Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

Diplomarbeit

Department Fahrzeugtechnik und Flugzeugbau

Assessment of Numerical Models for Thrust and Specific Fuel Consumption for Turbofan Engines

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Abstract

A study was undertaken to investigate the variation of thrust and specific fuel consumption due to different parameter changes. The main goal was to find empirical models describing the relationships in the public domain and to evaluate them by comparison with actual engine data. 31 models were found, 14 describing the variation of thrust and 17 describing the variation of specific fuel consumption (SFC). By using Excel spread sheets the equations where calculated for their different parameters and compared with actual engine data. Generally all models where found to be accurate but in this process a polynomial model over the Mach number was found to be most accurate for the description of the take-off thrust. An equation given by Denis Howe (in *Aircraft Conceptual Design Synthesis,* 2000) was found to be most accurate for the evaluation of the climb thrust. Additionally equations describing the variation of thrust with bleed air extraction, temperatures other than the international standard atmosphere (ISA) and for the evaluation of cruise thrust with take-off thrust for preliminary design were found and partly evaluated. Due to the lack of reference date the models describing the variation of specific fuel consumption could not be evaluated but their general trend was found to be correct. An equation also found in the book of Howe 2000 was found to be especially useful for the change over height and speed. The usually unknown starting point of all equations describing the specific fuel consumption was approximately given for his equation. There were also models found describing the variation of specific fuel consumption with reduced power, power off-take, bleed air extraction and temperature other than ISA.

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DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

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Task definition of a *Diplomarbeit* at Hamburg University of Applied Sciences

Background

Aircraft performance calculations are based on a) equations derived from first principles, b) a simplified representation of the aircraft engine and c) a simplified representation of the aerodynamics of the aircraft. The simplest way of representing a jet engine in cruise is to assume that relative thrust varies with relative air density $T / T_{SL} = a\,\sigma^n$ (a and n may be a function of bypass ratio) and that the specific fuel consumption (SFC) has a fixed value. The aerodynamics are often represented by the simple drag polar $C_D = C_{D0} + \mathrm{C}_L{}^2/(\pi\ A\ e)$. Better models for b) and c) are needed to improve the accuracy of performance calculations.

Objective

The thesis shall improve simple aircraft performance calculations by providing simple though more accurate turbofan engine thrust and SFC models.

Primary objectives:

- find models describing the thrust change with height and speed
- find models describing the change in SFC with height and speed
- compare the found models with reality (if possible)
- evaluate which is the most accurate / useful approach

Secondary objective:

- evaluate the influence of power off-take on thrust and SFC

The results have to be documented in a report. The report has to be written in a form up to internationally excepted scientific standards. The application of the German DIN standards is one excepted method to achieve the required scientific format.

The thesis is prepared at the University of Limerick, Department of Mechanical & Aeronautical Engineering. Supervisor is Dr. Trevor Young.

Declaration

This diplom thesis is entirely my own work. Where use has been made of the work of others, it has been fully acknowledged and referenced.

March 13, 2007

Date Signature

Contents

List of Figures

List of Tables

Nomenclature

a_0	Sonic speed at sea level
A	Variable factor
c	Specific fuel consumption
c_0	Variable constant for SFC calculation
C_1	Variable Constant
c_1	Variable constant for SFC calculation
C_2	Variable Constant
c_3	Variable constant for SFC calculation
c_4	Variable constant for SFC calculation
c_5	Variable constant for SFC calculation
c_6	Variable constant for SFC calculation
c_{bleed}	Specific fuel consumption with bleed air off-take
C_{bleed}	Bleed correction factor
c_{CR}	Specific fuel consumption for cruise conditions
C_{f1}	Specific fuel consumption factor
C_{f2}	Specific fuel consumption factor
c_{idle}	Specific fuel consumption at idle conditions
c_{ISA}	Specific fuel consumption for ISA conditions
$C_{mas,dry}$	Constant factor for maximum dry (no reheat) conditions
C_{OT}	Specific fuel consumption with power off-take
c_T	Specific fuel consumption due to temperature other than ISA
C_{Tc1}	Factor for thrust calculation
C_{Tc2}	Factor for thrust calculation
C_{Tc3}	Factor for thrust calculation
C_{Tc4}	Factor for thrust calculation
C_{Tc5}	Factor for thrust calculation
C_{TO}	Factor for take-off conditions
D	Drag
$\Delta F_N \%$	Thrust change in per cent
ΔT_{ISA}	Temperature change
$(\Delta T_{ISA})_{eff}$	Effective temperature change
F	Force

F_N	Net thrust
F_n	Force in *n*-direction
$F_{N,max,dry}$	Maximum net thrust for dry (no reheat) conditions
F_t	Force in t-direction
g_0	Acceleration due to gravity
H	Geopotential height
h	Height
k_1	Variable factor
k_2	Variable factor
k_3	Variable factor
k_4	Variable factor
k^*_{bleed}	Bleed off-take parameter
k_{bleed}	Bleed air extraction parameter
k_E	1/ Breguet-time-factor
k^*_P	Power off-take parameter
L	Constant temperature lapse rate
L	Lift
m	Mass
M	Mach number
m_f	Mass flow
n	Exponent
P	Pressure
P_{OT}	Power off-take
P_3 / P_2	Overall compressor pressure ratio
Q	Fuel flow
R	Specific gas constant
S	Exponent
t	Time
T	Temperature
V	Velocity
W	Weight
x	Constant
X	Variable factor
y	Exponent
Z	Variable factor

Greek

α	Angle of attack
γ	Angle of climb
δ	Pressure ratio
Δ	Difference to usual conditions
θ	Temperature ratio
λ	By-pass ratio
ρ	Density
σ	Density ratio

Subscript

0	Sea level conditions
amb	Ambiant conditions
CAS	Calibrated air speed
CL	Climb
core	Engine core
CR	Cruise
max	Maximum value
des	Descend
EAS	Equivalent air speed
eng	Engine
exit	Exit conditions
fan	Engine fan
fuel	Fuel
IAS	Indicated air speed
idle	Idle conditions
inlet	Inlet conditions
ISA	International standard atmosphere
OT	Off-take
TAS	True air speed
TO	Take-off
turb	Turbine

Superscript

*	Reference conditions

List of Abbreviations

BPR	By-pass ratio
ISA	International standard atmosphere
SFC	Specific fuel consumption
SOT	Stator outlet temperature
RR	Rolls-Royce

Terms and Definition

By-pass Ratio

The Bypass ratio (BPR) or λ is defined as the mass flow of the fan $m_{f,fan}$ divided by the core mass flow $m_{f,core}$ (**Davies 2002**, p.7.16).

$$\lambda = \frac{m_{f,fan}}{m_{f,core}}$$

Calibrated Airspeed

The calibrated airspeed is the speed the aircraft would have at sea level with the same dynamic pressure as it does for true airspeed at the actual altitude. Additionally the dynamic pressure is rectified so that the compressibility of the air at higher speeds does not falsify the outcome. According to **Davies 2002** (p.10.122) V_{CAS} can be calculated as

$$V_{CAS} = a_0 \sqrt{5\left(\left(\delta\left(\left(1+0{,}2M^2\right)^{3,5}-1\right)+1\right)^{\frac{1}{3,5}}-1\right)}$$

where a_0 = sonic speed at sea level
δ = pressure ratio
M = Mach number

Flat Rating

In order to enhance the life span of an engine, some of them are flat rated. The maximum take-off thrust for a temperature higher than ISA condition is used, e.g. +10 K, to set the maximum available thrust for an engine. The temperature is called flat rating temperature. At temperatures lower than the flat rating temperature the take-off thrust is fixed to the set value which reduces the maximum engine temperature and therefore increases the life span. Above the flat rating temperature the engine behaves like a non flat rated engine and the thrust reduces with rising ambient temperature.

Fuel Flow

The fuel flow Q is the amount of fuel consumed by an engine over a period of time. This can be defined in terms of weight

$$Q = \frac{W_{fuel}}{t} \text{ in } \left[\frac{\text{lb}}{\text{h}}\right]$$

or in terms of mass

$$Q = \frac{m_{fuel}}{t} \text{ in } \left[\frac{g}{s}\right]$$

where W_{fuel} = weight of the used fuel
m_{fuel} = mass of the used fuel

t = time

Indicated Airspeed

The indicated airspeed is the speed that can be read at an airspeed indicator. It is the same speed as the calibrated airspeed but for possible instrument, total pressure or position errors.

Relative Density

The relative density (σ) is defined as the ambient density (ρ) divided by the density at sea level for ISA conditions (ρ_0).

$$\sigma = \frac{\rho}{\rho_0}$$

Using the ideal gas law σ can be written as

$$\sigma = \frac{\delta}{\theta}$$

According to **Davies 2002** (p.10.110) σ, in the troposphere, can be calculated as

$$\sigma = \left(1 - \frac{LH}{T_0}\right)^{(g_0/RL)-1}$$

where T_0 = 288,15 K
L = 0,0065 K/m ≈ 0,0019812 K/ft
H = geopotential height in [m]
g_0 = 9,80665 m/s^2 ≈ 32,17405 ft/s^2
R = 287,059 m^2/(s^{2}K) ≈ 30089,811 ft^2/s^{2}K

Relative Pressure

The relative pressure (δ) is defined as the ambient pressure (p) divided by the pressure at sea level for ISA conditions (p_0).

$$\delta = \frac{p}{p_0}$$

According to **Davies 2002** (p.10.110) δ, in the troposphere, can be calculated as

$$\delta = \left(1 - \frac{LH}{T_0}\right)^{g_0/RL}$$

Relative Temperature

The relative temperature (θ) is defined as the ambient temperature (T) divided through the temperature at sea level for ISA conditions (T_0).

$$\theta = \frac{T}{T_0}$$

According to **Davies 2002** (p.10.110) θ, in the troposphere, can be calculated as

$$\theta = 1 - \frac{LH}{T_0}.$$

In the stratosphere θ = constant.

Stator outlet temperature (SOT)

The stator outlet temperature is the temperature that the air has when it leaves the combustion chamber and passes the stator to enter the high pressure turbine.

Specific fuel consumption (SFC)

The specific fuel consumption (c) (also known as thrust specific fuel consumption for jet engines) is the fuel flow (Q) divided by the net thrust (F_N). According to **Davies 2002** (p.10.129) SFC can be defined in terms of weight flow rate

$$c = \frac{(dW_{fuel}/dt)}{F_N} = \frac{Q}{F_N} \text{ in } \left[\frac{\text{lb}}{\text{lbh}}\right]$$

or mass flow rate

$$c = \frac{(dm_{fuel}/dt)}{F_N} = \frac{Q}{F_N} \text{ in } \left[\frac{\text{mg}}{\text{sN}}\right]$$

where F_N = net thrust

Engine manufacturers prefer to speak of the specific fuel consumption rather than the actual fuel flow because it is the inverse of efficiency of the engine. SFC increases with speed but decreases with height. There is a minimum of SFC at a certain thrust, higher or lower thrust causes an increase of SFC.

Thrust

Thrust F_N in [lb] or [N] is the net force that the engines produce for propulsion. According to Newton's second law (**Davies 2002** (p.7.1)) the equation for thrust (ignoring the pressure force) is

$$F = \frac{d(mV)}{dt}$$

or for jet engines

$$F_N = (m_{f,eng} + m_{f,fuel})V_{exit} - m_{f,eng}V_{inlet}$$

where $m_{f,eng}$ = mass flow through the engine

$m_{f,fuel}$ = mass flow of fuel

V_{exit} = speed at the exit of the engine

V_{inlet} = speed at the inlet of the engine

True Airspeed

The true airspeed V_{TAS} is the actual speed, with respect to the local (ambient) air mass.

1 Introduction

This work takes only commercial aircraft with a high to low BPR into account.

Due to confidentially, certain performance parameter have not been given in absolute terms, but rather in relative terms. For example, the actual thrust (F_N) is not given, but rather the ration of the thrust to a reference condition, usually the sea level condition ($F_{N,0}$). Furthermore, to enable the results of this study to be published without restriction, it was necessary that one of the cited references not be fully described.

1.1 Motivation

In order to evaluate the performance characteristics for an airplane it is vital to know several parameters such as thrust lapse rate or the variation of specific fuel consumption. Aircraft and engine manufacturers have access to this knowledge but to the large community of private performance engineers and lecturers at universities, this is usually unknown and kept secret by the manufacturers. To be able to conduct research in these areas, some relatively simple numerical models have been derived, but they are not all taking the same parameters into account and the outcome differs from each other. This thesis will collect and evaluate these models to show which fit best for the different parts of a flight (e.g. take-off, climb, cruise).

1.2 Objectives

Primary objectives:
- find models describing the thrust change with height and speed
- find models describing the change in SFC with height and speed
- compare the found models with reality (if possible)
- evaluate which is the most accurate / useful approach

Secondary objective:
- evaluate the influence of power off-take on thrust and SFC

1.3 Report Structure

Chapter 2 contains the literature review. The literature review contains all models found, describing the behaviour of thrust and SFC.

Chapter 3 contains the analysis of the performance models. The general behaviour of the models is shown as well as the comparison with actual engine data

.

Chapter 4 contains the conclusions plus some advice which model to use best.

2 Literature Review

2.1 Variation of Thrust

As described in the Terms and Definitions the thrust decreases with height and velocity. Several authors have dealt with the estimation of thrust, mainly with one of the two factors instead of both.

2.1.1 Variation of Thrust with Height

A very common way to describe the thrust variation with height is the approach used by **Eshelby 2000**, **Asselin 1997**, **Ojha 1995**, **Anderson 1999** and **Wikipedia 2006.**

$$\frac{F_N}{F_{N,0}} = \sigma^x \tag{2.1}$$

where $F_N =$ net thrust

$F_{N,0} =$ thrust at sea level

$x =$ variable coefficient

Anderson 1999 suggests roughly $x \approx 1$ but this can vary in either direction, **Wikipedia 2006** gives a value of $x = 0{,}85$ and all others give $x = 0{,}7$. According to **Eshelby 2000** x has to be unity for calculations in the stratosphere. He also states that the exact value for x can vary with characteristics of the engine cycle or bypass ratio (BPR).

Scholz 2007a has altered equation (2.1) to fit for cruise.

$$\frac{F_N}{F_{N,0}} = a\sigma^n \tag{2.2}$$

where $a = -0{,}0253\lambda + 0{,}7291$

$n = 0{,}0033\lambda + 0{,}7324$

Note that the equations for a and n are only valid for normal jet cruise Mach numbers.

Raymer 1999 approaches the variation of thrust with height in a linear way, by saying that at sea level the thrust is 100 % and at 55000 ft the thrust is 0 %. This approach is valid at heights below 40000 ft. Fitting these conditions in an equation leads to

$$\frac{F_N}{F_{N,0}} = 1 - Ch \tag{2.3}$$

where $C = 0{,}000018\ \text{ft}^{-1}$

$h =$ height in [ft]

Eurocontrol 2004 gives an approach to determine the climb thrust with height.

$$F_{N,CL,max} = C_{Tc1}\left(1 - \frac{h}{C_{Tc2}} + C_{Tc3}h^2\right) \tag{2.4}$$

where $F_{N,CL,max} =$ maximum climb thrust in [N]

$C_{Tc1}, C_{Tc2}, C_{Tc3} =$ constants given by **Eurocontrol 2004**

This approach is only valid for standard ISA conditions but a correction is given for other conditions. In addition, some correction factor for descend, maximum cruise and an equation for reduced power are also given. **Eurocontrol 2004** multiplies the climb thrust with a constant variable given by it to determine the maximum cruise thrust $F_{N,CR,max}$, and descent thrust $F_{N,des}$.

2.1.2 Variation of Thrust with Speed

One way to describe thrust variation with speed is

$$\frac{F_N}{F_{N,0}} = 1 - k_1 V + k_2 V^2 \tag{2.5}$$

where $V =$ Velocity (or Mach number)

$k_1, k_2 =$ variable factors

Mair & Birdsall 1992, **Anderson 1999** and **Young 2001** suggest this equation but give no clue of what value the *k* factors could be. All authors state that a general approach in this style is only valid for take- off or for a limited speed and height variation at a fixed thrust setting (rating). **Torenbeek 1982** uses a similar approach but goes into detail how to calculate the factors.

$$\frac{F_N}{F_{N,TO}} = 1 - \frac{0{,}45M(1+\lambda)}{\sqrt{(1+0{,}75\lambda)G}} + \left(0{,}6 + \frac{0{,}11\lambda}{G}\right)M^2 \tag{2.6}$$

where $F_{N,TO} =$ take-off thrust

$G =$ gas generator function

$G = 0{,}9$ for low λ

$G = 1{,}1$ for high λ

Torenbeek 1982 (Appendix H) also shows how to calculate the *G* factor but this is not possible without detailed knowledge of the engine component efficiencies.

It is useful to mention that **Torenbeek1982** gives actually two equations. Equation (2.6) is from chapter 4, and the following equation is from Appendix H.

$$\frac{F_N}{F_{N,TO}} = 1 - \frac{0,45M(1+\lambda)}{\sqrt{(1+0,75\lambda)G}} + \left(0,6 + \frac{0,13\lambda}{G}\right)M^2 \tag{2.6b}$$

A change has been made in the last term, 0,11λ becomes 0,13λ. It is not known if this happened accidentally or on purpose. It was decided to work with equation (2.6) because it was in the main part of the book and not in the appendix. Additionally, the difference in the outcome is not very large. This approach is only meant to be used for take-off.

Mair & Birdsall 1992 and **Anderson 1999** offer a way to determine the thrust at a constant height with varying speed, by the equation

$$\frac{F_N}{F_{N,0}} = AM^{-n} \tag{2.7}$$

where A and n are always positive constants and M is the Mach number.

2.1.3 Variation of Thrust with Height and Speed

Since the equation of **Torenbeek 1982** (based on data more than 25 years old) might not have been very accurate for modern engines **Bartel & Young 2007** (in a preliminary study of the subject) evaluated this topic and altered the equation into

$$\frac{F_N}{F_{N,TO}} = A - \frac{0,377(1+\lambda)}{\sqrt{(1+0,82\lambda)G}} Z \frac{p_{amb}}{p_{amb,0}} M + \left(0,23 + 0,19\sqrt{\lambda}\right) X \frac{p_{amb}}{p_{amb,0}} M^2 . \tag{2.8}$$

where
$$A = -0,4327\left(\frac{p_{amb}}{p_{amb,0}}\right)^2 + 1,3855\frac{p_{amb}}{p_{amb,0}} + 0,0472$$

$$X = 0,1377\left(\frac{p_{amb}}{p_{amb,0}}\right)^2 - 0,4374\frac{p_{amb}}{p_{amb,0}} + 1,3003$$

$$Z = 0,9106\left(\frac{p_{amb}}{p_{amb,0}}\right)^2 - 1,7736\frac{p_{amb}}{p_{amb,0}} + 1,8697$$

p_{amb} = ambient pressure

$p_{amb,0}$ = ambient pressure at sea level

They state that the original equation lead to good results below $M = 0,2$ but their alteration brings results within an accuracy of 1% up to $M = 0,4$. These results are validated only for two-shaft turbofan engines using a constant stator outlet temperature. In equation (2.8) is a height change included but if we calculate at sea level A, X and Z become 1 and don't have to be calculated. Note that $p_{amb,0}$ is the standard ISA pressure at sea level per definition

Howe 2000 describes a thrust variation with Mach number and height as

$$\frac{F_N}{F_{N,0,s}} = \left(k_1 + k_2\lambda + \left(k_3 + k_4\lambda\right)M\right)\sigma^S \tag{2.9}$$

where $\lambda =$ bypass ratio
$k_{1-4}, S =$ variable factors
only valid for $M \leq 0{,}9$

The factors are given for different Mach numbers and bypass ratios therefore it is possible to start calculations without further delay. The given factors are meant to be for maximum thrust.

Table 2.1 Factors for the thrust equation of **Howe 2000** (rewritten according to **Howe 2000**, p.67)

BPR	**M**	**k1**	**k2**	**k3**	**k4**	**S** h<11 km	**S** h>11 km
1	0 - 0,4	1	0	-0,2	0,07	0,8	1
	0,4 - 0,9	0,856	0,062	0,16	-0,23	0,8	1
3 to 6	0 - 0,4	1	0	-0,6	-0,04	0,7	1
	0,4 - 0,9	0,88	-0,016	-0,3	0	0,7	1
8	0 - 0,4	1	0	-0,595	-0,03	0,7	1
	0,4 - 0,9	0,89	-0,014	-0,3	0,005	0,7	1

2.1.4 Thrust Variation due to Temperatures other than ISA

Raymer 1999 states that for every 1 K above ISA conditions the thrust can be reduced by 0,75 %.

