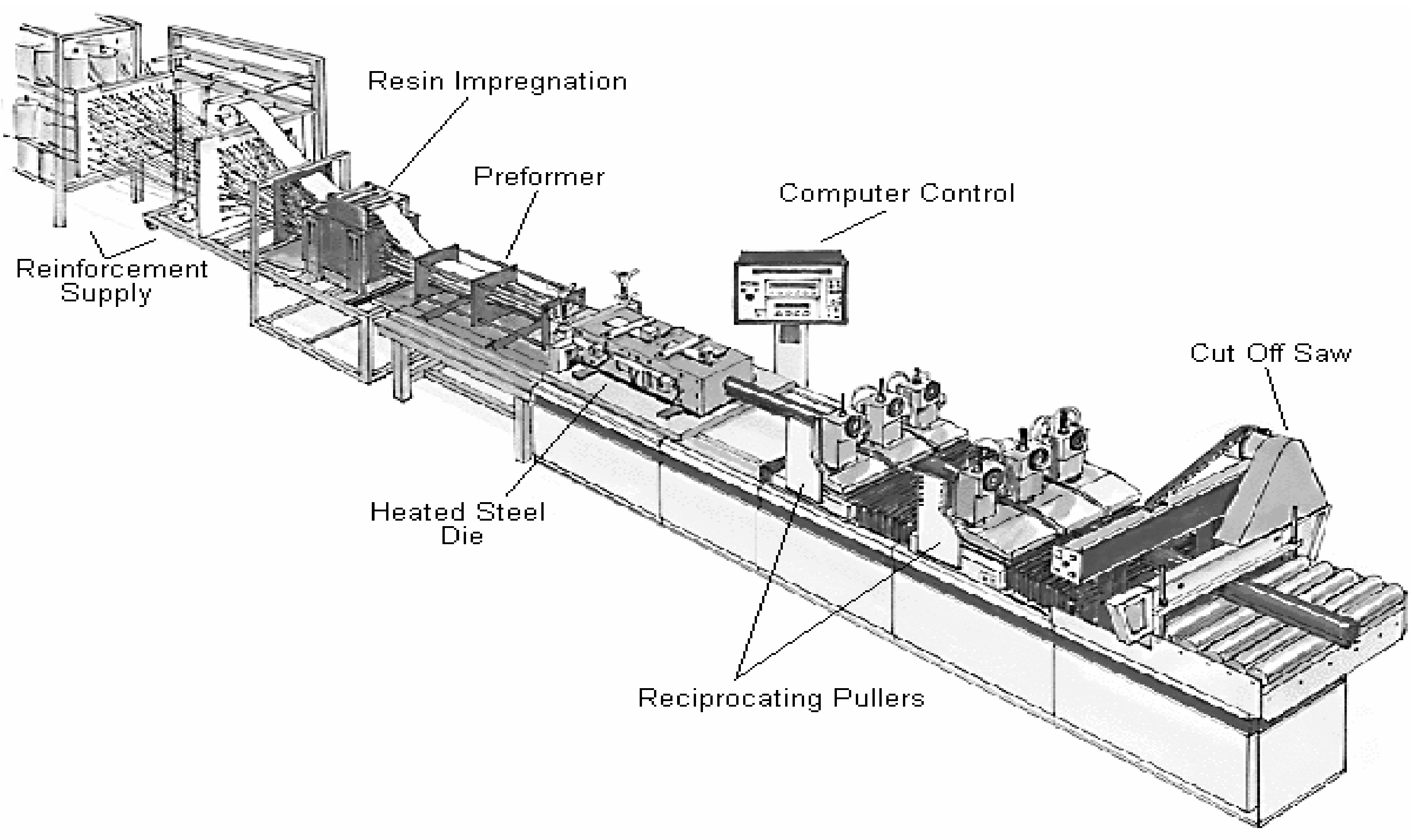
Hot Press Moulding



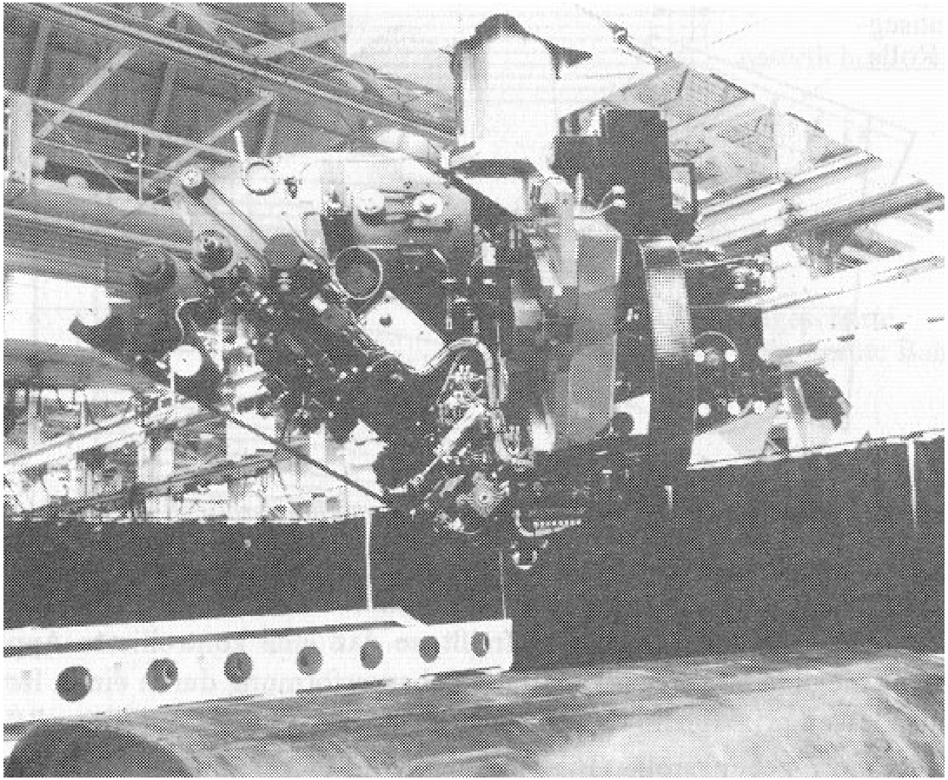
Resin Transfer Moulding / Resin Injection

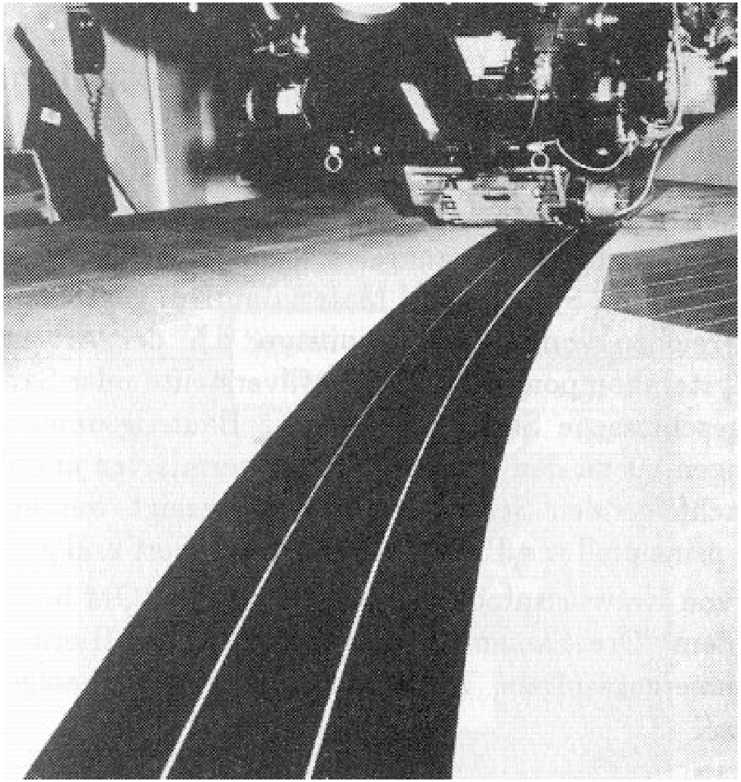


Profile Pultrusion

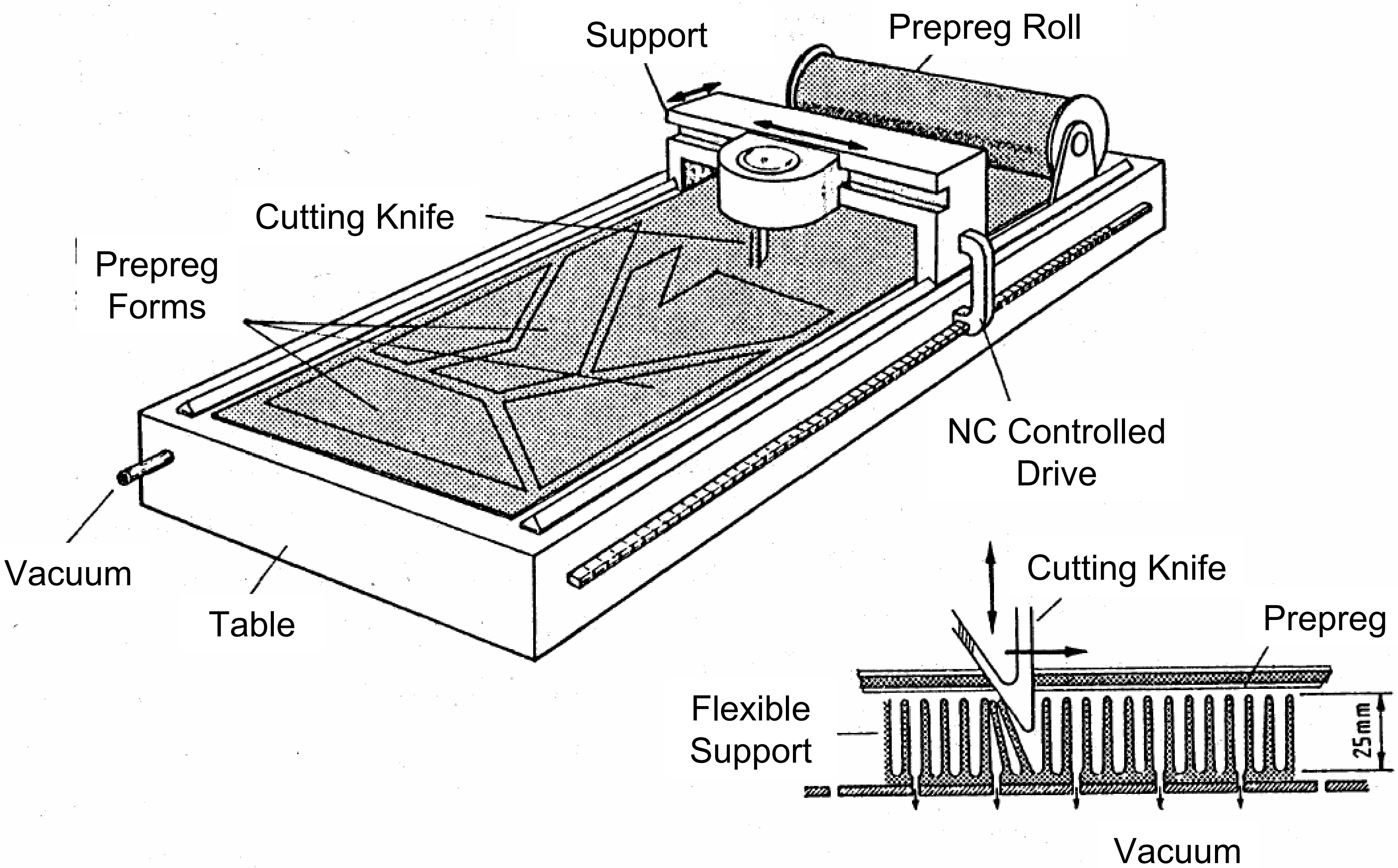


Automated Tape Laying (Prepregs)