Eurocontrol 2004 gives for the variation of thrust caused by temperatures other than ISA

$$F_{N,CL,max,\Delta ISA} = F_N\left(1 - C_{Tc5}\left(\Delta T_{ISA}\right)_{eff}\right) \tag{2.10}$$

where $F_{N,CL,max,\Delta ISA} =$ net thrust for temperatures other than ISA
$\Delta T_{ISA} =$ temperature deviation from ISA in [K]
$\left(\Delta T_{ISA}\right)_{eff} = \Delta T_{ISA} - C_{Tc4}$
$C_{Tc4}, C_{Tc5} =$ variable factors given by Eurocontrol

This model is only valid when $0 \leq \left(\Delta T_{ISA}\right)_{eff} \cdot C_{Tc5} \leq 0{,}4$.

2.1.5 Variation of Thrust due to Bleed Air Extraction

Not only the thrust but most of the energy required by the aircraft is generated by the engines. Doing so there are two ways of power extraction: off–take by shaft power or bleed air. Since the shaft power off-take usually does not exceed 150 kW for a 133 kN engine (**Raymer 1999**, p.389) it is very small and can usually be neglected. Bigger by far is the effect of bleed air off-take because the air conditioning/ cabin pressurization and anti ice systems are driven by this. Thrust losses due to bleed air cause a bigger loss of thrust than the actual loss of air. **Raymer 1999** gives an equation to estimate the effect of bleed air off–take, i.e.

$$\Delta F_N\,\% = C_{bleed}\left(\frac{m_{f,bleed}}{m_{f,eng}}\right)\cdot 100 \tag{2.11}$$

where $\Delta F_N\,\% =$ thrust change in %

$C_{bleed} =$ bleed correction factor is given by manufacturer

$C_{bleed} = 2$ if nothing else is given

$m_{f,bleed} =$ mass flow of bleed air, usually 1-5% of engine mass flow

$m_{f,eng} =$ mass flow of engine

2.1.6 Variation of Cruise Thrust with Take-off Thrust

According to **Svoboda 2000** the cruise thrust ($F_{N,CR}$) bears a statistical relationship with take-off thrust, which is approximately:

$$F_{N,CR} = 200 + 0{,}2F_{N,TO} \tag{2.12}$$

where the thrust is given in lb. This rough estimation, meant to be used for preliminary design, was derived from a survey of engine data.

Scholz 2007b gives for preliminary design

$$\frac{F_{N,CR}}{F_{N,TO}} = (0{,}0013\lambda - 0{,}0397)h_{CR} - 0{,}0248\lambda + 0{,}7125 \tag{2.13}$$

where h_{CR} = height for cruise in [km]

and

$$\frac{F_{N,CR}}{F_{N,TO}} = \left(3{,}962\cdot 10^{-7}\lambda - 1{,}21\cdot 10^{-5}\right)h_{CR} - 0{,}0248\lambda + 0{,}7125 \tag{2.14}$$

where h_{CR} = height for cruise in [ft]

2.2 Specific Fuel Consumption

The determination of the variation of specific fuel consumption was described by more authors than the variation of thrust. As described in the Terms and Definitions, the specific fuel consumption varies with speed and height.

2.2.1 Constant Specific Fuel Consumption

Eshelby 2000 offers the way of regarding SFC as constant but says that this is only valid as a rough estimation or over a very small range of speed or height.

2.2.2 Variation of SFC with Height and Speed

One approach to this topic given by **ESDU 73019**, **Mair & Birdsall 1992** and **Anderson 1999** is

$$c = c_2 \theta^{0,5} M^n . \tag{2.15}$$

where $n = 0{,}2$ for a $\lambda = 0$ and

$n = 0{,}6$ for a $\lambda = 10$ according to **Mair & Birdsall 1992**, **ESDU 73019** gives a graph to determine n (Fig. 2.1) but both ways are only for maximum cruise thrust

$c =$ specific fuel consumption

$c_2 =$ variable constant

Actually this approach is only valid for Mach No. of $0{,}6 \leq M \leq 0{,}9$ and only for a constant engine speed but since the engine speed varying with height for a cruise flight varies very little it can be used in this case as well according to **ESDU 73019**. Note that the values of n are for maximum cruise thrust and not for the usual used cruise thrust. It is not known how the values of n change with a reduction of thrust and this might lead to serious errors if n is used incorrectly.

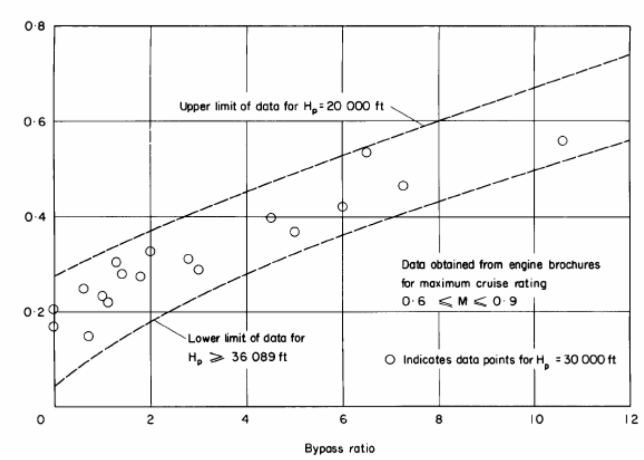


Figure 2.1 Graph of n over λ (**ESDU 73019**, p.37)

Howe 2000 says that with varying Mach number and altitude

$$c = c_1\left(1 - 0{,}15\lambda^{0{,}65}\right)\left(1 + 0{,}25\left(1 + 0{,}063\lambda^2\right)M\right)\sigma^{0{,}08} \qquad (2.16)$$

where $c_1 \approx 0{,}85\dfrac{\mathrm{N}}{\mathrm{Nh}}$ for a low λ if no manufacturer's information is available

$c_1 \approx 0{,}7\dfrac{\mathrm{N}}{\mathrm{Nh}}$ for a high λ if no manufacturer's information is available

Martinez-Val & Perez 1991 use for the determination of the SFC known reference conditions.

$$c = c^*\left(\frac{M}{M^*}\right)^n\left(\frac{\theta}{\theta^*}\right)^{0{,}5} \qquad (2.17)$$

where * = reference condition

$n = 0$ for $\lambda = 0$

$n = 0{,}2 - 0{,}4$ for a low λ

$n = 0{,}4 - 0{,}7$ for a high λ

valid only for $0{,}6 < M < 0{,}85$

This equation is given for cruise conditions from $0{,}6 \le M \le 0{,}85$.

Myose et al. 2005 and **Young 2001** use equation (2.15) without the height term to determine SFC at a given height

$$c = c_2 M^n \qquad (2.18)$$

Young 2001 gives the value of n = 0,45 to 0,5 for a modern high by-pass ratio turbofan.

Eshelby 2000 and **ESDU 73019** offer another way, dealing only with variation of speed, by giving

$$c = c_3 + c_4 M\ . \qquad (2.19)$$

This equation can be transferred into

$$c = c_5 + c_6 F_N \tag{2.20}$$

according to **Eshelby 2000**. In these model c_{3-6} are constants.

Mair & Birdsall 1992 and **Anderson 1999** approach the variation of SFC with speed by

$$c = c_0 (1 + k_4 M) \tag{2.21}$$

where $c_0 =$ SFC of static sea level thrust

$k_4 =$ variable constant

Eshelby 2000 also offers an approach with variation of height by saying

$$\frac{c}{c_0} = \theta^y \tag{2.22}$$

with $y \approx 0{,}5$ but varies with λ

Eurocontrol 2004 gives the equation

$$c = C_{f1}\left(1 + \frac{V_{TAS}}{C_{f2}}\right) \tag{2.23}$$

where $C_{f1}, C_{f2} =$ constant variables given by **Eurocontrol 2004**

This approach is valid for all flight phases except cruise and approach/ idle.

2.2.3 Variation of SFC with Thrust or By-pass Ratio

Svoboda 2000 did an analysis of existing engines and came up with some equations describing SFC for preliminary design.

$$c_{TO} = 0{,}49 - 0{,}0007\sqrt{F_{N,TO}} \tag{2.24}$$

$$c_{TO} = 0{,}71 - 0{,}15\sqrt{\lambda} \tag{2.25}$$

$$c_{CR} = 0{,}8 - 0{,}00096\sqrt{F_{N,TO}} \tag{2.26}$$

where c comes out in [lb/(lbh)]

$F_{N,TO}$ in [lb]

2.2.4 Variation of SFC due to Reduced Power

According to **Raymer 1999** a change in the specific fuel consumption due to reduced power with Mach number changes can be calculated by

$$\frac{c}{c_{max,dry}} = \frac{0{,}1}{\left(\frac{F_N}{F_{N,max\,dry}}\right)} + \frac{0{,}24}{\left(\frac{F_N}{F_{N,max\,dry}}\right)^{0{,}8}} + 0{,}66\left(\frac{F_N}{F_{N,max\,dry}}\right)^{0{,}8} + 0{,}1M\left(\frac{1}{\frac{F_N}{F_{N,max\,dry}}} - \left(\frac{F_N}{F_{N,max\,dry}}\right)\right) \tag{2.27}$$

The specific fuel consumption at idle can assumed to be

$$c_{idle} = 1{,}5c_{max\,dry}$$

if nothing else is given.

2.2.5 Variation of SFC due to Power Off-take and Bleed Air Extraction

The variation of SFC due to power off- take are described by **RR 1988** as

$$c_{OT} = cC_{OT}\frac{F_N}{F_{N,OT}} \tag{2.28}$$

where $c_{OT} =$ SFC after power off-take

$C_{OT} =$ off–take correction factor, varies between 0,9 and 1 with speed, height, temperature and amount of power off take

$F_{N,OT} =$ net thrust after power off-take

Young 2002 states that the change of SFC with shaft power off-take is a linear function that varies with the engine type. Generally it can be said that for engines with a higher thrust output the fuel penalty due to power off-take is lower than for engines with a lower thrust output. 100 kW power off-take cause roughly a SFC penalty of 0,5 – 1% of the old value. These values should only give a ballpark of the amount or rough estimate.

Ahlefelder 2006 agrees to a nearly linear behaviour of the SFC for power off-take but states that the gradient may vary strongly dependant on the engine configuration. **Ahlefelder 2006** calculates for the equation

$$m_{fuel,OT} = P_{OT}k_P^*t \tag{2.29}$$

where m_{fuel} = mass of used fuel due to power off-take

P_{OT} = Power off-take

$k_P^* =$ power off-take parameter

t = time

a $k_P^* = 0{,}176$kg/kWh. **Scholz 2006** uses this approach and gives different values for the power off-take parameter. **Scholz 2006** gives his own value as $k_P^* = 0{,}097$ kg/kWh and also values from other authors as $k_P^* =$ 0,125 or 0,167 kg/kWh. Equation (2.29) could be easily transferred into the change of the SFC by dividing by the time (t) and the thrust.

Ahlefelder 2006 also did some research for variation of SFC with bleed air off-take. According to him bleed air off-take results in a rise of the SFC. For engines with integrated nozzle the rise of the SFC is nearly linear but for engines with separate nozzles an exponential rise is to be expected. Generally the SFC rise due to bleed air off-take is strongly depending on the place it is taken. The rise in the SFC is higher at a high pressure stage of the compressor than it is at a lower pressure stage. **Ahlefelder 2006** and **Scholz 2006** give the equation

$$m_{f,fuel} = k_{bleed}^* m_{f,bleed} \tag{2.30}$$

where k_{bleed}^* = bleed air off-take parameter

Values for the bleed air off-take parameter are 0,028 (**Ahlefelder 2006**) or 0,0335. k_{bleed}^* can be calculated as $k_{bleed}^* = k_{BB}(P_3 / P_2)$ where $k_{BB} = 4{,}99 \cdot 10^{-3}$ K and y = 0,475 but the overall compressor pressure ratio P_3 / P_2 is not always known.

Scholz 2006 dereived equation (2.30) from

$$m_{fuel} = \frac{k_{bleed} T_{turb} m_{f,bleed}}{k_E} \left(e^{tk_E} - 1\right)) \tag{2.30a}$$

and

$$m_{f,fuel} = k_{bleed} T_{turb} m_{f,bleed} = k_{bleed}^* m_{f,bleed}$$

where $k_{bleed} = 3{,}015 \cdot 10^{-5}$ K^{-1}

T_{turb} = turbine inlet temperature (1100 K)

$k_E = cg_0 \left(\frac{cos\,\gamma}{L/D} + sin\,\gamma \right)$

L = lift

D = drag

γ = angle of climb (flight path angle)

2.2.6 Variation of SFC due to Temperature other than ISA

RR 1988 gives an equation describing the variation of SFC with temperature.

$$c_T = c_{ISA}\left(\frac{T}{T_{ISA}}\right)^{-0,6} \tag{2.31}$$

3 Analysis of Performance Models

3.1 General Behaviour of Models Describing Thrust

Thrust increases with decreasing temperature but decreases with decreasing air density. A rising of altitude causes a decrease of thrust, the density effect is dominant. The thrust decreases with rising speed due to drag, the ram effect raises the overall compression of the compressor which leads to an increase of thrust. For a turbofan engine the decrease of thrust due to drag loss is dominant but the level of dominance varies with bypass ratio. The higher the bypass ratio the more dominant the drag effect (see Fig.3.1). It is to mention that the upper curve characterizes the relative thrust of a turbojet engine or very low bypass ratio engine ($\lambda \leq 1$).

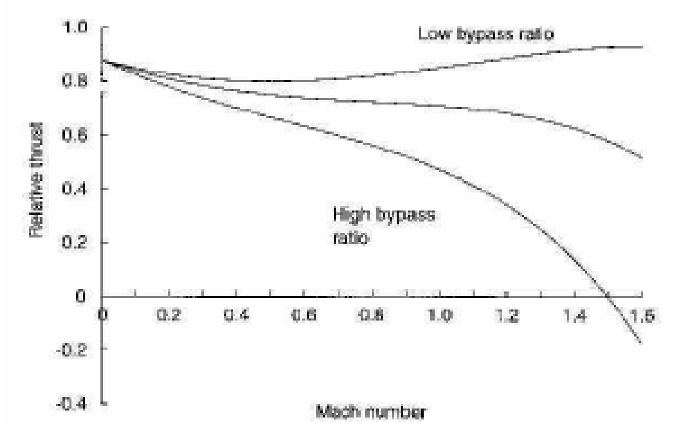


Figure 3.1 Thrust variation with Mach number and bypass ratio (**Eshelby 2000**, chap. 3.4)

Figures 3.2 and 3.3 show the general behaviour of the models over height and speed. Since the different models use different values as denominator it is difficult to compare the values for the thrust rate. To be able to show all models in one figure the thrust was calculated for all models and divided by the real thrust for sea level and V_{CAS} = 250 kts. The real thrust rate for climb was used as reference but the general behaviour of the models stays the same for take-off, regardless of the used thrust.

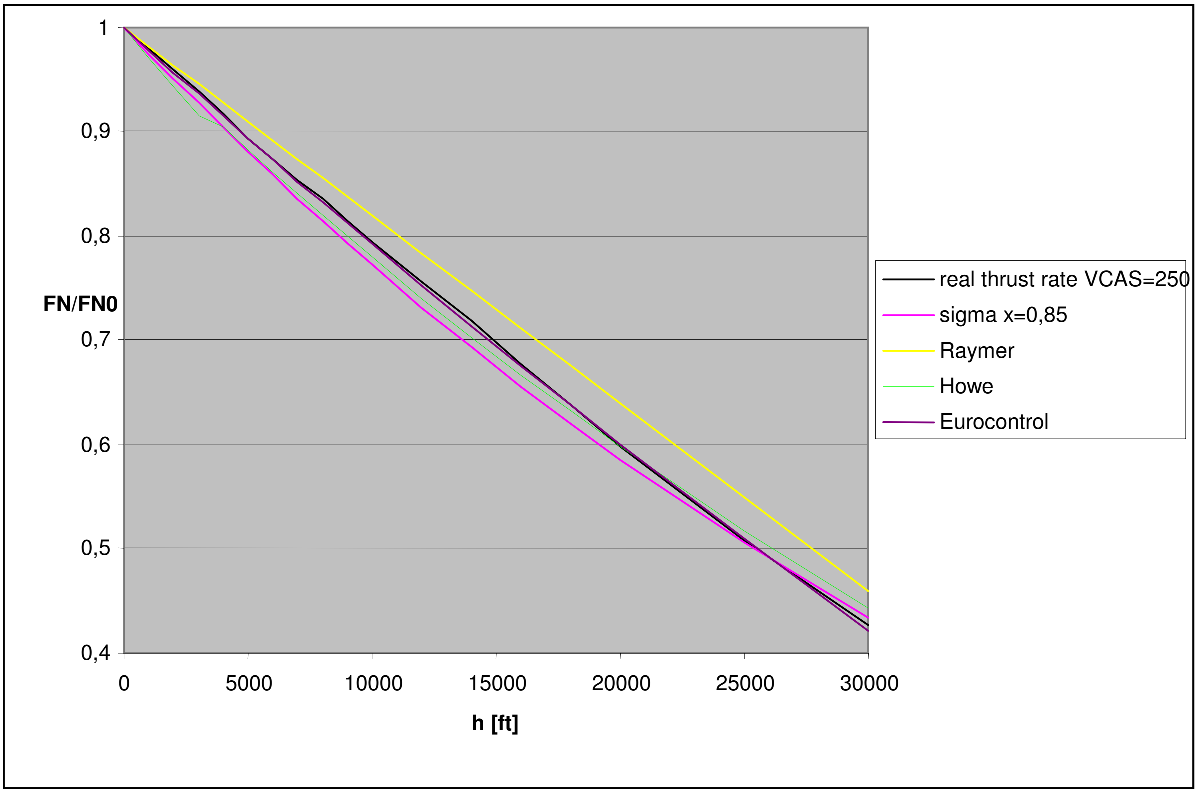


Figure 3.2 General behaviour of the models over height

Figure 3.2 shows that all models can follow the real thrust rate fairly well. The real thrust rate describes a slightly bended, nearly linear curve that can be produced by all shown models.

In Figure 3.3 the models do not reach $M = 0$. In fact they should go through this point but due to the way the graphs where made it was not possible to do so because the graph of equation (2.7) would not have looked the way it does now. This problem will be explained in the next section.

Figure 3.3 shows the thrust rate change with Mach number variation. It can be seen that the models of **Torenbeek 1982** and equation (2.7) do not follow the real thrust very well. For **Torenbeek 1982** this is the case because the equation was not meant to be used for cruise thrust. The fact that equation (2.5) matches the real thrust graph very well stresses that the deviation of the equation of **Torenbeek 1982** is due to not being calibrated for cruise. For Equation (2.7) it is obvious that the equation has problems to follow the thrust lapse. Since it is an exponential function it rises strongly the closer it gets to $M = 0$ and is zero when it reaches $M = 0$.

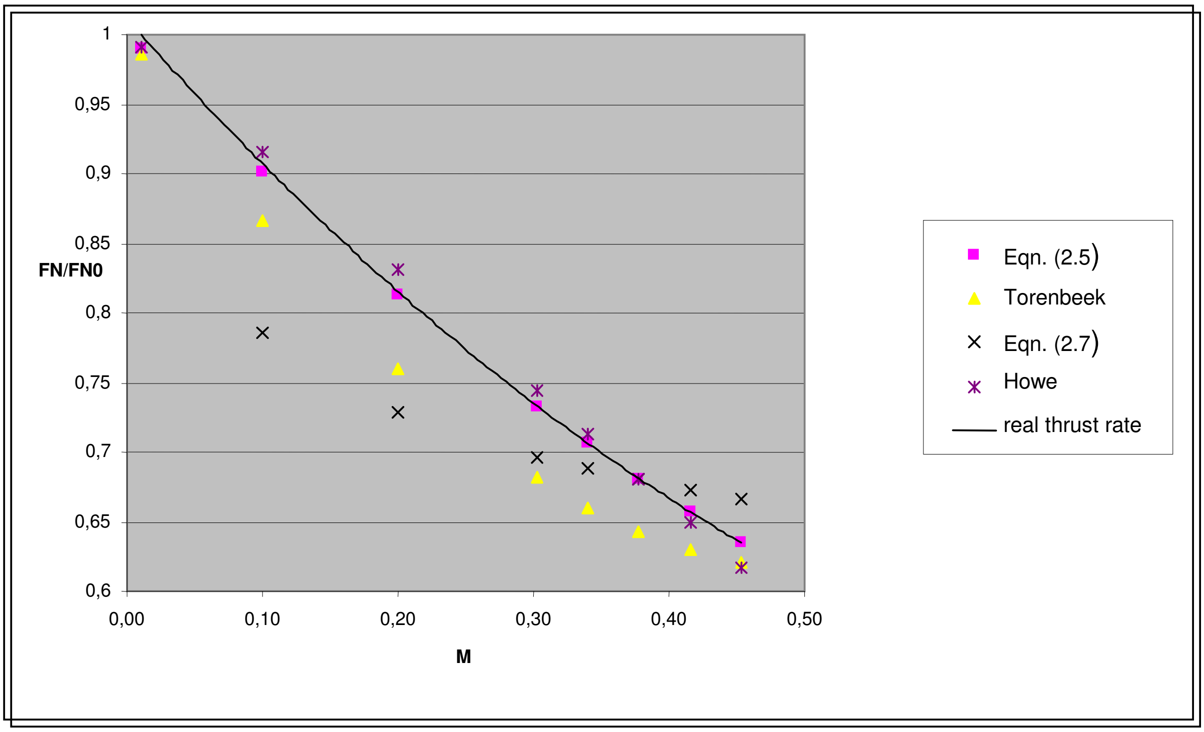


Figure 3.3 General behaviour of the models over Mach number

3.1.1 Variation of Thrust with Height

Equation (2.1) depends on the factor x and the relative density σ and is meant to be used if the thrust for any speed at sea level is known and the thrust for the same speed is to be determined for another height.

$$\frac{F_N}{F_{N,0}} = \sigma^x \tag{2.1}$$

Suggestions of the factor x range from 0,7 to 1 and the validity will be evaluated in section 3.3.3 but what happens when conditions other than ISA occur. The denominator in the equation for σ is the ISA density at sea level, so when there are other conditions than ISA equation (2.1) does not start with $\frac{F_N}{F_{N,0}} = 1$ but with something different. This is impossible because the thrust rate at sea level has to be one and therefore the equation has to include σ for ISA whatever conditions appear. The possibility of using σ for changing the thrust with temperatures other than ISA is discussed in section 3.5.2. The equation produces an exponential type of curve but since the equation for σ is already an exponential one it is possible to get a near linear curve by choosing the right value for x.