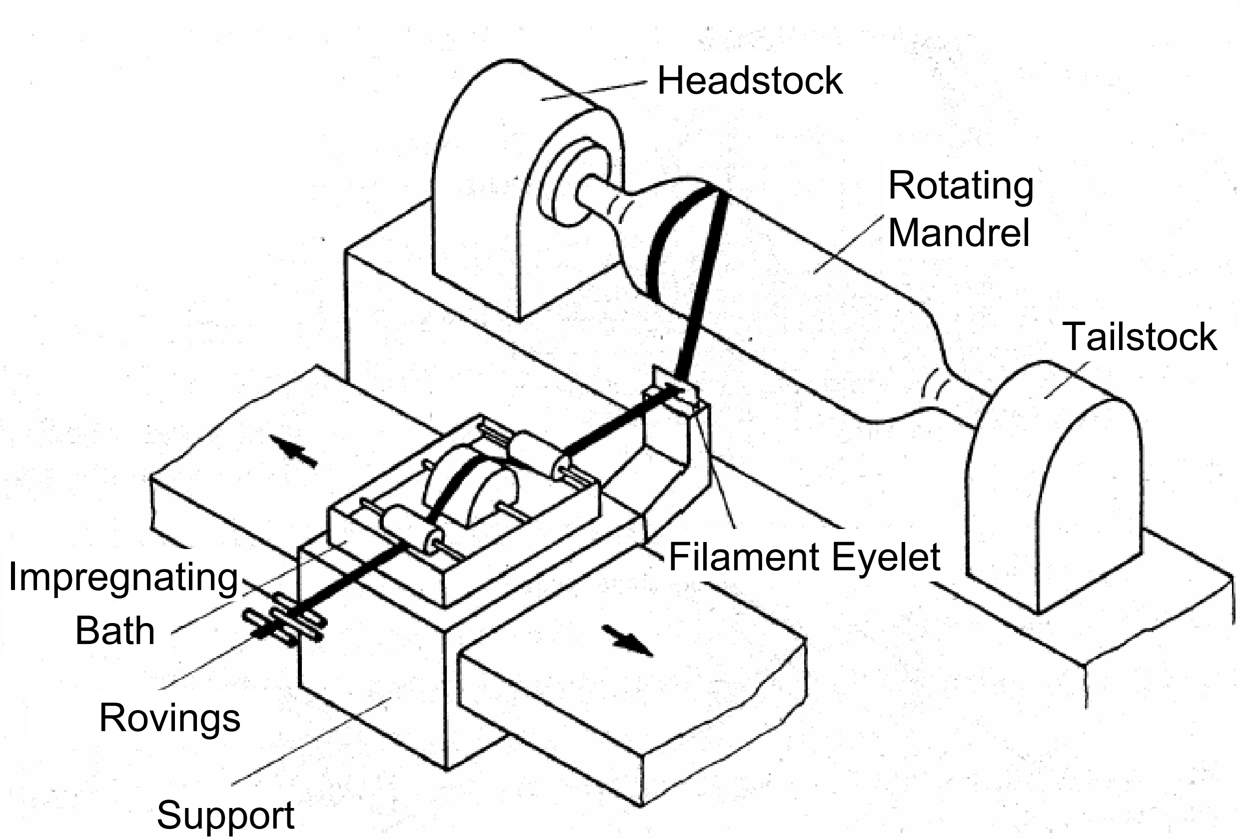


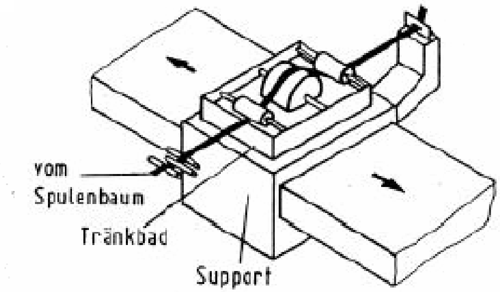


Cutting of Prepregs



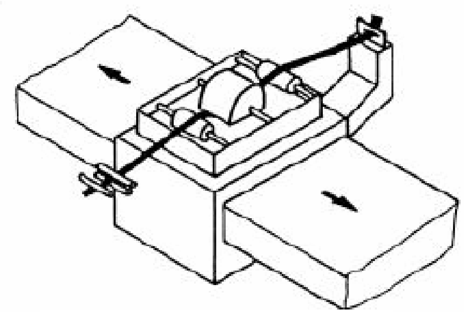
Filament Winding





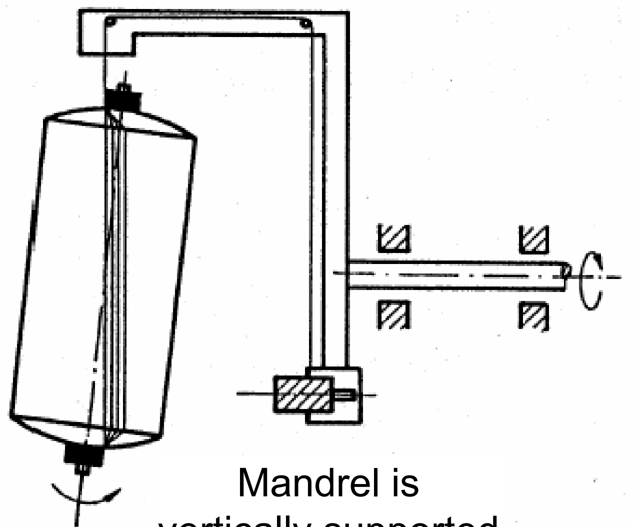
Indirect Impregnation

Roving Impregnation



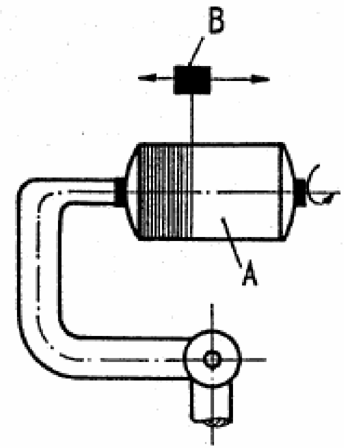
Direct Impregnation

Filament Winding



Mandrel is
vertically supported

Planetary System

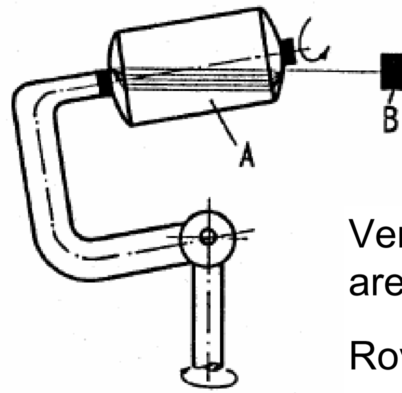


Vertical axis and mandrel
axis are perpendicular.

Vertical axis remains.

Mandrel rotates.

Roving eyelet moves.

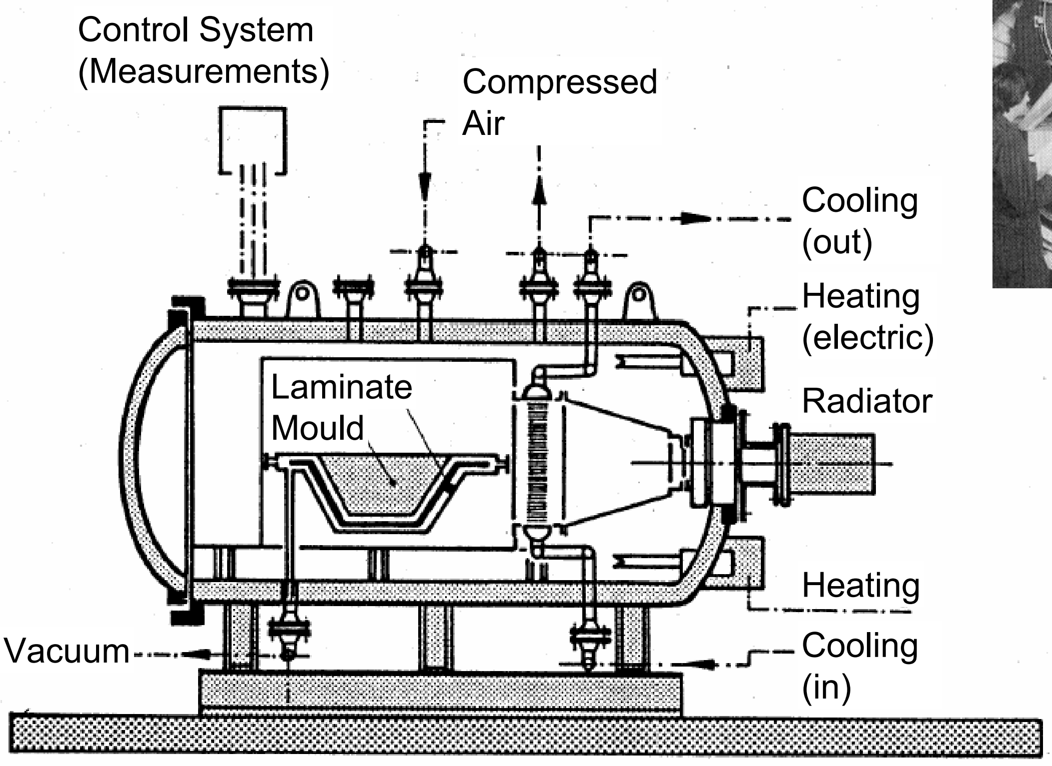


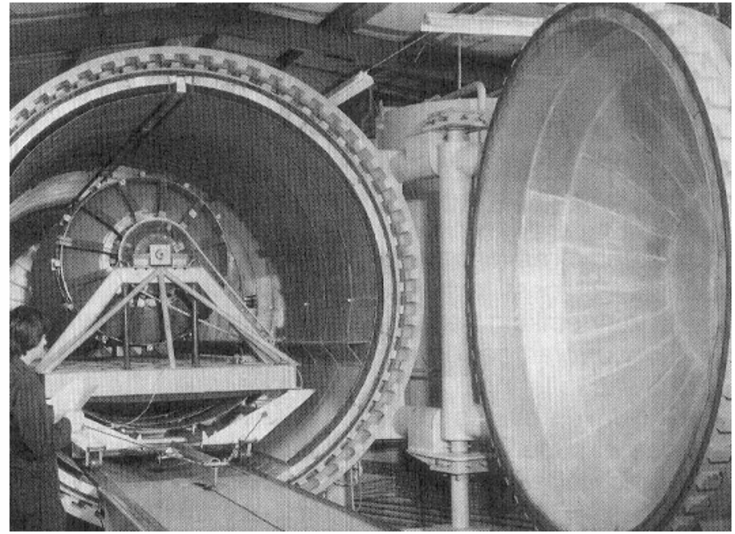
Vertical and winding axis
are inclined; both axes rotate.

Roving eyelet remains.

Polar System

Autoclave Application (Equipment)



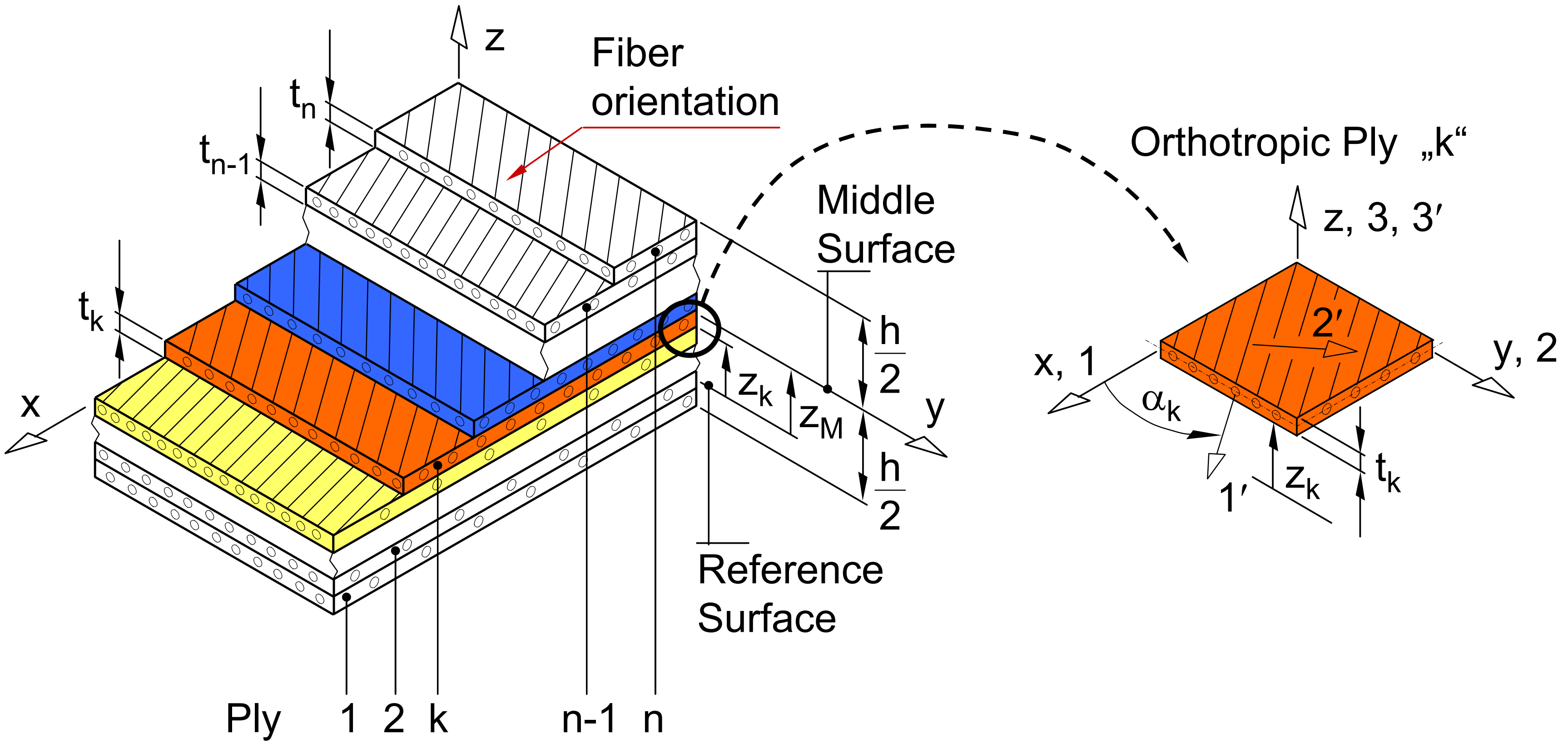


Modul: Faserverbund- und Sandwichtechnologie

Inhaltsübersicht

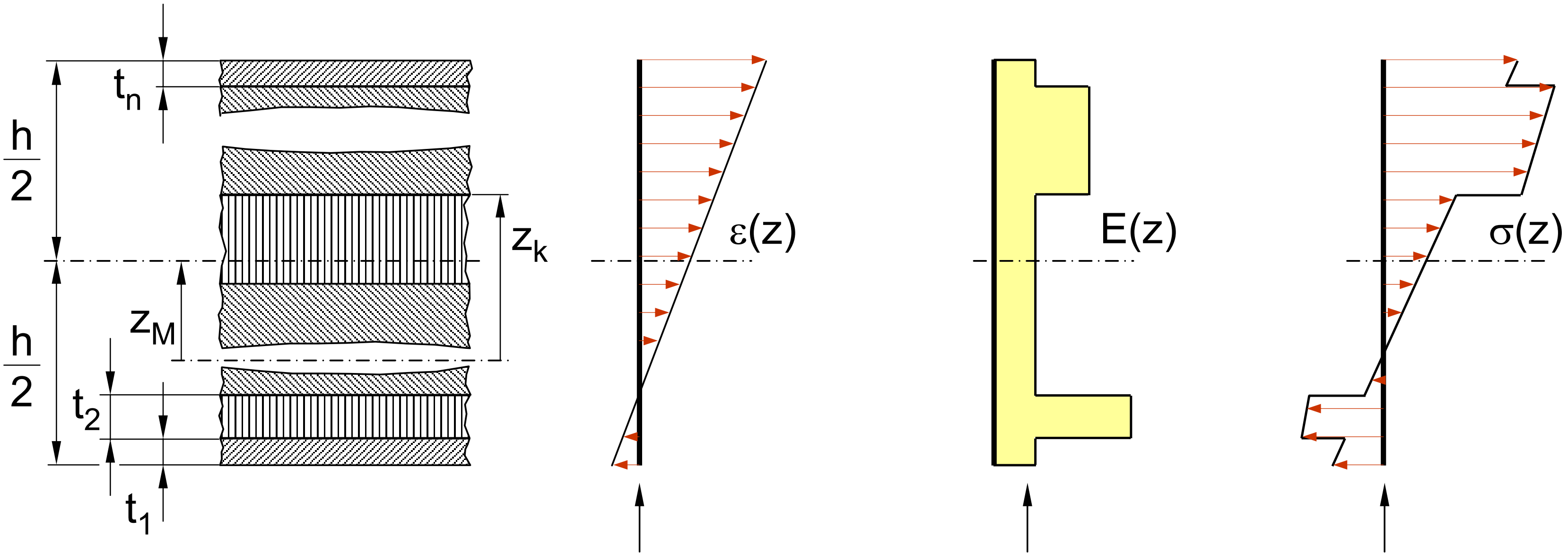
Laminate made of Composite Material

A laminate consists of two or more layers bonded together to act as an integral element



Laminate

Stacking Sequence | Strains | Module | Stresses



Linear strain distribution through the laminate (thickness direction)

Module depends on ply-angle and related material properties

Linear stress distribution within each layer;