Equation (2.3) by **Raymer 1999** is meant to be used in the same way as equation (2.1) but follows a linear approach.

$$\frac{F_N}{F_{N,0}} = 1 - c \cdot h \qquad (2.3)$$

Equation (2.4) is an empirical approach where the constants for different engine-aircraft configurations are given by **Eurocontrol 2004**. This is not a pure approach over height but also slightly with speed because only V_{CAS} is kept constant and therefore M varies. Since all necessary variables are given this is a useful way to determine the thrust variation during the climb. If a new engine is designed or **Eurocontrol 2004** hasn't included one engine in it's databank this is probably not the easiest way to evaluate an engine. Although it is not immediately obvious approach (2.3) is very similar to the one of **Eurocontrol 2004** equation (2.4).

$$F_{N,climb,max} = C_{Tc1}\left(1 - \frac{h}{C_{Tc2}} + C_{Tc3}h^2\right) \qquad (2.4)$$

By leaving the last term with C_{Tc3} away and divide by C_{Tc1} ,which is the net thrust at V_{CAS} = 250 kts, the same form is reached. **Raymer 1999** states that his factor $c = 0{,}000018 = 1/55556$, while the value of C_{Tc3} given by **Eurocontrol 2004** ranges from 40000 ft to 60000 ft. The value given by **Raymer 1999** lies somewhere in the middle because it is an average value.

By looking at the equation of **Eurocontrol 2004,** no speed variation is obvious, still it is used for varying velocities. Up to a height of 10000 ft the V_{CAS} = 250 kts is used and later V_{CAS} = 290 - 330 kts is used, depending on the engine/ aircraft combination. Since for a given calibrated airspeed the true airspeed changes with height there are not only two different speeds but all speeds differ from each other. This change in speed is already taken into account by the factors C_{Tc2} and C_{Tc3} given by **Eurocontrol 2004**. As mentioned above there are two different V_{CAS} and therefore two different curves of thrust over height if the factors are to be found for a new engine. **Eurocontrol 2004** uses one pair of factors, so between h = 10000 ft and h = 12000 ft two curves are combined to one curve. Since the equation above, describing the curve, cannot describe the kink where the two curves come together (see Figure 3.4) the flaw in this model is immediately recognizable. In addition, when two curves are connected the new equation describing both as one curve cannot be completely accurate. This report is not meant to question the accuracy of the work of **Eurocontrol 2004** but to mention the crux if one tries to decipher factors for a new engine. Another question is what C_{Tc1} really is. By looking at the equation it is evident that it has to be the maximum climb thrust, but at what speed? Since the starting point of the equation is the thrust at sea level and V_{CAS} = 250 kts it should be the thrust at these conditions due to the equation but the equation is also valid for V_{CAS} = 300 kts and the thrust at this speed differs from the one at 250 kts.

All these questions are not answered in the manual which leads to the conclusion that this equation is only meant to be used in combination with the factors given by **Eurocontrol 2004** and not for describing new engines on your own.

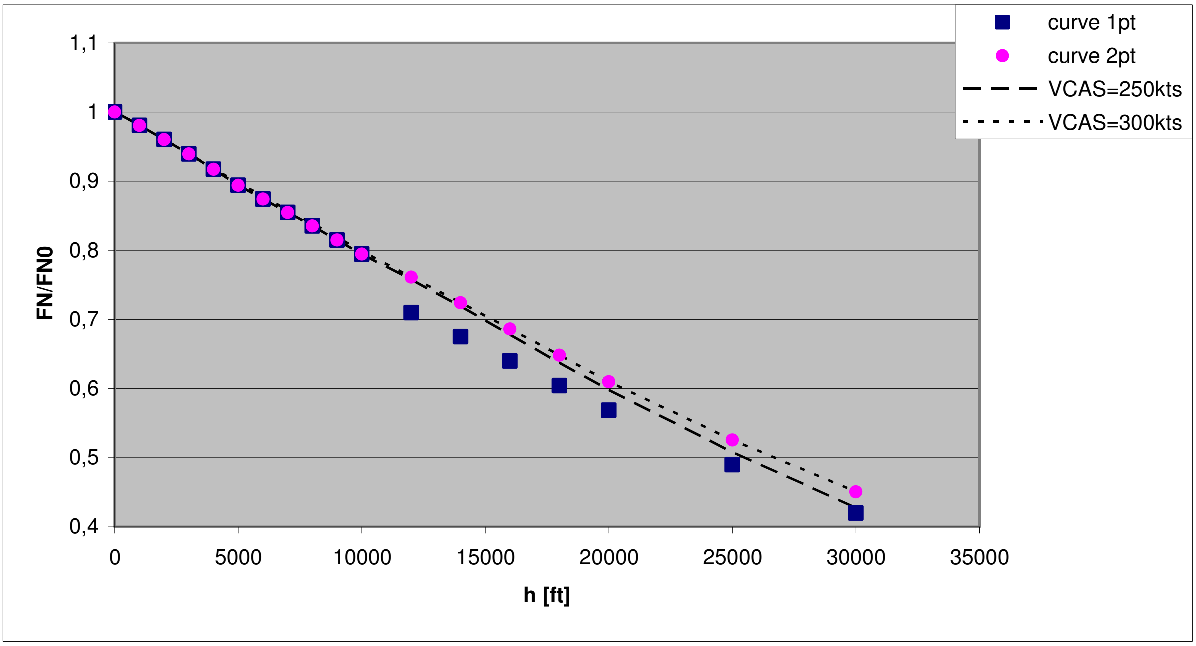


Figure 3.4 Different thrust rate curves over height by **Eurocontrol 2004**

3.1.2 Variation of Thrust with Speed

Equation (2.5) is widely accepted to be very accurate in describing the thrust variation for take-off but cannot be used immediately to determine the thrust variation because the *k*-factors have to be determined with the help of real engine data. **Torenbeek 1982** (equation (2.6)) and **Bartel & Young 2007** (equation (2.8)) have altered this approach into directly usable equations. They all produce a polynomial curve that can take a large variety of shapes.

Equation (2.7) is only meant to describe a known curve because the constants don't follow an obvious law and the authors don't go into detail how to determine them.

$$\frac{F_N}{F_{N,0}} = AM^{-n} \tag{2.7}$$

3.1.3 Variation of Thrust with Height and Speed

Equation (2.9) by **Howe 2000** could be seen as a combination of equation (2.1) describing the thrust change with an altitude variation and a well defined linear way describing the thrust change with a speed variation. A real thrust lapse rate does not follow a linear law but is slightly bended. **Howe 2000** takes this into account by dividing his linear approach into two sections giving different factors for every section. One section reaches from $0 \leq M < 0{,}4$ and the other from $0{,}4 \leq M \leq 0{,}9$. By doing this his method is can be more accurate than a simple linear approach. The model includes a variation with BPR. Since all factors are given the model can immediately be used to determine the thrust rate of an engine.

3.1.4 Thrust Variation due to Temperatures other than ISA

At first sight **Eurocontrol 2004** gives with equation (2.10) a very complicated way to vary the thrust with temperature changes. Upon closer inspection it is very similar to the one **Raymer 1999** offers by saying thrust changes 0,75 % with every 1 K. What makes the approach of Eurocontrol look so different and difficult is the consideration of an engine being flat rated. Equation (2.10) does not include the real temperature deviation but they introduce $(\Delta T_{ISA})_{eff}$ which is defined as $(\Delta T_{ISA})_{eff} = \Delta T_{ISA} - C_{Tc4}$. It is not mentioned by **Eurocontrol 2004** that this term has anything to do with a flat rated engine but by considering that C_{Tc4} = 6,75 (**Eurocontrol 2004**, p.C50) for the engine PW4158 it is the only explanation why the first 6,75 K does not change the thrust. The validity of this model is limited to $0 \leq \left(\Delta T_{ISA}\right)_{eff} \cdot C_{Tc5} \leq 0{,}4$. If we enter anything less than 6,75K, $(\Delta T_{ISA})_{eff}$ is negative and the model is not valid and therefore there is no thrust change. If equation (2.10) is reduced and the temperature deviation counted only after the maximum flat rating temperature, the outcome is

$$F_{N,climb,max,\Delta ISA} = F_N \left(1 - C_{Tc5} \Delta T\right) \tag{3.1}$$

At this point it is seen that the net thrust is simply a multiplication with a factor smaller than one. That is exactly the same approach of **Raymer 1999** but for the variable factor that is not fixed to 0,75 %.

3.1.5 Thrust Variation due to Bleed Air Extraction

Equation (2.11) a simple approach by changing the percentage of the thrust with a certain amount of bleed air extraction. The equation is ready to use and therefore quite useful.

3.1.6 Thrust Variation due to Take-off Thrust

Figure 3.5 shows the climb thrust over the take off thrust. The trend, given by **Svoboda 2000** (equation (2.12)) is clearly visible and the majority of the data points lies within a range of ±10 %.

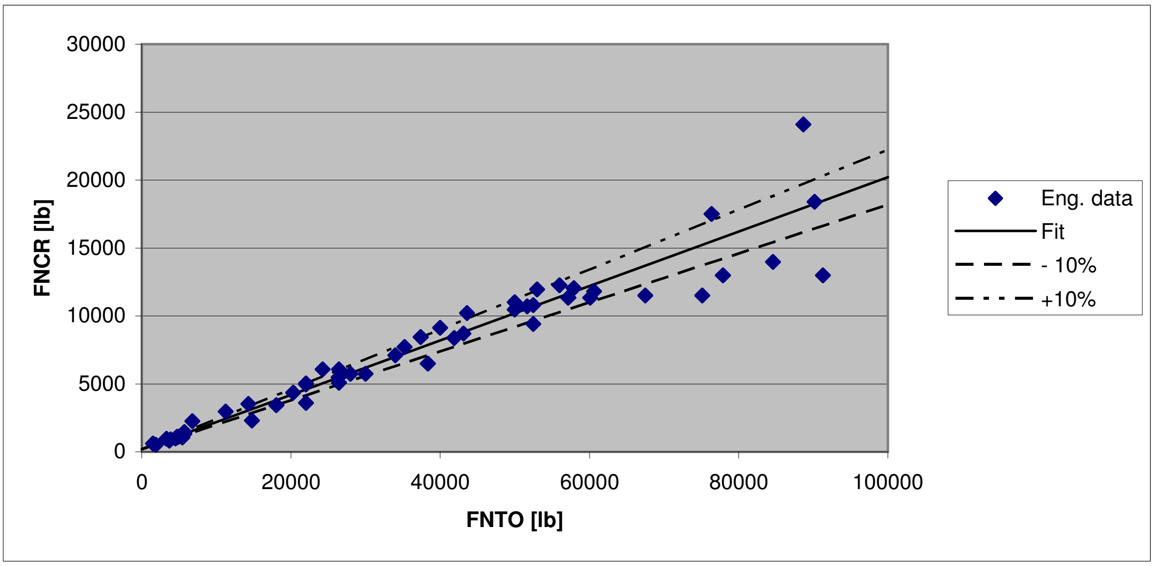


Figure 3.5 Climb thrust over take-off thrust (redrawn after **Svoboda 2000**)

Figure 3.6 shows the variation of $F_{N,CR}/F_{N,TO}$ over height according to **Scholz 2007b**. The thrust rate diminishes with rising height. This trend is correct and is due to the fact that with rising altitude the cruise thrust diminishes and the ratio $F_{N,CR}/F_{N,TO}$ as well. With rising BPR the thrust rate is diminished right from the starting point at sea level. This behaviour can be explained as the thrust for a high BPR engine diminishes stronger with rising speed than a low BPR engine. The lapse rate over height for the thrust rate of a high BPR engine is smaller than for a low BPR engine. The reason for this behaviour was not apparent from the literature, but the approach of **Howe 2000** produces the same tendency (see Figure 3.7). It is to mention that Figure 3.7 does not display $F_{N,CR}/F_{N,TO}$ but F_N/F_{N0} for any thrust setting. The tendency is still valid.

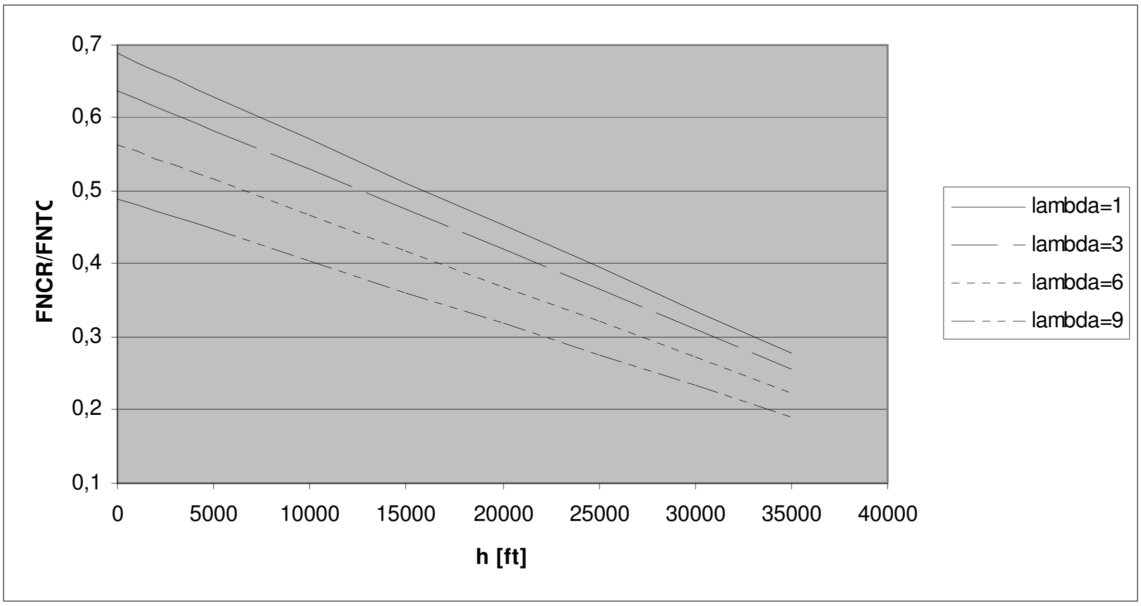


Figure 3.6 $F_{N,CR}/F_{N,TO}$ over height (according to **Scholz 2007b**)

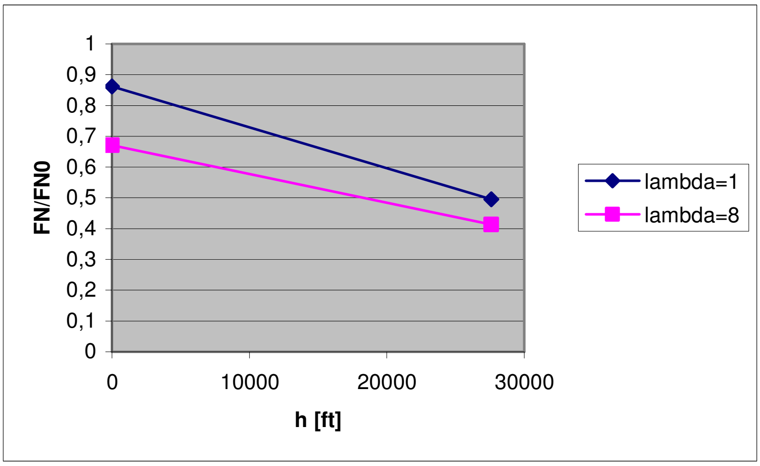


Figure 3.7 F_N/F_{N0} over height for $M = 0{,}8$ (drawn with equation (2.9) by **Howe 2000**)

3.2 Comparison of Thrust Models with Take-off Data

To evaluate different models calculating the thrust variation for take-off a figure obtained from **Engine Manufacturer 2006** was modified and the results compared with the models. The picture shows a graph describing the factors k_1, k_2 over $F_N / m_{f,eng}$. The factors are meant for the equation (2.5) but for every step there is a new pair of factors. The data was not meant to be used for comparing the reliability of models but due to a lack of other data it will be used as such anyway. The values of the Figure are shown in Table 3.1, together with the values of $F_{N,TO}$, λ and $m_{f,eng}$. Equation (2.5) was changed to get the Mach numbers.

Table 3.1 Data for take-off written down from **Engine Manufacturer 2006**

G=	1			
λ=	4,2			
FNTO =	known			
Mfeng =	known			
FN/mfeng	**k1**	**k2**	**FN/FNTO**	**M**
27	1,19	0,8	0,796	0,197
28	1,16	0,775	0,826	0,169
29	1,13	0,755	0,855	0,141
30	1,09	0,73	0,885	0,115
31	1,06	0,71	0,914	0,086
32	1,025	0,68	0,944	0,057
33	0,99	0,665	0,973	0,028
34	0,96	0,64	1,003	-0,003

With these data known the models of **Torenbeek 1982** (equation 2.6), **Howe 2000** (equation 2.9), **Bartel & Young 2007** (equation 2.8) plus equation (2.5) and (2.7) where compared. (see figure 3.8) The first three mentioned models where used with the factors given by the authors, the other two where modified to fit best the given data. **Bartel & Young 2007** recalculated the values of G = 0,8 for λ = 4 and G = 1,1 for λ = 8, therefore this was taken into account as well for the equation of **Torenbeek 1982**.