Stresses may jump at layer boundaries

Laminate Codes

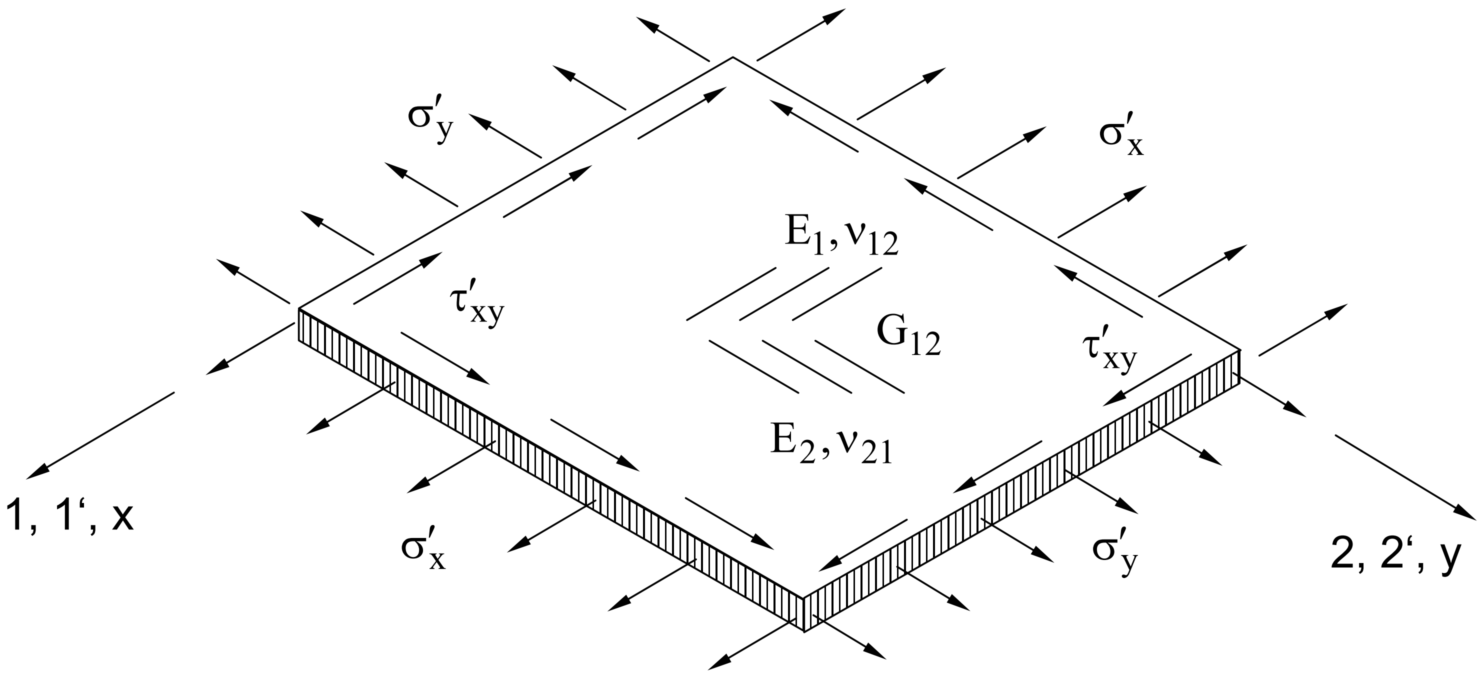
Three different laminate codes are commonly used to define ply orientations of laminates:

1. (25 / 50 / 25)	25%	0°-plies
	50%	±45°-plies
	25%	90°-plies
2. (25 / 25 / 25 / 25)	25%	0°-plies
	25%	+45°-plies
	25%	-45°-plies
	25%	90°-plies
3. [0° +45° -45° 90°]	1st ply is	0°
	2nd ply is	+45°
	3rd ply is	-45°
	4th ply is	90°

Code 1 and 3 are used here.

Orthotropic Materials = Single Ply

Plane Stress State
(only stresses within the plane: σ'_x , σ'_y , τ'_{xy})



Orthotropic Materials

Plane Stress State
(only stresses within the plane: σ'_x , σ'_y , τ'_{xy})

$$\varepsilon'_x = \frac{1}{E_1} \cdot \sigma'_x - \frac{\nu_{21}}{E_2} \cdot \sigma'_y$$

$$\varepsilon'_y = -\frac{\nu_{12}}{E_1} \cdot \sigma'_x + \frac{1}{E_2} \cdot \sigma'_y$$

$$\gamma'_{xy} = \frac{1}{G_{12}} \cdot \tau'_{xy}$$

$$\begin{bmatrix} \varepsilon'_x \\ \varepsilon'_y \\ \gamma'_{xy} \end{bmatrix} = \begin{bmatrix} S'_{11} & S'_{12} & 0 \\ S'_{21} & S'_{22} & 0 \\ 0 & 0 & S'_{33} \end{bmatrix} \begin{bmatrix} \sigma'_x \\ \sigma'_y \\ \tau'_{xy} \end{bmatrix}$$

$$S'_{11} = \frac{1}{E_1}; \quad S'_{22} = \frac{1}{E_2}$$

$$S'_{12} = S'_{21} = -\frac{\nu_{12}}{E_1}$$

$$S'_{33} = \frac{1}{G_{12}} \qquad \nu_{21} = \nu_{12} \cdot \frac{E_2}{E_1}$$

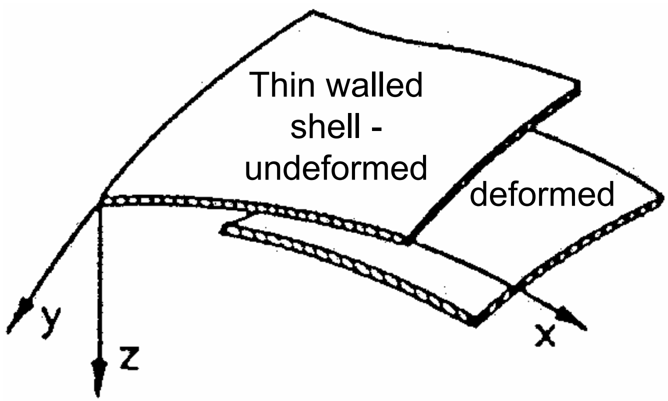
$$\begin{bmatrix} \sigma'_x \\ \sigma'_y \\ \tau'_{xy} \end{bmatrix} = \begin{bmatrix} Q'_{11} & Q'_{12} & 0 \\ Q'_{21} & Q'_{22} & 0 \\ 0 & 0 & Q'_{33} \end{bmatrix} \begin{bmatrix} \varepsilon'_x \\ \varepsilon'_y \\ \gamma'_{xy} \end{bmatrix}$$

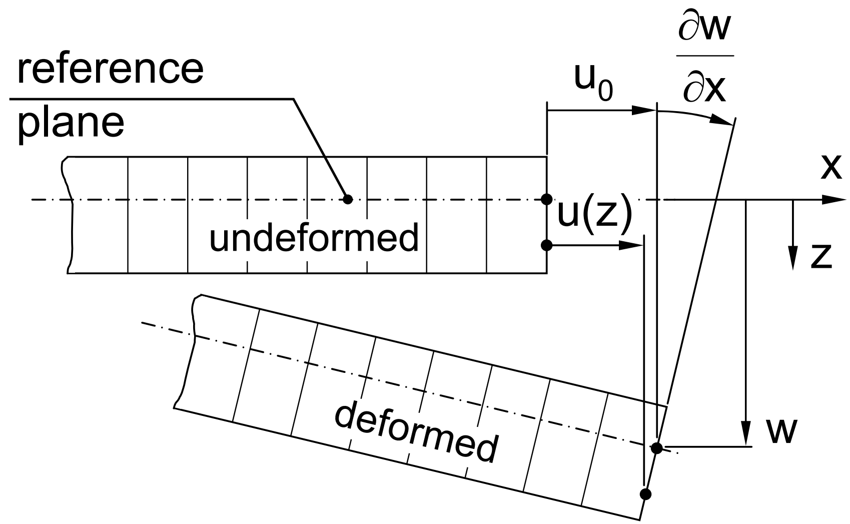
$$Q'_{11} = \frac{E_1}{1 - \nu_{12}^2 \cdot \frac{E_2}{E_1}} \quad ; \quad Q'_{22} = \frac{E_2}{1 - \nu_{12}^2 \cdot \frac{E_2}{E_1}}$$

$$Q'_{12} = Q'_{21} = \nu_{12} \cdot Q'_{22} \quad ; \quad Q'_{33} = G_{12}$$

Deformed Geometry of Shells

Kirchhoff – Love Hypotheses: $\gamma_{xz} = \gamma_{yz} = 0\,; \quad \varepsilon_z \ll 1$





In x – Direction: $u(z) = u_0 - z\dfrac{\partial w}{\partial x}$

In y – Direction: $v(z) = v_0 - z\dfrac{\partial w}{\partial y}$

Deformed Geometry of Shells

Strains – Displacements – Equations

$$\varepsilon_x(z) = \frac{\partial u(z)}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y(z) = \frac{\partial v(z)}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy}(z) = \frac{\partial u(z)}{\partial y} + \frac{\partial v(z)}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y}$$

$$\boldsymbol{\varepsilon}_0 = \begin{bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{bmatrix} \quad ; \quad \boldsymbol{\kappa} = \begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x\,\partial y} \end{bmatrix}$$

$$u(z) = u_0 - z\frac{\partial w}{\partial x}$$

$$v(z) = v_0 - z\frac{\partial w}{\partial y}$$

$$\boldsymbol{\varepsilon}(z) = \boldsymbol{\varepsilon}_0 - z\boldsymbol{\kappa}$$

Resultant Laminate Forces and Moments

Resultant forces and moments acting on a laminate are obtained by integration of the stresses in each layer through the laminate thickness:

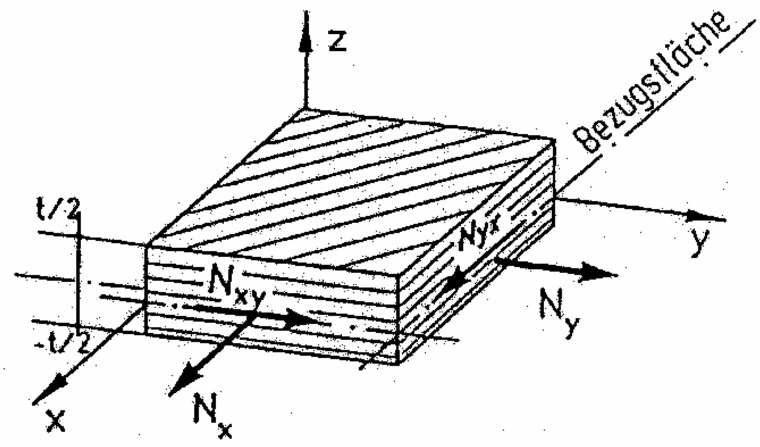
1. Resultant forces (section forces, membrane forces)
   Dimension: force per unit width

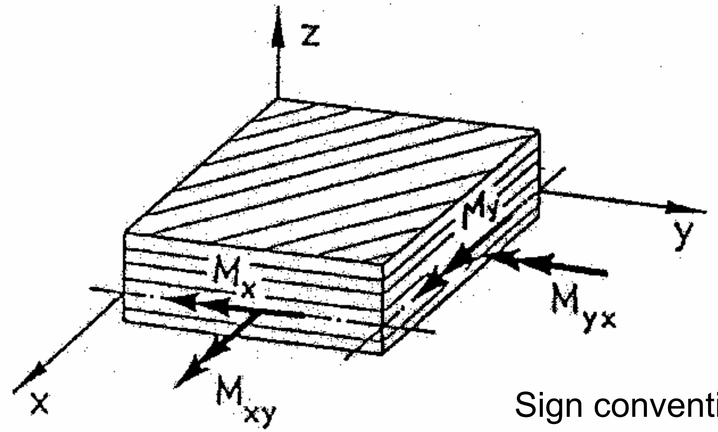
   That are:
   - Membrane forces: N_x, N_y
   - Shear flow N_{xy}

2. Resultant moments (section moments)
   Dimension: moment per unit width

   That are:
   - Bending moments M_x, M_y
   - Torsion moment M_{xy}

Resultant Laminate Forces and Moments





Sign convention:

Positive moments create positive curvatures

$$N_x = \int_{z_1}^{z_{n+1}} \sigma_x \, dz \qquad M_x = -\int_{z_1}^{z_{n+1}} \sigma_x \cdot z \, dz$$

$$N_y = \int_{z_1}^{z_{n+1}} \sigma_y \, dz \qquad M_y = -\int_{z_1}^{z_{n+1}} \sigma_y \cdot z \, dz$$

$$N_{xy} = N_{yx} = \int_{z_1}^{z_{n+1}} \tau_{xy} \, dz \qquad M_{xy} = M_{yx} = -\int_{z_1}^{z_{n+1}} \tau_{xy} \cdot z \, dz$$

Resultant Laminate Forces and Moments

Strains and Curvatures:

$$\boldsymbol{\varepsilon}_0 = \begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{bmatrix} \qquad \boldsymbol{\kappa} = \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Resultant Forces and Moments:

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} \qquad \mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

$$\mathbf{N} = \int_{z_1}^{z_{n+1}} \boldsymbol{\sigma}(z)\, dz \qquad \mathbf{M} = -\int_{z_1}^{z_{n+1}} \boldsymbol{\sigma}(z) \cdot z\, dz$$

$$\boldsymbol{\sigma}(z) = \mathbf{Q}(z) \cdot \boldsymbol{\varepsilon}(z) = \mathbf{Q}(z) \cdot \left[\boldsymbol{\varepsilon}_0 - z\,\boldsymbol{\kappa}\right]$$

(material law)

Stiffness Matrices

Resultant forces:

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot [\boldsymbol{\varepsilon}_0 - z \cdot \boldsymbol{\kappa}]\, dz = \underbrace{\left[\int_{z_1}^{z_{n+1}} \mathbf{Q}(z)\, dz \right]} \cdot \boldsymbol{\varepsilon}_0 + \underbrace{\left[- \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z\, dz \right]} \cdot \boldsymbol{\kappa}$$

$$\mathbf{A} = \int_{z_1}^{z_{n+1}} \mathbf{Q}(z)\, dz \qquad \mathbf{B} = - \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z\, dz$$

$$\boxed{\rightarrow \quad \mathbf{N} = \mathbf{A} \cdot \boldsymbol{\varepsilon}_0 + \mathbf{B} \cdot \boldsymbol{\kappa}}$$

Resultant moments:

$$\mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = - \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot [\boldsymbol{\varepsilon}_0 - z \cdot \boldsymbol{\kappa}] \cdot z\, dz = \underbrace{\left[- \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z\, dz \right]} \cdot \boldsymbol{\varepsilon}_0 + \underbrace{\left[\int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z^2\, dz \right]} \cdot \boldsymbol{\kappa}$$

$$\mathbf{B} = - \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z\, dz \qquad \mathbf{D} = \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z^2\, dz$$

$$\boxed{\rightarrow \quad \mathbf{M} = \mathbf{B} \cdot \boldsymbol{\varepsilon}_0 + \mathbf{D} \cdot \boldsymbol{\kappa}}$$

Law of Elasticity (Laminates)

$$\mathbf{N} = \mathbf{A} \cdot \boldsymbol{\varepsilon}_0 + \mathbf{B} \cdot \boldsymbol{\kappa}$$

$$\mathbf{M} = \mathbf{B} \cdot \boldsymbol{\varepsilon}_0 + \mathbf{D} \cdot \boldsymbol{\kappa}$$

$$\longrightarrow \quad \begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}}_{= \mathbf{C}} \begin{bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ \hdashline M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left[\begin{array}{ccc:ccc} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{11} \\ & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ & & A_{33} & B_{13} & B_{23} & B_{33} \\ \hdashline & & & D_{11} & D_{12} & D_{13} \\ & \text{Symmetry} & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{array}\right] \begin{bmatrix} \varepsilon_{x_0} \\ \varepsilon_{y_0} \\ \gamma_{xy_0} \\ \hdashline \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

[**A**] Extension-Matrix

[**B**] Bending-Extension-Matrix

[**D**] Bending-Matrix

„A – B – D“ Matrices

$$\mathbf{A} = \int_{z_1}^{z_{n+1}} \mathbf{Q}(z)\,dz = \sum_{k=1}^{n} \left[\mathbf{Q}_k \int_{z_k}^{z_{k+1}} dz \right] = \sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot \left(z_{k+1} - z_k \right) \right] = \sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot t_k \right]$$

$$\mathbf{B} = -\int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z\,dz = -\sum_{k=1}^{n} \left[\mathbf{Q}_k \int_{z_k}^{z_{k+1}} z\,dz \right] = -\sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot \frac{1}{2} \cdot \left(z_{k+1}^2 - z_k^2 \right) \right] = -\sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot t_k \cdot \frac{z_{k+1} + z_k}{2} \right]$$

$$\mathbf{D} = \int_{z_1}^{z_{n+1}} \mathbf{Q}(z) \cdot z^2\,dz = \sum_{k=1}^{n} \left[\mathbf{Q}_k \int_{z_k}^{z_{k+1}} z^2\,dz \right] = \sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot \frac{1}{3} \cdot \left(z_{k+1}^3 - z_k^3 \right) \right] = \sum_{k=1}^{n} \left[\mathbf{Q}_k \cdot t_k \left\{ \frac{t_k^2}{12} + \left(\frac{z_{k+1} + z_k}{2} \right)^2 \right\} \right]$$

Assumption:
Constant stiffnesses in each layer

$\mathbf{Q}(z)$: in layer k = $\mathbf{Q}_k$

Transformations - Summary

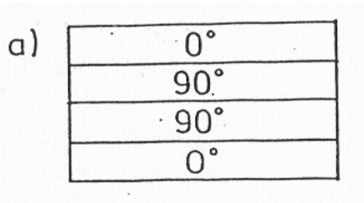
Summary of Transformations

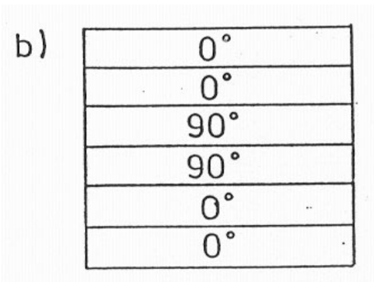
Local Coordinates (Material Principal Axes)		Global Coordinates (Laminate coordinate system)
$\boldsymbol{\sigma}' = \mathbf{Q}' \cdot \boldsymbol{\varepsilon}'$		$\boldsymbol{\sigma} = \mathbf{Q} \cdot \boldsymbol{\varepsilon}$
$\mathbf{Q}' = \mathbf{T}_{\varepsilon'}^{T} \cdot \mathbf{Q} \cdot \mathbf{T}_{\varepsilon'}$	Stiffnesses **Q, Q'**	$\mathbf{Q} = \mathbf{T}_{\varepsilon}^{T} \cdot \mathbf{Q}' \cdot \mathbf{T}_{\varepsilon}$
$\boldsymbol{\varepsilon}' = \mathbf{S}' \cdot \boldsymbol{\sigma}'$		$\boldsymbol{\varepsilon} = \mathbf{S} \cdot \boldsymbol{\sigma}$
$\mathbf{S}' = \mathbf{T}_{\varepsilon} \cdot \mathbf{S} \cdot \mathbf{T}_{\varepsilon}^{T}$	Compliances **S, S'**	$\mathbf{S} = \mathbf{T}_{\varepsilon'} \cdot \mathbf{S}' \cdot \mathbf{T}_{\varepsilon'}^{T}$

$$\mathbf{T}_{\varepsilon'} = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix} \qquad \mathbf{T}_{\varepsilon} = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix}$$

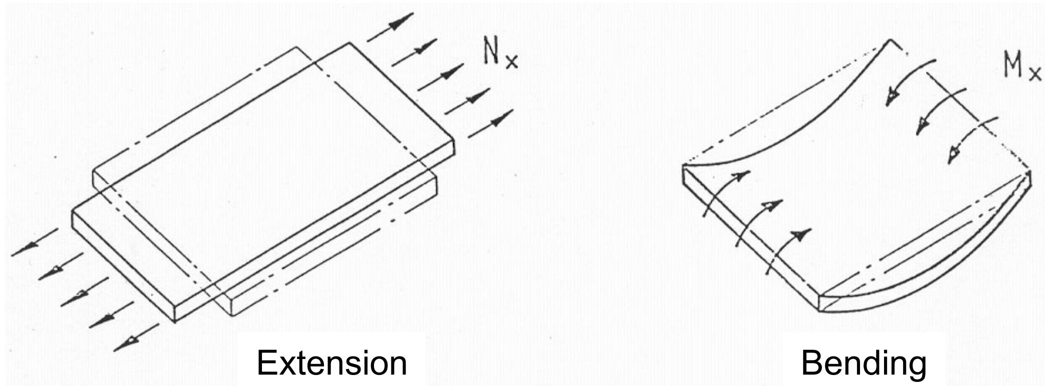
Orthotropic Material (Symmetrical)

Stacking Sequence





Deformed State

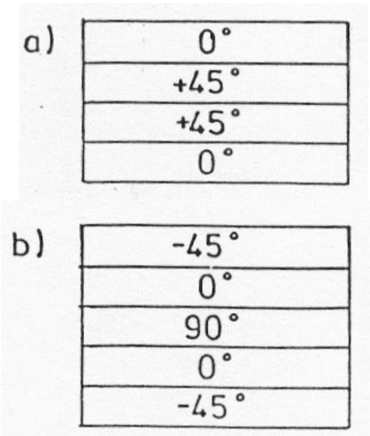


Stiffness Matrix

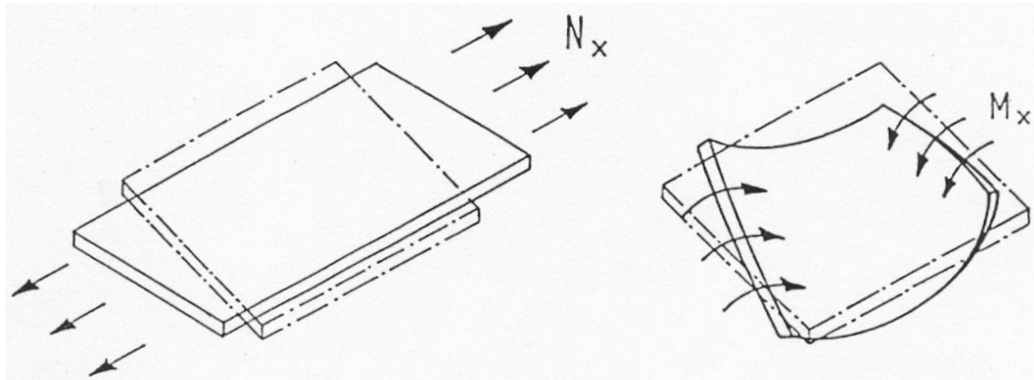
$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}
=
\begin{bmatrix}
A_{11} & A_{12} & 0 & & & \\
A_{12} & A_{22} & 0 & & 0 & \\
0 & 0 & A_{33} & & & \\
 & & & D_{11} & D_{12} & 0 \\
 & 0 & & D_{12} & D_{22} & 0 \\
 & & & 0 & 0 & D_{33}
\end{bmatrix}
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varkappa_x \\ \varkappa_y \\ \varkappa_{xy} \end{bmatrix}
$$

Anisotropic Material   (Symmetrical)

Stacking Sequence



Deformed State



Extension and Shear

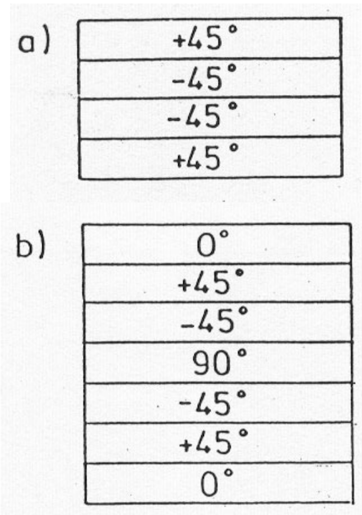
Bending and Torsion

Stiffness Matrix

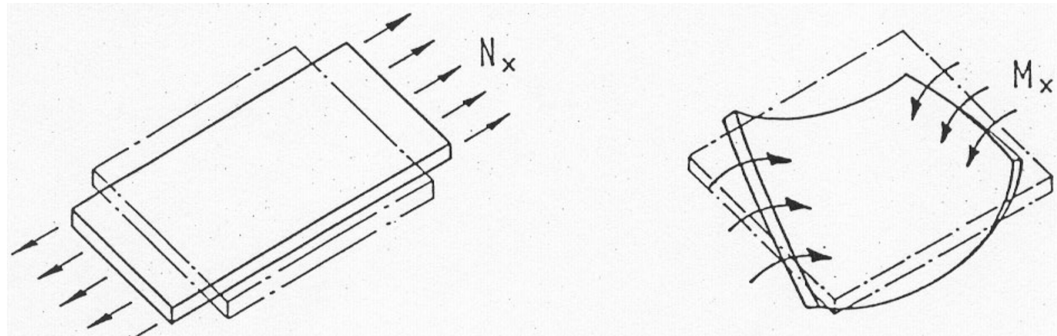
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ A_{12} & A_{22} & A_{23} & & 0 & \\ A_{13} & A_{23} & A_{33} & & & \\ & & & D_{11} & D_{12} & D_{13} \\ & 0 & & D_{12} & D_{22} & D_{23} \\ & & & D_{13} & D_{23} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varkappa_x \\ \varkappa_y \\ \varkappa_{xy} \end{bmatrix}$$

Pseudo – Orthotropic Material (Symmetrical)

Stacking Sequence



Deformed State



Extension

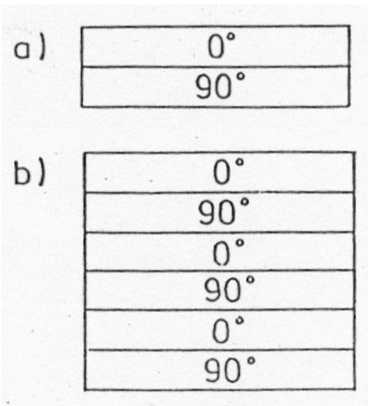
Bending and Torsion

Stiffness Matrix

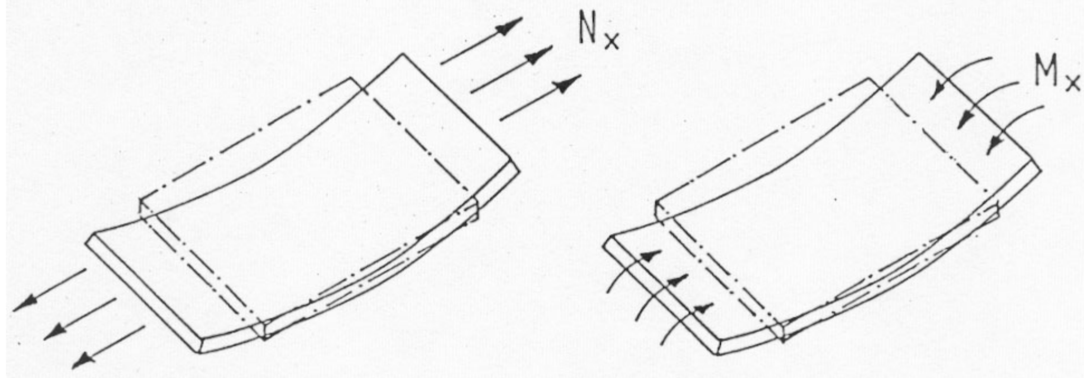
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & & & \\ A_{12} & A_{22} & 0 & & 0 & \\ 0 & 0 & A_{33} & & & \\ & & & D_{11} & D_{12} & D_{13} \\ & 0 & & D_{12} & D_{22} & D_{23} \\ & & & D_{13} & D_{23} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varkappa_x \\ \varkappa_y \\ \varkappa_{xy} \end{bmatrix}$$

Unsymmetrical Cross-Ply

Stacking Sequence



Deformed State



Extension and Bending

Bending and Extension

Stiffness Matrix

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & B_{22} & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ B_{11} & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varkappa_x \\ \varkappa_y \\ \varkappa_{xy} \end{bmatrix}$$

Unsymmetrical Angle-Ply

Stacking Sequence

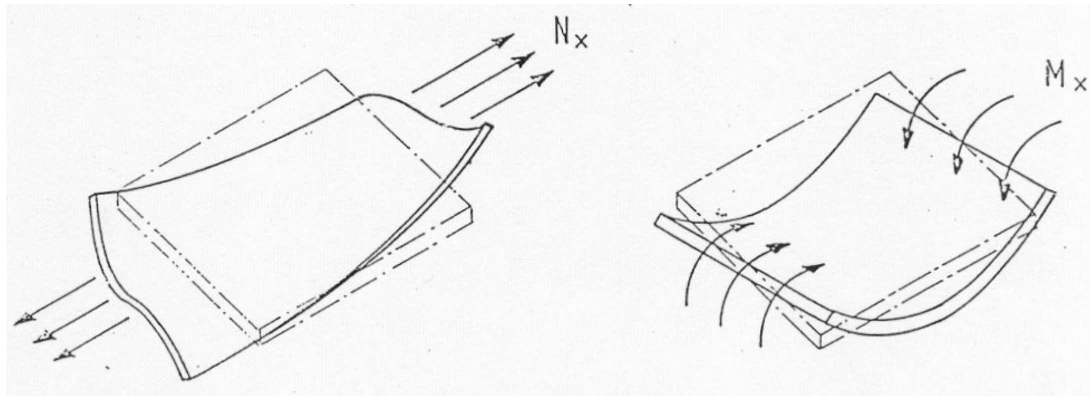
a)

+60°
-60°

b)