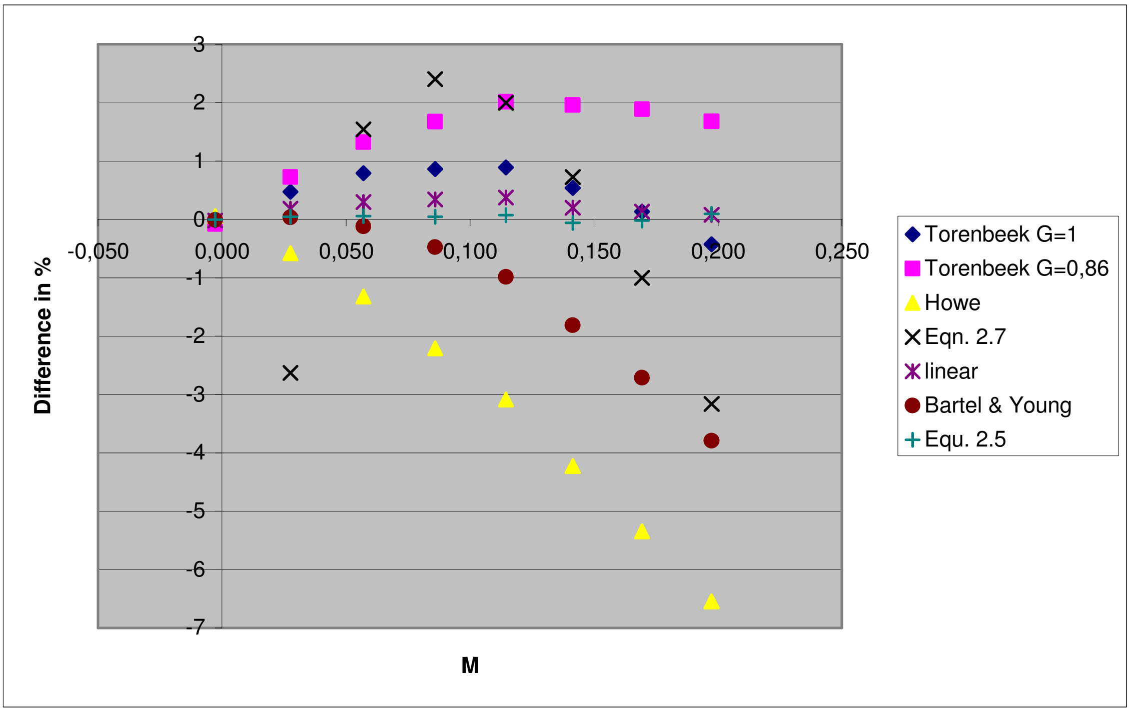


Figure 3.8 Difference of the models over Mach number

It can be seen that all models produce reasonable results. The most accurate result is given by equation (2.5) since it is modified to fit perfectly to the data. This supports the general assumption that this equation matches the true thrust lapse rate best. Equation (2.7), although modified to fit best as well does not bring the same perfect results. It is obvious that the equation cannot describe the thrust rate very well. The unmodified equation of **Torenbeek 1982** fits well within a range of 1 % to the data and is therefore even better than the new equation of **Bartel & Young 2007** that fits within a range of 4 %. The reason for this is more likely to be the way how the actual data set was generated than an inaccuracy in the equation. Additionally the boundary conditions for the given data are not completely known. If the control parameter was not a constant stator outlet temperature a disparity might be caused by this. The *G*-modified equation of **Torenbeek 1982** also lacks the accuracy of the other one which might be due to the fact that the new values of G are not meant to be used with this equation. The model of **Howe 2000** produces the worst results. It can only describe a linear thrust lapse rate and since the Mach number stays below 0,4 the second pair of factors changing the linear direction mentioned in Chapter 3.1.3 are not used. To put it in a nut shell, equation (2.5)

$$\frac{F_N}{F_{N,0}} = 1 - k_1 V + k_2 V^2 \tag{2.5}$$

is definitely the best way to describe the thrust lapse rate for take-off and the equations of **Torenbeek 1982** and **Bartel & Young 2007** are using the same approach. They are both very accurate.

3.3 Comparisson of Thrust Models with Climb Data

3.3.1 General Explanations

Explanation of climb thrust

The climb thrust is not as dependent on engine parameters (such as maximum engine temperature) as the take-off thrust. Usually the customer demands a certain thrust at a certain height, usually about 30000 ft to 35000 ft. The engine manufacturer sets a certain SOT at this height to achieve exactly the demanded thrust (**Engine Manufacturer 2006**). With this SOT known the thrust is scaled down to sea level, the SOT is reduced in this process. The engine data mentioned in section 3.2 was obtained in such way. The SOT was set for a high or low climb rate. The thrust lapse rates for the same engine running with different SOTs are not the same. There is also the way of controlling the engine over the rotation speed of the shafts but this was not the case here.

Climb thrust problem

The data sets of Engine 3 to 5 (**Engine Manufacturer 2006**) are not complete for the static climb thrust at sea level and the sea level thrust for $V_{CAS} = 300$ kts were missing. Since a data set without these data points could not be used for comparison of some of the models **Engine Manufacturer 2006** suggests to multiply the known sea level thrust at $V_{CAS} = 250$ kts with

1,42 to gain a very close result for the static sea level thrust for climb and then to extrapolate the missing thrust for $V_{CAS} = 300$ kts. By comparing this approach with approach with the complete data sets of Engine 1 and 2 it becomes obvious that the idea is close, but maybe not close enough to function as validated data for comparison. The two factors for the given engines range from 1,41 to 1,47 and due to the lack of further data this could not be changed into a law following BPR for example. The equation of **Howe 2000** seems to be very accurate, especially at low altitudes therefore the static sea level thrust for climb is calculated by this approach and the sea level thrust for $V_{CAS} = 300$ kts is then extrapolated. Although the error is minimized by this method it is very likely that there is one. Since the error for the static sea level thrust is five times higher than the one for $V_{CAS} = 300$ kts (distance from 0 kts to 250 kts and 250 kts to 300 kts equals 5 to 1) the data was not uses for models using the static sea level thrust for climb as the denominator in every thrust rate. The data was not used for the models of **Howe 2000** and **Torenbeek 1982**.

Climbing speed

Engine or airplane manufacturers recommend to climb at a constant V_{IAS} (**Swatton 2000**, chapter 14.5.1) until at a certain height a certain Mach number is reached. This point is referred to as speed change point. Afterward the climb is performed at the Mach number until the cruise altitude is reached. **Eurocontrol 2004** uses this technique, using V_{CAS} = 250 kts at h < 10000 ft and afterward V_{CAS} = 290 – 330 kts (see difference between V_{CAS} and V_{IAS} in the terms and definitions) until M = 0,78 – 0,82 is reached. **Engine Manufacturer 2006** uses V_{CAS} = 250 kts at h < 10000 ft and afterward V_{CAS} = 300 kts until the speed change point is reached. **Engine Manufacturer 2006** agrees to the approach given by **Swatton 2000** and adds that usually the velocity for best climb is used and varies therefore from aircraft to aircraft. Additionally there is a FAA regulation that the climb has to be performed at VCAS = 250 kts below 10000 ft (**Young 2007**). This regulation is only valid where the FAA is accepted as lawgiving authority. In other parts of the world, there is no single policy but some air traffic control zones (e.g. for many European airports), this is also enforced. Furthermore airlines can have their own policy and in many cases, these use the 250 kts if nothing is enforced.

3.3.2 Introduction

To evaluate the models describing the climb thrust rate five data sets (**Engine Manufacturer 2006**) for five different engines where used as reference data.

Some of the models include the BPR in their equations. The authors have probably meant a constant BPR to be used for usually the variation of the BPR is not known without detailed engine data. To evaluate this assumption a constant BPR as well as the true, changing BPR where used and compared. Figure 3.9 shows the constant and variable thrust rate for the model of **Howe 2000** (equation (2.9)) as an example. It can be seen that the difference between the two graphs are not very big since the variation in BPR is not very big but does not follow an obvious law. This is the case for all models having the BPR included. It is therefore recommended to use a constant BPR.

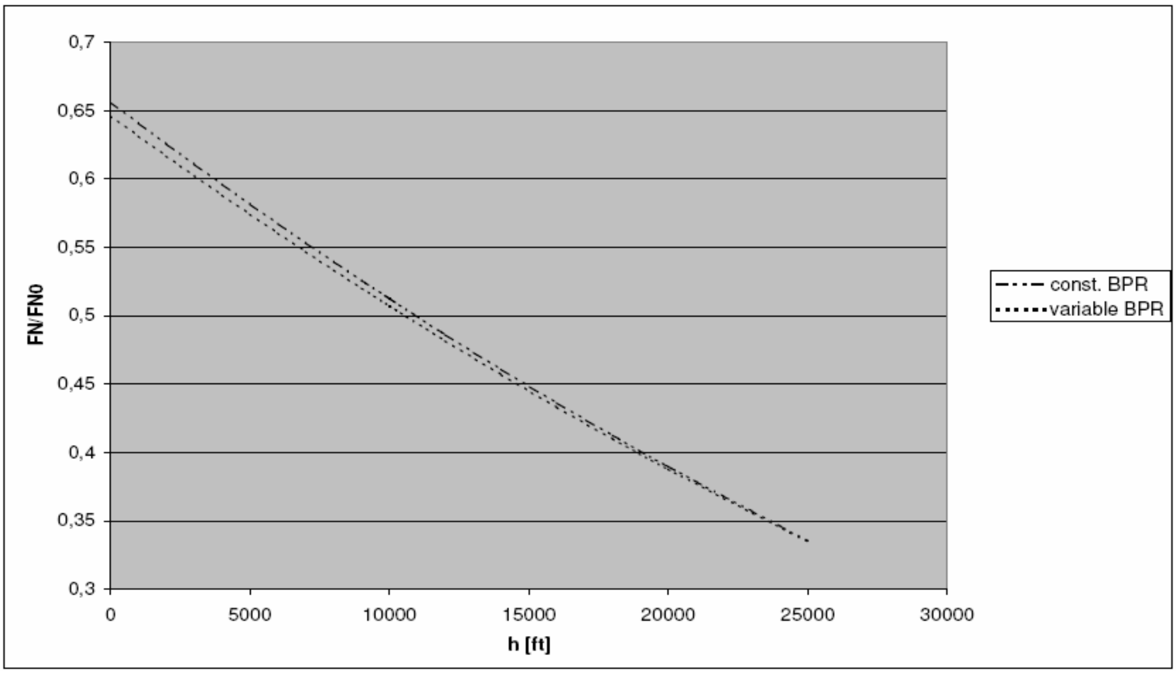


Figure 3.9 Thrust rate with constant and variable BPR according to **Howe 2000**

3.3.3 Thrust Variation with σ only

Equation (2.1) was used to define the thrust rate with different values for the factor *x*.

$$\frac{F_N}{F_{N,0}} = \sigma^x \tag{2.1}$$

Figure 3.10 shows the thrust rate for the three different values of *x*. It can be seen that in this case the value of $x = 0{,}85$ matches the real thrust curve very well.

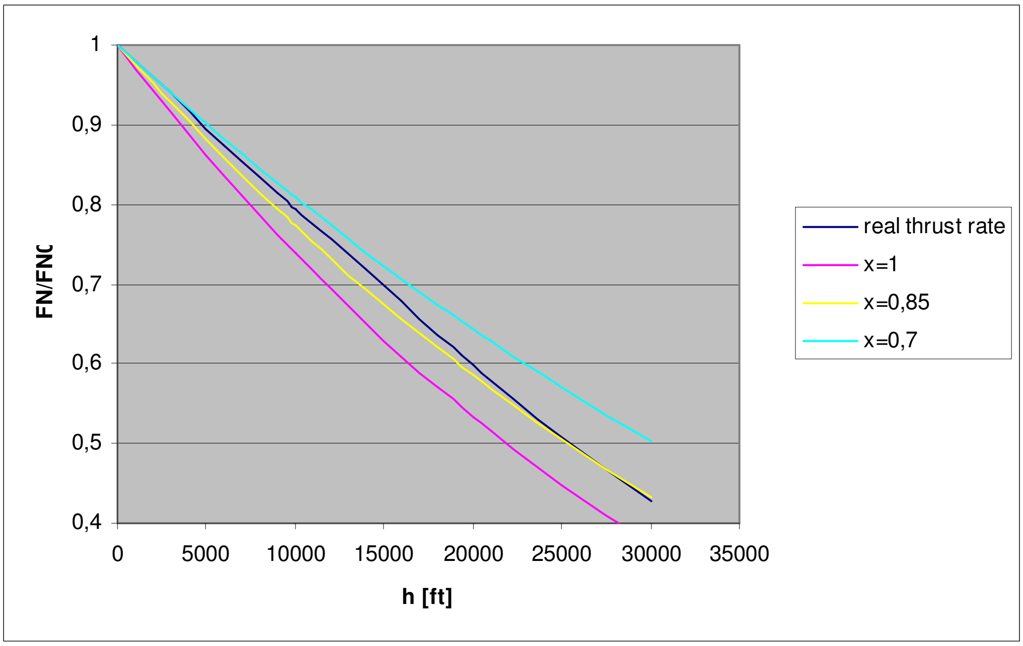


Figure 3.10 Thrust rate over height for different values of x at a speed of V_{CAS} = 250 kts

Figure 3.11 shows the calculated values for five engines of which three where run at two different SOTs, one for a low climb rate and one for a high climb (cl) rate. There are no obvious trends with velocity or BPR. The values for x lie between 0,86 and 0,59. It is suggested to use an x between these two values and the models suggesting x = 0,7 and x = 0,85 lie between them. The suggestion of **Anderson 1999** to choose x = 1 cannot be confirmed but since only the calibrated airspeed was kept constant this suggestion cannot be called false either.

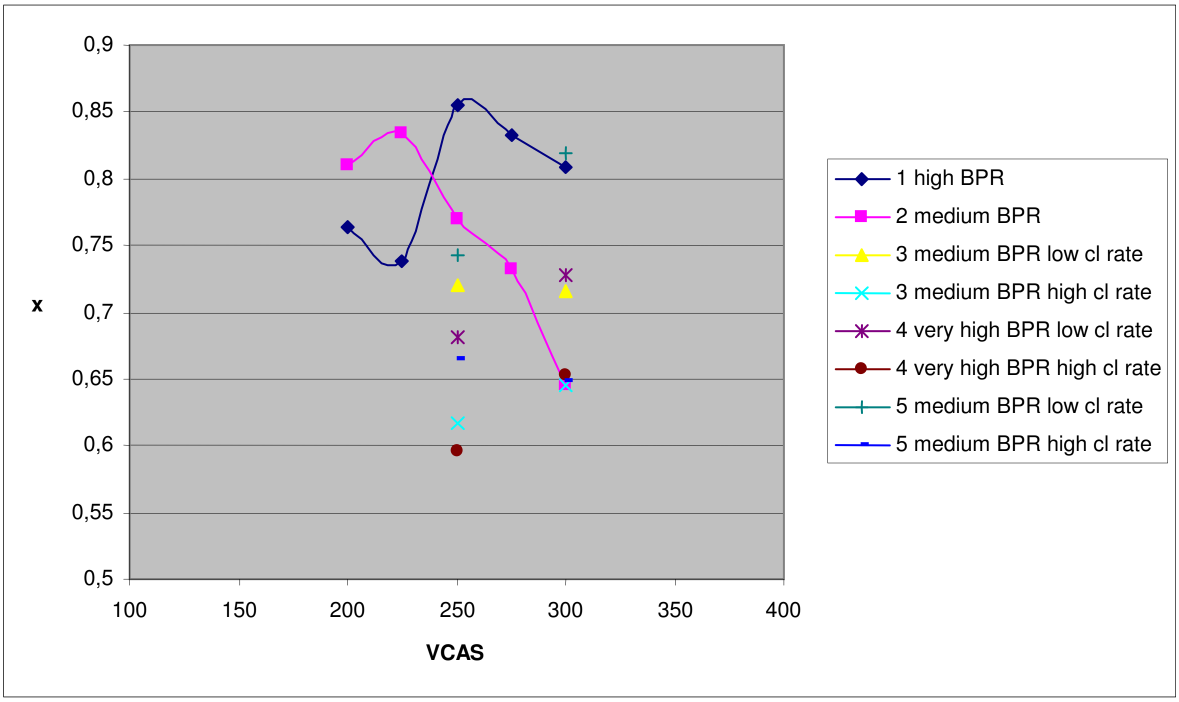


Figure 3.11 Calculated values for different V_{CAS}

3.3.4 Torenbeek

The model of **Torenbeek 1982** was not meant to be used for the calculation of climb thrust rates but was used anyway. **Bartel & Young 2007** are trying to modify their equation to fit into their data but up to now haven't come to a final solution. Only Engine 1 and 2 are evaluated with this model because the data for the other engines where not sufficient to compare (see section 3.3.1 climb thrust problem).

The equation of **Torenbeek 1982** was used with two different values for *G*, one according to **Torenbeek 1982** and the other altered by **Bartel & Young 2007**. For Engine 1 the value is $G = 1{,}1$ according to both authors and for the second engine $G = 1$ according to **Torenbeek 1982** and $G = 0{,}85$ according to **Bartel & Young 2007**. Figure 3.12 shows the differences of the equation of **Torenbeek 1982** with the real thrust. The maximum difference for Engine 1 is 7,3 % and for Engine 2 is -4,7 % with $G = 1$ or 3,2 % with $G = 0{,}85$. Although the outcome with altered *G* for Engine 2 is the best approach it is still obvious that the general approach of **Torenbeek 1982** should not be used in this way. In addition to the fact that this is climb thrust comes the fact that the equation is only valid to $M = 0{,}3$ while this test reaches $M = 0{,}45$. A difference of 7,3 % at sea level and $M = 0{,}45$ (≈ 300 kts) is simply too much . Using this model up to 30000 ft brings a difference of up to 200 % and up to 40000 ft a difference of 318 %.

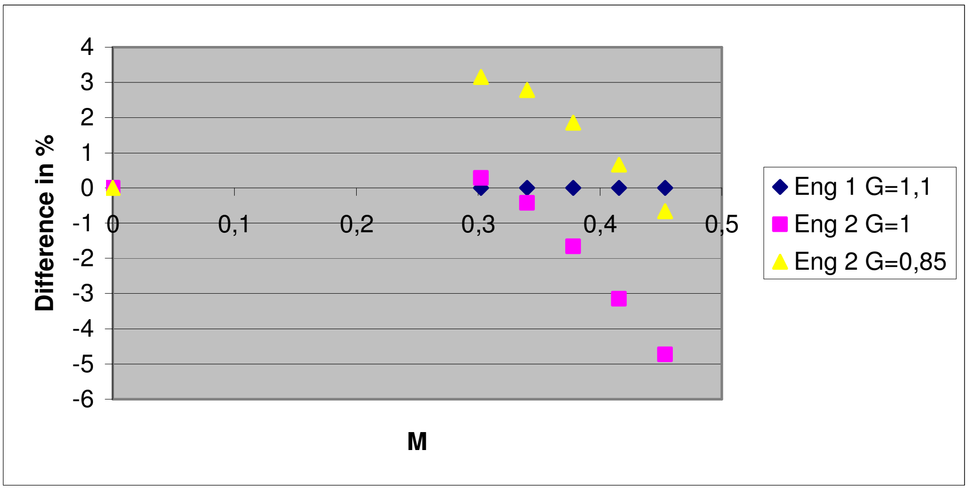


Figure 3.12 Differences of the model of **Torenbeek 1982**

3.3.5 Howe

For the model of **Howe 2000** (equation (2.9)) the same problem with Engine 3 to 5 occurs as with **Torenbeek 1982** (see section 3.3.1 climb thrust problem) therefore only Engine 1 and 2 were used as reference data.

Figure 3.13 shows the differences in thrust rate of the model of **Howe 2000** to the real thrust rate over height with varying velocities. It can be seen that the model is very accurate for low altitudes and compounds with rising altitudes. Up to a height of 14000 ft the differences are below 5 % which is a very good result. Up to a height of 20000 ft the differences are below 11 % which is still a good result. Up to a height of 35000 ft the differences stay within a range of 20 %. At 40000 ft the differences are up to 43 %. Although **Howe 2000** changes the value of his exponent *s* for heights in the stratosphere this change does not enhance the accuracy of the model for these altitudes. Although these results are better than nothing they cannot match the accuracy at lower altitudes. Since the standard flight mission uses climb only up to 31000 ft the inaccuracy of this models at high altitudes is not too significant for climb evaluation.

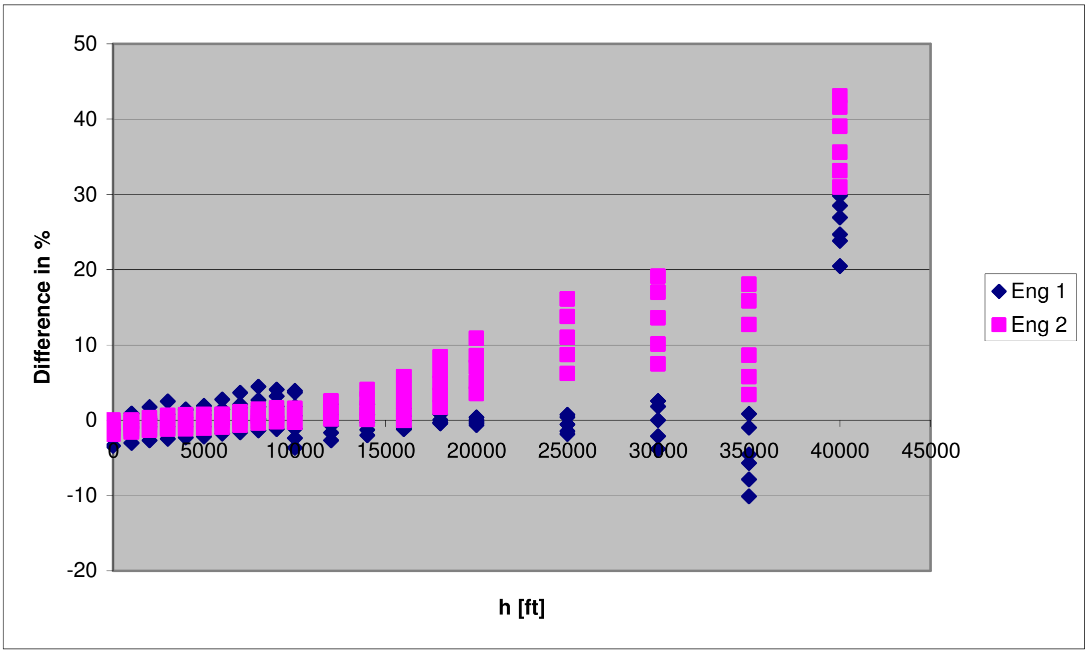


Figure 3.13 Differences of the model of **Howe 2000**

3.3.6 Raymer

Figure 3.14 shows the differences of the thrust rate calculated after **Raymer 1999** (equation (2.3)) compared with the real thrust rate over height for different velocities. The differences at lower altitudes are lower than at high altitudes. The differences up to a height of 10000 ft are below 6 % which is a very good result and even up to a height of 30000 ft the difference stays below 10 %. If we take into account that the model was given for all BPR and all speeds this model seems to be very accurate and even enhanceable. This assumption is definitely correct but the good results are also achieved by the way the equation works (see Chapter 3.1.1) The thrust rate uses the sea level thrust for a certain velocity as denominator. To be able to use the equation this value has to be known or calculated, in this case it was known and therefore the reliability of the model is enhanced. If these values have to be calculated the result will also depend on this equation and the outcome might not be that good. Still, this model will be investigated further in section 3.3.8. One might think that since the data sets for the evaluation of all models are the same, the height up to the evaluation is done should be the same as well. The reason why this model was not evaluated up to heights of 40000 ft is that the values for the thrust at sea level used for as the denominator are not known for these areas or the thrust levels at these heights are not known.