0°
+60°
-60°
0°

Deformed State



Extension and Torsion

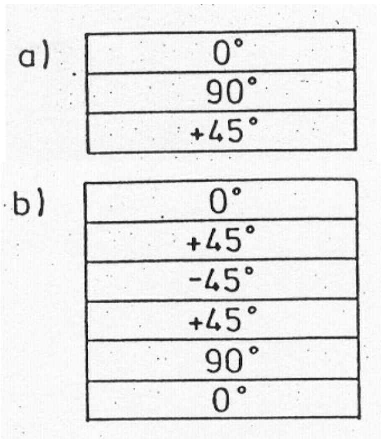
Bending and Shear

Stiffness Matrix

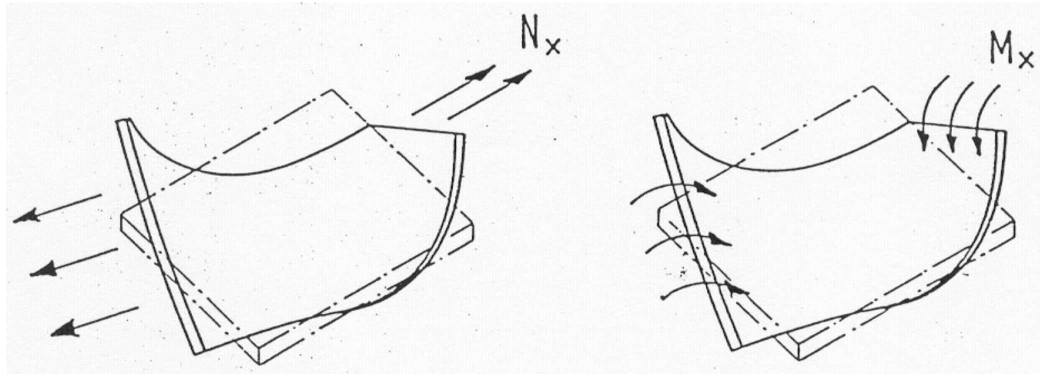
$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}
=
\begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & B_{13} \\
A_{12} & A_{22} & 0 & 0 & 0 & B_{23} \\
0 & 0 & A_{33} & B_{13} & B_{23} & 0 \\
0 & 0 & B_{13} & D_{11} & D_{12} & 0 \\
0 & 0 & B_{23} & D_{12} & D_{22} & 0 \\
B_{13} & B_{23} & 0 & 0 & 0 & D_{33}
\end{bmatrix}
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \varkappa_x \\ \varkappa_y \\ \varkappa_{xy} \end{bmatrix}
$$

General Laminate (Unsymmetrical)

Stacking Sequence



Deformed State



Extension, Shear, Bending and Torsion

Bending, Torsion, Extension and Shear

Stiffness Matrix

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Special Cases of Laminate Stiffnesses

Symmetric Laminates	The geometry and the material properties are symmetric about the middle surface
Antisymmetric Laminates	- Symmetry about the middle surface of geometry - Reversal or mirror image of material properties
Cross-Ply Laminates	The laminate consists only of layers with principal material directions alternating at 0° and 90° to the laminate axes
Angle-Ply Laminates	Alternating stacking of layers with $+\alpha°$ and $-\alpha°$ to the laminate coordinate system

Physical Significance of Stiffness Terms

$$
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\
 & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\
 & & A_{33} & B_{13} & B_{23} & B_{33} \\
 & & & D_{11} & D_{12} & D_{13} \\
 & \text{symm.} & & & D_{22} & D_{23} \\
 & & & & & D_{33}
\end{bmatrix}
$$

Shear-Extension

Bending-Extension

Bending-Twisting

Special Cases of Laminate Stiffnesses

	General Laminate	Angle-Ply Laminate	
	anisotropic	quasi-orthotropic	orthotropic
unsymmetrical	$\begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{11} \\ & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ & & A_{33} & B_{13} & B_{23} & B_{33} \\ & & & D_{11} & D_{12} & D_{13} \\ & & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{11} \\ & A_{22} & 0 & B_{12} & B_{22} & B_{23} \\ & & A_{33} & B_{13} & B_{23} & B_{33} \\ & & & D_{11} & D_{12} & D_{13} \\ & & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ & A_{22} & 0 & B_{12} & B_{22} & 0 \\ & & A_{33} & 0 & 0 & B_{33} \\ & & & D_{11} & D_{12} & 0 \\ & & & & D_{22} & 0 \\ & & & & & D_{33} \end{bmatrix}$
symmetrical	$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\ & A_{22} & A_{23} & 0 & 0 & 0 \\ & & A_{33} & 0 & 0 & 0 \\ & & & D_{11} & D_{12} & D_{13} \\ & & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ & A_{22} & 0 & 0 & 0 & 0 \\ & & A_{33} & 0 & 0 & 0 \\ & & & D_{11} & D_{12} & D_{13} \\ & & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ & A_{22} & 0 & 0 & 0 & 0 \\ & & A_{33} & 0 & 0 & 0 \\ & & & D_{11} & D_{12} & 0 \\ & & & & D_{22} & 0 \\ & & & & & D_{33} \end{bmatrix}$

Failure Criteria – Isotropic Material

Isotropic Materials

Maximum load carrying capacity $\rightarrow$ defined via single strength limits derived from: tension, compression, or shear tests (by means of simple test specimen)

$\rightarrow$ For uniaxial stress state:
unambiguous definition of failure limits

Failure occurs, if the acting stress level (uniaxial) is above the stress limit (uniaxial) that was derived in a corresponding material test (e.g. tension test)

$\rightarrow$ For multiaxial stress state:
Transformation of the multiaxial stress state into an equivalent uniaxial stress state, e.g. von MISES stress, which can finally be compared with the single strength limits.

Failure occurs, if the acting equivalent uniaxial stress is above the limit that was derived in a corresponding material test (e.g. tension test)

Failure Criteria – Isotropic Material

Isotropic Materials

The definition of the equivalent stress state depends upon the selected failure hypothesis.

Usually applied failure hypotheses for isotropic materials (metals) are:

- Maximum distortional energy
- Maximum shear stress
- Maximum normal stress
- Maximum strain

Calculated from:

Normal- and shear stresses or principal stresses

For plane stress states the design failure envelopes (surfaces) can be defined as functions of the principal stress axes. Each stress state within the envelope can be sustained by the material.

Failure Criteria – Isotropic vs. Anisotropic Material

Anisotropic Materials

Anisotropic Materials have direction/fiber depending strength values, i.e. the strength value in longitudinal direction differs from the one in transverse direction. Thus, the traditional failure criteria of isotropic materials can not be used.

Example:
A quasi-isotropic $[0° \pm 45° \ 90°]_s$ laminate is loaded in 0°-direction only (=maximum load direction). However, the 90°-layer is the first layer that fails (=not highly loaded, but very little strength limit)!

Failure Criteria – Anisotropic Material

Anisotropic Material

The multiaxial stress state is dependent on each single layer of the laminate, and as such from each single ply-angle, ply-thicknesses and number of plies the laminate could consist of . . .

Procedure of the "Classical Lamination Theory"

Consideration of the individual layers of a laminate

Orthotropic Layers

Assuming thin walled structures, effects in thickness direction can be neglected. Thus, the failure assessment of a laminate can be done for each orthotropic layer of the laminate.

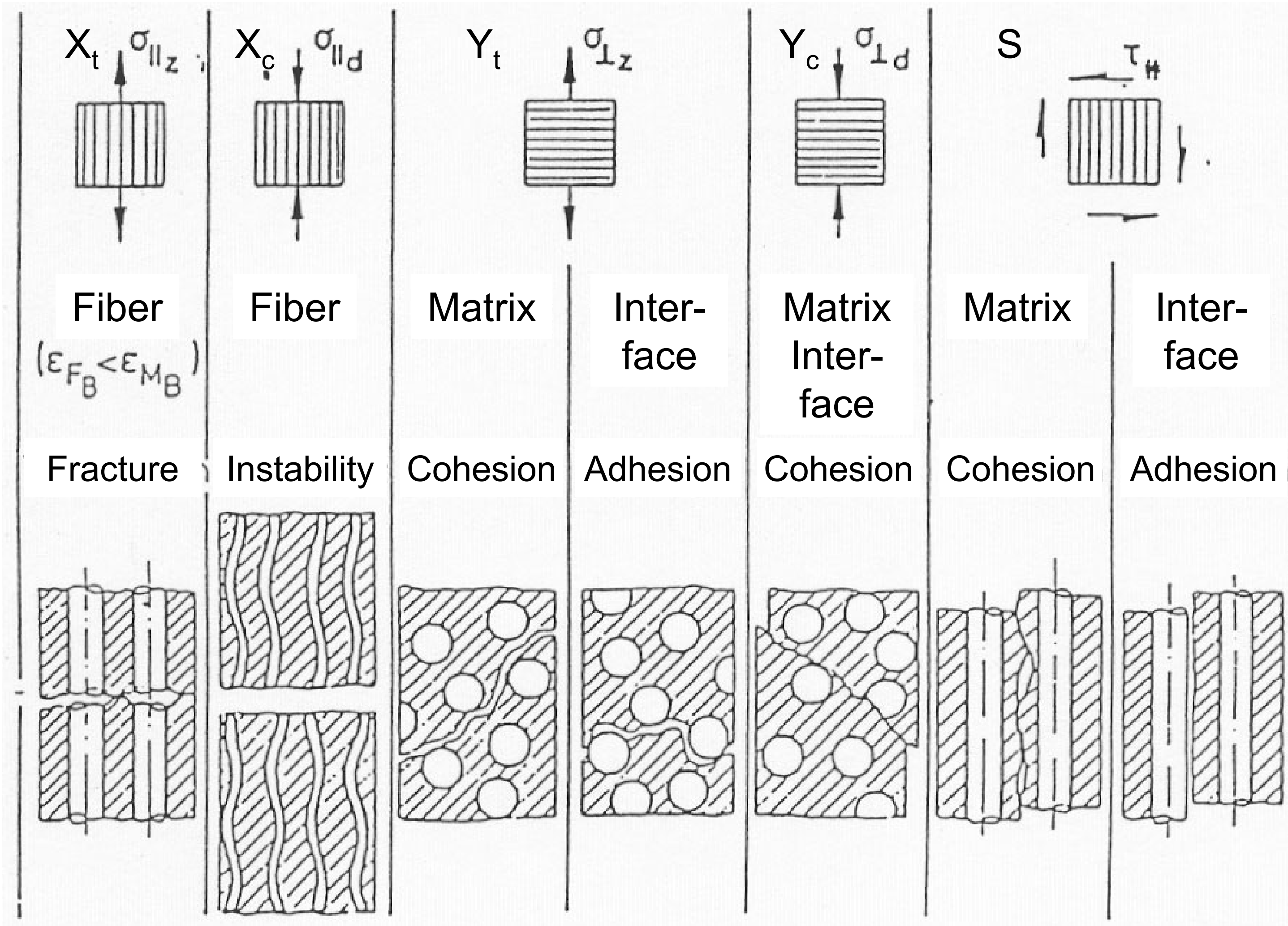
Failure Criteria – Anisotropic Material

Orthotropic Layers (cont.'d)

For each orthotropic layer the analysis must be done using the normal and shear stresses that run parallel to the principal axes of the orthotropic layer.

Finally these stresses have to be compared with the strength values of the considered orthotropic material. These values can again be determined by simple uniaxial tests.

Failure Cases – Orthotropic Material



Failure Cases – Orthotropic Material

Five (5) independent parameters are necessary to describe the failure cases of an orthotropic material layer in a plane stress state. The parameters are defined in the principal orthotropic material axes:

X_t Allowable tension strength of orthotropic layer in longitudinal direction (1-direction; fiber direction)

X_c Allowable compression strength of orthotropic layer in longitudinal direction (1-direction; fiber direction)

Y_t Allowable tension strength of orthotropic layer in transverse direction (2-direction; transverse to the fiber direction)

Y_c Allowable compression strength of orthotropic layer in transverse direction (2-direction; transverse to the fiber direction)

S Allowable shear strength of orthotropic layer (identical for tension and compression)

Failure Criteria – Anisotropic Material

Maximum Stress Criterion

Failure occurs, if one of the following conditions is violated:

$\sigma'_1 < X_t \quad ; \quad \sigma'_1 < X_c$

or

$\sigma'_2 < Y_t \quad ; \quad \sigma'_2 < Y_c$

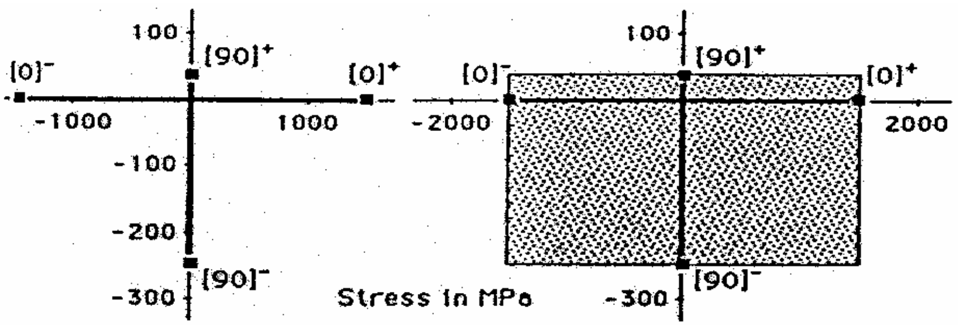
or

$\tau'_{12} < S$

No distinction between fiber and matrix fracture

T300 / 5208

Attached is the Maximum Stress Criterion in principal axes for $\tau = 0$. For a single layer made of Carbon / Epoxy (T300 / 5208), the failure envelope is a rectangle with its limits $X_{t,c}$ and $Y_{t,c}$.



Failure Criteria – Anisotropic Material

Maximum Strain Criterion

Failure occurs, if one of the following conditions is violated:

$$\varepsilon'_1 < \frac{X_t}{E_1} \quad ; \quad \varepsilon'_1 < \frac{X_c}{E_1}$$

or

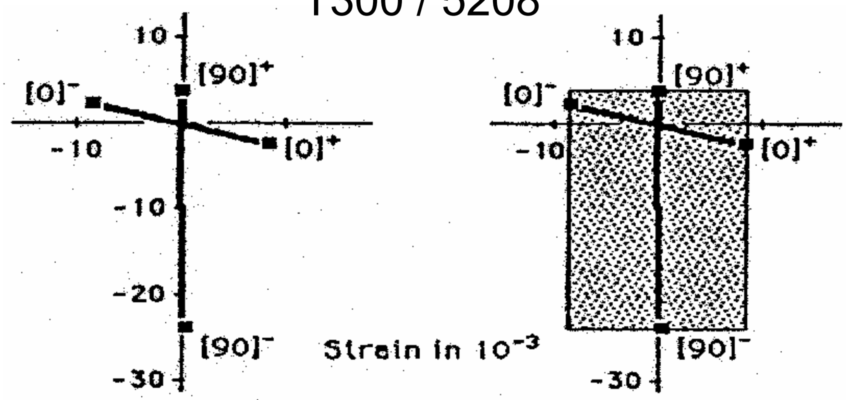
$$\varepsilon'_2 < \frac{Y_t}{E_2} \quad ; \quad \varepsilon'_2 < \frac{Y_c}{E_2}$$

or

$$\gamma'_{12} < \frac{S}{G_{12}}$$

No distinction between fiber and matrix fracture

T300 / 5208



Attached is the Maximum Stress Criterion in principal axes for t = 0.