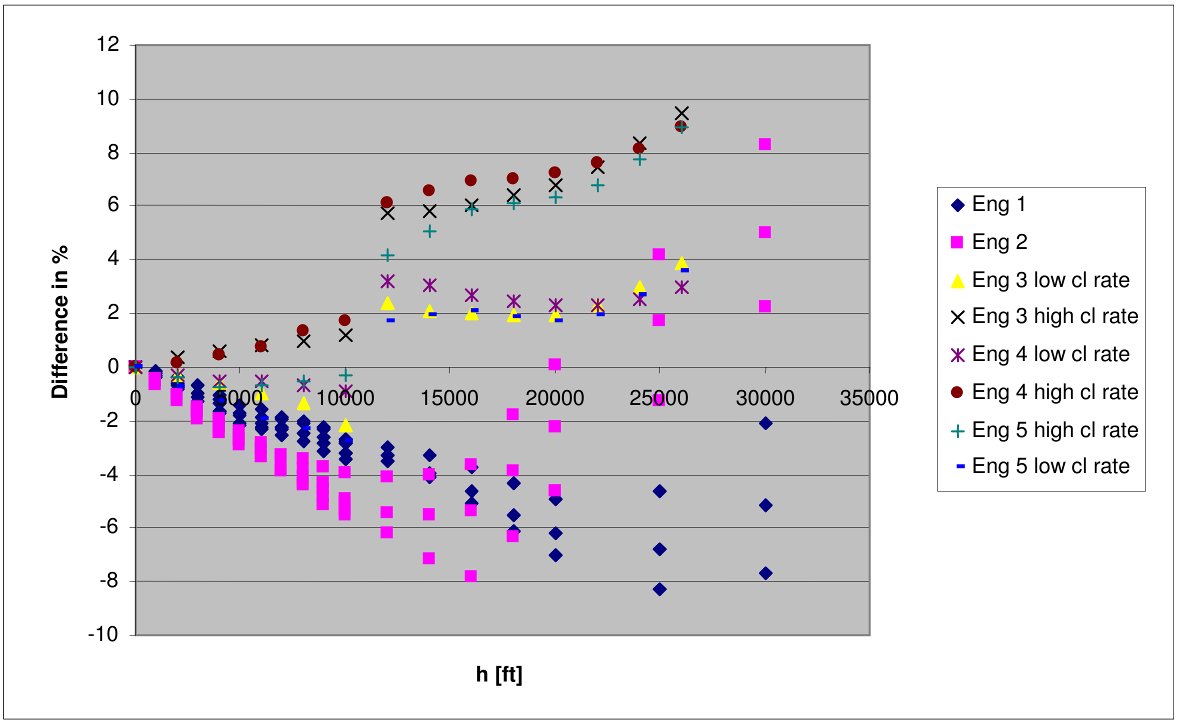


Figure 3.14 Difference of the model of **Raymer 1999**

3.3.7 Eurocontrol

The approach of **Eurcontrol 2004** (equation (2.4)) only gives the general equation and the necessary factors for the engines they have evaluated. In this project none of the **Eurocontrol 2004** used engines is evaluated since no reference data was available. This project evaluates the general behaviour of the model and calculates the factors for the engines.

As described in chapter 3.1.1 it is not completely clear how to get the factors for a curve describing both speeds. Figure 3.15 shows the accuracy of the methods by giving the difference over height. It is obvious that the differences of the two methods describing the thrust for the actual calibrated airspeed give the best accuracy. This is to be expected since they don't have to be accurate for two different curves connected to one. Generally the curve using the two different sea level thrusts as denominators is a little bit more accurate than the one using only one, but the large disparity between the two differences is unusual. There are cases when the method using only one value as denominator is more accurate than the one with two values. The disadvantage of the method using two denominators is that two different thrust values have to be known. Note that the reasonable good accuracy is only valid for the thrust at $V_{CAS} = 250$ kts from 0 to 10000 ft and for the thrust at $V_{CAS} = 300$ kts from over 10000 ft to the end of cruise. If one tries to calculate the thrust at sea level and $V_{CAS} = 300$ kts with any of the two equations the outcome might not be very accurate. A suggestion at this point is to describe the thrust lapse rate in single equations if one tries to describe the thrust lapse rate on one's own. Due to a lack of further data, similar to the one mentioned in section 3.3.6, the maximum height evaluated is 30000 ft.

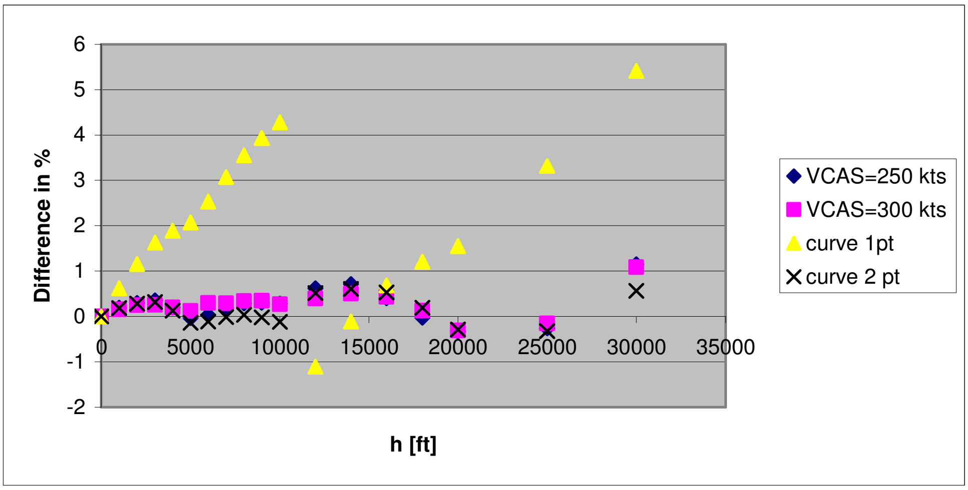


Figure 3.15 Difference of the method of **Eurocontrol 2004**

Figure 3.16 gives values for the factors C_{Tc2} over the BPR. It can be seen that the factors don't follow a trend with BPR.

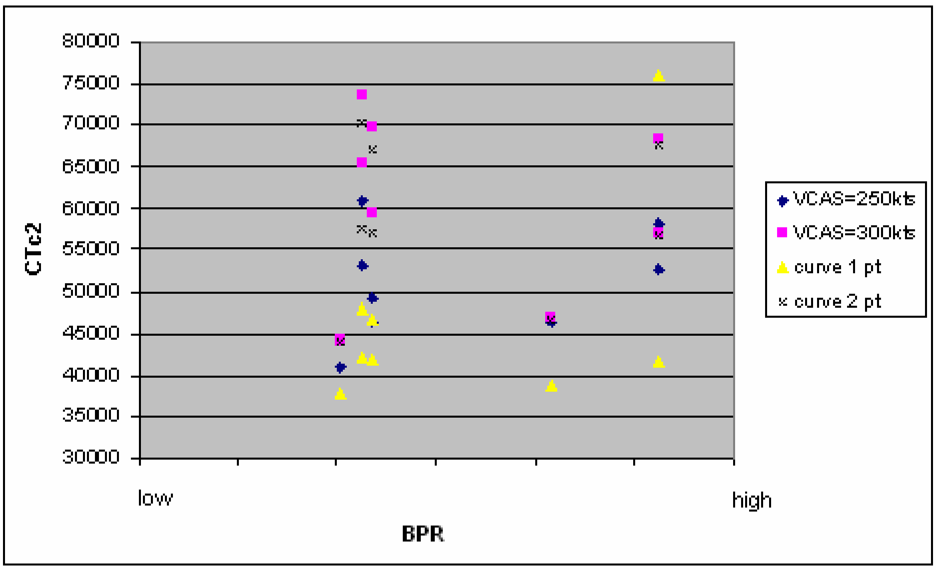


Figure 3.16 Values of C_{Tc2} over BPR

Figure 3.17 shows the values of C_{Tc3} over the BPR. The factors don't follow a trend with BPR.

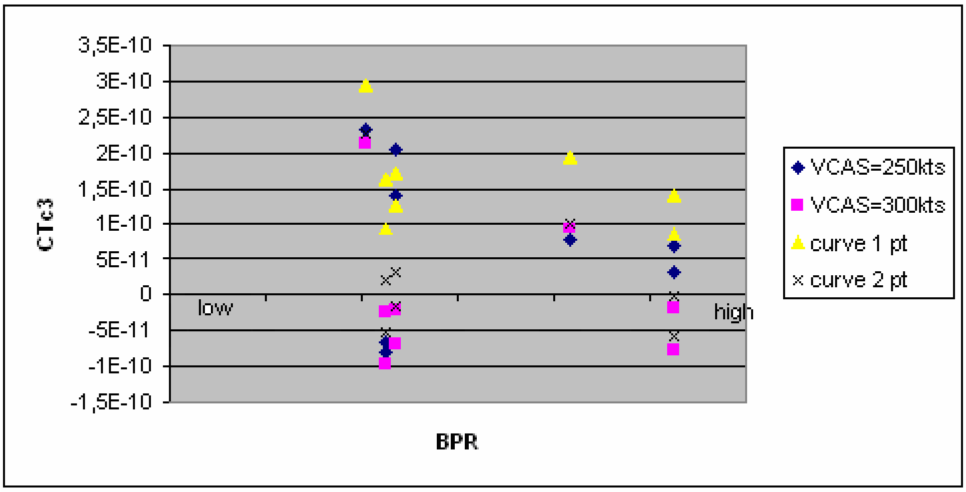


Figure 3.17 Values of C_{Tc3} over BPR

Both factors don't follow an obvious trend, it is therefore impossible to predict the factors for unknown engines. The assumption made in Chapter 3.1.1 that this method is possibly not meant for calculation on one's own is confirmed by this fact.

3.3.8 New Model

As discovered in section 3.3.6 the approach of **Raymer 1999** (equation (2.3)) seems to be very promising therefore some further investigation in this area are being made.

$$\frac{F_N}{F_{N,0}} = 1 - \frac{h}{C} \tag{3.1}$$

where $C =$ variable factor

In section 3.1.1 it was discovered that the equations of **Raymer 1999** and **Eurocontrol 2004** (equation 2.4) have a common base which leads to the reason why the form *h*/*C* was used instead of *Ch*. It was decided that it is easier to handle full numbers instead of fractions of one. Figure 3.18 shows the maximum differences of this model for Engine 2. All other engine data fitted better in the equation. The used value of the *C* was the calculated best fitting. It can be seen that the differences up to 30000 ft are less than 8 %. Although this result is good it has to be taken into account that this is the result for ideal conditions. Errors for the evaluation of the sea level thrust at a certain speed and for a possible trend might still be added. The up to now error occurs because of the bending of the graph that this linear approach cannot cope with.

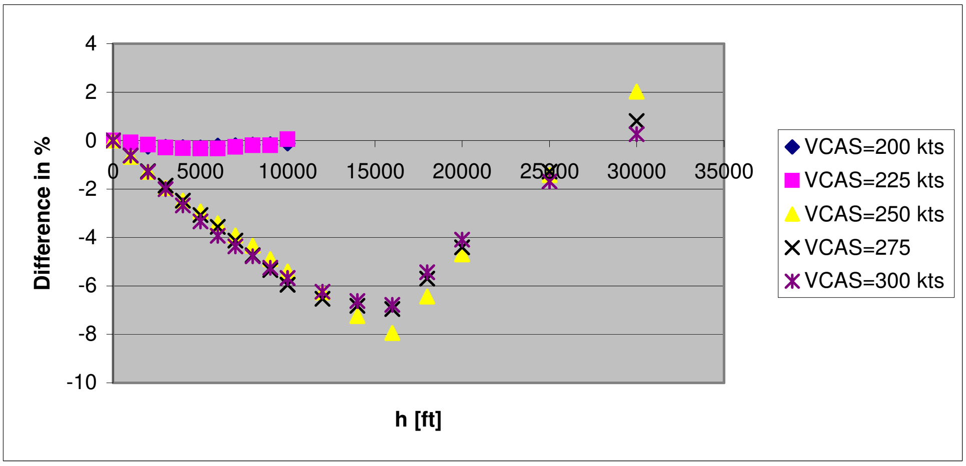


Figure 3.18 Maximum differences for Engine 2

Figure 3.19 shows the values of C over a variety of BPR. Obviously the values of C don't follow a common trend with BPR which is bad because it is difficult to predict the outcome and give the right equation to give a good result. This behaviour is not completely unexpected since the thrust rate from the same engine does change when the SOT is changed and therefore several values of C can occur for the same BPR. The equation of **Howe 2004** suggests an influence of the BPR but by having a closer look this influence is extremely small or non existent because the factors are zero or close to it. This very easy approach with only one factor cannot possibly cover such fineness.

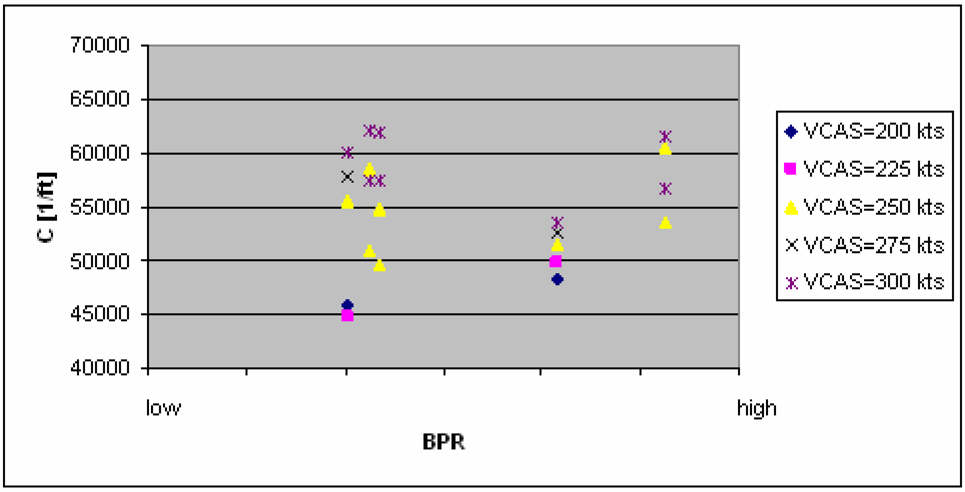


Figure 3.19 Values of *C* over BPR

Figure 3.20 shows the values of C over velocity. It can be seen that the values follow a general trend, they rise with rising speed. For a V_{CAS} = 200 kts the factor has the average value of C = 47000 ft^{-1} and C = 57000 ft^{-1} for V_{CAS} = 300 kts.

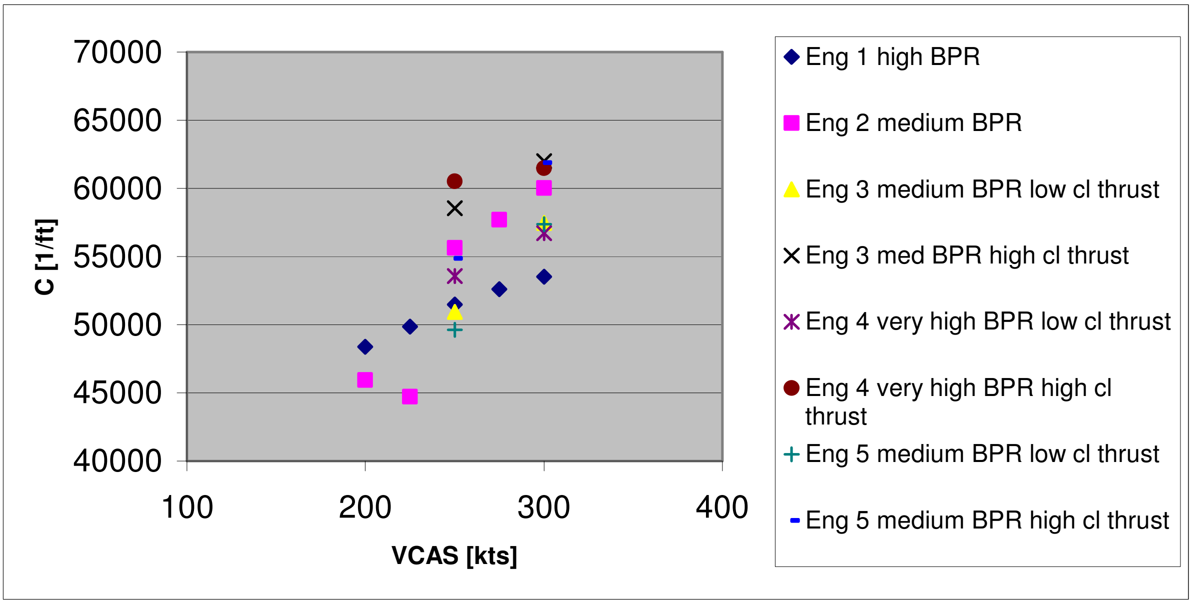


Figure 3.20 Values of *C* over velocity

Again Engine 2 brings the worst results for this equation; the result can be seen in Figure 3.21. The maximum difference of 10 % occurs at 30000 ft. If we compare this results to the accuracy of the approach of **Raymer 1999** it seems that his approach, although no variability at all brings slightly better results. Technically this is correct but as can be seen in Figures 3.14 and 3.18 there is a high difference for all engines whereas the general differences in Figure 3.21 are below 5 % and only some peaks occur that compound the overall result.

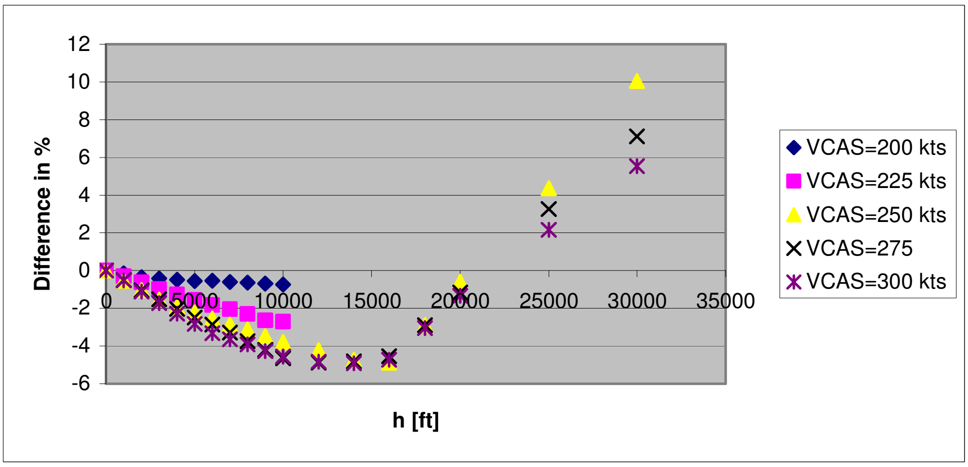


Figure 3.21 Maximum differences for Engine 2 calculated according to the trend of *C*

3.4 Thrust Change due to Bleed Air Extraction

3.4.1 General Approach and Information

Data provided by **Bartel & Young 2007** was analysed and the equation of **Raymer 1999** describing the thrust variation with bleed air extraction was evaluated for accuracy.

$$\Delta F_N\,\% = C_{bleed}\left(\frac{m_{f,bleed}}{m_{f,eng}}\right)\cdot 100 \tag{2.11}$$

where $\Delta F_N\,\% =$ thrust change in %

$C_{bleed} =$ bleed correction factor is given by manufacturer

$C_{bleed} = 2$ if nothing else is given

$m_{f,bleed} =$ mass flow of bleed air, usually 1-5% of engine mass flow

$m_{f,bleed} =$ mass flow of engine

Since the data did not provide any mass flows or thrust values but instead the percentage of the bleed air extraction and the thrust rate $F_N / F_{N,0}$ the equation was altered into

$$100 - \frac{F_N - \Delta F_N}{F_N} = C_{bleed}\left(\frac{m_{f,bleed}}{m_{f,eng}}\right)\cdot 100 \tag{3.2}$$

where $\left(\frac{m_{f,bleed}}{m_{f,eng}}\right)\cdot 100 = \Delta B$ bleed air off – take in %

$\Delta F_N\,\% = 100 - \frac{F_N - \Delta F_N}{F_N}.100$

$\Delta F_N =$ thrust change in reality

$$\left(\frac{F_N - \Delta F_N}{F_{N,0}}\right) = \frac{F_N}{F_{N,0}}\left(\frac{100 - C_{bleed}\cdot \Delta B}{100}\right) \tag{3.4}$$

$$\left(\frac{F_N}{F_{N,0}}\right)_{OT} = \frac{F_N}{F_{N,0}}\left(\frac{100 - C_{bleed}\cdot \Delta B}{100}\right) \tag{3.5}$$

3.4.2 Discussion

Figure 3.22 shows the variation of the thrust rates with Mach number and different amounts of bleed air extraction for take-off.

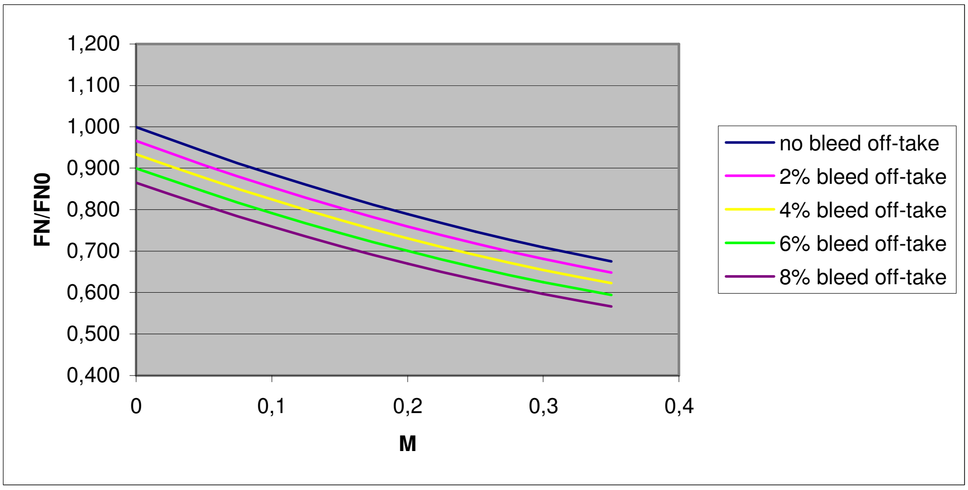


Figure 3.22 Thrust variation with bleed air extraction (redrawn according to **Bartel & Young 2007**)

In Figure 3.23 the differences of the calculated thrust rate to the real thrust rate can be seen. For a $C_{bleed} = 2$ the maximum deviation is less than 3 %, which is a good result. The deviation seems to be very unstable though so the best fitting C_{bleed} was calculated. The result can be seen in Figure 3.24.

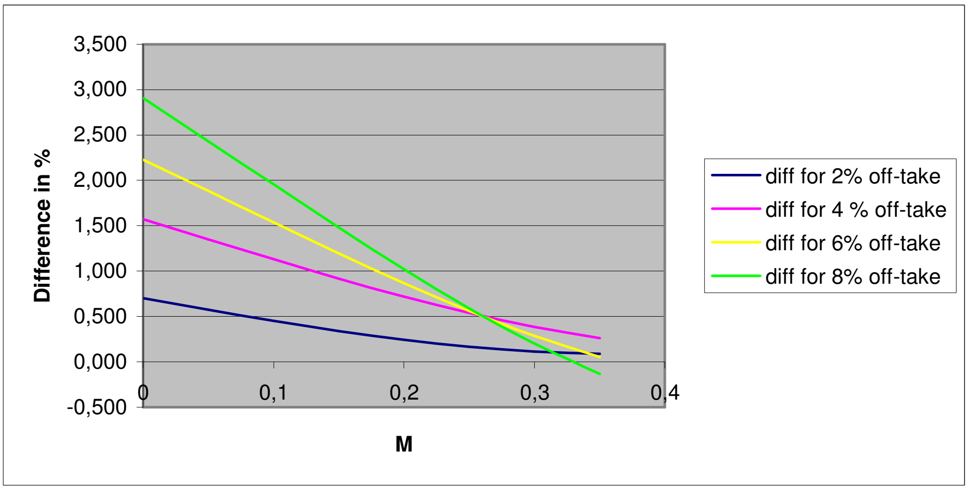


Figure 3.23 Calculated difference with $C_{bleed} = 2$

Figure 3.24 shows what the actual C_{bleed} value would have been and it becomes obvious that the value is not fixed but varies.

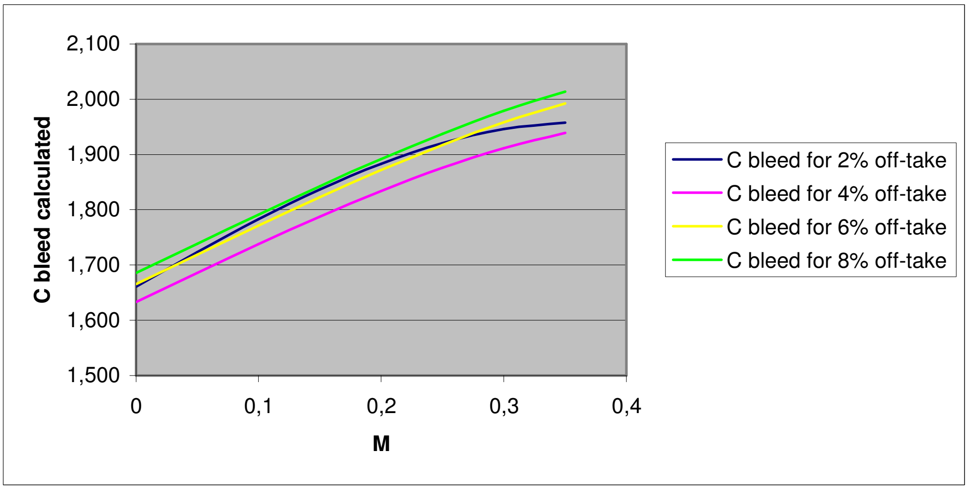


Figure 3.24 Calculation of the real value of C_{bleed}

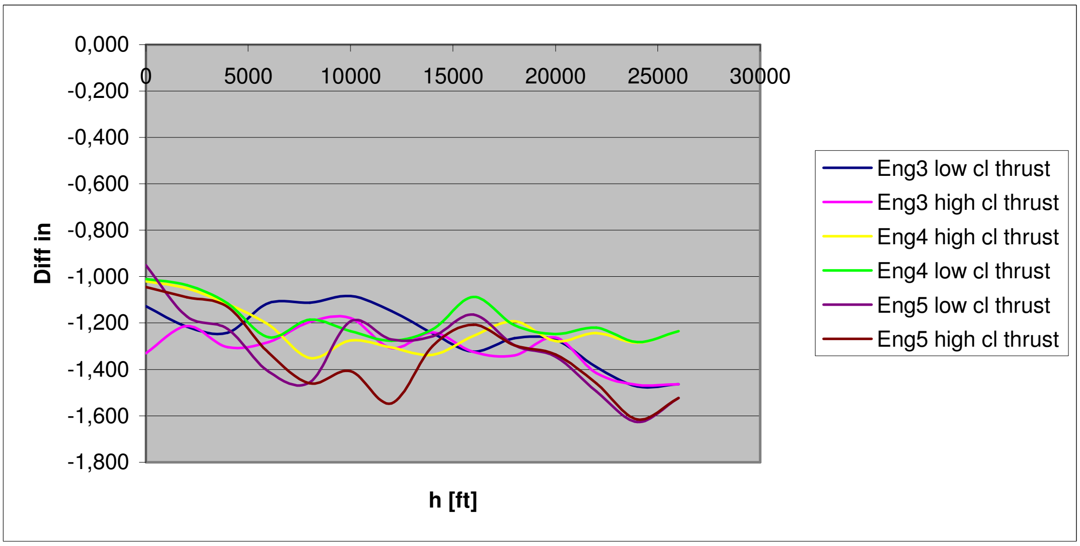
Another data set (**Engine Manufacturer 2006**), this time for climb, was evaluated. It consisted of 3 engines, each running at 2 different stator outlet temperatures (SOT) for high or low climb thrust, which lead to six different data sources to be evaluated. The reason for six instead of three data sources is explained in section 3.3.1.

To evaluate the data equation (2.11) was altered to

$$F_{N,OT} = F_N \left(100 - C_{bleed} \left(\frac{m_{f,bleed}}{m_{f,eng}} \right) \cdot 100 \right) \tag{3.6}$$

where $F_{N,OT}$ = net thrust with bleed air off-take

Note that F_N and $F_{N,OT}$ have to be for the same Mach number.



Figurer 3.25 Difference between F_N calculated and real for bleed air extraction for C_{bleed} = 2

Figure 3.25 shows that the differences between the actual data and the empirical model are below 1,7 %, which is very good. After calculating the C_{bleed} factor (see Figure 3.25) it becomes obvious that this good result is only due to the fact that the bleed air off-take of 0,5 lb/s is roughly 0,1 % of the engine mass flow. The calculated values for C_{bleed} lie between 9 and 26. This result stresses the fact that C_{bleed} is not fixed for an engine and the value can also vary a lot. It is very important to stress that for the same engine with a different SOT the calculated values for C_{bleed} are different because this leads to the conclusion that there is no easy way to give absolute values of this factor. Still C_{bleed} tends to rise with a rise of BPR which can also be seen in Figure 3.26 by noting that Engine 1 has a λ = medium, Engine 2 has a λ = very high and Engine 3 has a λ = medium (but higher than Engine 1).

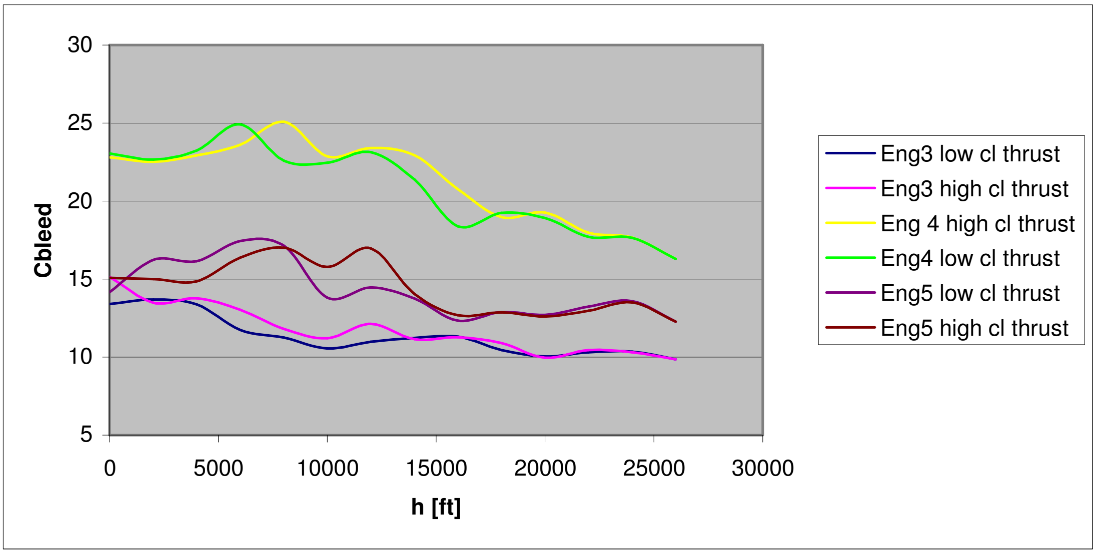


Figure 3.26 Calculated C_{bleed}

A final suggestion to the user of this method is to follow the advice of **Raymer 1999** and use the value of $C_{bleed} = 2$ if nothing else is given or do some research for the engine if possible.

3.5 Thrust Variation due to Temperatures other than ISA

3.5.1 General Information about the Approach

Since no actual data is available to evaluate this topic the data produced by **Eurocontrol 2004** will function as reference values. Although it is unusual to evaluate the accuracy of a model by using another model as a reference it is probably acceptable, taking into account that **Eurocontrol 2004** used validated data to derive their equations.

3.5.2 Comparison of two Approaches

In Chapter 3.1.4 it was shown that the models of **Eurocontrol 2004** (equation (2.10) and **Raymer 1999** are very similar. Now the data given from **Eurocontrol 2004** is used as reference date and used to evaluate the accuracy of the model of **Raymer 1999** (Table 3.2).

Table 3.2 Calculation of the reduced thrust for an A 300-600

ΔT =	7,75	K	
(ΔTisa)eff =	1		
0<	0,00426	<0,4	
h [ft]	**FN [N]**	**FN red. [N]**	**Diff in %**
0	304000,00	302704,96	0,426
1000	297249,55	295983,267	0,426

Table 3.3 Calculation of the thrust change due to ΔT=1 K in %

Aircraft	**Diff in %**
Aircraft 1	0,8349
Aircraft 2	0,67981
Aircraft 3	0,91868
Aircraft 4	0,73165
Aircraft 5	0,958
Aircraft 6	0,804
Aircraft 7	0,426

Table 3.3 shows the calculated thrust percentage change for several aircraft engine combinations. It can be seen that the thrust change due to a temperature variation of 1 K above the maximum flat rating temperature leads to a range between 0,92% and 0,43%. If we compare this to the 0,75% proposed by **Raymer 1999** it is obvious that this is roughly in the middle of the range and therefore well suited to provied a good estimation if nothing else is given. Additionally it can be said that the value for C_{Tc5} multiplied with 100 is the thrust change in % per K and C_{Tc4} seems to be the maximum flat rating temperature.

Some of the equations introduced in Chapter 2 include σ which is dependant on the ISA conditions. One might be tempted to say that due to this dependency the thrust change due to temperatures other than ISA are already included in these equations. Figure 3.27 shows the difference in thrust change due to ΔT = 1 K. By comparing the difference in per cent with the data from Table 3.3 it is obvious that such an assumption is not true. The thrust change rises with height and the difference for sea level is 0,242 % per K. By assuming that the data from **Eurocontrol 2004** is correct there is no change over height and the difference is way too small and therefore incorrect to be an average value.

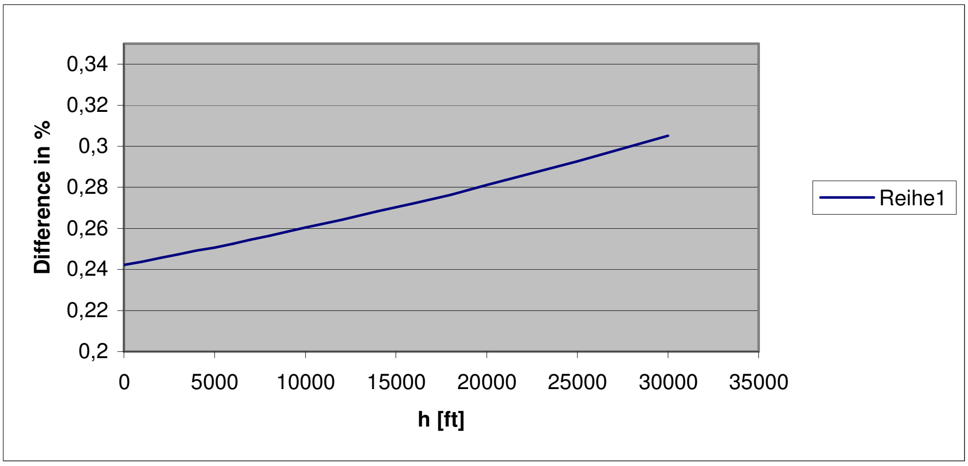


Figure 3.27 Change in difference of thrust due to a change of σ with ΔT

3.6 Variation of Cruise Thrust with Take-off Thrust

Equation (2.14) by **Scholz 2007b** is compared with the engine data used by **Svoboda 2000** (see figure 3.28) It can be seen that the majority of the engines fits well within a 20 % range of the equation. Large differences occur if the take-off thrust is very high or very low.

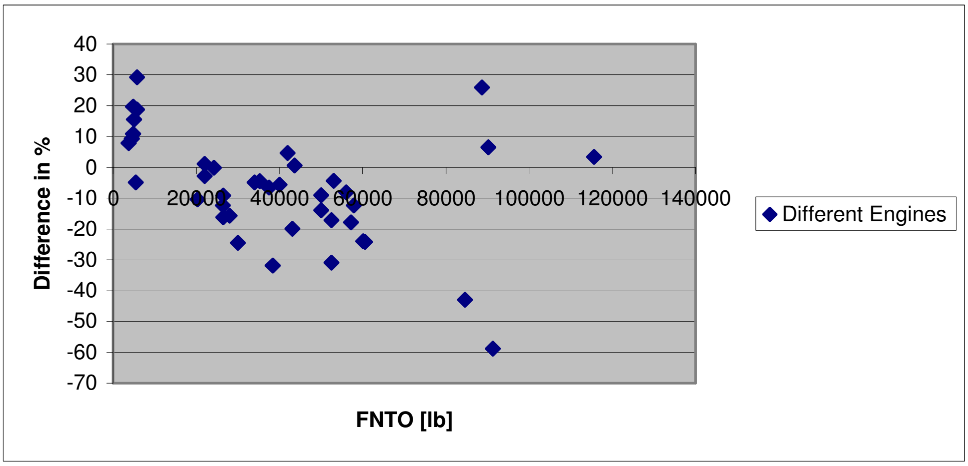


Figure 3.28 Differences of equation (2.14) over take-off thrust

3.7 Thrust Variation during Cruise

For ideal conditions with still air and the angle between thrust line and flight line negligible small the equations for static equilibrium are (**Young 2001**)

$$\sum F_t = F_N \cos\alpha - D - W \sin\gamma = \frac{W}{g}\frac{dV}{dt} \tag{3.7}$$

and

$$\sum F_n = F_N \sin\alpha + L - W \cos\gamma = \frac{W}{g} V \frac{d\gamma}{dt} . \tag{3.8}$$

where D = drag
L = lift
V = velocity
γ = angle of climb
α = angle of attack
F_t = Forces in tangential direction
F_n = Forces in normal direction

An ideal cruise flight is performed without height changes or if with level steps without height changes between the steps. By setting $\gamma = 0$ the level flight is ensured. The equations change into

$$\sum F_t = F_N \cos\alpha - D = \frac{W}{g}\frac{dV}{dt} \tag{3.9}$$

and

$$\sum F_n = F_N \sin\alpha + L = W . \tag{3.10}$$

Equation (3.10) gives two ways to support the weight of the aircraft in the air. Either the angle of attack is zero and the lift equals the weight or we fly with an angle of attack and the combination of lift and thrust fraction equal the weight. As the angle of attack in cruise is small $\cos\gamma \approx 1$ and hence equation (3.9) can be written as

$$\sum F_t = F_N - D = \frac{W}{g}\frac{dV}{dt} . \tag{4.6}$$

With no acceleration the drag equals the thrust. The standard cruise is therefore performed with thrust equalling drag. To evaluate the actual cruise drag detailed knowledge of the aircraft's drag polar has to be gained which is not part of this project. A cruise flight could also be performed at a constant Mach number. This would lead to a change in height because the aircraft loses weight (fuel burn) but the lift stays constant or a change of the angle of attack to reduce lift accordingly. The last option would result in a higher SFC. Equation (2.2) deals with height changes for cruise thrust but due to a lack of reference data this could not be evaluated.

3.8 General Behaviour of Models Describing SFC

3.8.1 SFC in General

The specific fuel consumption of an aircraft engine depends on a variety of factors. Basically these are thrust, speed, height (atmospheric conditions) and throttle setting of which some are connected. A diagram giving the SFC over thrust/ delta is shown in figure 3.29. For the throttle setting "cruise" and for a height of 35000 ft several curves with different Mach number are shown but not all the thrust levels can be flown. For a given airplane weight a certain amount of lift is necessary which leads to a certain speed and height and therefore amount of thrust. This leads to the fact that only a very limited area of the curves can actually be flown. To determine these factors knowledge of the drag is necessary. Usually neither these diagrams are given by the engine manufacturer nor the knowledge about the drag or aerodynamic by the airplane manufacturer. What all equations describing the SFC are trying to do is shaping a curve for a given SFC data point.