Failure Criteria – Anisotropic Material

ZTL Criterion (according to HSB 51301 – 01)

Condition 1: Matrix breakage (identical to Tsai-Wu)

$$F_{11} \cdot \sigma_1'^2 + 2 \cdot F_{12} \cdot \sigma_1' \cdot \sigma_2' + F_{22} \cdot \sigma_2'^2 + F_{ss} \cdot \tau_{12}'^2 + F_1 \cdot \sigma_1' + F_2 \cdot \sigma_2' = 1$$

Strength values

$$F_{11} = \frac{1}{X_t \cdot X_c} \quad ; \quad F_{22} = \frac{1}{Y_t \cdot Y_c} \quad ; \quad F_{ss} = \frac{1}{S^2}$$

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c} \quad ; \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c} \quad ; \quad F_{12} = -\frac{1}{2} \cdot \sqrt{F_{11} \cdot F_{22}}$$

Condition 2: Fiber breakage

$$\left(\frac{\sigma_1'}{X_t}\right)^2 = 1 \quad \text{oder} \quad \left(\frac{\sigma_1'}{X_c}\right)^2 = 1 \longrightarrow$$

Prevents inadmissible stress states (may be up to factor 1.9 in fiber direction), caused by strength term F_{12} according to condition 1.

Failure Criteria – Anisotropic Material

ZTL Criterion Reserve Factors

Due to condition 1: Matrix Breakage

$$RF_{MB} = \frac{2}{R_i + \sqrt{R_i^2 + 4 \cdot R_{ii}}}$$

with:

$$R_{ii} = F_{11} \cdot \sigma_1'^2 + 2 \cdot F_{12} \cdot \sigma_1' \cdot \sigma_2' + F_{22} \cdot \sigma_2'^2 + F_{ss} \cdot \tau_{12}'^2$$

$$R_i = F_1 \cdot \sigma_1' + F_2 \cdot \sigma_2'$$

Due to condition 2: Fiber Breakage

$$RF_{FB} = \frac{X_t}{\sigma_1'} \quad \text{or} \quad RF_{FB} = \left| \frac{X_c}{\sigma_1'} \right|$$

$\rightarrow$ The critical value is the smaller one of RF_{MB} and RF_{FB} !

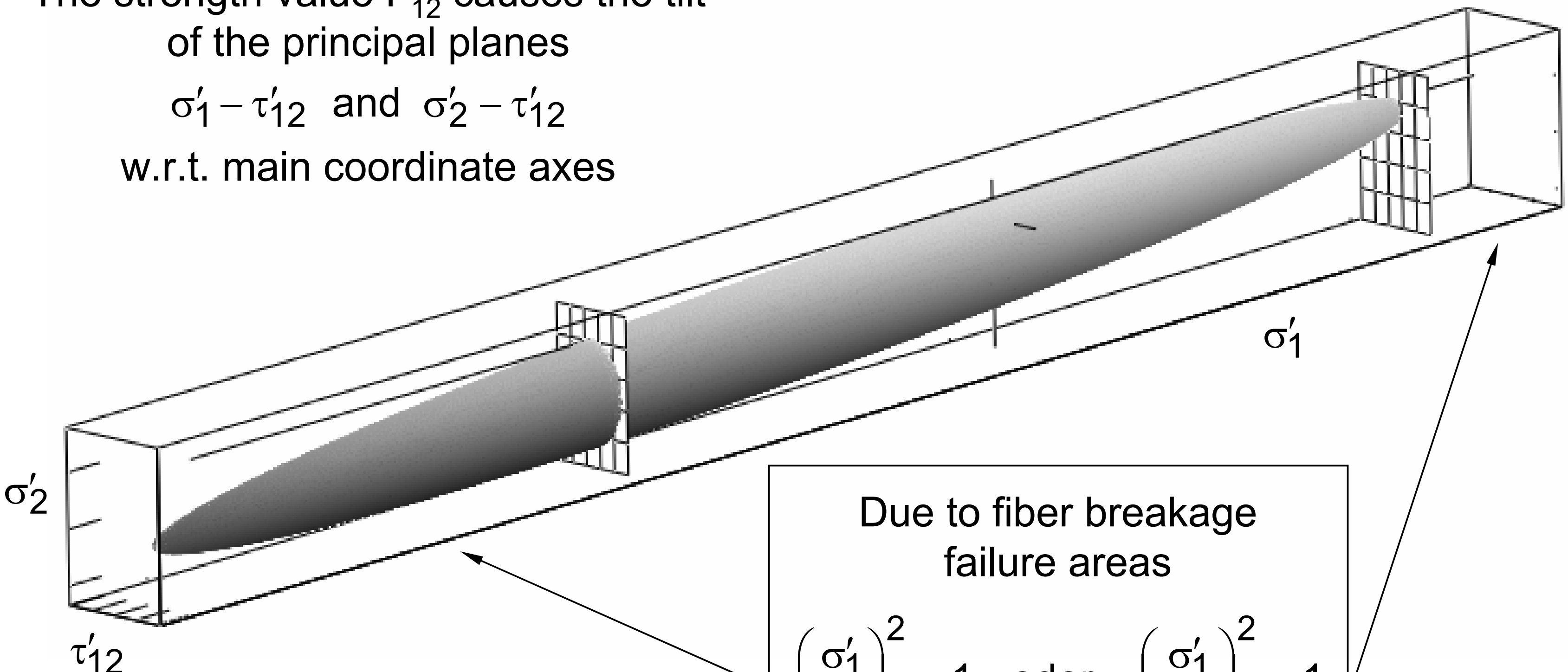
Failure Criteria – Anisotropic Material

ZTL – Envelope:
(all axes same scale)

The strength value F_{12} causes the tilt of the principal planes

$\sigma'_1 - \tau'_{12}$ and $\sigma'_2 - \tau'_{12}$

w.r.t. main coordinate axes



Due to fiber breakage failure areas

$$\left(\frac{\sigma'_1}{X_t}\right)^2 > 1 \quad \text{oder} \quad \left(\frac{\sigma'_1}{X_c}\right)^2 > 1$$

Standardized Analysis Procedure for Laminates

Laminate description:

n, t_k, z_k, φ_k, material data

↓

Calculation of stiffness matrix **Q‘** from …

micromechanis or test data

↓

Transformation of stiffness matrix **Q‘** (local coordinate system) into stiffness matrix **Q** (global coordinate system)

↓

Calculation of global stiffness matrices:

- Extension matrix **A**
- Bending-Extension matrix **B**
- Bending matrix **D**

↓

Structural analysis

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{D} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \end{bmatrix}$$

providing:
- strains $\boldsymbol{\varepsilon}_0$ and curvatures $\boldsymbol{\kappa}$
- section forces **N** and section moments **M**

↓

Standardized Analysis Procedure for Laminates

For each layer k of the laminate

Strain vector (global coordinate system) at layer boundaries

Lower boundary of layer k: $^{-}\boldsymbol{\varepsilon}_k = \boldsymbol{\varepsilon}_0 - z_k \cdot \boldsymbol{\kappa}$

Upper boundary of layer k: $^{+}\boldsymbol{\varepsilon}_k = \boldsymbol{\varepsilon}_0 - z_{k+1} \cdot \boldsymbol{\kappa}$

Transformation of strains into local coordinate system

$$^{-}\boldsymbol{\varepsilon}'_k = \left(\mathbf{T}_\varepsilon\right)_k \cdot {}^{-}\boldsymbol{\varepsilon}_k$$

$$^{+}\boldsymbol{\varepsilon}'_k = \left(\mathbf{T}_\varepsilon\right)_k \cdot {}^{+}\boldsymbol{\varepsilon}_k$$

Calculation of stresses at layer boundaries (global co-system)

$$^{-}\boldsymbol{\sigma}_k = \mathbf{Q}_k \cdot {}^{-}\boldsymbol{\varepsilon}_k$$

$$^{+}\boldsymbol{\sigma}_k = \mathbf{Q}_k \cdot {}^{+}\boldsymbol{\varepsilon}_k$$

Calculation of stresses at layer boundaries (local co-system)

$$^{-}\boldsymbol{\sigma}'_k = \mathbf{Q}'_k \cdot {}^{-}\boldsymbol{\varepsilon}'_k$$

$$^{+}\boldsymbol{\sigma}'_k = \mathbf{Q}'_k \cdot {}^{+}\boldsymbol{\varepsilon}'_k$$

Transformation of stresses into local coordinate system

$$^{-}\boldsymbol{\sigma}'_k = \left(\mathbf{T}_\sigma\right)_k \cdot {}^{-}\boldsymbol{\sigma}_k$$

$$^{+}\boldsymbol{\sigma}'_k = \left(\mathbf{T}_\sigma\right)_k \cdot {}^{+}\boldsymbol{\sigma}_k$$

Sandwich Structures

“Work Share“

- Facesheets are loaded in-plane, within the sandwich plane, by tension, compression and shear loads (so called “In-plane Loads”).
- Core is loaded out-of-plane, perpendicular to the sandwich plane, by tension, compression and shear loads (“Out-of-plane Loads”).

This “Work Share” is ensured, if

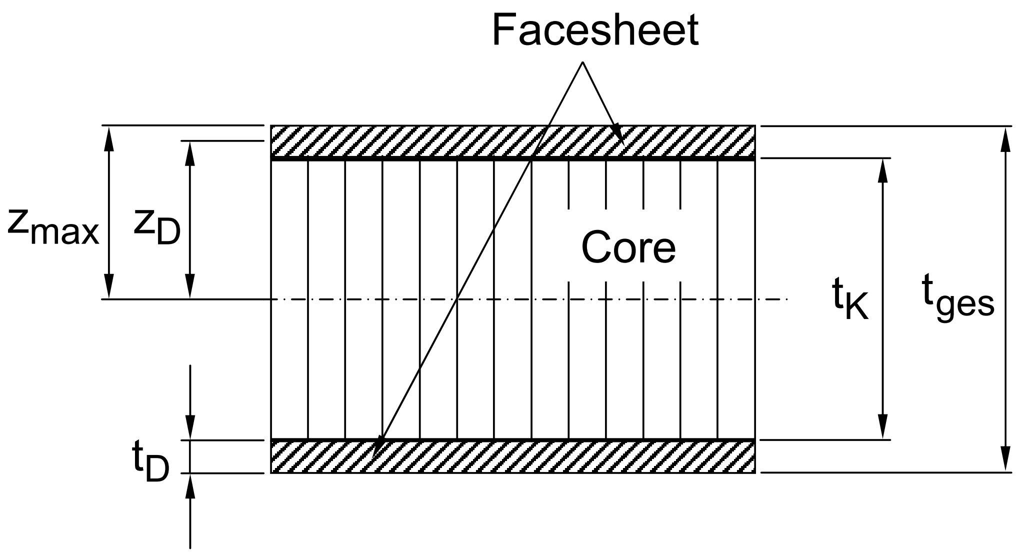
- The bonding between facesheets and core is given for
  - all relevant loads and/or
  - combinations of load cases

Sandwich Structures

Advantages	Disadvantages
Very low weight up to a medium load level	High investments (tooling, models)
Surface can be formed according to the required shape	Core materials may show adverse effects w.r.t. moisture and corrosion
Fatigue behaviour	Load introductions, cut outs, fittings cause costly design considerations and are associated with additional weight impacts
Fail-safe characteristics	Modifications difficult to implement
Isolation is integrated (partly)	Costly repairs
Dynamic behaviour (e.g. high eigenfrequencies)	Inspections need experienced personnel and special tools (e.g. delaminations)
	Susceptible to impacts

Sandwich Structures

Geometry parameters of sandwich structures



Thickness of facesheet t_D

Thickness of core t_K

Distance to facesheets $z_D = \frac{t_{ges} - t_D}{2}$

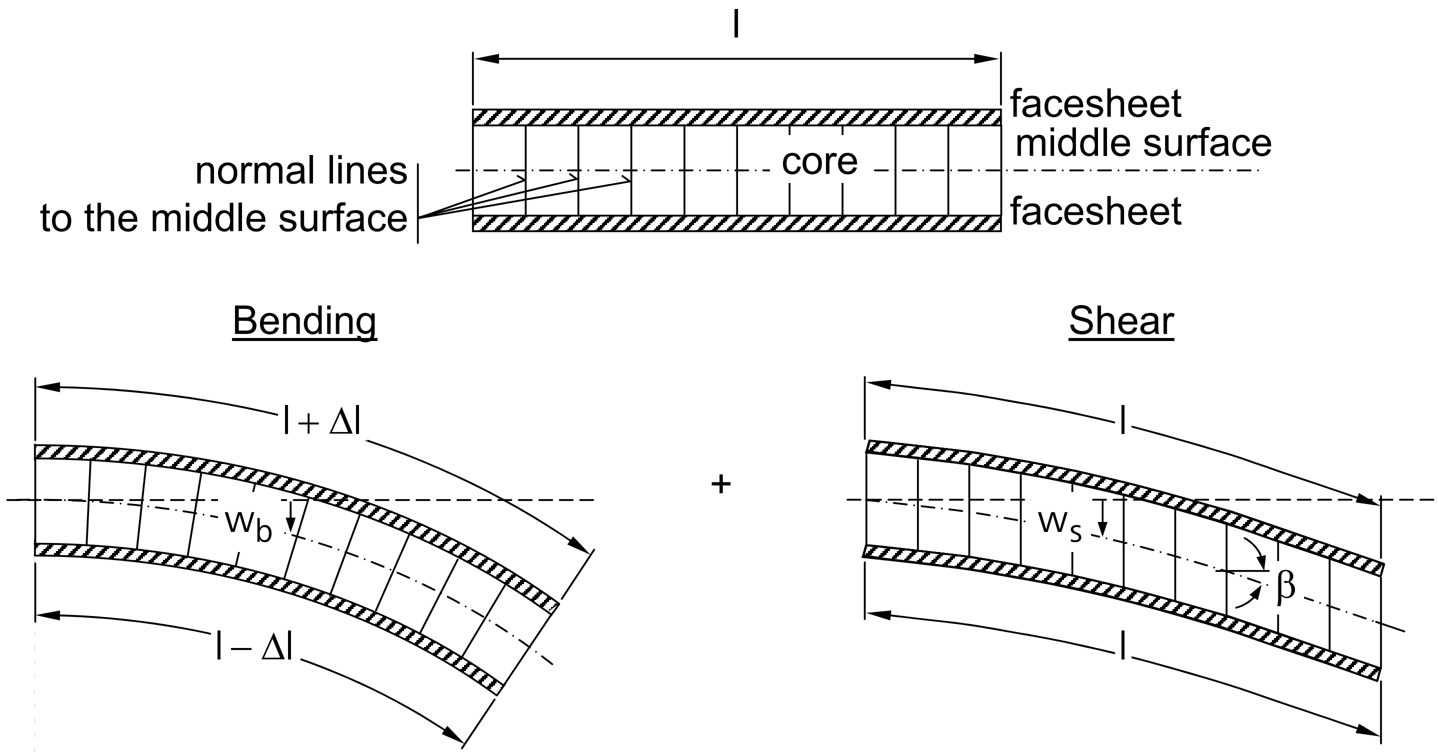
Edge distance $z_{max} = \frac{t_{ges}}{2}$