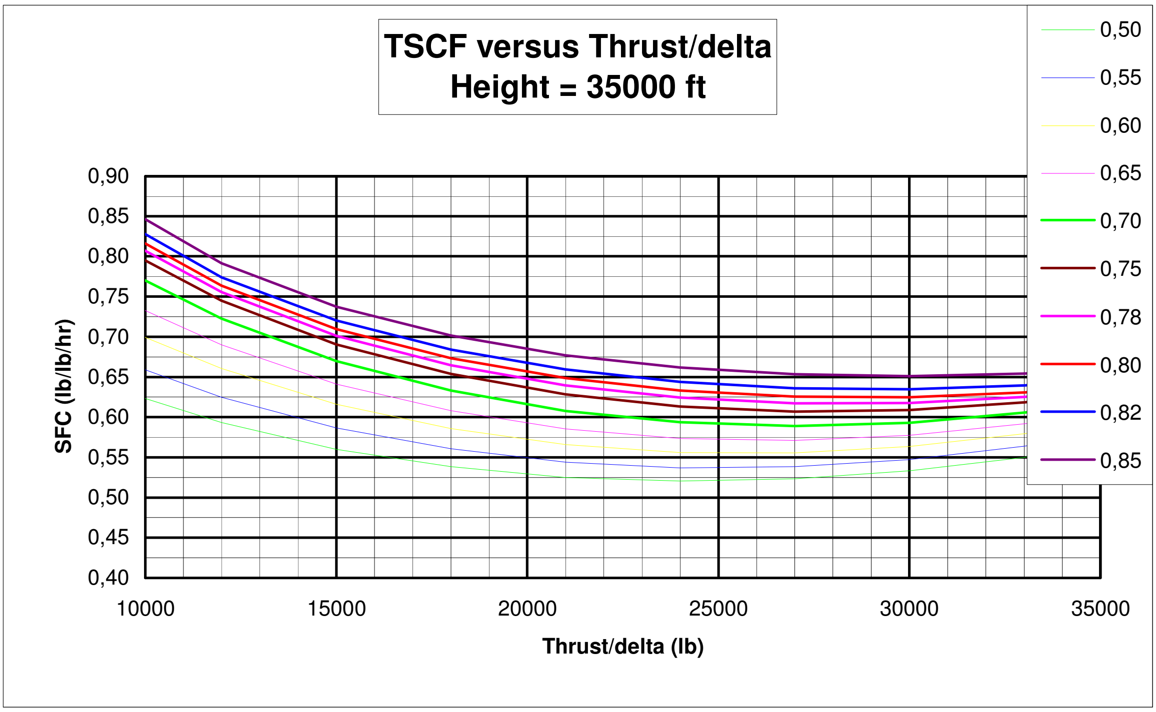


Figure 3.29 SFC variation over thrust/ δ for an B757-200 class aircraft with an RB211-535E4 class engine (from **Young 2007**)

3.8.2 Variation of SFC with Height and Speed

A large number of models describing the variation of SFC was found in the literature but upon closer inspection the number can be reduced to smaller number because some of them use a similar approach. Equation (2.15) could be used as a good example of the combination of two approaches.

$$c = c_2 \theta^{0,5} M^{n} \tag{2.15}$$

Equation (2.22) is in fact the height term θ^y where the value of y = 0,5 is suggested but may vary a bit. Equation (2.18) is the term giving the variation with Mach number c_2M^n. The question of the value of the factor c_2 is not answered. The given values of n are accurate only for maximum cruise thrust. The cruise is usually not flown at this thrust but at reduced thrust. A reduction of thrust results in a change of SFC and therefore the values of n cannot be used for reduced cruise thrust if certain accuracy is wanted. Equation (2.15) takes variation of height and Mach number into account, but a good estimation of the starting point c_2 is necessary. The disadvantages of equation (2.18) are similar.

Martinez-Val & Perez 1991 used a similar approach to equation (2.15) but use reference data in their equation.

$$c = c^* \left(\frac{M}{M^*} \right)^n \left(\frac{\theta}{\theta^*} \right)^{0,5} \tag{2.17}$$

If this equation uses reference data for M = 1 at sea level the equation become equation (2.15). The advantage of this version is that you use a known actual value for the SFC and scale them in the way the Mach number rate and theta rate change. This method could be more accurate than equation (2.15).

Equation (2.19) and (2.21) use a linear approach with Mach number and could be called the same equation. **Howe 2000** (equation (2.16)) also uses a linear approach over Mach number but includes the SFC change with BPR in a different way. He does not use an exponent of M, changing with BPR.

$$c = c_1 \left(1 - 0{,}15\lambda^{0,65}\right)\left(1 + 0{,}25\left(1 + 0{,}063\lambda^2\right)M\right)\sigma^{0,08} \tag{2.16}$$

This is a very detailed equation since it is not restricted or limited to a certain range of height or speed. Even without any further data a rough estimation could be done. Figure 3.30 shows the variation of c/c_1 with BPR. Note that the denominator is not the same for both models, the values of the SFC rate is therefore different. In spite of the different way including the change over BPR, the SFC rates behave in a similar way. It can be said that the higher the BPR, the lower the SFC rate and the lower the Mach number, the higher the lapse rate. If the value for the denominator is accurately adjusted the outcome of SFC would be very close for both methods. The fact that the SFC rate and therefore SFC itself goes down was to be expected and speaks in favour of the accuracy of the equation.

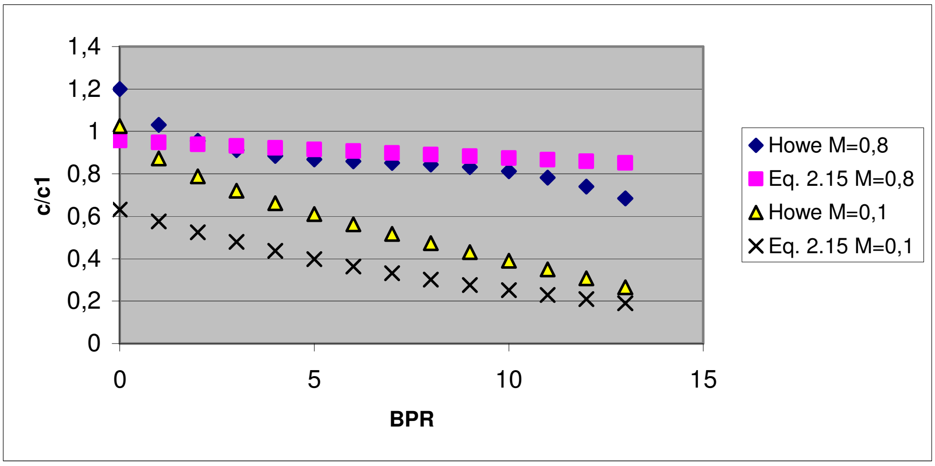


Figure 3.30 Variation of c/c_1 over BPR for two models

Figure (3.31) shows the variation of c/c_1 with the change of height. Equation (2.15) also shows the behaviour of equation (2.22). As can be seen the SFC rate diminishes with altitude which was also to be expected.

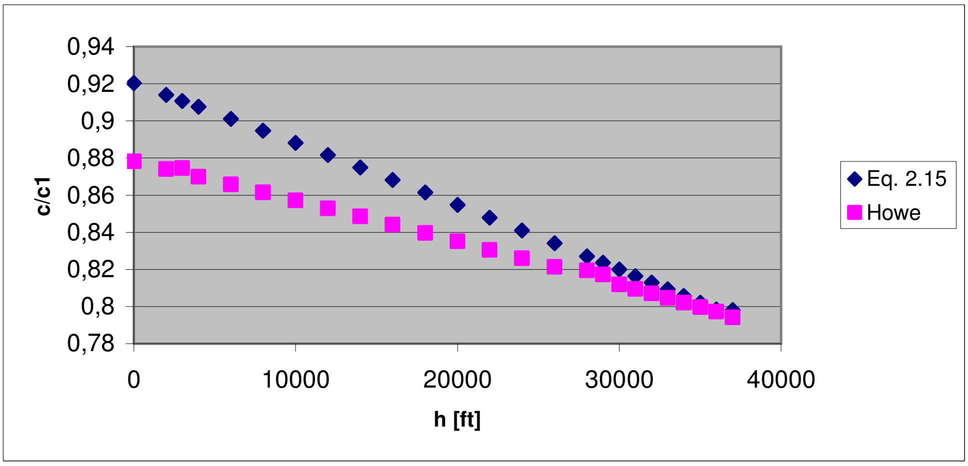


Figure 3.31 Variation of c/c_1 over height for two models

Figure (3.32) shows the variation of c/c_1 over Mach number. Equation (2.15) also shows the behaviour of equation (2.18). All equations show a rise in the SFC with increased Mach number. The general trend is expected and correct but obviously the models cannot display a reduction of SFC with reduced thrust and therefore Mach number. A general problem of equation (2.15) is the reduction of SFC rate to zero when $M = 0$. At Mach number from $0{,}4 \le M \le 0{,}9$ the trend is very similar to the approach of **Howe 2000**. This fact stresses the statement from **Mair & Birdsall 1992** that the equation is only accurate for $0{,}6 \le M \le 0{,}9$.

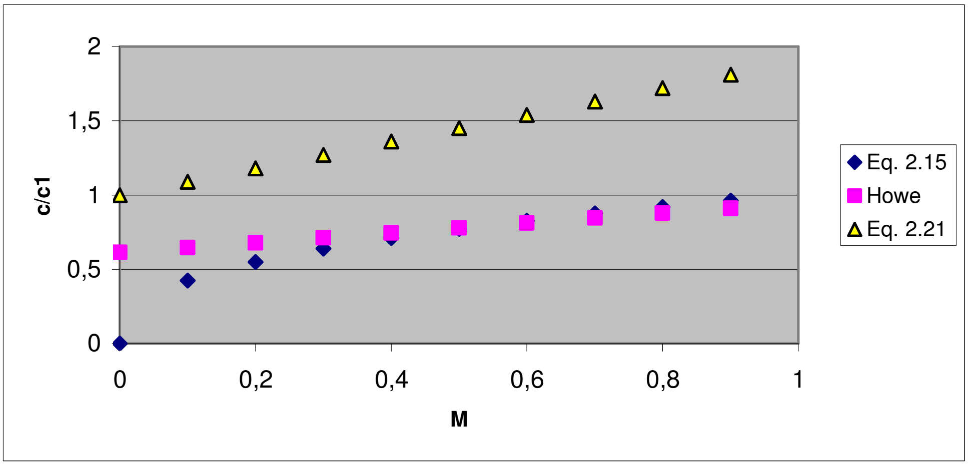


Figure 3.32 Variation of c/c_1 over Mach number

Equation (2.20) changes the SFC relationship to a linear one with thrust. This approach could be used within a limited range of height and speed change.

Eurocontrol 2004 states that their model describing the SFC can be used in all flight phases except cruise, descent or idle. This leaves us with climb where this equation is valid. By having a look at equation (2.23) no term describing the height change for climb conditions is obvious.

$$c = C_{f1}\left(1 + \frac{V_{TAS}}{C_{f2}}\right) \tag{2.23}$$

That means that the height change must be included in the factors C_{f1} and C_{f2} in order to be valid for climb. For a standard climb phase the height and speed depend on each other. Since there is a height change for the SFC that has to be taken into account to be accurate it seems logical that equation (2.23) is only accurate for a certain speed – height combination. The standard climb is defined by **Eurocontrol 2004** using V_{CAS} = 250 kts until h = 10000 ft and V_{CAS} = 290 - 330 kts (depending on the aircraft engine combination) until Mach transition altitude and the true airspeed correlates with these parameters. It is not possible to enter any V_{TAS} but it has to be the correct one, following these laws. **Eurocontrol 2004** doesn't state these limitations in the paragraph dealing with the SFC at all. They do give an explanation of the standard flight parameters they used to derive their model in a different chapter but even there a statement is missing that the model is to be used exclusively with their flight procedures. Figure 3.33 shows the trend of c/C_{f1} over height, figure 3.34 shows the trend of c/C_{f1} over true airspeed. Since the change of speed and height are connected there is a rise of c/C_{f1} over both of them. This is to be expected since the rise in speed brings and increase of SFC that is larger than the decrease the height change brings.

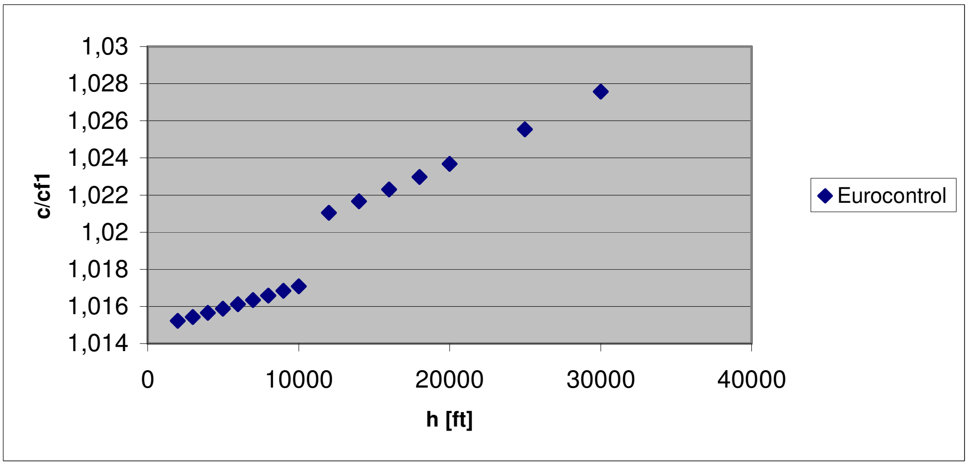


Figure 3.33 General trend of c/C_{f1} over height with **Eurocontrol 2004**

Actually c/C_{f1} is not really SFC ratio because it is not really known what C_{f1} is. Probably C_{f1} is the specific fuel consumption of climb for h = 1500 ft and V_{CAS} = 250 kts but this is certainly not the best possible denominator of a thrust rate. It was only used to give a general idea of the behaviour of this method.

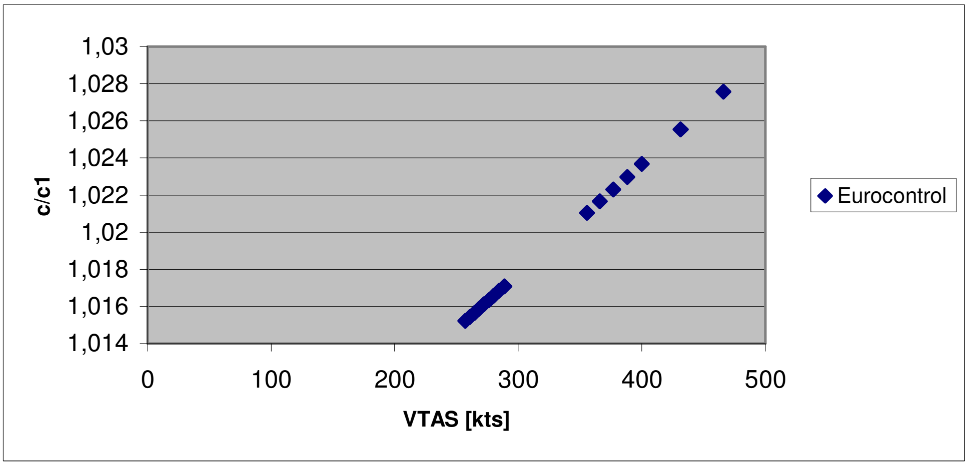


Figure 3.34 General trend of c/C_{f1} over V_{TAS} with **Eurocontrol 2004**

Generally it is to say that a large number of equations only describe the general shape of the SFC curve. A problem of the equations describing the variation of SFC is that without data giving a starting point of SFC it is not always possible to get any result. This problem is even compounded by the fact that engine manufacturers are not always willing to share the necessary information.

3.8.3 Variation of SFC with Thrust or By-pass Ratio

Svoboda 2000 calculated his equations ((2.24), (2.25), (2.26)) with the data shown in figures 3.35, 3.37 and 3.38. The trend of figure 3.35 is obviously a weak one. A large number of data points lies outside of the 10 % range. Additionally an explanation for this trend is rather difficult to establish. The real trend is a dependency of SFC for take-off with BPR. Engines with a small amount of thrust have statistically a smaller BPR than high thrust engines (compare figure 3.36). If we compare figure 3.35 with figure 3.38 they don't seem to have much in common but this is due to the fact that for some engines not all data was available. By recalling how the SFC is defined, (as fuel flow divided by thrust) it gets even more obvious that the fuel flow for take off divided by the take-off thrust is unlikely to follow a trend with the same, at least for this reason.

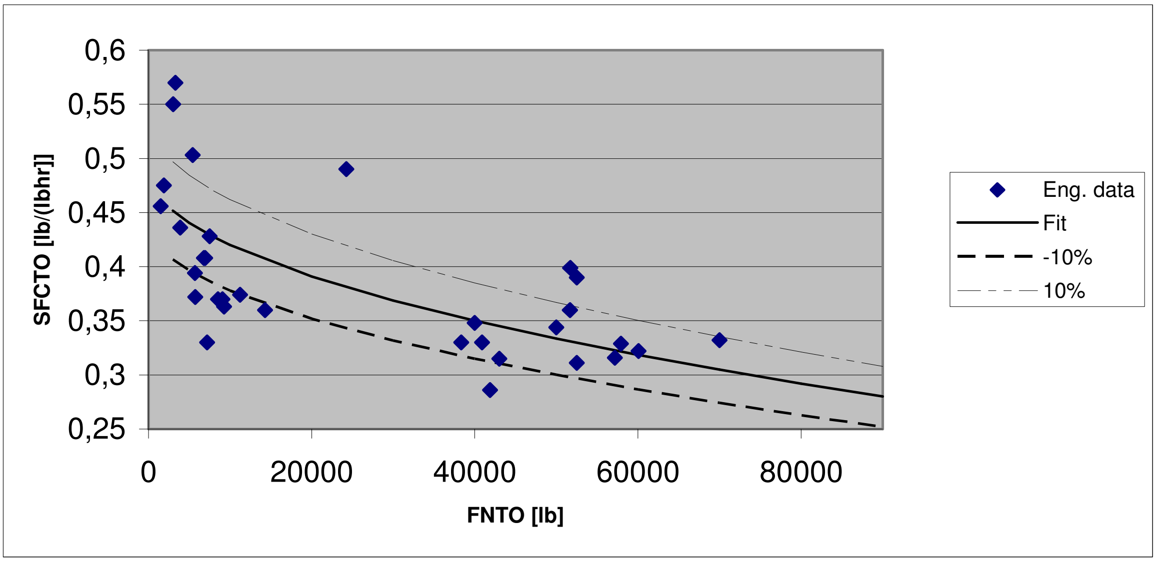


Figure 3.35 Trend of SFC for take-off over $F_{N,TO}$ (redrawn after **Svoboda 2000**)

Although the data points for the cruise SFC (figure 3.37) fit very well to equation 2.26 the same argumentation as just done could be repeated.

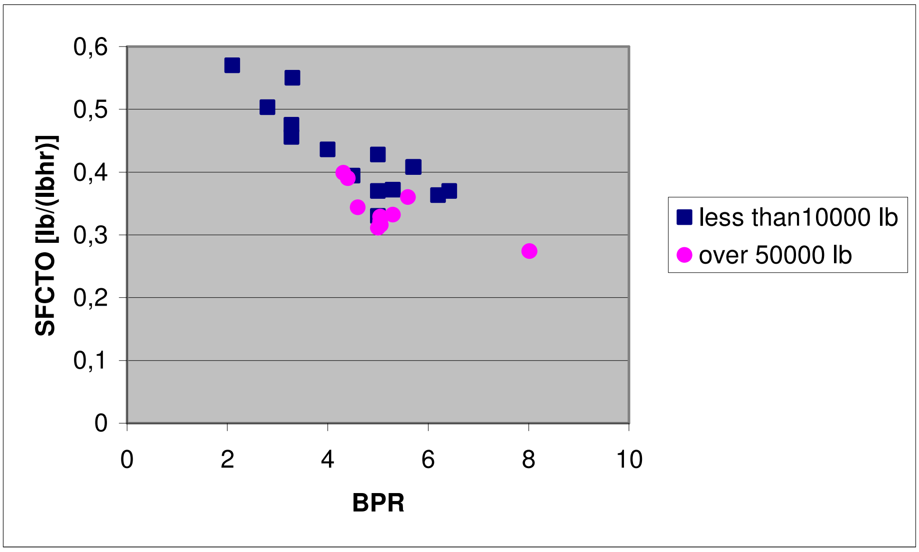


Figure 3.36 SFC over BPR sorted by take-off thrust (data from **Svoboda 2000**)

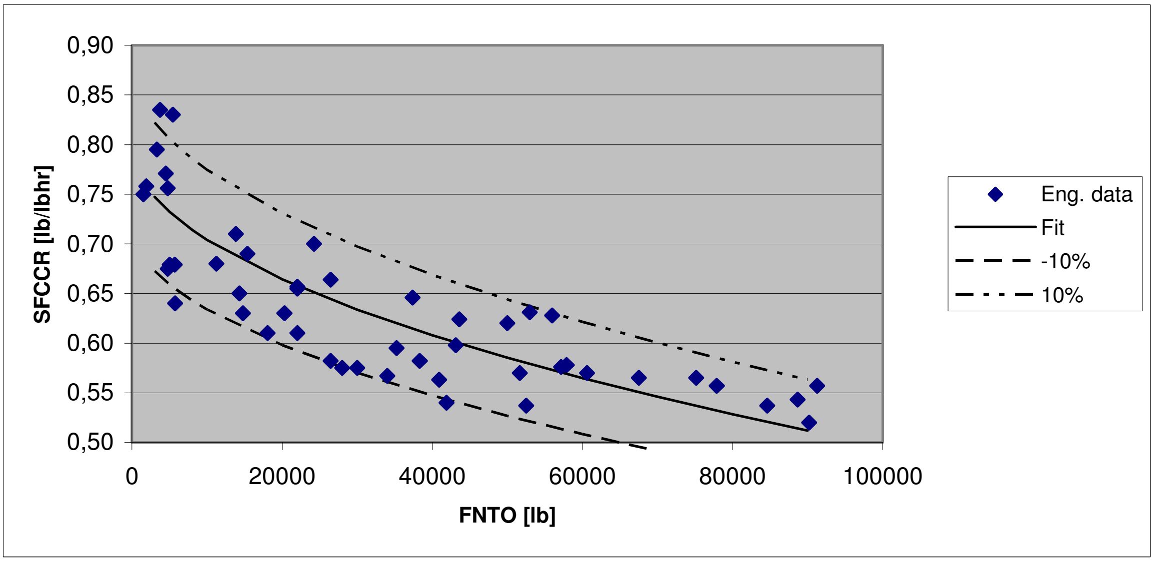


Figure 3.37 Trend of SFC for cruise over $F_{N,TO}$ (redrawn after **Svoboda 2000**)

The real trend, SFC for take off diminishing with a rising BPR can be seen in figure 3.38. The method is not very accurate but it is only meant to be used for preliminary design.

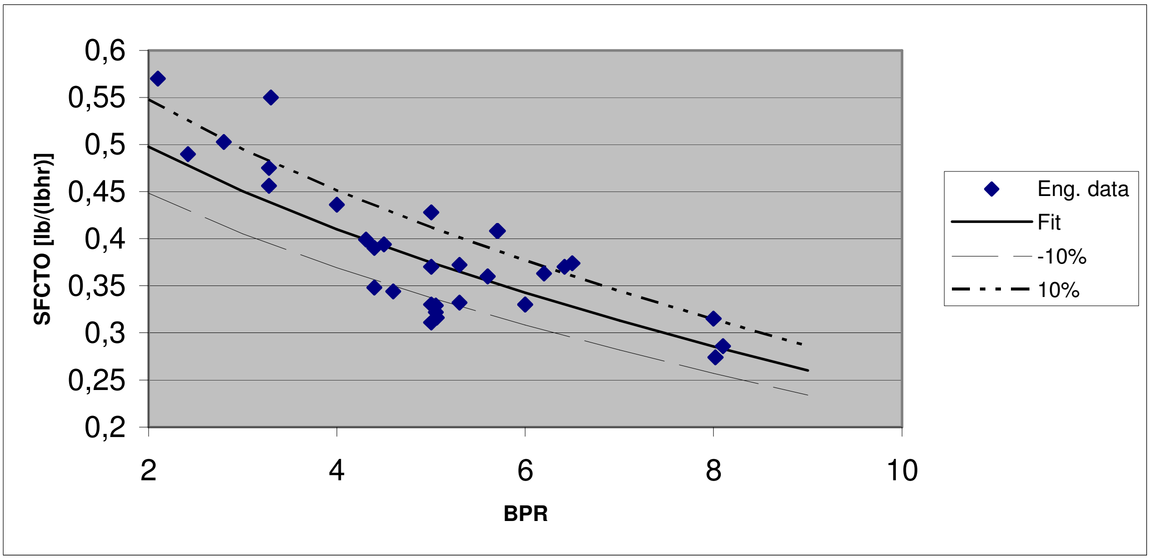


Figure 3.38 Trend of SFC for take-off over BPR (redrawn after **Svoboda 2000**)

3.8.4 Variation of SFC due to reduced Power

Figure 3.39 shows the variation of SFC rate for reduced power with Mach number after **Raymer 1999** (equation (2.27)). With rising Mach number the SFC ratio rises and diminishing thrust rate the angle of the linear graph rises. The general trend, a lower SFC ratio and therefore SFC at lower Mach numbers, is correct.

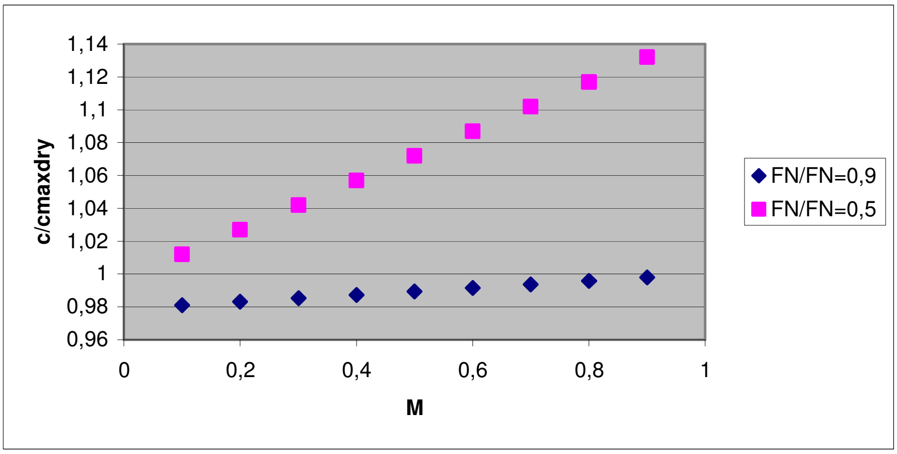


Figure 3.39 Variation of SFC ratio with Mach number

Figure 3.40 shows the SFC rate for reduced power having a minimum and rises from this point with both, rising and falling thrust rate. The general trend is correct. By reducing the thrust a little bit the SFC rate does reduce. When the reduction of thrust is too much the compressor does not give the best performance because the angle of attack for the compressor blades is not perfect. A rapid increase of the SFC is the consequence.

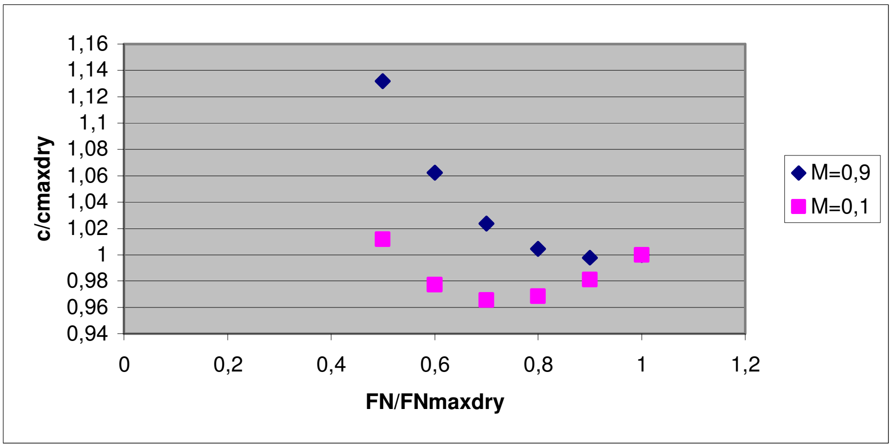


Figure 3.40 Variation of SFC ratio over thrust rate

3.8.5 Variation of SFC due to Power off-take and Bleed Air Extraction

Equation (2.28) (by **RR 1988**) for the variation of SFC due to power off-take is a simple linear approach for the change of the SFC with power off-take. With reduced thrust due to the power off-take the SFC rises in a linear way, depending on the amount of thrust change and a variable factor. A linear function is also suggested by **Young 2002**. **Scholz 2006** and **Ahlefelder 2006** give a simple linear approach for the variation of the SFC with power off-take as well as for bleed air off-take. The fact that the SFC rise due to bleed air extraction can result in an exponential rise for engines with a separate nozzle is not included in the linear approach.

Table 3.4 shows the behaviour of equation (2.29). Since **Raymer 1999** stated that for an engine with 133 kN thrust the power off-take does not exceed 150 kW this boundary condition was used to evaluate the amount of fuel flow due to power off-take in comparison to the regular fuel flow. Equation (2.29) was used to calculate the fuel flow due to P_{OT} = 150 kW with the highes value of k_P^*. The SFC for an imaginary engine with λ = 8 was calculated for cruise conditions with equation (2.16), the value for take-off was estimated to be a little higher. Equation (2.16) can only be used for one power setting with the given data. The take-off thrust was said to be 133000 N, the cruise thrust was estimated by equation (2.9). Note that this example only shows a general behaviour and the actual values are not correct. The fuel flow due to power off-take is 1,24 % of the fuel flow for cruise conditions and 0,33 % of the one for take-off conditions. This is very little and might be neglect able but since the equation (2.29) is very easy and ready to use the use of this equation is recommended. The larger influence of bleed air off-take at cruise conditions is due to the fact that the fuel flow for cruise is lower than the fuel flow for take-off.

Table 3.4 Calculation with equation (2.29)

P_{OT}=	150	kW	
k_P=	0,176	kg/kWh	
$m_{ffuelOT}$=	26,4	kg/h	
m_{ffuel}=	7980	kg/h	Take-off
c=	0,06	kg/N/h	
F_N=	133000	N	
m_{ffuel}=	2119,362	kg/h	Cruise
c=	0,05564	kg/N/h	
F_N=	38090,61	N	

It is to mention that the primary equations given by **Scholz 2006** (e.g.(2.30a) need detailed engine/ aircraft data and are therefore difficult to solve. Equation (2.30) is therefore the simplest way to get results with this approach. Table 3.5 shows the fuel flow due to bleed air extraction in comparison to the regular fuel flow. **Raymer 1999** states that the usual extracted bleed air is roughly 1 % to 5 % of the engine mass flow. In this case it is assumed that the imaginary bleed air off-take is 1 % of $m_{f,eng}$ = 500 kg/s and therefore $m_{f,bleed}$ = 18000 kg/s. The other engine parameters are the same as for the power off-take calculation, k_b^* = 0,028. It can be seen that the additional burned fuel due to bleed air extraction is nearly 24 % of the regular burned fuel for cruise and over 6 % for take-off. It is obvious that the additional fuel flow cannot be neglected and the equation (2.30) should definitely be used. The larger influence of bleed air off-take at cruise conditions is due to the fact that the fuel flow for cruise is lower than the fuel flow for take-off.

Table 3.5 Calculation with equation (2.30)

$m_{f,bleed}$=	18000	kg/h	
k_b^* =	0,028		
$m_{f,fuelbleed}$=	504	kg/h	
c =	0,06	kg/N/h	TO
F_N=	133000	N	
$m_{f,fuel}$=	7980	kg/h	
c =	0,05564	kg/N/h	Cruise
F_N=	38090,61	N	
$m_{f,fuel}$=	2119,362	kg/h	

3.8.6 Variation of SFC due to Temperatures other than ISA

The variation of SFC due to temperatures other than ISA (equation (2.31)) is shown in figure 3.41. Although the equation is an exponential function the outcome is nearly linear. A rise in ambient temperature is followed by a reduction of SFC ratio. The trend is correct, a temperature rise leads to a reduction of thrust. This reduces the SFC accordingly.

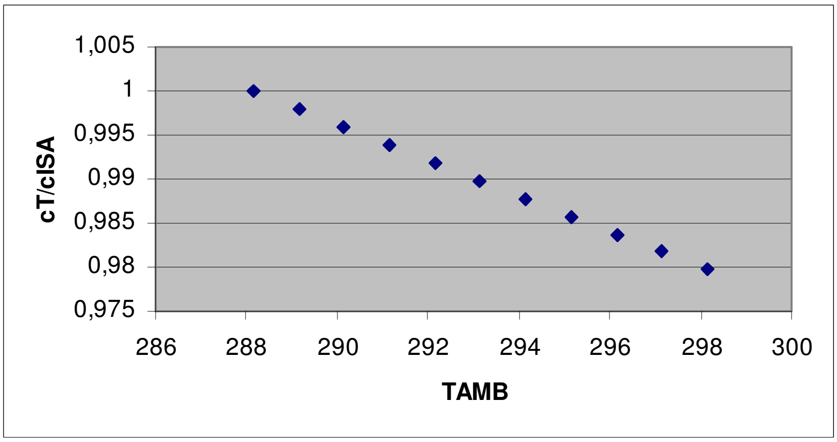


Figure 3.41 Variation of SFC ratio due to temperatures other than ISA

3.8.7 Final Statement concerning the Variation of SFC

Due to a lack of data about very little can be said other than what was said in section 3.8. Data concerning the SFC are obviously the most treasured and best guarded information of the engine manufacturers.

4 Conclusions

4.1 Thrust

4.1.1 Take-off

The conclusion to this topic is only based on one set of engine data therefore it cannot be seen as an all comprising result. Equation (2.5) is definitely the right equation for the description of thrust variation for take-off. If the equation is adjusted to the engine the difference could be below 1 %. Without the knowledge of engine data to adjust the equation it is difficult to achieve this accuracy. **Torenbeek 1982** and **Bartel & Young 2007** have partly solved these difficulties (see equation (2.6) and (2.8)) by providing method for estimating the coefficients for equation (2.5). Equation (2.6) brings results within a range of 2 % where as equation (2.8) brings an accuracy of less than 4 %. **Bartel & Young 2007** state that they can reach a better accuracy for the tested engines that they used to validate their work. The equations of **Torenbeek 1982** and **Bartel & Young 2007** are very similar and both give good results.

4.1.2 Climb

Equation (2.1) can describe the thrust rate over height very well if adjusted to the engine data. Adjustments for an unknown engine are problematic since the exponent x does not follow a general trend. If the equation is used with an x between 0,7 and 0,85 the result is still good (see figure 4.1). Up to 20000 ft the equation brings an accuracy of less than 8 % but afterward the accuracy reduces close to 18 %. If the values for the thrust at sea level for different velocities are not known there might be a reduced accuracy when calculating them. If this equation is to be used, use a value for x between 0,7 and 0,85.

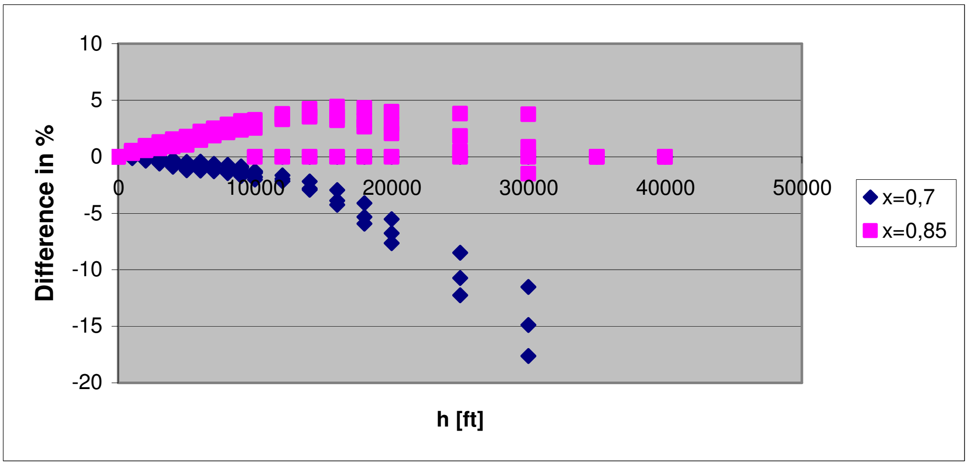


Figure 4.1 Difference of equation (2.1) over height

Equation (2.3) brings results within 10 % up to 30000 ft, which is a very good result especially if the simplicity of the equation is taken into account. The scatter within these 10 % is very big. And additional error might occur if the starting values for the thrust at sea level for different velocities are not known.

Equation (4.1) by **Raymer 1999** is an adjustment of equation (2.3) and takes the different velocities into account.

$$\frac{F_N}{F_{N,0}} = 1 - \frac{h}{C} \tag{3.1}$$

The value of C was calculated to be 47000 ft^{-1} for a V_{CAS} = 200 kts and 57000 ft^{-1} for a V_{CAS} = 300 kts and behaves linear in between. The results of this modification bring the results within 10 % accuracy, but the general scatter is better than for the pure model of **Raymer 1999**.

The model of **Eurocontrol 2004** (equation 2.4) is not meant to be used for engine evaluation on its own. The model brings results within 6 % if adjusted, depending on the way to adjust it a result within 1,5 % is possible. Together with the coefficients given by **Eurocontrol 2004** the accuracy could be very good but this could not be evaluated due to a lack of reference data.

With equation (2.9) by **Howe 2000** it is possible to calculate the thrust change over height and speed with only the value of the static thrust at sea level known. The results stay within an accuracy of 5 % up to 14000 ft, 11 % up to 20000 ft and 20 % up to 35000 ft. The results above this height are highly unreliable. This result is very good.

4.1.3 Cruise

Equation (2.2) is the only equation adjusted especially for cruise. The tendency of the equation is correct but due to a lack of reference data no further investigation could be done.

4.1.4 Bleed Air Extraction

The model of **Raymer 1999** (equation 2.11) lies within an accuracy of 3 % which is a very good result. The suggested value of $C_{bleed} = 2$ was found out not to be always correct since the value varies slightly with the amount of bleed air extraction and strongly from engine to engine.

4.1.5 Temperature other than ISA

The model of **Eurocontrol 2004** (equation (2.10) is similar to the approach of **Raymer 1999** to reduce the thrust 0,75 % per 1 K. **Eurocontrol 2004** suggests a thrust drop of 0,42 to 0,92 depending on the engine. The suggestion of **Raymer 1999** could be seen as an average. Since the values by **Eurocontrol 2004** refer to validated data the result could be seen as an affirmation of the approach of **Raymer 1999**.

4.1.6 Variation of Cruise Thrust with Take-off Thrust

Svoboda 2000 (equation (2.12)) and **Scholz 2007b** (equation (2.14)) both offer equation to evaluate the behaviour of cruise thrust with take-off thrust for preliminary design. **Svoboda 2000** gives a trend according to actual engine data but has no height or BPR term included. The equation of **Scholz 2007b** includes height and BPR. The majority of reference engines stays within a range of 20 %, the equation is therefore sufficiently accurate for preliminary design.

4.1.7 Final Statement regarding Thrust Equations

The investigated models are nearly all reasonable and accurate within certain limits. Since some topics contain only one or similar approaches it is suggested to use them if no other information is given. Equation (2.5), (2.6) or (2.8) should be used for take-off calculation. For the climb thrust variation with speed and height equation (2.9) by **Howe 2000** is certainly the most accurate model and only one thrust value has to be known. In case additional data points like sea level thrust for certain velocities are known the model of **Raymer 1999** (equation (2.3)) is certainly the easiest way to get good results. With a little bit more effort equation (3.1) brings more focused results. Equation (2.1) also brings very good results but knowledge of the values of the density rate has to be gained.

4.2 SFC

4.2.1 Variation of SFC

Due to the lack of reference data very little is to say about the variation of SFC. The general trend of the models is correct but the given limitations (e.g. Mach number limitation for equation (2.15)) have to be acknowledged.

In case no reference value (starting point) is given, equation (2.16) by **Howe 2000** is certainly the right choice. The general trend of the equation is correct and therefore it is suggested to use this equation if SFC over height and speed change is to be calculated.

The pure linear models (equations (2.19), (2.20) and (2.21)) are certainly correct if the correct starting points are used but the higher the distance to this starting point, the higher the possible error.

Equation 2.29 used for calculation of additional fuel burned due to power off-take is ready to use and although the additional burned fuel is small in comparison to the overall burned fuel this equation should be used.

Equation 2.30 used for the calculation of fuel use for bleed air off-take is a linear approach but the rise of SFC is not necessarily linear (**Ahlefeler 2006**). This fact might lead to a larger difference from engine to engine. Since the amount of additional burned fuel can be nearly 24 % it is suggested to use the equation anyway since the error might be huge in the other case.

4.3 Variation of BPR

An observation in the behaviour of the BPR was noted by looking at the usable engine data. The BPR is not constant but changes with height and speed. The BPR rises with rising speed and declines with an increase of height.

Acknowledgements

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