Sandwich Structures

Assumptions:

- Symmetrical facesheets
- Young's-Modulus of facesheets is E_D
  Isotropic or quasi-isotropic material behaviour assumed (required)
- Loads:
  Normal forces N_x , N_y
  Shear forces Q_{xz} , Q_{yz}
  Moments M_x , M_y
- Related (width b) Loads
  n_x , n_y
  n_{xz} , n_{yz}
  m_x , m_y

Sandwich Structures



Beam due to bending deformation

a) Upper facesheet is lengthened, lower facesheets is shortened

b) Normal lines to the middle surface remain straight and normal

Beam due to shear deformation

a) Length of the facesheets is constant

b) Normal lines to the middle surface remain straight (…but are no longer normal to the middle surface…)

Sandwich Structures

Stiffnesses of sandwich structures:

- Extension Stiffness

$$D = E_D \cdot A \qquad \Big| \; \text{Cross Section } A = 2 \cdot b \cdot t_D$$

$$= E_D \cdot 2 \cdot b \cdot t_D$$

- Bending Stiffness

$$B = E_D \cdot I \qquad \Big| \; \text{area moment of inertia } I = 2 \cdot \left[b \cdot t_D \cdot z_D^2 + \frac{1}{12} \cdot b \cdot t_D^3 \right]$$

$$= E_D \cdot 2 \cdot \left[b \cdot t_D \cdot z_D^2 + \underbrace{\frac{1}{12} \cdot b \cdot t_D^3}_{\text{negligible}} \right]$$

- Shear Stiffness

$$K = G_K \cdot S$$

$$= G_K \cdot b \cdot \left(2 \cdot z_D \right)$$

Sandwich Structures

Stresses of sandwich structures:

- Facesheets (tension- / compression stresses)

$$\sigma_x = M_x \cdot \frac{z_{max}}{I} + N_x \cdot \frac{1}{A} \qquad \sigma_y = M_y \cdot \frac{z_{max}}{I} + N_y \cdot \frac{1}{A}$$

$$= m_x \cdot \frac{z_{max}}{2 \cdot \left[t_D \cdot z_D^2 + \frac{1}{12} \cdot t_D^3 \right]} + n_x \cdot \frac{1}{2 \cdot t_D} \qquad = m_y \cdot \frac{z_{max}}{2 \cdot \left[t_D \cdot z_D^2 + \frac{1}{12} \cdot t_D^3 \right]} + n_y \cdot \frac{1}{2 \cdot t_D}$$

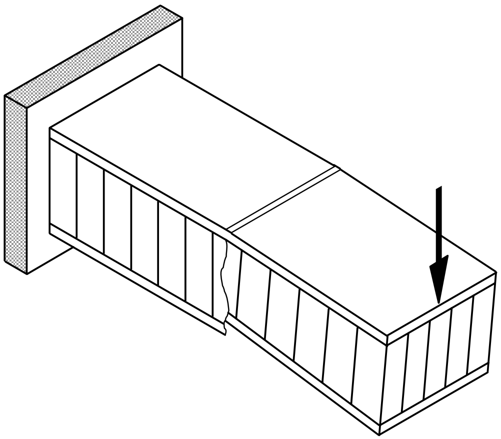
- Core (shear stresses)

$$\tau_{xz} = \frac{Q_{xz}}{b \cdot 2 \cdot z_D} \qquad \tau_{yz} = \frac{Q_{yz}}{b \cdot 2 \cdot z_D}$$

$$= \frac{n_{xz}}{2 \cdot z_D} \qquad = \frac{n_{yz}}{2 \cdot z_D}$$

Sandwich Failure Modes

1. Strength
   The skin (facesheet) and core materials should be able to withstand the tensile, compressive and shear stresses induced by the design load.



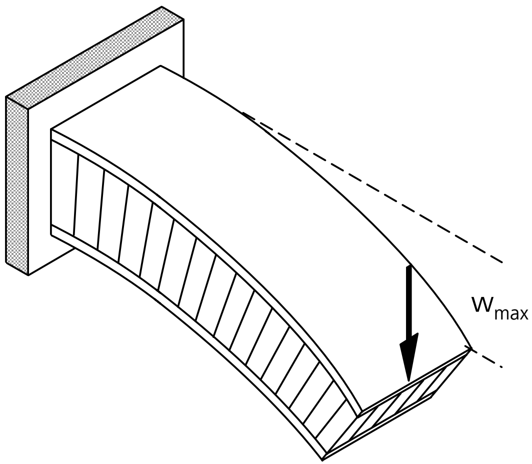
Facesheet: $\sigma_{face} \leq \sigma_{allowable}$

Core: $\tau_{Core} \leq \tau_{allowable}$

2. Debonding
   The skin to core adhesive must be capable of transferring the shear stresses between skin and core.

Sandwich Failure Modes

3. Stiffness
   The sandwich panel should have sufficient bending and shear stiffness to prevent excessive deflection.



Maximum deflection limit:

w_{max} to be analyzed from mechanical model

Sandwich Failure Modes

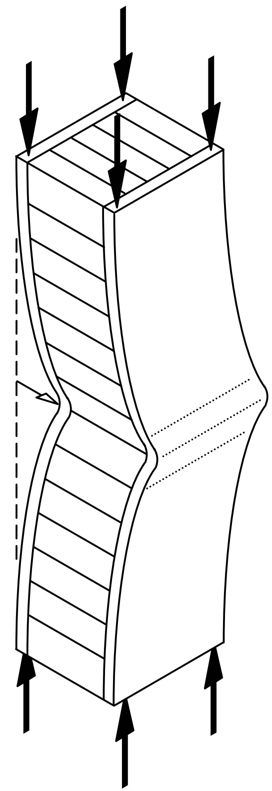
4. Panel buckling
   The core thickness and shear modulus must be adequate to prevent the panel from buckling under end compression loads.

→ Beam $\sigma_{cr} = \pi^2 \cdot E_D \cdot \left(\frac{z_D}{l}\right)^2$

→ Plate

$$\sigma_{cr} = \kappa \cdot k_d \cdot E_D \cdot \left(\frac{2 \cdot z_D}{b}\right)^2$$

$$\tau_{cr} = \kappa \cdot k_s \cdot E_D \cdot \left(\frac{2 \cdot z_D}{b}\right)^2$$

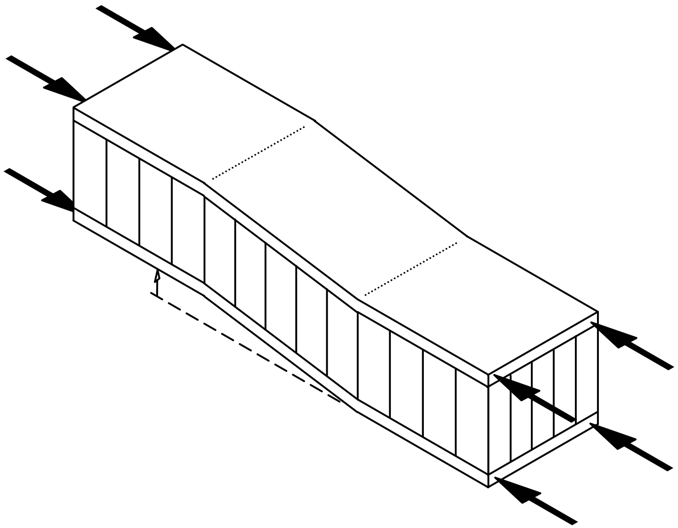


$\kappa = 3$ for identical facesheet thicknesses

k_d, k_s Buckling coefficients

Sandwich Failure Modes

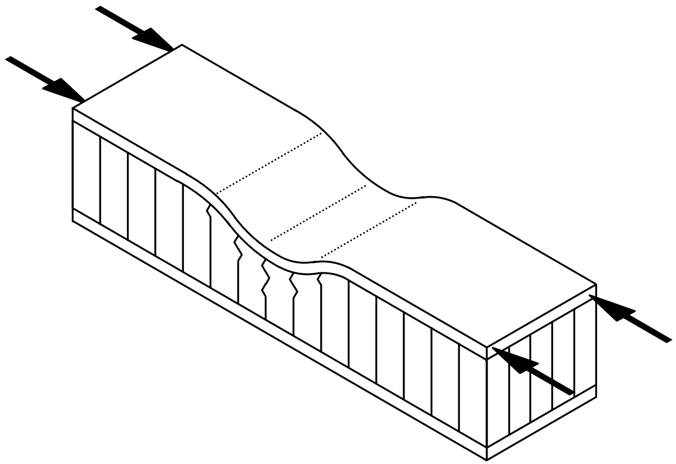
5. Shear crimping
   The core thickness and shear modulus must be adequate to prevent the core from prematurely failing in shear under end compression loads.



→ Beam: $F_{cr} = G_K \cdot b \cdot t_K$

Sandwich Failure Modes

6. Skin wrinkling
   The compressive modulus of the facing skin and the core compression strength must both be high enough to prevent a skin wrinkling failure.

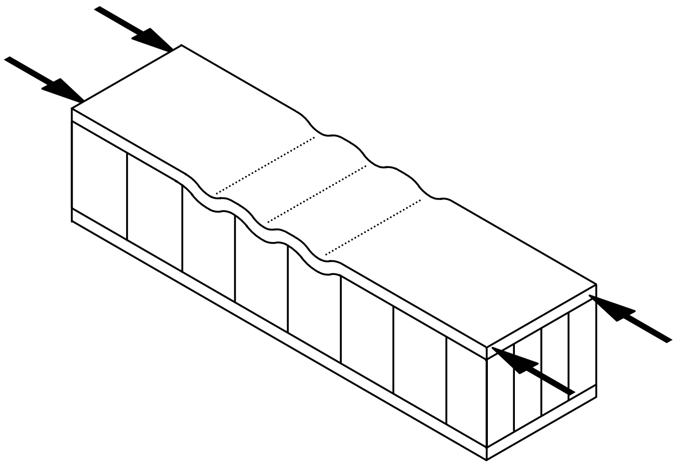


→ Critical stress limit: $\sigma_{wr} = \underline{0.5} \cdot \sqrt[3]{G_K \cdot E_K \cdot E_D}$

Theoretically a value of 0.82 can be achieved

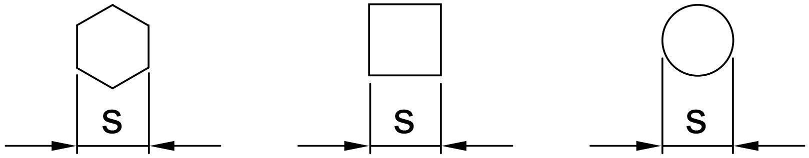
Sandwich Failure Modes

7. Intra cell buckling (Dimpling)
   For a given skin material, the core cell size must be small enough to prevent intra cell buckling.



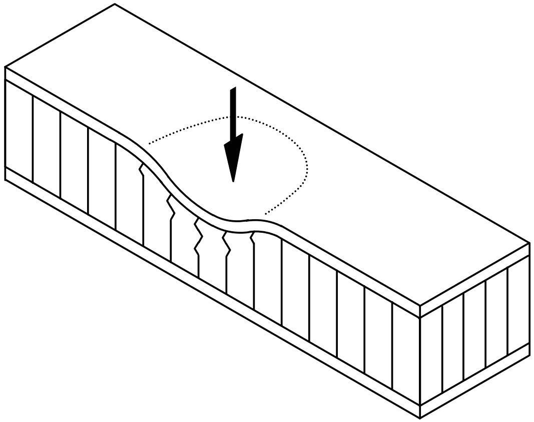
→ Critical stress limit: $\sigma_{dp} = \frac{2 \cdot E_D}{1-\nu_D^2} \cdot \left(\frac{t_D}{s}\right)^2$

Cell size „s“:



Sandwich Failure Modes

8. Local compression
   The core compressive strength must be adequate to resist local loads on the panel surface.



→ Critical stress limit: $\sigma_c = \frac{F}{A} = \frac{q \cdot l}{A}$

Design Guidelines: Honeycomb Sandwich Panel

1. Define loading conditions
   e.g. Point loading, uniform distributed load, end loads

2. Define panel type
   e.g. Cantilever, simply supported
   This is determined by the type and extent of the panel supports. Fully built in support conditions should only be considered when the supporting structure has adequate stiffness to resist deflection under the applied loads.

3. Define physical/space constraints
   This should include an assessment of the requirements including:
   - deflection limit
   - thickness limit
   - weight limit
   - factor of safety

   Preliminary materials selection should be based on the above criteria in conjunction with the mechanical properties of the considered materials.

Design Guidelines: Honeycomb Sandwich Panel

4. Preliminary calculations
   - Make an assumption about skin material, skin thickness and panel thickness (Ignore the core material at this stage)
   - Calculate stiffness
   - Calculate deflection (ignoring shear deflection)
   - Calculate facing skin stress
   - Calculate core shear stress

5. Optimize design
   - Modify skin thickness, skin material and panel thickness to achieve acceptable performance
   - Select suitable core to withstand shear stress

6. Detailed calculations
   - Calculate stiffnesses and stresses (face sheets and core)
   - Calculate deflection, including shear deflection
   - Check all applicable failure criteria
   - Check for local compression loads on core

Modul: Faserverbund- und Sandwichtechnologie

Inhaltsübersicht

Design Rules for Aerospace Vehicles

- Use composites where weight reduction can be achieved at a reasonable cost.
- Use toughened-resin materials or additional gage to increase resistance to service threats.
- Provide for a wide range of repair options, including mechanically fastened repairs.
- Provide access for maintenance, inspections, and repair.
- Design for visual in-service inspections only.
- Design to maximize automated fabrication.

Design Considerations

General Aspects

- Laminates shall be designed that all loads (load directions) can be carried through the fibers. The matrix only positions the fibers.
  A high loading of the matrix or huge matrix deformations should be avoided.

- A laminate should have at least three different ply orientations. For an arbitrary in-plane load scenario, the fibers can carry all loads – even assuming a damaged matrix.

- Due to that reason, all „Cross-Ply“ laminates should be avoided (two different ply orientations only).

- Thus, usually $\left[0^\circ{}_n, 90^\circ{}_m, +45^\circ{}_k, -45^\circ{}_k\right]$ stacking sequences are applied, because they provide 4 different ply orientations.

Design Considerations

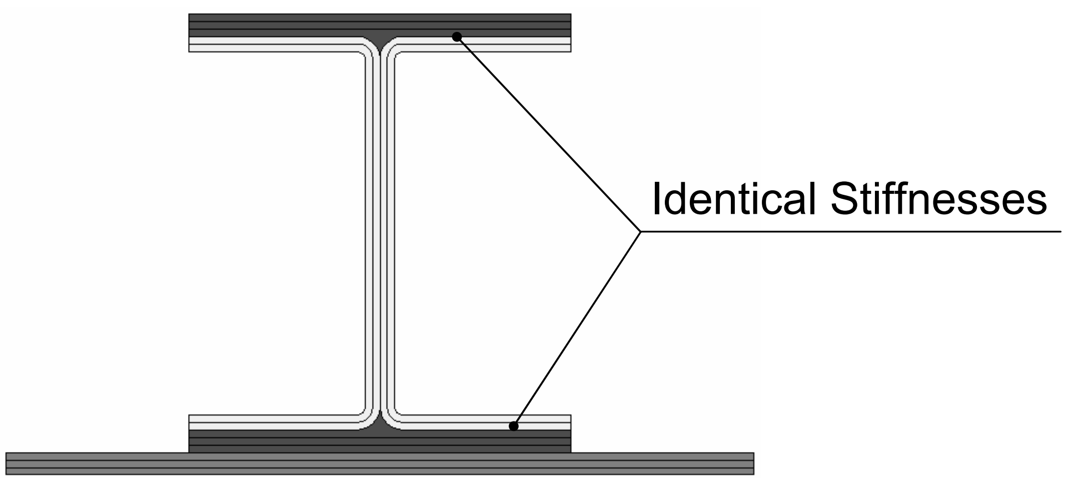
General Aspects

- 3 ply orientations shall only be applied for structures having one preferred load direction only: minimum layer proportion (each direction) 15-20%
- 4 ply orientations shall be applied otherwise:
  minimum layer proportion (each direction) 6-10%
  90°-plies: reduction of Poisson ratio and coefficient of thermal expansion
  45°-plies: in order to carry secondary loads
- Small ply orientation changes (from layer to layer) shall be preferred:
  - interlaminar shear strength is improved
  - deformations due to the cure cycle are reduced
  to be preferred: 0°, 45°, 90°
  to be avoided: 0°, 90°, 45°

Design Considerations

General Aspects

- Symmetrical laminates (angle-ply arrangements) shall be used.
- Unsymmetrical (or unbalanced) laminates shall only be used, if the anisotropic behaviour is intended.
  Unintended coupling effects may occur, e.g. bending twisting coupling.
- Symmetrical stiffness arrangements within one structural group shall be used, e.g. profiles:



Design Considerations

General Aspects

- Buckling / Instability
  Use $\pm 45°$-plies, in particular outer plies

- Bending-Tension-Loading
  Use $0°$-plies, in particular outer plies

- Excessive direct stacking of identical ply-angles shall be avoided
  (Separation of identical plies recommended)
  Compromise: maximum of 4 plies per block (same fiber direction)

Design Considerations

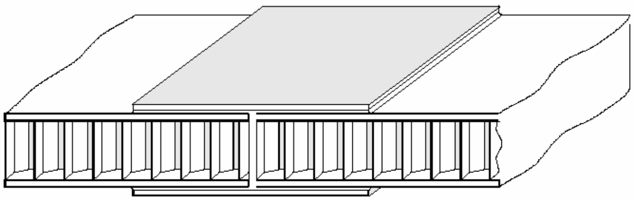
Tailor fiber arrangement to optimize resistance to loads:

- ±45° plies give buckling stability and carry shear
- 0° plies give column stability and carry tension or compression
- 90° plies carry transverse loads and reduce Poisson's effects

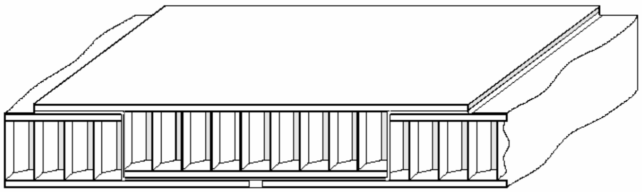
Configuration	Load	Percent Fiber		
		0°	±45°	90°
Rod	axial	50 - 100	0 - 25	0 - 25
Stiffener	axial, transverse, moment	25 - 50	25 - 50	0 - 25
Cylinder	internal pressure	25 - 33	0 - 25	50 - 67
Cylinder	external pressure	25 - 33	0 - 25	25 - 67
Cylinder	torsion	0 - 25	50 - 100	0 - 25
Plate	tension, moment	25 - 50	0 - 50	0 - 25
Plate	shear	0 - 25	50 - 100	0 - 25
Plate	compression	25 - 50	25 - 50	0 - 25

Sandwich: Typical Flat Joining Methods

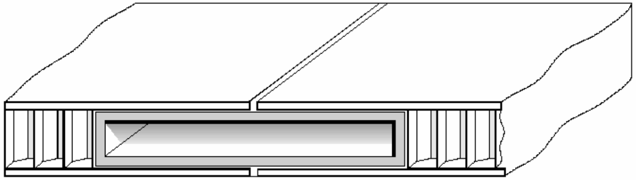
Bonded face supported butt joint



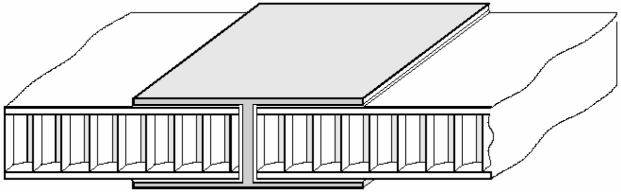
Panel section insertion method, using same panel material



Flush faced bonded joint, supported by a special internal extrusion (or wood block)

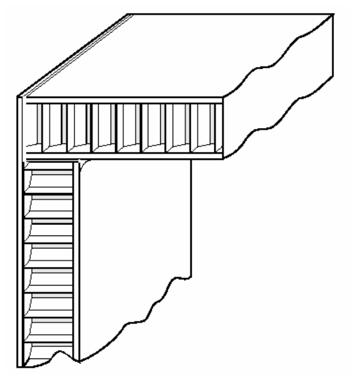


Bonded butt joint using 'H' section extrusion

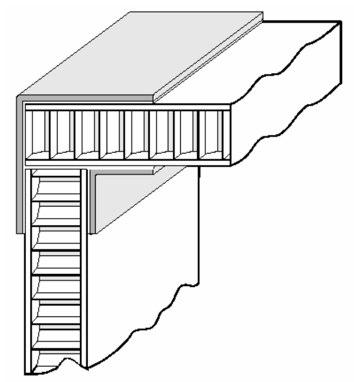


Sandwich: Typical Corner Joining Methods

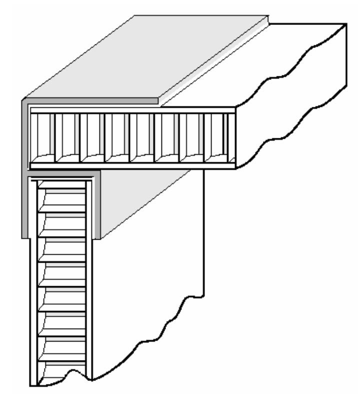
Rebated and bonded,
low strength



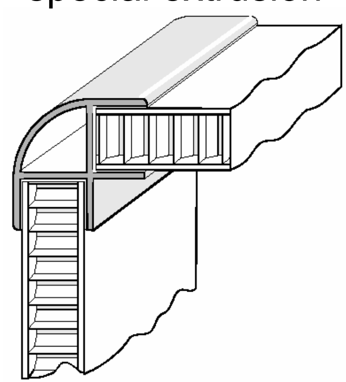
Supported by bonded
L-section extrusions



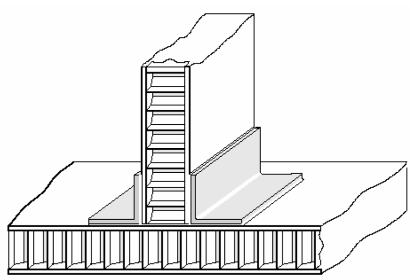
Supported by bonded
special extrusion



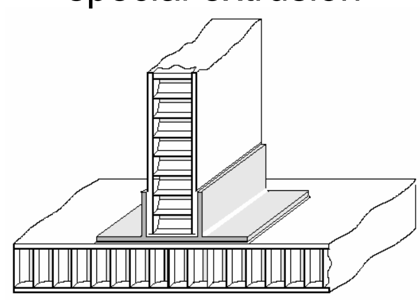
Supported by bonded
special extrusion



Supported by bonded
L-sections

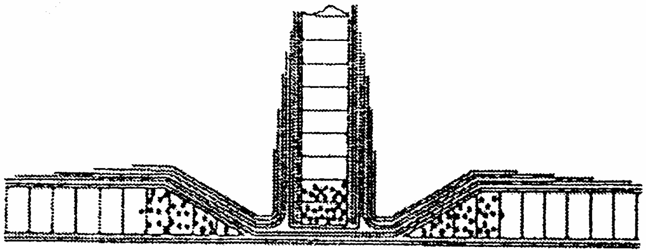
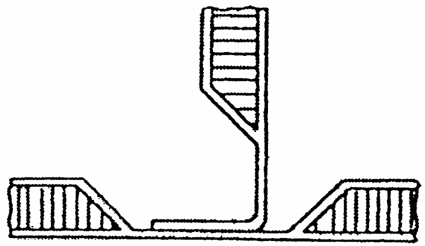
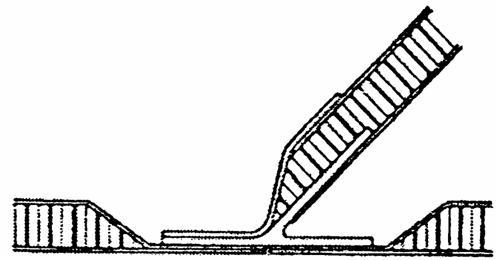


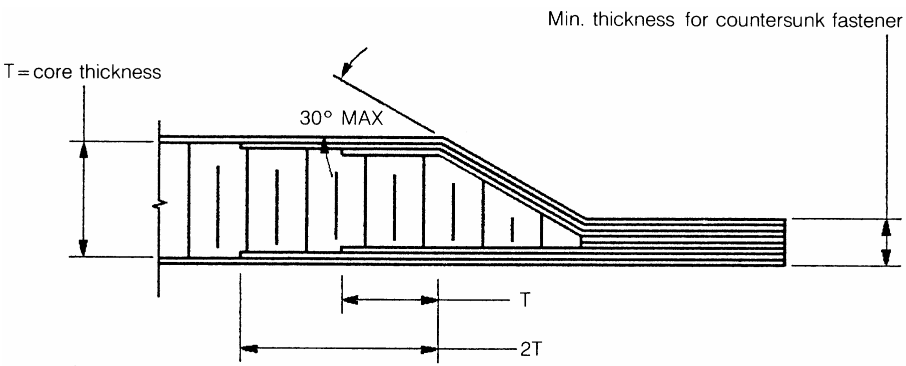
Supported by bonded
special extrusion

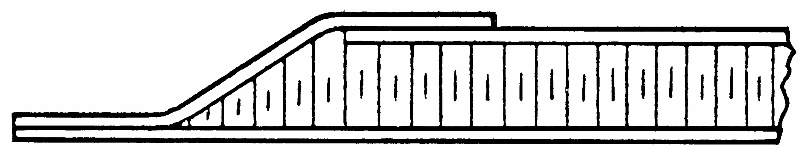


Sandwich: Typical Corner Joining Methods

T-Joints with Panel Edge Closeout

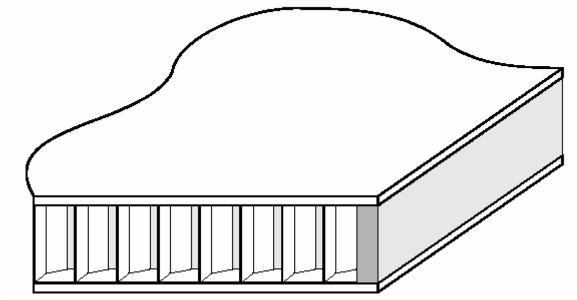






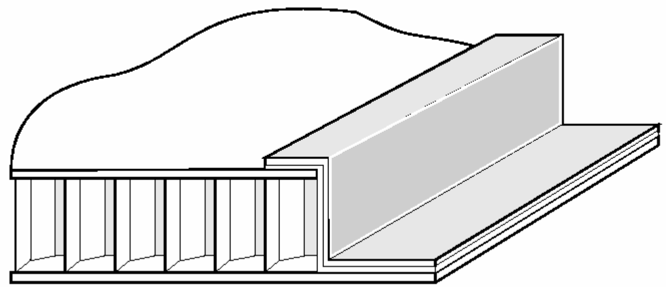


Sandwich: Typical Edge Closure Methods

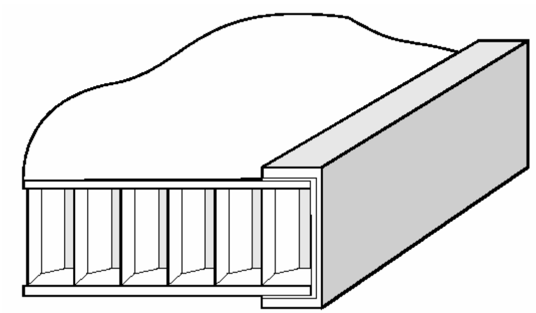
Edge filler



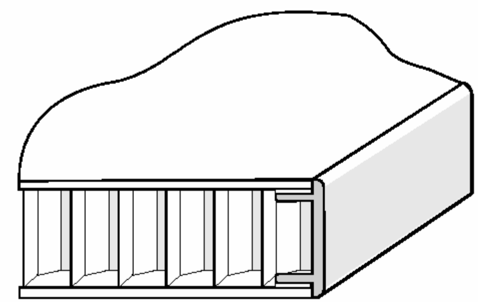
Bonded 'Z' section



Bonded 'U' section

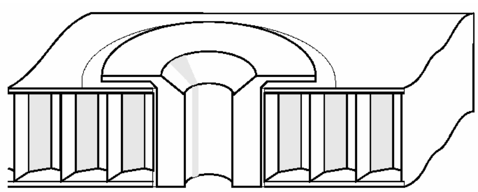


Bonded edge closure section
(suitable for thicker panels)

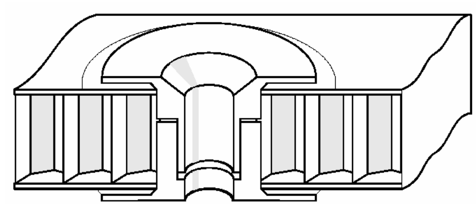


Sandwich: Typical Panel Fixings

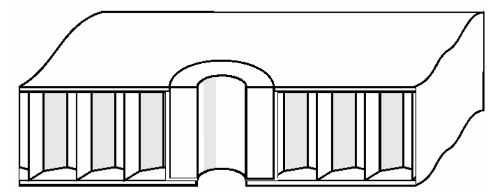
Single part ferrule



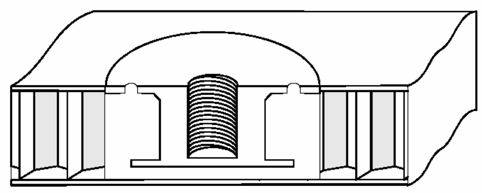
Two part ferrule



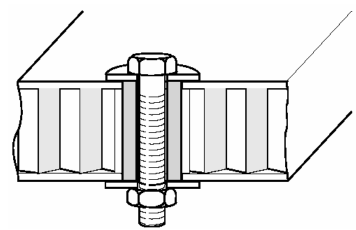
Distance tube



Threaded insert



Through panel distance tube using penny washer



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